Chapter 6
Visuospatial Reasoning in Other Cultures

> *What avail is it to win prescribed amounts of information about geography and history, to win ability to read and write, if in the process the individual loses his (or her) own soul: loses his (or her) appreciation of things worthwhile, of values to which these things are relative; if he (or she) loses the desire to apply what he (or she) has learned and above all, loses the ability to extract meaning from his (or her) future experiences as they occur.*
>
> (John Dewey, 1938, p. 49)

The Challenge

An ecocultural perspective of visuospatial reasoning was established in Chap. 5 through first-hand and recorded experiences in PNG. Geometry and measurement cover a broad spectrum in terms of locating, comparing, and measuring of different attributes and notions, experiencing the physical world in all its manifestations, and going beyond the physical to abstract ideas. Is it possible to capture more of the wealth of visuospatial reasoning if an ecocultural perspective is taken across recorded experiences in other parts of the world?

This chapter will draw on studies influenced by cultures in the Americas, Australia, Pacific, Africa, Middle East, and Asia. In particular we note how visuospatial reasoning is enriched by locating and experiencing in the physical world and beyond and how visuospatial reasoning is used in creativity of different people.

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Early Studies of Visuospatial Reasoning with an Ecocultural Perspective

One of the earliest and continuing studies in this area was by Berry (1966, 2003, 2011). His work indicates that individual differences arising from psychological, socioeconomic, and ecocultural experiences promote better adaption in an intercultural environment. He advocates multicultural frameworks at the classroom and national curriculum level to produce mutuality and reciprocity, equity in participation, and maintenance of cultures and identities. However, in this chapter we will look at his early study of perceptual and reasoning skills that compared Scots from UK, Temne from Sierra Leone, and Eskimos from Alaska. The study was carried out at a time when quantitative analysis prevailed and relatively culture-free tests were used (he noted that socioeconomic disadvantage in lack of visual materials did impact on test results when culture was kept constant). One interesting aspect was his comparison of words and environments in which he noted that the whiteness of the Eskimo environment seemed to contribute to high ability in noticing detail and more language words (rather than English transliterations). He also attributed this higher ability to child-rearing practices in which the stricter upbringing was associated with lower scores on perception and reasoning and women's dependence more field-dependent perceptual characteristics. He noted that western education reduced transitional communities' perceptual scores. He "concluded that ecological demands and cultural practices are significantly related to the development of perceptual skills; it has been shown that perceptual skills vary predictably as the demands of the land and the cultural characteristics vary" (Berry, 1966, p. 228).

Navajo Knowledge

Another long-term study that shows the strengths of traditional community living comes from Pinxten and colleagues with a strong anthropological/sociological analysis of mathematical thinking (François & Pinxten, 2012; François, Pinxten, & Mesquita, 2013; Pinxten, 1991; Pinxten & François, 2011, 2012a; Pinxten, van Dooren, & Harvey, 1983; Pinxten, van Dooren, & Soberon, 1987). Part of Pinxten's argument for a strong, rich geometry that differs from western mathematics is reflected in the following passage, taken from a small story he tells:

> Coming out of the pass he paused to eat, and then moved on in the direction of Badger Rock. He did not see the rock for a good time, since it was hidden by the black wall of Snake Rock extending right across you, from the south to the north. It had to be followed for a long time until one could see the small pass, which was hidden behind three juniper trees. They were the only trees sitting together on the edge of Snake Rock. When one did not know there was a pass behind them, one would keep following the rock for miles, without being able to cross it. Chee had found the pass and started climbing the steep path, while Chuck barked at the herd, which was wary of following Chee up the slide. After a remarkably

> short climb, Chee reached the flat top of the rock, where all of a sudden he saw Eagle's Nest, the small flat stone, only a few yards away from him. At that point, he started looking for Badger Rock. A whole range of rock formations was spreading out before him, as far as the eye could see. He stood above the canyon of his parents, unable to see anything of it beneath him. He carefully looked at the rocks in the distance in front of him and recognized Badger Rock after a while. He could walk at ease now, almost on a flat surface until he would reach a small canyon he had to cross, straight to Badger rock. When he would make a turn to the right at this standing rock called the Badger, he would walk away from the sun and reach the arroyo of Salt Water in a short time. (Pinxten & François, 2011, p. 262)

Pinxten and François then attempt to summarise the richness of Chee's mathematics for non-Navajo:

> It is almost impossible to picture the real "landscape" Chee is tracking through; one is dwarfed by the enormous red mesas and other rock formations one wanders through, and no "straight" line will be definable as the shortest distance between two points one can see.
>
> Chee works with the following elements in order to orient himself:
>
> 1. The sun; different positions function as a clock for Chee. If his spatial markers mismatch with the sun's positions, he knows he is in serious trouble.
> 2. Certain conspicuous rock formations are major references or markers.
> 3. The general topological notion of path is essential, as are the cardinal directions.
> 4. The need for water and feed for the animals is essential in the notion of path Chee uses.
> 5. Adjacency and separation are two topological notions that have an important status in Chee's movements.
> 6. Going up and down, front and behind, narrowness and wideness as nonmetrical spatial notions play a primary role in Chee's orientation.
>
> If we make abstraction of most of the actual rocks and mesas, dips, and canyons, we could draw a little map to represent the major features of the orientation system Chee works with. However, that would take Chee out of his context and into the schooled context we expect from him at the expense of the home knowledge. (Pinxten & François, 2011, p. 203)

Pinxten with colleagues developed a geometry curriculum for the Navajo based on their rich cultural knowledge embedded in place and survival. This is considered further in Chap. 8 to illustrate that mathematics education can and should take account of the rich knowledges of communities. The dynamics of spatial boundaries is further elaborated by Ascher (1994). If the activity can continue on the other side, then the boundary is on Navajo land but if it has to be reversed or modified then it is a boundary with non-Navajo land.

> The mountain ridge itself is an interrelated system of parts that are in motion and in the process of change. Further, the entire earth, of which the mountain ridge is an integral part, is in motion as well. The earth and sky are always undergoing expansion and contraction. … to the Navajo, the significance is the processes of which the boundary is a part and how it affects and is being affected by those processes. …space should not be segmented in an arbitrary and static way." (Ascher, 1994, p. 129)

Thus the worldview is impacting on visuospatial reasoning of the Navajo providing relationships between places. The richness of visuospatial reasoning in an eco-cultural context includes mental processing of richly connected places adding to the mental and emotional "mapping" associated with visuospatial reasoning.

Pacific Navigation

Navigating on land where there are some signs such as rocks and dips is one thing but navigating on the sea requires other extraordinary skills. Some of the most intricate ways of describing place and space have come from the descriptions of navigating in the Pacific Ocean (Oceania). The following comments are based on earlier discussions by Akerblom (1968) on navigating in the Caroline Islands, Marshall Islands, and Gilbert Islands; and on the Caroline Islands by Lewis (1973); from Worsley's (1997) journalistic style of writing; and from the Penn Museum of the University of Pennsylvania (1997) whose website provided dynamic images to assist understanding. Further work on the Marshall Islands navigation is available from Bryan (1938), Davenport (1960), and Spennemann (1998). However, Hutchins (1983, 1995) provided a detailed explanation of the unique systematic visuospatial way of thinking of space used by Marshall Islands navigators. Uses of star charts in "wayfinding" also occurred in the large Polynesian routes such as from Hawaii to Tahiti (Davis, 2009; Polynesian Voyaging Society, ~2003). Islands are out of sight and without a magnetic north compass, these sailors have sophisticated and skilled ways of travelling. Some sailors travelled thousands of kilometres and returned.

In the Caroline Islands, the star positions vary over the course of the year as the earth is on a tilt. A star chart had 32 positions. Sailors know an island's position on a particular star direction and they have sea roads that are taken regularly which take account of the swells and currents. When sailors start off on a trip, they assess the strength of the swells and currents by noting how far off course they move. They then adjust their direction. According to Worsley, the star positions are less important than the sea roads but they do provide a holistic visuospatial mental map on which to superimpose the sea paths. When travelling, the stars are kept between the halyards of the mast. Around each island there are usually two concentric circles for the limits of two kinds of birds. Swells also depict the position of islands when the boat is closer. In the Marshall Islands, stick charts represent the flow of swells around islands and divide up the plane so that a sailor can tell whether the canoe is on track for the island, or when coming from the other side has reached the doldrums behind the island. Curves illustrate the routes and sail positions to take account of the currents, refracted and deflected swells. Close islands have different swell patterns. Davenport (1960) notes that more than one kind of stick chart may represent the island and sea information (Fig. 6.1b). Furthermore, there are variations between the existing charts and the information they provide. Some provide more detail, others specific travel times or distances. Bryan (1938) noted there were three kinds of charts which he associated with the whole group, a part of the group, or as a general instruction (Fig. 6.1a). Some are specific to given islands whereas others are more generic while others vary on the importance placed on either island locations or the swells.

Most important is the recognition of the sailing canoe being the reference point so that sailors give directions, along star lines including those yet to rise, of the starting and finish islands and places, whether an island or a place on the sea, to the north or south of the travelling line (Fig. 6.1c, from Hutchins, 1995). As the canoe moves

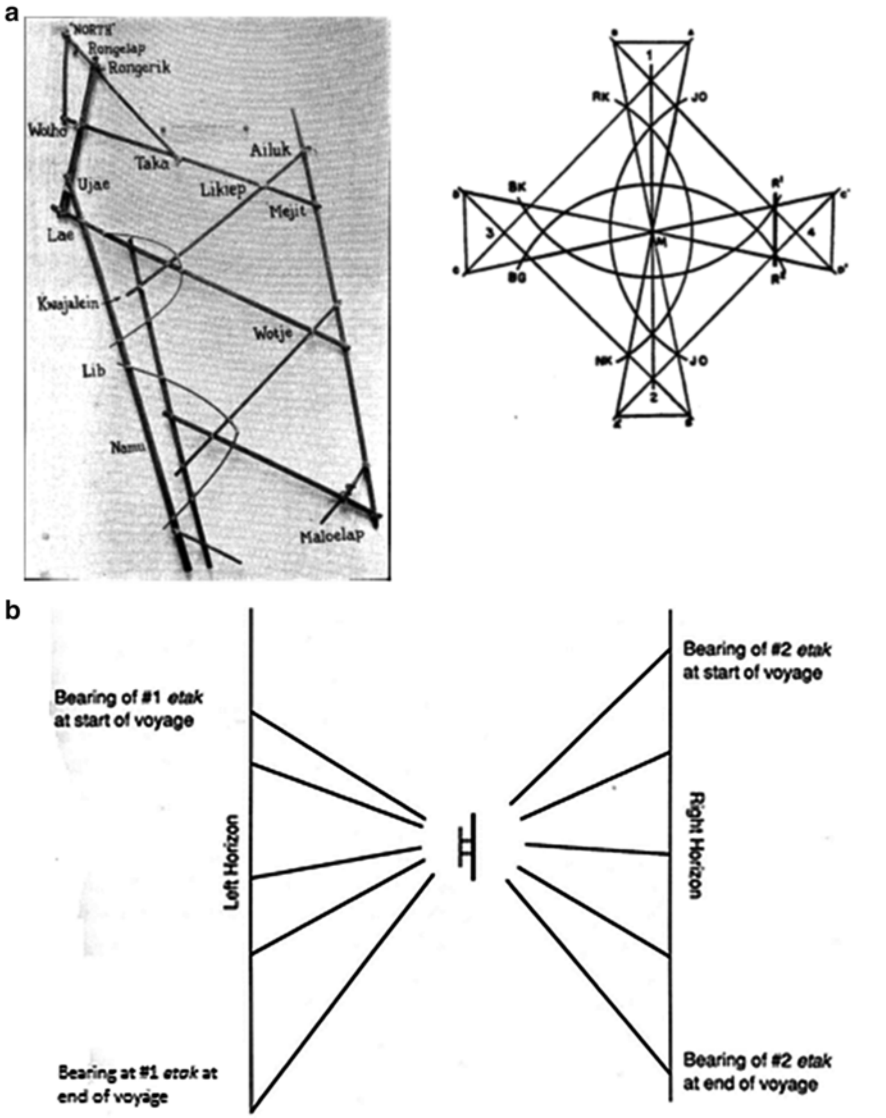

Fig. 6.1 Representations from the Marshall Islands. (**a**) Swells diagram (Davenport, 1960, p. 22). Stick chart for learning places (Bryan, 1938, p. 13). (**b**) Representation by Hutchins of the navigating system with the moving boat being the reference point (Hutchins, 1995, p. 89)

along the travelling line they know from experience their speed, time, and likely distance. Predictions are adjusted accordingly as the stars confirm position of the places in reference to the canoe. Sailors will wait for dawn and the bird flights before travelling on so they do not over sail their destination (Hutchins, 1995). Sailors have sea roads that are taken regularly and that take account of the swells and currents. Sea places are represented by a day's sail (*etak*) or by being directly south of an island, and are named, for example, from Puluwat to Eauripie, whale 1–6. The idea of a right-angle turn (breadfruit picker symbol), the trigger fish "map" also systematises the visuospatial images (Penn Museum, 1997).

The star charts with the moving stars around them provide visuospatial representations to assist the Caroline navigators to reason about "motion, relative position, and relative direction" (Ascher, 1994, p. 149). While games are used to learn the main star positions initially using stones or coral, later their position from different sea places and islands are learned. Gradually the various games become more complex. One game, island hopping, requires the names of islands on a particular star direction. Routes are combined and reversed in the games which have various nautical names. Dragging is a game in which the children give the position of places from a place which is not their home (Penn Museum, 1997).

However, these navigation systems are more than locating. They are a way of problem solving. They are elegant and effective ways for thinking in the head about position and movement. Interestingly the positional accuracy provided by these systems is embedded in the activity with an emphasis on connections and routes rather than lengths unlike scale maps in other navigation systems (Barton, 2008). The e*tak* brings together the navigator's knowledge of rate, time, geography, and astronomy and used appropriately (not as a linear measure) to determine destination. It is a logical construct or cognitive map.

Watson-Verran and Turnbull (1995) argue that the cognitive map permits the knowledge to go beyond the local even though the Marshalles used wave swells which the Puluwatans did not. It can be extended to new situations. The importance of the visuospatial reasoning is that the knowledge is used contingent with the place and the environment at the time. The vast body of knowledge can be passed on encoded in song, ritual, testing, mnemonics, group learning, connecting knowledge, and represented visuospatially with stick charts and stones on maps. The tacit knowledge and skills are also linked to the cognitive map for reasoning. The sets of information are coordinated but not necessarily in a unitising way as western measurement expects.

Ascher's (2002) thorough analysis of the stick charts and sailing techniques illustrate that the models are not intended to represent what is there like a map.

> [They] encapsulate and explain the system. When they are used by the Marshall Islanders for teaching, … they elaborate such depictions with words, but words alone would be insufficient. Particularly for dynamic systems, diagrams play a crucial role. They not only provide a way to visualize the interrelationships of the parts, but enable us to keep the entire system in mind while mentally manipulating or focusing on some part of it.
>
> The essence of an explanatory model is its simplicity. … Essentials are phrased in terms of the geometric characteristics of the ocean phenomena—the substances of the land and sea and wind are recast into points, lines, curves, and angles, and the interplay of the phenomena is recast into how these geometric aspects change and interact. (Ascher, 2002, p. 114)

Ascher (2002) also notes that the young navigators lie in the water to feel the swells and that sailors will lie on the bottom of the boat to feel the swell. These spatial imageries like the tilting of the head to see the stars between the halyards mentioned by Worsley (1997) illustrate the importance of spatial, bodily imagery given meaning ecoculturally in visuospatial reasoning.

> Numerical, spatial, and linear concepts in Melanesia, Polynesia, and Micronesia vary in accordance with distinctive physical environments and the social and cultural histories embodied in these Western assigned boundaries of the Pacific. (Goetzfridt, 2012)

One aspect that Goetzfridt mentions is the use of mnemonics for remembering star or geographic positions but in fact much of the memorisation is of a visual nature, the four points associated with a trip that forms a quadrilateral initially or trigger fish with different areas represented by the head, dorsal fin, ventral fin, and tail, together with places on the backbone for real or imagined places to assist with the navigation. The parts of the fish stylised as joined rhombus act as a metaphor for position and traversing the sea-lane. The use of fish names and many other distinctive objects or imaginary creatures also aids memory. Thus is the scheme to assist the navigator from Polowat in the central Caroline Islands to Guam in the Mariana Islands. Similarly, the parrot fish probing at the reef hole enables the travel from one island to the next until the sailor arrives back at first and catches the fish. The i-Kiribati envisaged the heavens as a giant roof with purlins associated with specific stars and through a story of travel over 150 stars could be named. Furthermore, it is in the visuospatial reasoning using trees and/or roots to discuss people in a group's relationship with the land that metaphor is important (Goetzfridt, 2012).

Australian Aboriginal Astronomy

There is growing evidence that Aboriginal Australians knew of the movement of the stars and used them for calendric purposes and other practical purposes such as measurement of distance (Norris & Norris, 2009). Across the 400 or more Australian Indigenous languages and cultures, there are some similarities in stories related to the sun and moon. One story indicates that there was a long association between tides and moon and variation in tides. This is visuospatial reasoning in large space. From the Northern Territory comes this story.

> The Warlpiri people explain a solar eclipse as being the Sun-woman being hidden by the Moon-man as he makes love to her. On the other hand, a lunar eclipse is caused when the Moon-man is threatened by the Sun-woman who is pursuing him and perhaps catching up. These two stories demonstrate an understanding that eclipses were caused by a conjunction between the Sun and Moon moving on different paths across the sky, occasionally intersecting. (Warner, 1937)

Norris and Norris (2009) suggest that a boomerang-shaped stone carving because of its juxtaposition to a man standing in front of the woman actually represents an eclipse. If so, Aboriginal Australians have provided some very early astronomy (these cultures are considered to have existed for more than 60,000 years). The stars are also used in calendars which are complex, with several seasons, and the appearing of stars rising in a part of the sky may indicate the beginning of a cold season and for deciding when to move for different food sources. The Mallee-fowl constellation (Lyra) appears in March when the birds build their mound nests and it disappears in October when the eggs are laid and could be collected (Boorong people of Victoria). Similarly, the rising of Scorpius tells the Yolngu of Northern Australia that the Macassans from Indonesia will soon arrive to fish for Trepang. The dark patches of the milky way are called the emu in the sky by many Aboriginal groups (not all), and in Sydney can also be found the engraving of an emu (feet back) lined up with sky emu at the time when emus lay their eggs. The Yolngu also track Venus knowing it is "held" to the sun but also that it only rises in the morning on a few days each year when they hold their morning star ceremony for which they prepare over some time. Though information is now lost, the engravings by the Nganguraku people on the Murray River cliff face are about the moon and sun, recording their movement in some way. The Wathaurung people of Victoria also built stone arrangements, one line being east–west, other stones placed to be in line with the setting of the sun at the equinox (Norris & Norris, 2009). Thus visuospatial representations are used for astronomical purposes by Aboriginal people in Australia as early astronomers.

Circle Geometry and Straight-Edged Shapes

Not long after I first began to explore the tile work of Portugal (Fig. 6.2) and Spain and the influences of the Middle East on these countries, I met Moustafa, a refugee in Australia from Afghanistan. He painted both landscapes and abstraction in miniature in circles. I asked him how he worked within the circle. Often he repeated his abstract design as rotational symmetry around the circle but other times he varied each sector. He first divided 360° by the number of sectors, drew a radius, and literally used a protractor to draw in each radius for the number of sectors he wanted whether that was 12 or 16. However, the next step required considerable visuospatial reasoning to ensure there was balance and no section of a sector dominated. Mostly he worked with curves carefully positioned on the sector. Colour too was carefully selected to ensure there was no dominance.

Figure 6.2 shows the basic circle constructions for developing a large range of tiled or painted spaces. Because of the interlocking spaces on the circle, one or more shapes are repeated and fill the space without gaps. Tessellations are the tilings of the same shape. The tiles join together without gaps or overlaps and with a pattern that allows the tessellation to continue in both directions. If two shapes are used it is a semi-tessellation. Some examples are shown in Fig. 6.2a but the intriguing thing is how an artist can picture the shapes within the myriad of construction lines and create another design for straight-edged tiles or painted shapes. The circle designs are not all Islamic, but for the Moors and other Muslims they contained no images.

The extraordinary hollowed patterned arched ceilings of La Alhambra (Fig. 6.2c) are the most beautiful and amazing three-dimensional tessellations that illustrate extraordinary visuospatial reasoning. Architecturally, hollowed panels in ceilings in La Alhambra and the pantheons (Rome and Paris) make them lighter but still strong.

Fig. 6.2 Visuospatial difference reflected in tiles from around the world. (**a**) Circle construction lines and resulting tessellations La Alhambra, Spain. (**b**) Firenze duomo, Italy. (**c**) La Alhambra ceiling. (**d**) Tiles in Portuguese palace, 3D illusion. (**e**) 3D tiles on cheda Thailand

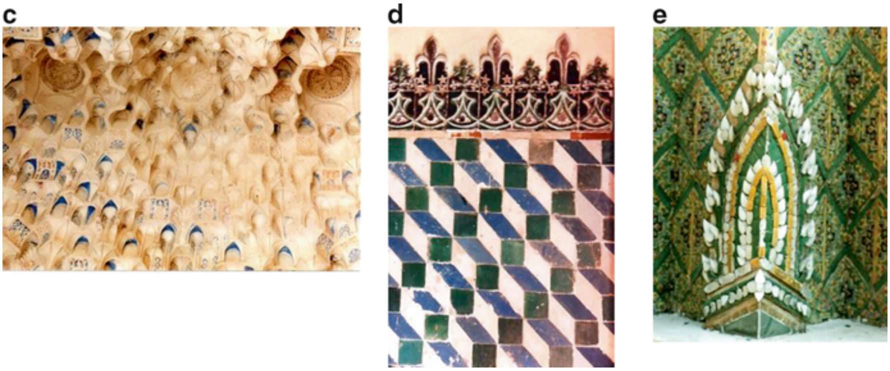

Fig. 6.2 (continued)

Domes and arches represent other marvellous architectural designs that portray visuospatial reasoning. Fine examples are the Sancta Sophia Turkey and the cleverly constructed duomo of Firenze by Brunelleschi with its internal spiral connecting the inner and outer walls and providing a means of construction. The external, more recent tiling of the duomo (Fig. 6.2b), the tiled floors of Pisa's belltower, and the three-dimensional tiling of walls of chedas in Thailand (Fig. 6.2e) and other parts of Asia must be some of the most exquisite tiling in the world illustrating a strong link between visuospatial reasoning and creativity. In modern times, the "sails" of the Sydney Opera house with its modularised construction provides further evidence of visuospatial reasoning in practice.

Creative Designs Across the World

Strong images from the Pacific are the sand drawings from Vanuata. These continuous curves are carefully crafted and associated with cosmology. The order in which each part of the figure is drawn indicates a strong overall visual image together with memory of order but the exactness is remarkable as shown in the figure from Deacon and Wedgwood (1934) shown in Fig. 6.3a. A strong sense of reflective symmetry is evident. However, Goetzfridt (2012) emphasises the associated story encouraging a recognition of the emplacement of myth (Rumsey & Weiner, 2001).

Ascher (1994) points out that some designs are repeated curves that may come from rotations or symmetries as shown in Fig. 6.3b and which she associated with some of the *kolam* prepared by Tamil Nadu women (e.g. Fig. 6.4b) although they also have other symmetrical reflected designs that can be incorporated in large designs (one or two reflected symmetries) (Ascher, 2002). Interestingly she notes that the Siromoney have used the Tamil Nadu designs in computer rules such as in Logo, albeit in more straight rather than curved formats. It is noted that Ascher divided the *nitus* sand drawings into those with even-degreed graphs and those with

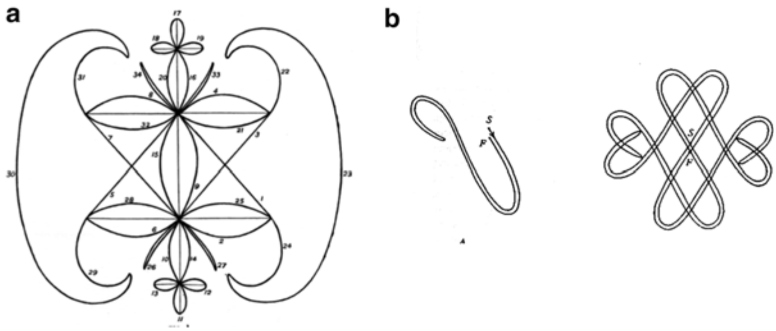

Fig. 6.3 Creating designs in the Pacific. (**a**) One of at least 91 sand drawings recorded in Vanuatu (Deacon & Wedgwood, 1934, p. 148). (**b**) Rotational and reflective symmetry in creating sand drawing using a base tracing, note the start (s) and finish points (f) (Ascher, 1994, p. 53)

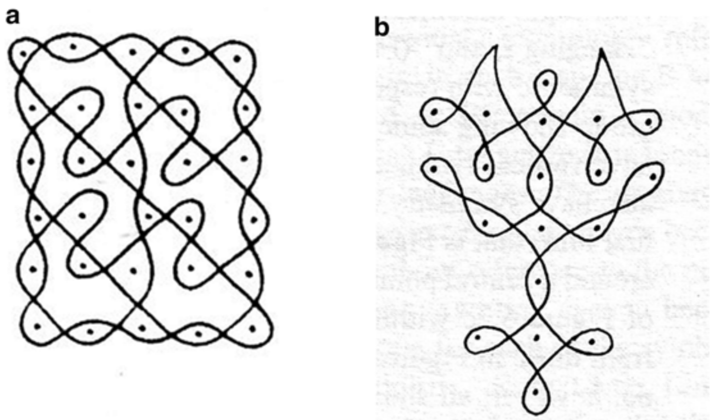

Fig. 6.4 Rows of dots used as markers for the curved lines to create the design. (**a**) Tshokwe (Central Africa) *Sona:* "the marks on the ground left by a chicken when it is chased" (Ascher, 1994, p. 42). (**b**) Tamil Nadu *kolam*: nose jewel that is embedded in other *kolam* (Ascher, 2002, p. 65)

a pair of odd vertices, by the systems used to trace them as well as by the presence or lack of visual symmetry created by the tracing procedures. The *sona* decorations on embroidered cloth or sculpted wooden objects, masks, woven belts, hats, and other objects produced by the Bushoong for the Kuba exchange system of mainly Angola are copied as sand drawings by children (Ascher, 1994). They can also be analysed in similar ways. The men of Tshokwe in West Central Bantu area create similar continuous line designs for significant cultural purposes. These are topologically analysed by Ascher (1994). The Tshokwe place dots in rectangular format to divide the space and provide the structure (Fig. 6.4a) but size may vary, and in most cases the vertices are of degree 4. (Some with odd vertices are actually being joinable outside the rest of the structure.)

Fig. 6.5 Fijian tapa, 1977

Many studies of the symmetries of the beautiful Polynesian strip patterns such as those found in New Zealand *Aotearoa* have been made but visuospatial reasoning is evident in some of the designs described by Ascher (1994). Decisions were sometimes made so that colour usage did not follow the carved symmetry but rather created a new idea in terms of symmetry loosely described as "juxtaposing one symmetry with another" (p. 170). Importantly while the ecology has influenced the designs,

> the harmonies, balances, rhythms, symmetries, and asymmetries …[are] related to and expressive of the structures that underlie the Maori belief systems. Complementarity, the relatedness of pairs through difference, and symmetry, the relatedness of pairs through sameness, are seen as organizing principles in much of Maori myth, religion, social life, and economies, … [including] the male-female complementary relationship, …the world of humans and the world of the gods. (Ascher, 1994, p. 171)

Tapa is often decorated with symmetrical designs. In Fiji, stencils or stamps are used to repeat a design as individual blocks together with lines in regular patterns (Fig. 6.5) while in Tonga a board *kupesi* is prepared tying on curved and straight sticks into different patterns, using different triangular forms but also a plant motif with curved lines. Once the repeated pattern is rubbed directly on to the tapa placed above the kupesi and shifted along, then it is carefully painted along the rubbings. This is a group process so the making together of the *papa koka*'*anga* with the repeated pattern built along it using coconut sticks, then rubbing and painting together is an important part of pattern and design relationships. In Fig. 6.6, various designs with rotational and translation symmetry are shown. Finally different colourings are applied. Finau (Finau & Stillman, 1995; Stillman & Balatti, 2001) linked this work to matrices for different translations to make a connection to

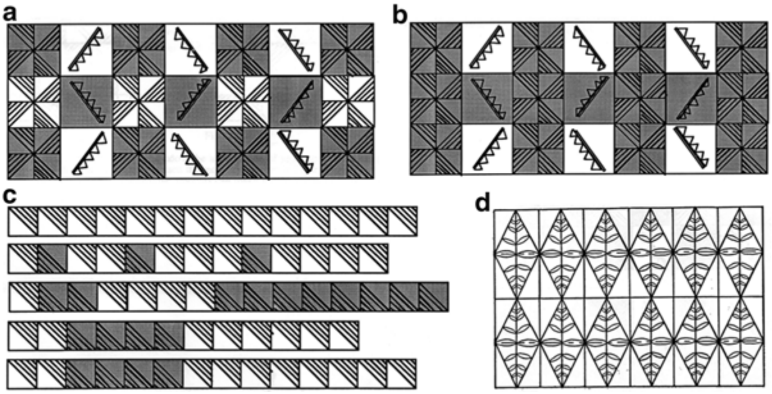

Fig. 6.6 Tongan tapa designs. *Source*: Finau and Stillman (1995). (**a**) Repeated rotated design with symmetries and a "perfect" colouring. (**b**) Repeated rotated design with symmetries and "imperfect" colouring but still attractive. (**c**) Various patterns incorporated into the repeated motif design. (**d**) *Kupesi* board coconut fronds bend to form plant designs

academic mathematics and to a variety of growing patterns in the background colourings or otherwise, all of which are pleasing and provide for creative variation in the tapa. As in PNG where group connections and group activity are just as important in the house construction or design, so it is in the creation of these Pacific designs. Much of the tapa of PNG is unpainted but in Oro Province, free-hand designs with some repeated curves are common (Fig. 5.20).

Symmetry is one of the more obvious results of the designs created and often repeated by Indigenous cultures. It is evident that the artisans use systematic ways of making, creating, and recreating. The following, discussing embroidery and weaving of the Hñähñu: the Otomies, Central America, is typical of most Indigenous artists in which detail, complexity, and cultural significance in terms of relationships ecoculturally are important.

> First, to become a traditional Otomi weaver is a life long process. A common situation is one in which there are several generations involved in creating woven or embroidered projects. That is, there is often a gathering that includes girls, mothers, aunts, grandmothers, and so on. Second, Otomi weavers and embroiderers must keep track of many counts of threads, and must make precise measurements. They must know the entire design from memory. The impressive part of the weaving or embroidery process is that the artist typically is not using diagrams for the patterns, nor is that person using a ruler to measure distances. A third consideration is that the products finished by the artists (as is the case in many traditional contexts) often have important cultural significance. Finally, it is important to mention that many kinds of weaving and embroidery designs are very time consuming to make. It is not unusual for traditional textile artists to take several weeks to make a work of their art. (Gilsdorf, 2009, p. 91)

Symmetries are evident in the decorations from southern Africa. Significant is the spatial embodied aspects of visuospatial reasoning in creating the designs. The *litima* designs

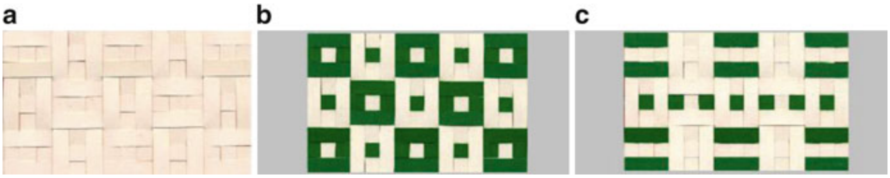

Fig. 6.7 The artisan and learners' designs. *Source*: Cherinda (2012). (**a**) Artisan's basic mat. (**b**) Highlighting with colour. (**c**) Another colouring

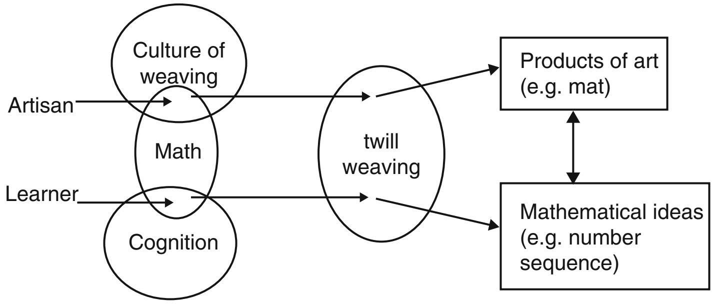

Fig. 6.8 Cherinda's diagram of ethnomathematics in terms of the mathematical learner. *Source*: Cherinda (2012)

> function as an extension of human action, and echoes the structure of the body. A painter, when drawing on the ground in explanation or preparation for mural painting, often draws with both hands. She begins at the top of an imaginary vertical, and the resultant forms on either side of this are simultaneously realized and are mirror images of each other. (Changuion, p. 35, cited in Gerdes, 1998)

The litima designs consist of a basic square that is repeated or reflected in the vertical axis and then a further two squares below are repeated or reflected in the horizontal axis. In other cases, the change is also accompanied by a reversal of the dark and light areas. Many designs involve curved lines but there are also straight-edge designs. Despite the amazing diversity, each is beautiful, technically well done and well integrated in terms of the overall four-square design forming a new design unit. For example, some curved lines do not finish at a corner but join to a curve on the side of the next square.

Cherinda (2001, 2002) noted that colour can transform the same pattern of weaving (a design motif) into different appearances (see Fig. 6.7a–c). He provides us with a way forward in connecting artisan's work to that of the mathematician as shown in Fig. 6.8.

> The premise is that mathematical objects (ideas) produced by the learner in the process of weaving… can be applied for stimulating new mathematical ideas and for producing new patterns in the weaving of the mats as well. … It is the instances where I say that both are 'weavers', versus both are 'mathematicians'. (Cherinda, 2012, p. 932)

Importantly, Cherinda details the visuospatial reasoning of students. For example, he notes that one student said that you count over and under to copy a line of weave, that you look at what is there overall to know whether it is a good look or not. You look at what is above or below. To start a new phase, the student isolated the first line of the weaving to copy. This is an important selective attention but also inductive reasoning. Cherinda noted that the student was able to create his own design, albeit by combining ones he had probably seen. These were created out of his mind. Another student too was able to count and reverse rows to create what they called beautiful designs. This affective aspect may or may not suggest recognition of symmetry as Cherinda claimed but it does indicate how affect is a critical aspect of self-regulating for a mathematical identity developed from an ecocultural perspective (Cherinda, 2012).

Cherinda perceived the connection between artisan and mathematical learner as bringing their own contextual knowledge to bear on their weaving and thus producing both inquisitive artisans and learners creating design in their own right (Fig. 6.8). Further argument on the process of learning visuospatial reasoning in schools is elaborated in Chap. 8.

Weaving often creates three-dimensional objects. Adam (2010) worked with weavers of food covers indicating the ethnomathematics of the artisans and the visuospatial reasoning evident in deciding what modifications were appropriate. Adam provided some suggestions that she had generated from advanced academic mathematics. Weavers rejected one that would be too steep but considered one that lengthened the food cover. Gerdes showed how some African weavers took a flat piece of woven mat and folded it to produce a cone shape container. The connection between flat surface and final product is not the same as the western view of surface area of a cone but there is embedded a cultural perception of size and purpose and strategies for modification. Many cultural groups make fish traps (Fig. 5.14c), storage containers (Fig. 5.15a), baskets (Fig. 5.14b–d), and masks (Fig. 5.14c, d). The visuospatial reasoning in producing a slightly smaller woven container compared to the woven lid is evident in the Timor Leste container (Fig. 5.24b) or vice versa in the Buka basket (Fig. 5.14c, bottom). This is not just a counting exercise nor a slight difference in length but one in which experiences provide visual images of what is required and what happens during weaving. A similar reasoning occurs with ikat weaving from Timor Leste (Fig. 5.24a) where the lengths of thread are carefully determined so that the tie dying of sections of the skene of threads means that in the weaving process, the colour starts slightly before or after the previous row. A similar reasoning occurs with making a box and lid from a sheet of paper. The sheet for the lid will be about 6 mm shorter on adjacent sides (Owens, 2006a).

Geometries are not necessarily linked to number as we found in PNG where visuospatial reasoning did not always occur with number and measurement although in some cases of gathering and counting food items there was a strong connection (see Chap. 5 and, for example, Paraide, 2010). Weaving and bilum making patterns may be, but are not always, counted to create designs. The visuospatial design imagery and subitising are often sufficient for the maker.

However, there are examples from ethnomathematics studies that show the visuospatial representations support numerical ideas especially if numbers are associated with spiritual values. In Mayan cosmology, following the diagonals down and up a mat of three divisions to count 1–6 results in each row equalling seven which was associated with God in divine power and the resultant diamond pattern is linked to the significance of the rattle snake. Priests traversing the steps of a temple also formed the diamonds that are associated with all of life in Mayan culture. The diamond was positioned representing space with blue diamonds for the seas on either side of the land in middle America, and for a sun on the sea from which it comes with a trajectory to the other sea. Like many Indigenous cultures, the rising and going down of the sun provided key reference points.

> Through the development of a sacred number system using mats with divine patterns, Mayan people possessed a sophisticated geometric and numerical creation story of their universe, whose first record is related to sacred numerical values. Numbers, symbols, and words could direct the priests to corresponding numerical values. A study of Mayan practices demonstrates one use of an ethnomathematical—global perspective. Ethnomathematics serves as an academic counterpoint to globalization, and offers a critical perspective to the internationalization of mathematical knowledge through attempts to connect mathematics and social justice. (Rosa & Orey, 2007)

It is important for ethnomathematics to take up the issue of some amazing mathematics existing in Indigenous cultures especially in terms of design. Their particular mathematical processes including visuospatial reasoning are to be valued, respected, and should be part of mathematics curriculum and education for the rights of learners as it is so important for their cultural and hence mathematical identities.

Settlement Patterns and Shelters: Place, not Just Position

In the Pacific, the patterns of settlement vary. However, in much of Polynesia ranking, symmetry and position of housing is evidently significant to the people. In Palau, the long history of symmetry in the meeting house and in the village gave visuospatial representation to power in decision-making processes (Wickler, 2002). In Chap. 5, the organisation of Trobriand Island villages was mentioned; the Trobriand Islands are relatively unique culturally in PNG in terms of hierarchy. In the Sepik PNG, other settlement patterns were evident. Land in PNG communities, like the Yolngu map (Fig. 4.4a) is often in patches, not only for sacred reasons. It may be due to intermarriage, status, the need for a variety of soils and environments such as fertile gardens, forest resources, grasslands, or food gathering resources such as swamps. Housing may be found on the various lands or even over the water. Hence the sections of housing at Tubusereia PNG over the water connected by a walkway, sections on land nearby and further away could belong to the one family group.

Place markers for different sacred sites evident on Lamotrek Atoll, Yap, Micronesia provide another aspect of geometry (Metzgar, 1991, 2004). Such points indicate lines and enclosure, a basic aspect of geometry in schools as well as traditionally. Similarly markers occurred on boundaries in most highland areas of PNG and in Hela and other areas of PNG drains demarcated the boundaries. The straightness and slope of drains is another example of visuospatial reasoning (Fig. 5.9, Edmonds-Wathen, Owens, Sakopa, & Bino 2014). Bellwood's (1979) historic study drew on examples from all parts of Polynesia. He concluded that ecological factors including the nature of the environment (often relatively poor soil on coral bases), people living in that environment, social and political factors determine individual settlement location in terms of clustering and dispersal.

In west and central Africa, Eglash (1999, 2007) found that there was intentional design with social meanings mapped onto the scaling architectural patterns of houses. Circular houses in circles of circles and rectangular houses in ever-diminishing rectangles occurred. However, he found these scaling patterns, that he associated with fractals, in a wide variety of designs, for example, textiles, paintings, sculptures, hairstyles, and religious symbols. The depth and richness of culture behind these is "lost" in summarising. However, one thing that Eglash has provided are useable tools for creating patterns on the computer and this will lead us into the next chapter. For example, he used the hairstyle cornstalks to assist in engaging Afro-American youth, and fourfold "beadwork" for creating a number of Native American designs. He notes:

> The presence of four-fold symmetry in Native American design is not a trivial geometric feature; rather it provides deep cultural connections spanning many facets of life, from religion to astronomy. Moreover its mathematical implications go far beyond that of reflection symmetry, allowing exploration of processes ranging from transformational geometry to iterative computation. This is just one example of the more general need in ethnomathematics to expand from a focus on static images to include process-oriented frameworks that illuminate design in the making, and that offer students a creative medium they can appropriate, for the purpose of expressing their own mathematical and cultural ideas. (Eglash, 2009)

In Brazil, de Castello Branco Fantinato (2006) found that belonging to a place and showing this by countering the expected way of driving on the road made it difficult to assess by observation people's spatial knowledge and skills. Furthermore, women felt the quality of a garment to decide if they could buy it when unable to read the price. Thus in her brief report on the study, she illustrated how ecocultural, especially from a living and socioeconomic situation, impacted on people's visuospatial reasoning to solve mathematical situations.

In Chap. 5, the role of visuospatial reasoning related to house building established a strong case for culture and ecology being considered in discussions on visuospatial reasoning. In the USA, a similar situation occurs. Partly related to prevailing winds and other factors, the Sioux tipi has three poles each planted at the vertex of the equilateral triangle (this triangle is not hard to achieve with two equal sticks as we saw in PNG). The long poles are tied at the top so the rope attached at the top falls to the centre of the triangle. While Rosa and Orey (2012) point out the significance of the centre culturally, they do not make it clear whether the centre is actually found by using the medians (lines from vertex to middle of the opposite side) or whether this centre is estimated or determined by the three equal poles tied at the top of the tipi and the rope falling. The centre is for the fire for cooking, heating, and burning incense in the centre of holiness. It seems that their use of etic (outsider perspective) and emic (insider or empathetic perspective) ethnomodelling could be simplified by talking of an ecocultural perspective of mathematics, especially visuospatial reasoning. Sioux setting up their tipi is one further example to add to the collection discussed in this book. Thus the links between ethnomathematics, ethnomodelling, and school experiences are seen to exist and be valuable. They are embedded in understanding visuospatial reasoning from an ecocultural perspective.

Moving Forward

One principle considered by the Alaskan universities for cultural competence is that the curriculum "respects and validates knowledge that has been derived from a number of cultural traditions" (Alaskan Universities Council, ~2012). It then becomes evident that visuospatial reasoning taken from an ecocultural perspective, especially from Indigenous communities must be a consideration of curriculum. Can this be achieved in schools and in teacher education? In the next chapter, the arguments for the importance of this perspective for mathematics and mathematics education are presented. In Chap. 8 programmes that take an ecocultural perspective on education are illustrated showing the role visuospatial reasoning plays in the development of space, geometry, and measurement.