Chapter 7
The Impact of an Ecocultural Perspective of Visuospatial Reasoning on Mathematics Education

At its best, schooling can be about how to make a life, which is quite different from how to make a living.

(Neil Postman, 1996)

The Challenge

Over the years there have been differences in the way researchers have viewed ethnomathematics (Shirley, 1995). Historical studies and studies of large societies (e.g. India) have referenced and evaluated non-western mathematics in terms of western mathematics rather than just referring to difference. This is evident in books and papers like those of Joseph (1991, 2000) who had grounds for emphasising the non-European bases of much mathematics to counter the Eurocentric view of mathematics. Another group of studies have looked at the mathematics behind the products of culture and made links with the western mathematics. These include studies by Eglash (2007), Fiorentino & Favilli (2006), Vandendreissche (REHSEIS-UMR7219, 2005; Vandendreissche, 2007) and Gerdes (1998, 1999). Some of these studies have used anthropological approaches and mathematical modelling to describe, for example, kinship relationships. This is evident in studies showing reciprocity and recursive patterns as shown in the Garma Project for the Yolgnu people in Australia (Thornton & Watson-Verran, 1996) or Eglash's (2007) studies on self-similarity. However, other studies by Saxe (Esmonde & Saxe, 2004; Saxe, 1991, 2012), Wassmann (1997) and Dasen (Wassmann & Dasen, 1994a) used both anthropological and psychological approaches to illustrate the social cognitive psychology of knowing. This approach may result in assessment tasks that are itemised and not necessarily part of thinking in an actual cultural activity. Another group of studies have focussed on equity and expectations in learning in context including studies by Civil (Planas & Civil, 2009), de Castello Branco Fantinato (2006), Knijnik (2002),

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Owens (1992b, 1999b), Restivo et al. (1993), and Voigt (1985). Some of these studies of situated cognition (Lave, 1988) such as Carraher's (1988) work on street vendors or Millroy's (1992) work on carpenters have led to the idea of explicating tacit knowledge (Aikenhead, 2010; Frade & Falcão, 2008) as a key to enriching western mathematics with that of other cultures.

There is an overlap in ethnomathematics theory-building and work on languages in mathematics education (Adler, 2002; Barton, 2008; Clarkson & Presmeg, 2008; Setati & Adler, 2000). In a study of language, culture, and mathematics education, Matang (2008; Matang & Owens, 2014) has shown the main difference between learning in the vernacular, a Creole (Tok Pisin or Papua New Guinea (PNG) Melanesian Pidgin English), or English is in terms of language of formal instruction. The children in vernacular schools spend at least 80 % of their classroom time learning to read and write in their own mother tongue unlike those in the Tok Pisin and English schools. The higher performance by children in vernacular schools is due to longer length of time spent by children in learning early number knowledge embedded in the counting number words. The digitally counting systems (e.g. Kâte), automatically reinforce the idea of composite units assisting children to construct larger numbers through the use of cyclic pattern numerals (i.e. 2, 5, 20) that are physically expressed through the use of fingers and toes, and hands and feet. The measurement study highlighted visuospatial reasoning in mathematical activities related to distance, volume, and area (see Chaps. 4 and 5; Owens & Kaleva, 2008a, 2008b).

Having considered in depth the value of ethnomathematics in terms of visuospatial reasoning in various place-based situations and the ecocultural nature of these activities, it is important to take a critical look at what this means for mathematics, mathematics education, and mathematical identity. Each of these will be addressed in this chapter.

It is often stated that over the ages people developed mathematics and expressed their ideas in various ways, in language, diagrams, and actions. People shared their ideas about representation of systems and gradually a fairly dominant European mathematics was developed (Menghini, 2012). In some cases, mathematical ideas such as the relationship of sides of right-angled triangles were noted by many ancient cultural groups. However, it is also known that there are complex systems of mathematics that have often remained outside the European-influenced school systems such as Vedic mathematics. This book reinforces the importance of recognising diversity in mathematics and the creation of valuable mathematical ideas by sharing often unwritten mathematical processes and systems. By strengthening the importance of visuospatial reasoning in mathematics, I explore the richness of mathematics beyond the arbitrary symbols used in relationship and logic mathematics. It helps to establish the value of collaborative thinking in visuospatial reasoning much as Fermi[1] suggested as a means of problem solving.

While other authors have emphasised the importance of a consistency between culture and school for strong identity, I explicate how this occurs in practice.

[1] A collaborative estimate is made to solve these problems. For a brief explanation see http://www.edu.gov.on.ca/eng/studentsuccess/lms/files/fermiproblems.pdf

The model of identity as a mathematical thinker draws on ecocultural identity and establishes a strong mathematical identity by virtue of strong visuospatial reasoning being accepted in the learning. Data from some ethnomathematics projects in PNG and other places will be used to illustrate the challenges and values of this perspective.

Part of the purpose of encouraging an ecocultural perspective of visuospatial reasoning is to improve education especially in this important area of visuospatial reasoning. There is a justification for both an emphasis on visuospatial reasoning and an ecocultural perspective in mathematics education. Many of the limited conceptions and lack of visuospatial reasoning can be attributed to the lack of links between ecocultural backgrounds and the mathematics of the classroom. By interpreting school mathematics in terms of the students' background mathematics, it is possible to engage students in improved spatial sense and visuospatial reasoning.

In essence this chapter will develop the arguments presented and justified in the previous chapters to show the importance and impact of an ecocultural perspective on understanding and valuing visuospatial reasoning. This chapter will bridge to the next chapter where an ecocultural perspective is established as significant in terms of Indigenous cultures and transcultural/multicultural education and evidence of visuospatial reasoning in practice in these contexts is given. Let's first turn to the impact of an ecocultural perspective and visuospatial reasoning on mathematics itself.

Impact on Mathematics

Mathematics can be considered as a way

> to explain and understand the world in order to transcend, manage, and cope with reality so that the members of cultural groups can survive and thrive (through) techniques such as counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring, and modeling. (Rosa & Orey, 2012, p. 3)

Visuospatial reasoning has a strong role to play in mathematics when people use techniques that are strengthened by visuospatial representations and skills and reasoning to make decisions, no matter how simple or complex. As Amos (2007) said about the many PNG women past and present who make bilums, "when making this bilum a lot (of) imagination is involved with vitalizing (sic) of the design or pattern". This imagination in visuospatial reasoning brings life to the design.

Rivera (2011) argued that visuospatial reasoning was a legitimate way of thinking in mathematics. For example, π can be visually verified by initially measuring the length of the diameter of a circle with a piece of string and then showing that its circumference is slightly more than three times the diameter by tracing three copies of the string on the circumference. This is not a difficult task or unknown in PNG where round houses are common and people need to obtain materials for the walls (they also use six times the radius as the circumference, Fig. 5.10). Phi, φ, whose exact value is $(1+\sqrt{5})/2$, is another example of an irrational number that could be

Fig. 7.1 Visuospatial reasoning about ratios represented by irrational numbers in western mathematics. (**a**) Extending the house, half as much again. Malalamai, Madang Province, PNG. (**b**) Two ropes to form a right angle in PNG (Yamu, ~2000)

visualised by obtaining the ratio of the length and width of a golden rectangle[2] and whilst this shape does not seem particularly valued in PNG for deciding shapes of houses, the golden rectangle does occur. For example, when a house that has 4×3 posts has half as much more floor area than the rectangular floor of the house with 3×3 posts (Figs. 7.1a and 5.13; Malalamai houses discussed in Chap. 5), then this is approximately in the form of the golden ratio (≈1.66). Visualising such ratios requires inferring particular relationships between parts. Rivera (2011) made the point that every square root of a non-perfect square number could be depicted by taking the length of the segment corresponding to the hypotenuse of the relevant right triangle. In some places in PNG, this was visualised in practice. For example, where rectangular houses are generally of the same size, the diagonal length as well as the side lengths are well known in steps and in visualised lengths of ropes or saplings. In one area it was recorded that the villagers keep a rope with three knots (one at each end and in the middle obtained by folding exactly in half) and the diagonal rope with two knots (Fig. 7.1b). These ropes are used to form right angles (Yamu, ~2000).

While these are specific cultural examples, a general argument can be established. Culture may be seen as a way of life in its entirety for a particular cultural group or society while mathematics, on the other hand, is a systematic problem-solving method purposely developed to solve the everyday problems of the existence of its members. Mathematics education can be seen as the processes and organisation that enable cultural knowledge whether that be in school or in the family to develop and survive. The type of content knowledge is determined by the existing conceptual-knowledge frame of a particular cultural group including developments from various groups over time. This is further refined by the individual needs of learners as a prerequisite requirement to becoming an effective everyday problem solver.

An ecocultural perspective suggests that learning is not only about personal accomplishments and growth but also about the person's function in a community's activities and how they develop identities as mathematical learners and thinkers (Greeno, 2003). In particular, concern is for the interaction of learners with the ecocultural context—people and resources. Key to development of visuospatial

[2] For a brief note on the golden rectangle and associated golden ratio, see respectively http://www.mathopenref.com/rectanglegolden.html; http://www.goldennumber.net/golden-ratio/

reasoning is the opportunity for students to engage in constructing meaning of concepts and problems by engaging, trying ideas, visuospatially representing ideas mentally and physically in a way that they contribute collaboratively to each others' communications verbally, materially, and visually. Ways of doing this need to be developed within the community of learners in an ecoculturally appropriate way. Thus there is no dichotomy between school learning of procedures and practical or applied procedures but rather the learning draws on the ecocultural experiences and approaches to develop the mathematics throughout the curriculum or learning sequences and to develop abstract mathematical reasoning. Visuospatial reasoning, explaining, and justifying are keys to this in terms of learning concepts and what it means to carry out these mathematical processes. It is likely that there will be a development from intuitive reasoning to developing more sophisticated and integrated ideas and proofs where the oral and visuospatial precede and guide any later written reasons and proofs. Language plays a key role in these oral and shared visuospatial reasoning as well as what the community accepts as appropriate.

One of the purposes of mathematics is to solve problems and an ecocultural perspective encourages mathematics to be appropriate to the ecological and cultural context. Navigation required the use of felt motion, visual stars and sea, a memory and application of the star maps, and a mental ready reckoner in visuospatial terms of the distance travelled in a period of time. These complex pieces of information formed the inputs into the visuospatial reasoning pursued by the navigators. Through practice, judgments over time made such decisions more reliable but I claim the reasoning was visuospatial although numbers may have assisted in minor ways such as counting days. There were representative maps held in the mind on which islands, sea places, and stars could be placed but if the navigator was at the centre then other places were dynamically moving. Dynamic imagery is in fact an important part of visuospatial reasoning not only in navigation but also in other activities. For example, a person adjusting the position of posts, rafters, and other parts of a house, a bilum-maker growing a pattern towards the desired connecting shapes on a bilum (continuous string bag), or a person taking the next step in a string design (cat's cradle) will all use visuospatial imagery even if they have used counting or measurement to support their imagery. Similar imagination occurred with knotting, fishing, and carving.

Another significant type of visuospatial reasoning was the recognition and use of patterns. When the Elders were discussing the weaving of the diamond, they reasoned about where to start the row, knowing from past experience and in the example that they were working on what the pattern of overs and unders would result in the colour turning up in the right place to continue the slanting line of the diamond (see picture in prologue). Other patterns occurred, for example, in one place where a specific plant marked every fifth pair of *kaukau* (sweet potato) mounds. The marker may have had other purposes such as a food, shade, or for decoration. However, the marker may not have been used all the time.

Another critical part of the patterns that may not have been as regular as we might expect from a western education are those embedded in the mental ready reckoners that matched certain spaces with others. These links were established through experience in the ecocultural context. For example, they occurred when men knew how much kunai grass to cut to cover a house whose floor area was half as much again as another one, or when women said they would need two *tulip*[3] trees to make the 30 balls of string for a fishing net, or the Elder went from describing the size of his house to that of the size of the garden needed to provide a feast for all the helpers required to build such as big house (Fig. 5.10, Chap. 5, Owens & Kaleva, 2008a). These forms of reckoning were substantially in terms of related visual imagery associated with the place and aspects of the place. Like other tools in the visuospatial arena, decision making was based on best practice or probability of success. Experience largely informed the development of the mental reckoner and of its use in decision making.

One aspect of visuospatial imagery embedded in ecocultural contexts that has been given little attention in western mathematics is that of spatial imagery resulting from physical involvement in the activity. The ability to make decisions without necessarily explicating the reasons has been a part of air pilots skills but it is also a part of knowing the safety of a structure being built out of bush materials, the effectiveness of an arrow or a trap or the balance of a canoe. Such visuospatial reasoning does not seem to have an equivalent in school mathematics. Odobu (2007) from Manumanu village, Central Province, PNG provided a number of examples illustrating the spatial aspects of visuospatial reasoning derived from cultural activities (Fig. 7.2).

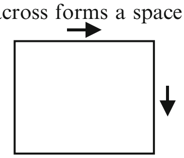
Odobu provided similar examples of how people make sectors of circles especially in sharing and house building, make decisions about volumes (mostly thought of in terms of liquid and food for containers such as clay pots and baskets), and tell time especially for the Hiri trading and other activities that cover long periods like weeks and parts of a day rather than hours, minutes, and seconds. Odobu matched each aspect of geometry and measurement with typical examples from school textbooks which used PNG contexts but not necessarily from one place. These were similar to Jannok Nutti's (2010, 2013) examples in which Sámi contexts were used but the basic curriculum is western as discussed in Chap. 8. Throughout each of the discussions and examples given above, there is a bodily movement associated with the visuospatial reasoning. Odobu was less clear about the size of a kilometre and how that related to a hectare as this was school mathematics and his school experiences had not associated this space with cultural embodied spatial experience. Nevertheless, his ecocultural mathematics was strongly associated with visuospatial reasoning—something that was commonly found in the teachers' projects (some referred to in Chap. 5 and others in Chap. 8). His concepts were strongly established for ecocultural mathematics.

The question remains about whether visuospatial reasoning has the so-called logic of mathematics. I contend that it does when used by skilled people. Even the use of 'by eye' decisions in achieving a straight wall or a right angle between the walls of a house have logical connectors. There is logic in knowing that looking from further away along three or more sticks or people provides better for deciding

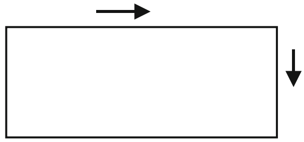
[3] Tok Pisin for two leaves coming from one leaf stalk; inner bark also used for tapa.

There are no clear methods of calculating areas. Area is a matter of practical experience usually done by comparison. People know what areas look like and so they can construct them through activities. For example, constructing base of a house, door of a house, base of a canoe, canoe floor surface, clearing a piece of land for garden and so on.
The sides of surfaces are done in hand-spans or leg-spans.

For instance, one leg-span forward and one-leg span across forms a space like this.



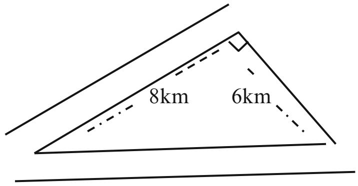
Unequal leg-span or hands-spans give a shape like this



One can form many different shapes in the similar way. The shapes may represent many different objects in traditional cultures.

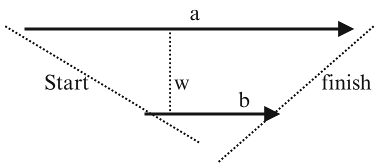
The measure of the area bounded by towns, cities or plantations are too large to be measured in mm, cm, or m. The area of large dimensions are measured in metres (sic) or hectares

Calculate the area of a triangular coffee plantation in hectares.



Parallelogram is a shape obtained by tying two longer sticks and two shorter sticks end to end and then skewing them to the right or left. Similarly, skewing a shape formed by four equal sticks, you form rhombus. Parallelograms do not have a name in culture. However, these shapes tend to tell people that house is going to fall.

Do activity done by people to resemble this shape. Example: Two people walking in the same direction. They are ‘w’ apart. One walks a total of 'a' distance, while another walks 'b' distance. The former walks longer than the latter. If the distance between paths these two people take remain same (w) and two paths are parallel than area formed is a trapezium.



For the circle, kundu, base of highlands kunai house, clay pot, cut end of round object like log. Measures are made by comparison. The designer could use the hand span, a piece of rope or visual comparison. Sometimes a designer of the object thinks about the contents it could hold say more food to feed larger family.
To draw a circle using a traditional method, hold with two hands a piece of rope, holding one end of the rope firmly on the ground. Move another end around to form a round shape. You are forming a circle.

Fig. 7.2 Excerpt from student project on traditional measurement and shape construction (Odobu, 2007)

the straightness of a line or corner right angle. The decision making in itself is logical as seen in the following situations:

- Looking from two or more directions for positioning the central pole of a house
- Noting which slopes for a roof are sufficient for run off

- Knowing that the closeness of the *morata* roof material depending on the length of the leaves makes a difference for waterproofing
- Knowing whether roof poles are close enough for tying on kunai thatching
- Preparing baskets of food that are large enough to satisfy an acceptable exchange
- Mixing materials in the right proportions to make a good paint, soup, or pancake

Hence I argue that an ecocultural perspective on visuospatial reasoning is mathematical and it enhances mathematical applications, ways of reasoning, and decision making.

Impact on Understanding Mathematical Learning

By taking the idea of situated cognition as a beginning premise, we can then begin to explore visuospatial reasoning in terms of ecoculture. For example, to what extent may experiences related to weaving impact on the reasoning of students about spaces and shapes. This may be quite different to the way a western student from an environment with television and structured blocks (such as pattern blocks with specific shapes) might both develop intuitive reasoning, conceptual reasoning, and purpose for reasoning. Furthermore, cultural values may become widespread in a society leading to reasoning from the cultural values. De Abreu refers to this as valorisation.

> Cole's [1998] version of cultural psychology has as a strength an emphasis on tool mediated action rather than pure cognition. Emphasis on action enabled the recognition of the heterogeneity of psychological processes of groups engaged in distinct social and cultural practices. It also allowed recognition of the distributed nature of human cognition. Thus, the action of a person does not need to be situated in one mind, but rather in an activity system. … However a weakness of Cole's type of approach is that it does not yet provide a satisfactory account of within-group diversity (de Abreu, 2002, p. 174)

There is no doubt that the impact of various cultural tools used for mediating thought may indeed produce diversity within the group if values are considered. What one student or their family values may impact on the degree to which a particular tool is used. This is exacerbated by the many cultures that may be impacting on students and in some cases producing dissonance in thinking. In particular, perceptions and apprehensions of visual representations or reasoning from visual representations may take quite distinctly different perspectives. Owens and Clements (1998) illustrated this by a scenario in which two people are viewing a house and while one views it from the perspective of being situated historically at a specific period and notices features related to this perspective, the other views it in terms of its suitability for their particular family. Again we see a link between diversity and the more psychologically embedded notion of attention which is a key to visuospatial thinking and reasoning.

In her thorough analytical review of papers presented at conferences of the International Group for the Psychology of Mathematics Education, Presmeg (2006) noted that education to encourage visualisation was critical for quality visualisation in learning and mathematical practice. To "fold back" to a visuospatial basis as Pirie and Kieren (1994) purported for the learner to then go forward with deeper understanding is a critical perspective to take on learning. Interestingly, one of the studies reviewed by Presmeg that illustrated this point was about a plumber having limited strategies but the studies discussed in Chap. 2 (Owens, 1999c, 2005) were about classrooms in which visuospatial reasoning improved with activities involving spatial problems.

Informal mathematics learning takes place outside the formal classroom environment but within the boundaries of a particular cultural group's everyday activities and worldview and subsequent epistemology. Such learning has the advantage that it is not only familiar to the learners, but importantly, it provides the necessary contextual meaning to many abstract mathematical ideas and concepts taught in the formal classroom. It is also applying locally derived mathematics to solve the problems encountered in the particular ecology and culture. For example, in PNG, utilising the Indigenous knowledge within the formal classroom will not only enable the learners to construct meaningful mathematical relationships, but also provide an opportunity for interactions to occur between the learners themselves, as well as the teacher in explaining the many contrasting differences that exist across different cultural groups. The students can be learning in their home languages and using the words that hold powerful concepts embedded in the languages and using the wisdom that is thousands of years old (Trudgen, 2000). The teacher can learn from the discussions if they are teaching cross-culturally and learning the language and culture. They can enhance school mathematics with the funds of knowledge available from forming a community of practice with the families of the students (González, Moll, & Amanti, 2005). There are advantages for the learner if formal education takes seriously the notions of informal education, and in particular ideas that arise from ethnomathematics.

The Context for the Current School System

Most school systems are dominated by a financially powerful and controlling government machine. These systems often stipulate a curriculum which is to be taught although the means of controlling the actual classroom practice may vary. In some cases, external examinations and testing regimes provide high stakes for successful completion. These examinations are creeping into lower years of schooling. The examination items can control the curriculum in practice. In some cases, examination items are as close as possible to the curriculum intentions given the restrictions of the type of testing (usually restrictive paper-and-pencil testing) and the facilities for marking and assessing. Nevertheless, in spite of the examination regime, the curriculum generally remains dominant especially in the lower years of schooling although it may be interpreted by a textbook for teachers and students. The curriculum is generally developed by people in central government offices working under guidelines from governments with varying degrees of input from the mathematics education researchers and teachers in the classroom. The impact of the government's agenda may be less controlling in less authoritarian states but systems with neoliberal tendencies, consumerism and globalisation as westernisation may still prevent the diversity of mathematics developing (Atweh, Barton, & Borba, 2007). The centralised curriculum may prevent full potential in any system of mathematical reasoning for the student from a minority, with a first language background different to the official school language, from a class or gender group that prevents full opportunities for learning in some countries, or in a country that has been colonised.

Current schooling is strongly influenced by views of geometry and research into mathematics education that looks mainly at early geometry learning centred on naming, classifying, and defining shapes that were of interest to Euclid, the Greek mathematician two and a half thousand years ago. Some researchers mainly see geometry as a way of strengthening the area of proof (Mariotti, 2006). Visuospatial reasoning is seen mainly as a way of reasoning spatially as part of mapping or locating (Clements, 2004) and later graphing in algebra to reason about relationships (Meira, 1998). However, visuospatial reasoning has a role beyond positioning as indicated by both NRCCG's (2006) and Shah and Miyake's (2005) comprehensive studies.

Space and geometry curricula in the primary school tend to focus on the shapes introduced in Euclid's geometry. These begin with shapes like triangles, circles, rectangles, and squares. Curricula could begin with three-sided, four-sided general shapes and then name the subsets (Dreyfus & Eisenberg, 1990). Objects are also given names like cylinder, prism, and pyramid. This emphasis on shapes and on classification is reflected in the commonly emphasised levels as outlined by the van Hieles (Battista, 2007a; van Hiele, 1986) based on their studies of secondary school students. These have formed the basis of many curricula (Halat, 2007; Owens & Perry, 1998) and many research studies (Owens & Outhred, 2006). A look at these studies will indicate the approach taken to geometry education.

Many studies have used van Hiele's levels to indicate how students learn to name, classify, and define shapes. Researchers have suggested that the development from Level 1 (recognition based on global perspective of a figure) to Level 2 (analysis of the properties of figures) is not straightforward. This transition can be understood in terms of students' responses based on a realisation that aspects of a figure are important (identification of features), an attempt to document more than one feature, and grouping of figures based on a single property (Pegg & Baker, 1999; Pegg & Davey, 1998). The transition from Level 2 to Level 3 is also problematic (Clements & Battista, 1991). Researchers focussed on class inclusion (for example, squares as a subset of rectangles) as a distinguishing feature of Level 3 (Currie & Pegg, 1998; De Villiers, 1998; Matsuo, 1993). However, Matsuo (1993) suggested students' classification of a square as a rectangle seemed to depend on the property that they focussed on. This might suggest alternative perspectives on classification in which sociocultural experiences should be taken into account. Students might use known definitions or definitions they develop from perceiving certain properties (Shir & Zaslavsky, 2001, 2002) or procedural definitions that develop from constructing shapes, for example in dynamic computer environments (Furinghetti & Paola, 2002) or simple thin elastic. It is at this point on constructing shapes and creating definitions that sociocultural experiences may lead to totally different perspectives on shapes. For example, beginning with weaving, curves, or paths as the dominant basic feature of various cultural groups discussed in Chaps. 5 and 6, different properties and definitions may result in quite different systems of geometry.

Impact on Mathematics Education

Geometry is grasping by exploring the space in which the child lives, breathes, and moves (National Council of Teachers of Mathematics, 1989, referencing Freudenthal). In this statement, geometry is firmly situated in the space and place where the child is. The child explores this place both from the exploration based on his own perceptions and intuitions and from the influence of those around him. Through exploration, relationships of objects, their positions, their similarities, differences, and features are established and generalisations drawn.

If you watch young children play, they will tell stories and establish relationships and roles. They may use trains with personalities, dolls, animals, cars, lumps of wood, or themselves in imaginative or adult roles to tell their stories. They are imagining creatively and talking their own stories in their own language. Despite intonations and long sentences, it is not generally standard school language. They will draw in any adults willing to participate by a few recognisable words or gestures. These imaginary activities will involve some form of mathematics such as the object that represents food being placed on a plate and held horizontally to bring it to another person or to a toy for tasting. It is clear that children have a strong sense of horizontal and vertical developed at least from experiences with gravity. Ness and Farenga (2007) noted that young children have a sense of location in space that has horizontal and vertical axes but also movement forward or back. Their world is not just that of topology as Piaget mentioned with spaces inside and outside or with proximity being considered as a significant aspect of the space being explored by children. Furthermore, such imaginative play occurs in a space but that space is a place to these young children. As a result, the space takes on meanings that go beyond the simple geometric relationships. These meanings will become involved in the reasoning that the children use in play.

Early childhood contexts are mainly families (often extended) and in some cases institutions such as prior-to-school settings in which they may have contact with only a few other people on a regular basis. Children are developing relationships with significant others whose interaction with them is often dominated by cultural mores about these relationships, what is considered appropriate ways of permitting children to learn and what are appropriate content for them to learn. This includes the spatial experiences provided to children and the various representations associated with these experiences. It will influence the way in which children can interact with the spaces and objects around them.

Language is a dominant aspect of these contexts. Multilingual contexts can both enrich but also hinder development especially when the child fails to learn any language well (Clarkson, 2009; Valdés, 1998). Values are a strong influence in determining the selection of experiences that significant others provide for their children (Walden & Walkerdine, 1982). These are particularly relevant in experiences that may differ by gender but they also impact on the dominance of labels for shapes in western education home and school situations. They are also important in the value put on place and the visuospatial reasoning associated with valuing place. The sociocultural background of the child will be influencing the way in which these relationships are described and perceived. Geometry is the abstraction of these relationships which are then used for further exploration and development of ideas. Geometric reasoning then is "the invention and use of formal conceptual systems to investigate shape and space" (Battista, 2007a, p. 843). However, "formal" may be specific to a cultural group.

Visual and language knowledge are combined to provide an understanding of shape and space agreed upon together as people interact over time from an early age (Clements & Sarama 2007a, 2007b). Descriptions of specific experiences may be pragmatic arguments but they are interpretations of mathematical activity and social construction of concepts (Patronis, 1994). For example, a class of 16-year-olds took adjacent angles whose sum was a straight angle and showed that the bisectors of the angles were perpendicular, relying on angle measurements for "proof". Even in dynamic geometry environments, students continuously move from "spatio-graphic geometry" to "theoretical geometry" when elaborating a proof. The student uses the figure to make conjectures or to control results, then shifts to using definitions and theorems, then goes back to the figure and so on (Laborde & Capponi, 1995). This approach appears to occur early. Students apply different cognitive actions such as attending to features like "it's pointy", to decide on prototypical images (such as the equilateral triangle for triangles), to dynamic changes like sliding or pushing images to transform into another shape (Clements & Sarama 2007a, 2007b; Lehrer, Jacobson et al., 1998; Owens, 1996b). This pragmatic activity-based approach to arguments is closely akin to that found in Indigenous communities who discuss their house construction to make decisions. No single person is expected to make the decisions or to have a too restrictive rule-bound approach to the geometry involved (villages in PNG).

Tessellations can be found in many cultural groups. These may result in designs that involve Euclidean shapes but they are frequently developed from circle constructions as in Middle Eastern art (Critchlow, 1992) or weaving (Cherinda, 2001, 2002). The progression of western students' knowledge of tessellations is not well understood (Callingham, 2004), with the exception of an array of square units (Outhred & Mitchelmore, 2004; Owens & Outhred, 1998). There is a development based on visualising the coverage of the area with tiles which results in seeing the structure of the tessellation. Callingham (2004) reverted to using the van Hiele levels as a way of describing students' understanding of tessellations. Most students could describe an array of squares giving the name, informally or more technically describing the array and explaining the transformation to make the array. For other shapes students were at the visualisation level and could only recognise and name shapes. Whether this was a sociocultural result or not was not explored. Nevertheless, the studies referred to in Chap. 2 show how students can develop a sound pattern imagery understanding of tessellations while studies of weaving indicate an alternative understanding of tessellations.

The South African curriculum has emphasised development of visualisation as critical for geometry in the early years (Kuhn, personal communication, 1998). Another programme that moved away from the van Hiele development was provided by the *Count Me Into Space* studies (NSW, Australia, Department of Education and Training) which emphasised developing strategies for investigating, visualising, and describing rather than just classifying shapes. Nevertheless, the same shapes such as triangles featured strongly in these programmes. Transforming shapes was seen as a strategy for exploring shapes and visualising rather than as a way of using rotation and reflection to prove congruence, as many curricula do (Gutiérrez, 1996).

A large study illustrates this interaction between formal school and the child's intuitive learning. Appropriate classroom experiences were designed

> around children's everyday activity related to (a) perception and use of form (e.g. noticing patterns or building with blocks), leading to the mathematics of dimension, classification, transformation; (b) wayfinding (e.g. navigating in the neighborhood), leading to the mathematics of position and direction: (c) drawing (e.g. representing aspects of the world), leading to the mathematics of maps and other systems for visualizing space; and (d) measure (e.g. questions concerning how far? how big?), leading to the mathematics of length, area, and volume measure. (Lehrer, Jacobson et al., 1998, p. 170)

The emphasis here is on activities that are discussed in terms of using form, wayfinding, drawing, and measuring which are all seen as everyday childhood experiences. These are universal activities but practiced differently in different societies (Bishop, 1988; Dehaene, Izard, Pica, & Spelke, 2006). Space and geometry at this stage consists of concepts such as dimension, classification and transformation, position and direction, maps and other systems, and measuring which become both the geometry and the tools for exploring space. Again each of these appears to be universal but within each there is latitude for cultural difference and difference over time.

Processes for exploration in early geometry include inventions of ways to represent space, conversations that fix mathematically important elements of space such as properties of figures, argument, and justification around activities that involve manipulative tools or images, and narrative around what learners have done in stages, what they know informally and intuitively, and what they then own as part of their mathematical knowledge (Lehrer, Jacobson et al., 1998). Such learning requires increasingly more sophisticated investigating and visualising; describing and classifying (Owens, McPhail, & Reddacliff, 2003). The link between visualising and investigating and geometry is visuospatial reasoning.

There is no shortage of studies that have now taken cultural competence of teachers seriously in education (e.g. Averill et al., 2009). We discussed this earlier in reference to Indigenous education especially in Australia in supporting the role of Elders (see Chap. 3, Owens et al., 2011, 2012). The Sámi in Sweden established a Sámi

Handicraft School for adults to specialise in the various crafts such as knives with handles carved from reindeer horn for men and traditional clothing for women, and ways of thinking in their culture supported by revitalising their language. At the same time, Sámi schools for children were established “to pass on norms, values, traditions, and cultural heritage” (Jannok Nutti, 2013, p. 58) with teachers who were fluent in at least one of the Sámi dialects. The Sámi Handicraft Centre, supported by the museum, also established a programme to bring to schools. With model reindeers (on wheels) and sleighs, the traditional equipment included ear-marking tools (each reindeer is marked by the owner’s geometric mark), ladles from beech tree boles used for drinking, reindeer skin pouches for coffee that folded as the coffee was drunk, ropes, and easily transported *lávvu* (cone-shaped tent).

In the school hall, the children simulate a trek to the reindeer and set up camp learning about

- The lightness and minimal space needed for the trek
- How to tie different knots for the shapes
- Sizes, spaces, and uses of the different artefacts

The children learn to recognise different geometric marks by making them on “soft” (recycled foam) ears attached to the model reindeers. Adults and children learn how to cut to size the desired clothes and shoes, how to sew them and how to make the important decorative patterns. In practice, reindeer herders and their families learn to track the reindeer and to map their routes, and are able to lasso their own reindeers and place them in their own corral when the reindeers are herded in summer. From the hundreds moving around quickly the skilled reindeer herders can recognise the markings on the skins of the calves of their own herd. This is a quite extraordinary visuospatial skill. The Sámi have their own approach to mapping based on the north, the rivers, and the reindeer routes. Much of the skill of the Sámi is in the ecocultural perspective taken to visuospatial reasoning around activities that relate to geometry and measurement (based on two personal visits, several oral presentations by Jannok Nutti, and Jannok Nutti, 2008, 2010). Jannok Nutti provided a summary of her earlier research Jannok Nutti (2007) into the mathematics of Sámi reindeer herders and handicrafters as follows:

> there are several conceptions, for example different names for reindeer herds based on the approximate number of animals. Unusual reindeer, for example animals with distinctive colours, function as support in counting or approximation of the wholeness of the herd. This is because reindeer herders easily recognise and identify this reindeer and if some of them are missing the herd is incomplete. The number of branded reindeer calves was counted by making marks on a wooden stick, by saving part of the ears of the branded calves, or by making notes on a piece of paper. Locating was made possible by well known objects in the natural environment, by the wind, or by rivers. The cardinal points were based on the landscape, the rivers, or lakes and the valleys around them. Body measurements were used. Depth of snow and water was measured with a stick or a rope and body measurement units, or with the help of the complete body. Distance was measured by the time it took to walk, by sound, or by sight. The concept of *beanagullan* is an example of a unit of measurement of distance. *Beanagullan* can be translated as the distance at which a dog’s bark can be heard. Eight seasons divided the year and time was regulated by heat, light, or seasonal activities. The designing activity involves designing of buildings and artefacts. (Jannok Nutti, 2013, p. 61)

Each of the mathematical activities given in this paragraph involve a considerable amount of visuospatial reasoning as well as skills, and most relate to geometry or measurement of the environment, a cultural activity, or ecological response. Thus an ecocultural perspective of visuospatial reasoning is well exemplified.

Jannok Nutti showed from an action research study that:

> teachers changed from a problem-focused perspective to a possibility-focused culture-based teaching perspective characterised by a self-empowered Indigenous teacher role, as a result of which they started to act as agents for Indigenous school change. The concept of 'decolonisation' was visible in the teachers' narratives. The teachers' newly developed knowledge about the ethnomathematical research field seemed to enhance their work with Indigenous culture-based mathematics teaching. (Jannok Nutti, 2013, p. 57)

The self-empowerment of teachers who had to creatively make use of Sámi cultural knowledge and through those experiences develop students' mathematical knowledge came from working together and their interest to attend seminars about Sámi mathematics, mathematics education, and Indigenous education. Thus the notion of ecocultural context becomes important in the self-regulating, affective learner directly, and through the social competencies of the learner and ecocultural identity (Fig. 1.2). Furthermore, the teachers were responsive and affect was a part of their developing mathematical identity:

> The teachers' active engagement, and visions of culture-based teaching and its implementation were central. They tried to rediscover or reinvent Sámi culture in a mathematics school context. The concept of "rediscovery" led to joy and dreams, but also to mourning for lost knowledge and made the concept of "mourning" visible. (Jannok Nutti, 2013, p. 69)

It seemed that the Sámi cultural theme lessons with ethnomathematical learning were more productive than providing standard textbook type problems with a Sámi context. Teachers who followed this latter strategy were concerned that they needed to teach the students the national curriculum for them to become independent. Furthermore, there is other evidence to suggest that using cultural activities to teach mathematics can result in improvements in national assessments (Lipka & Adams, 2004; Meaney, Trinick, & Fairhall, 2013).

Other ecocultural situations have also illustrated the importance of visuospatial reasoning in mathematics education. Lipka, Wildfeuer, Wahlberg, George, and Ezran (2001) illustrated how to introduce elastic geometry, or topology, into the elementary classroom through visuospatial reasoning using intuitive, visual, and spatial components of storyknifing[4] as well as other everyday and ethnomathematical activities.

Tacit Knowledge in Visuospatial Reasoning

Frade and Falcão (2008) discussed the issues of making implicit knowledge explicit. We can generalise to say that people have a sense of area (tacit knowledge) developed through sleeping, gardening, and house building in particular. People are able

[4] For an example, see http://aifg.arizona.edu/film/storyknifing

to use this idea of area to make judgements such as the estimated amount of material needed for a house of a particular floor size. Many participant researchers referred to the pacing of (the length of) a garden as a measure of a garden. However, people would visualise a garden by knowing its length. Some visualised the number of kaukau mounds, others visualised a garden with a common width. Similar comments could be made about floor plans and roof areas. The static environment provides some mathematical examples whereas mathematical thinking occurs during the process or activity. By making these points explicit, teachers can reduce the discontinuities in knowledge and hence build a firm basis for school mathematics.

In PNG's measurement study, quantities were provided on numerous occasions to indicate amounts but these were frequently indicative of approximations or possibilities like round numbers are used in western societies. Dehaene et al. (2006) noted a similar effect among the Mundurukú speakers of the Amazon, South America when they mentioned "five" or "a handful" to refer to displays of five up to nine dots or using "four" or "a few" when five dots were presented.

Mathematical features such as shapes of bilums, pigs, holes, and houses were not mentioned in reference to volume; they were assumed by sight. However, length was seen as an important "rule-of-thumb" way of determining volume. For example, a length of string or part of the forearm was linked to the volume of a bilum, or the girth of a pig to its volume and hence its mass. Nomographs and ready-reckoner tables are possible equivalents to these mentally stored Indigenous knowledges. Nevertheless, the lengths that were referred to in describing a house did recognise the basic shape of the house. Thus radii for round houses or lengths of the sides for rectangles were mentioned together with heights.

One should also note the sophisticated ability of people to estimate needed amounts, for example, water for *mumus*[5] or garden areas. In each case, a good sense of comparative rates is applied, based on previous experience rather than on a mathematical calculation of volume. For this reason, comparisons are frequently made and confirmed by a group of people when payment or decisions involving sizes of mass or volume are made. The visual reasoning dominates over the numerical reasoning although numbers will be called upon to support a discussion. In other cases, the number rather than the size will dominate so long as items are roughly equal in size.

If we turn to the issue of engaging students in school mathematics,

> in reality, many students do not see the need to learn school mathematics further adding to barriers of meaningful learning of mathematics as many of these formal mathematical methods are viewed by students to be inappropriate in solving many everyday practical problems at hand … Ethnomathematics, unlike the school mathematics, is both context-relevant and problem-specific thus provides the necessary linkage between the everyday cultural practices of mathematics and the teaching of school mathematics. … Recognition of students' ethnomathematical knowledge also increases their self-esteem, which in turn increases their performance on school mathematics. (Matang, 2001, pp. 2, 4)

D'Ambrosio (1990) also raised the relevance of mathematics and the importance of self-worth as significant for mathematics education and aspects of tacit knowledge

[5] Food cooked in the ground using hot stones (see Fig. 5.12).

in visuospatial reasoning. A greater appreciation of the thinking behind out-of-school mathematics and school mathematics will bridge not only the conceptual-knowledge barriers but also the motivational barriers to learning. The teacher is no longer the sole transmitter of knowledge but knowledge resides within the community and community knowledge is valued. The teacher is a learner in the community of the contextualised classroom. As D'Ambrosio said in 2004 "ethnomathematics is the backbone of mathematics"[6] without which mathematics will not stand up and be of assistance to society.

Around the world, including PNG, curricula have become more proscriptive in the last 15 years. Little attention was paid to ethnomathematics. However, current mathematics curricula under reform began the process to include a "wide variety of rich problems that: (a) build upon the mathematical understanding students have from their everyday experiences, and (b) engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations" (Masingila, 1993, p. 19). The details and analysis of PNG research provide sound evidence that ethnomathematics should be taken into account in curricula development and implementation. Mathematics teaching that is contextualised and concept oriented (rather than procedural) implies that teachers must incorporate students' ethnomathematical knowledge into the planning of learning experiences. "In the long term this will not only make mathematics to be a meaningful and reflective subject but relevant to solving everyday problems found in a complex and an evolving technologically-oriented society" (Matang, 2001, p. 7).

Language and Concepts in Mathematics Education

Concepts are established by language so it is important to recognise the range of ways by which groups indicate measurement attributes. The diversity and uncertainty by which speakers provided words for the commonly used school terms of volume, mass, unit, and composite unit (Chap. 4) indicated that most communities need to consolidate their Indigenous knowledge and determine how best to refer to these ways of thinking and acting in their language and then to appropriately link to school mathematics either by a clause, phrase, or single word. The mostly oral languages are rapidly changing and being overtaken by Tok Pisin (the main creole). In areas with long contact with English around Port Moresby, the language Motu has many transliterations (English terms sounding like Motu words) for mathematical concepts. This was also prevalent in Maori in New Zealand *Aotearoa* prior to the establishment of *te reo Māori* terms for mathematics (Meaney, Trinick, & Fairhall, 2012). Linguistic ways of comparing vary (Smith, 1984). For example, there may be a limited number of comparative adjectives or very general concepts like size.

[6] Keynote address: International Congress on Mathematics Education 10, Copenhagen, Denmark, 2004.

Other languages have a wide variety of terms for “smallness” or “largeness” which was evident in our PNG data (see Chap. 4).

In each situation, the use of visuospatial reasoning will assist the community to determine what might be a good way of describing school mathematical terms in local languages. For example, if the visuospatial mental image for a preposition indicates one idea, how might it be modified for another related concept. If number words are used for different groups of objects, then how can the language provide alternative abstractions for school arithmetic. Alternatively, as you will find in the discussion on elementary schools described below, only the treasures from the language are used for mathematics rather than taking a whole language approach. In other words, the strengths of cultural practice associated with language that provide alternative abstractions or related abstractions for school mathematics might be used. An alternative abstraction might be the importance of “heaps” in spatial arrangements. Another abstraction might be the idea of “a complete group”. In measurement, the idea of estimate might be strong as well as ratios for comparative lengths, volumes, and areas. The idea of comparing and measuring with a unit might be evident even if the word for unit still needs to be derived.

There is no doubt that there are culturally different concepts that are in some ways more complex but abstracted. The number for deciding equality may always be associated with quality. Yet it is used consistently. Number might be an initial approximation for equality when volume is the main equaliser. This is evident in exchanges based on pigs. Exchange systems are complex yet well established mathematically without reliance just on number. Does this go beyond Davidov’s ideas of abstraction (White & Mitchelmore, 2010) in which concepts in mathematics associated with different objects or constructs are eventually abstracted to a general mathematical term such as angle? It seems that western abstractions will dominate discussions on this issue due to global influences. However, it is important to recognise that there are equally important worldviews and abstractions in different ecocultural situations.

Impact on Mathematical Identity

The main purpose for taking an ecocultural perspective to both mathematics and visuospatial reasoning is its impact on the person. The studies on mathematical identity cited in this book relate to adults and teachers. However, if a teacher has developed their identity as a mathematical thinker from an ecocultural perspective, then it is likely, as Fig. 1.2 suggests that the teacher will influence the students in a similar way. For example, the lecturers at UoG had a significant impact on their students—the preservice and in-service teachers (see examples in Chap. 8). The teachers in turn were preparing classes taking an ethnomathematics, often ecocultural, perspective as they made many references to the landscape, the environment, and people’s ways of living within the environment.

Kono (2007) illustrates the impact of culture on his planning for teaching. Interestingly, he developed a number of mathematical ideas that were not from a standard textbook. First his cultural identity is expressed:

> Mathematical concepts and principles are involved in most of the handiwork of the indigenous people of Papua New Guinea. These works are often overlooked and hence alienate the thoughts of mathematics learner. This makes the learner to think that mathematics has a foreign origin and has no relevance in our socio-cultural contexts.

Then he illustrates the weaving patterns including the "three up—three down [*diagonal*], chevron, V [vertical reflections of each other] and block" (similar to the *diamond*). He notes the lines of symmetry of each (0, 1, and 2) and says

> Students should be told to look at the pattern of the weaves rather than the shape of the complete work. The symmetry lines are being darkened. Instead of asking the students to draw what is in the textbooks the teacher can encourage the students to improvise strips of materials such as papers, bamboos, … to create patterns for themselves from weaving'

He suggests they could do these in other subjects if necessary. He then continues to produce some mathematics which is original to his own exploring of the visuospatial representations of shapes. His report continues in Fig. 7.3.[7]

It is evident that the teacher was developing his mathematical reasoning from the visuospatial representations established through cultural learning and about which he was able to reason in an original fashion. These ideas were not covered in his own education.

One aspect that was strongly developed in the different teachers' projects was the use of visuospatial representations in village objects (houses, traps, bilums, carvings) and village activities (selling food, playing games, imitating parents in hunting, making objects, building houses, even arguing) to recognise mathematics. The teachers often said the mathematics was not necessarily recognised by the Elders as mathematics. The teachers only thought of mathematics as school mathematics up until they developed their projects but they continued to think in terms of the school syllabus in terms of topics. However, they now extended their conceptualisations to incorporate traditional or contemporary cultural ways of thinking and doing. Pepeta (2007) espoused this for his Enga community in Hawks' land[8] around Wapenamanda, noting his cultural identity. He listed the activities he then developed from two kinds of houses (round house for men and rectangular house for women) (Fig. 7.4).

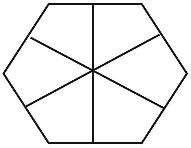
He also provided a description for a game *bras flaua* (like Happy Families) associated with algebra, and woven bands associated with shapes and corresponding angles. It was very common for teachers to apply the ecocultural representations to school geometry shapes but behind this was knowledge of how the shapes were made. For example, how the surface area of the cone roof of a round house is covered with kunai grass, bundle by bundle, or planks placed vertically around the

[7] Some modifications to the text were made to describe angles rather than to use letters on the diagrams. Some diagrams have been omitted.

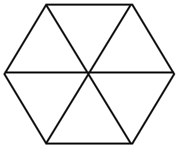
[8] He referred to his totem and his ecocultural links with the land.

The lines can be constructed in two ways: one, by drawing the lines to meeting the corners, and two, by drawing any perpendicular lines from the centre of any side meeting the opposite side right at the centre … .

For a regular hexagon



(a) Lines subtended from sides.



(b) Lines subtended from corners

If you should combine the number of symmetry lines, you will notice that the number of lines of symmetry in any regular polygon is equal to the number of sides of that polygon. Therefore we can write as

N = L Where N = number of sides of a polygon
L = number of symmetry lines in that polygon.

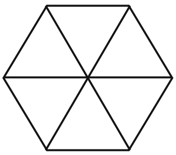
Example:For a pentagon, N = 5 $\therefore$L = 5, Nonagon, N = 9 $\therefore$L = 9 and so on.

The number of sides is the equivalence of the number of bamboo strips that are required to make one of these shapes. This also holds true for any equilateral triangle or a regular quadrilateral. Ask your students to try it out.

Rotation and total interior angle of a polygon

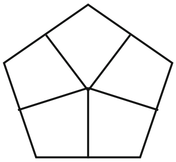
This section will give any mathematics teacher an insight into his/her approach of teaching geometry that the normal way of finding the total interior angle is not the only mechanism of determining solutions. Lines of symmetry can also be employed in any polygons to determine the total interior angles.

As in the example on regular hexagon, there are six (6) equilateral triangles.



However, in an equilateral triangle the measure of each angle is 60° two angles = 120°. There are six sides so 120° x 6 = 720° the total of the interior angles. Or 60° x 12 (total angles formed at the corners) = 720°.

In another dimension (sic), say the symmetry lines are perpendicular to the sides. In a pentagon, for instance. Pentagon has five symmetry lines. The lines are drawn perpendicular to the sides to meet at the centre. Then, angles at the centre sum to 360°. Since all the angles are equal and there are five angles, denote $5x$ for all five angles, and hence $5x = 360°$. $\therefore x = 72°$. You can be able to find obtuse angles of the triangles because the other two angles are right angles.



Hence 360° – (72 + 180)° = each obtuse angle, that is 108°. These sum to 540°. These are the angles of the pentagon. If we isolate one of the shapes in the large pentagon, it looks like a kite. But all the kites are similar. Therefore it can be deduced that a pentagonal shape is formed by a full *revolution* of only one kite. Any teacher should be aware that as long as any two angles in a kite are known the others could be calculated by *rotating the* kite for one revolution. This idea can be applied in other regular polygons too.

Fig. 7.3 Cultural identity reflected in mathematical identity (Kono, 2007)

The traditional way of doing things especially designs reflect back the wisdom and knowledge of the people though there was no knowledge of mathematics known to them as today but mathematics was also used in the types of activities they performed every day.

In the traditional society of PNG, adults take the responsibility for teaching children about the cultural values and resources. The children learn by watching adults by trying things out by correcting where necessary. Children spend lot of their time with the same age group. They would build, make-believe, houses and gardens, pretend to hunt birds and pigs … They acted out feasts, dancing, making *moka* (*tee* in Enga), building houses, *bras flaua*, selling of food stuff, making bilums and string bags for different purposes, belts, paying bride prices, and imitate the older people in many different ways. The girls also imitate their mothers working in the gardens, carrying bags, washing babies. These are the shapes which are identified from the two traditional houses.

Shapes:	**What part to be calculated:**
Triangle,	its area, base, height and the angles.
rhombus,	its sides, height, area and angles.
rectangle,	its area, width, height and the angles.
semi-circle,	its circumference, diameter and radius. Sector, area.
cylinder,	its height, area of the base, radius, volume and diameter.
circle,	its radius, diameter, area and the volume.
cone,	its area of the circular base, height. diameter and radius.
square.	its area, angles, height and the width.
Sphere	its area and the radius.
Trapezium	its area length and the height.
Cuboid	its area, volume, height, length and width.

Fig. 7.4 Examples of built environment linked to school mathematics (Pepeta, 2007)

wall or the sticks arranged in the frame of the roof of the rectangular house making triangles and trapezium (Figs. 5.11 and 5.13).

Cultural identity and recognition of mathematical knowledge as having a basis in culture were effective drivers in producing quality mathematics. Much of this can be credited to self-regulation for the following reasons (Fig. 1.2; Owens, 2007/2008, 2014; Wilson, 1997):

- The projects were on topics that the teachers selected themselves
- They applied their own goals
- They explained the mathematics
- They solved the problems of sourcing the details of the activities and connecting them to school mathematics
- They structured learning environments for their school students
- They evaluated their successes in their conclusions

Affective aspects were revealed by the degree of engagement with the task, portrayal of ownership of the mathematics in the culture, their imagination to prepare examples, resilience in problem solving, and the quality of their reporting.

Values about improving social cohesion if students found mathematics more relevant and less students might become “rascals” (criminals). Hope and aspiration was evident in some of the projects prepared at UoG.

There was a synergy between ecocultural mathematics and representations of school mathematics. Teachers valued the abilities or mathematical processes of their ancestors and Elders but their understanding was enhanced by school mathematics. The teachers recognised the importance that an ecocultural pedagogy had in terms of learning and developing their students’ sense of worth.

Moving Forward

By taking an ecocultural perspective on visuospatial reasoning in mathematics, especially geometry and measurement, the meaning of mathematics is extended to incorporate more of the cultural ways of thinking mathematically. Indigenous cultures in particular have strong spatial experiences associated with mathematical concepts. Visuospatial reasoning is embodied in activity, often in a group situation. Furthermore, the few examples provided in this chapter support the view that cultural mathematics has a stronger basis for learning than school mathematics, creating visuospatial representations, and ways of thinking to which the learner can fold back during problem solving. Furthermore, despite concerns by teachers for students to perform on the national stage, it is clear that the learners were highly motivated by culture and that cultural identity could be harnessed for mathematics through an ecocultural approach in geometry and measurement education. However, can this extension of mathematics to value ethnomathematics address the social justice issues and the global issues of education for Indigenous communities? The next chapter delves into how an ecocultural perspective on visuospatial reasoning for geometry and measurement assists learning for the Indigenous student and the curriculum.