Chapter 4
Place, Culture, Language, and Visuospatial Reasoning

One doesn't understand concepts and 'then' solve the problems, one understands concepts 'by' solving problems. One doesn't communicate mathematical ideas 'as well as' reason about mathematics, rather, 'through' communication one refines one's mathematical reasoning.

(Elizabeth Badger, 1992)

The Challenge

In Chap. 2, psychological theories were discussed establishing that people store information that is accessible as either verbal or visual information either mentally or physically. Language was established as a representation both mentally and externally. Visual information was perceived as diagrams although the physical look of formulae, for example, can contain a pattern for the memorisation of material (Presmeg, 1986). There were different strategies for visual imagery in the mind including emerging strategies, perceptual strategies, concrete pictorial imagery, dynamic and pattern imagery, and efficient strategies. The last two were closely linked to reasoning with imagery. Furthermore, dynamic and pattern imagery may have been associated with bodily movement. However, a question arises about whether a language approach to education leads to a cognitive psychology focused on representation or does it relate to other cultural aspects or place-based aspects of learning.

Language development occurs as students learn names of shapes through family, television, pre-school experiences, blocks used in play, and in school activities where the dominant approach is to use western school mathematics. Schooling, home facilities, and language/culture influence the making of designs and arranging shapes. Everyday experiences impact on spatial language, so social inequities (Cross, Woods, Schweingruber, & National Research Council, Committee on Early Childhood Mathematics, 2009) and cultural differences for learning geometry may be substantial.

K. Owens, *Visuospatial Reasoning*, Mathematics Education Library,
DOI 10.1007/978-3-319-02463-9_4

One issue is that of support for homework and another is the availability of reading material and encouragement for reading (Sukon & Jawahir, 2005).

Questions arise about whether language is a significant aspect of visuospatial reasoning and whether language changes the way one thinks about visuospatial experiences. Is it possible to have a window into this reasoning from a linguistic perspective? How do languages show different ways of visuospatial reasoning? How might this relate to identity?

From an ecological perspective, a place is a space inhabited by people with values and relationships and, as a result, the place takes a significant role in meanings and specific ways of thinking (Chap. 1) at a personal and a societal level (Chap. 3). Place is local with local ecologies but it is part of a global place. In Chap. 3, I argued for the importance of a sociocultural perspective on visuospatial reasoning and that societies influence individuals' perspectives on space and place. Language is the way in which people in a society communicate, and ideas are generated. What do we learn from language studies in terms of space and measurement understandings? Furthermore, we argued for a critical perspective of place in terms of social justice. Visuospatial reasoning is understood in terms of an ecocultural perspective permitting children to keep and build on the strengths of their place and culture.

This chapter draws on linguistic and anthropological research to assist in understanding these bases of education. It establishes how important land is to the thinking of cultural groups as portrayed in communicating in various ways. It provides evidence of visuospatial reasoning varying according to ecocultural contexts. In particular, it notes that position, size, shapes, and patterning are unique to different language groups. Language structures indicate that ways of denoting all of these aspects differ from western or school mathematics. It shows that different cultural groups value different shapes and kinds of patterns in different ways and that worldviews impact on the place of these mathematical concepts and their uses in different ways. To explore this issue, I look specifically at language for measuring space and locating in space but first I provide some background to languages.

The Role of Language

Within this critical perspective of place, Setati and Adler (2000) note the role of language in a critical ecocultural pedagogy:

> Firstly, the political and pedagogical issues in rural and urban multilingual mathematics classrooms in South Africa are different, and this *contextual diversity* needs to be recognised in language-in-education policy, research and practice. Secondly, moving between languages (e.g. English and isiZulu) is only part of the process of learning mathematics in multilingual classrooms. There are numerous, distinct mathematical discourses that require navigation at the same time. (p. 244)

Thus permitting code switching between languages and leaving the choice of mathematical register to the students in the co-learning of mathematics can "privilege the students' competence in all their languages. … code switching can be used as a mark of solidarity empowering the students in the classroom" (Prediger,

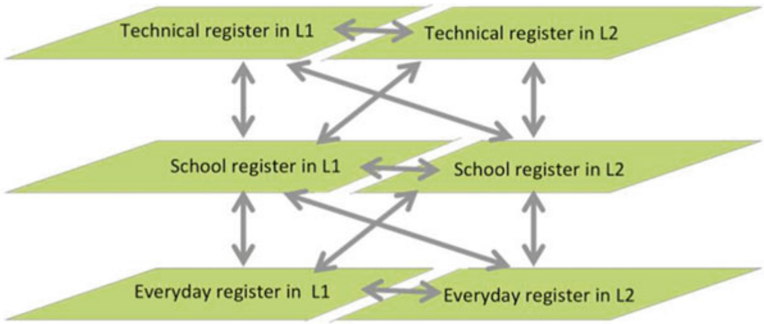


Fig. 4.1 The interplay of registers in first language (L1) and second language (L2). *Source*: Prediger et al. (2012, p. 6216)

Clarkson, & Bose, 2012, p. 6124). While informal language may assist students to understand the mathematical concepts and informal contexts permit gestures to support the language and interpersonal relations, additional language skills are needed to communicate in the school and technical registers. The interplay of registers is illustrated in Fig. 4.1.

> For mathematics classrooms, it is hence important to facilitate transitions between all these registers. It also becomes important for teachers to understand the possibilities that are afforded to students to gain deeper understanding if they are encouraged to use their languages effectively. (Prediger et al., 2012, p. 6215)

In addition gestures, diagrams, and other visuospatial representations are important in these transitions. One way of achieving transitions between registers is through a teacher "revoicing", for example, using adequate technical/mathematical language to say again what a child has said as a part of increasing the second language understanding. Revoicing a child's idea through the use of repetition, rephrasing, and expansion allows the child to be seen as the authority of the idea and an agent in the action of mathematical problem solving (Turner, Dominguez, Maldonado, & Empson, 2013). The boundaries between the contexts of the registers are necessarily fluid, so teachers can develop the registers to advantage through different activities. The register (technical L2) is important in skilling multilingual students through activities of translating from one register into another, finding and fitting registers for consolidating vocabulary, examining or aligning when registers are not aligned, explaining how to find a mathematical relation or structure in a certain register, and collecting and reflecting different means of expression within one register (Prediger et al., 2012).

Representation is not a direct correspondence but reflective of "context, functions, and social embeddedness" (Prediger et al., 2012, p. 6218). Furthermore, changing representations also indicates meaning in itself and the way it can be used and the properties it provides. For example, a graphical or algebraic expression of a straight line yields different communications. Similarly, changes in language and visuospatial representations can also explicate meaning, purpose, and techniques of the two forms. For example, Saxe's (2012) study on money showed numbers, monetary terms, and the term *fu* (complete group of plenty) varied in representation with different contexts, personal backgrounds, and time. The social purpose influences the register represented in the content field and text structure (Derewianka & Jones, 2012). Language analysis then is important for social justice in the teaching situation.

A critical social analysis concerned exclusively with human relationships fails to demonstrate ecological thinking whereas an ecocultural pedagogy seeks the twin objectives of decolonisation and reinhabitation, important particularly for Indigenous students (Barnhardt, 2007), but it also takes account of different geographies. This ecocultural perspective challenges all educators to reflect on the relationship between the kind of education they pursue and the kind of places we inhabit and leave behind for future generations and in which the learner takes initiative, makes decisions, is accountable for the results, poses questions, experiments, and solves problems, all regarded as central to mathematics education. All require visuospatial reasoning. Places are visualised. Actions in places are visualised. People reason holistically about places that they visualise (Pinxten et al., 1983).

I call this an ecocultural perspective in order to emphasise culture linked to land as embedded in relationships and language. There is a growing body of research that illustrates alternative ways of viewing position in Indigenous communities (Owens, 2013b; Owens et al., 2011; Pinxten & François, 2011; Senft, 2004b). Language and other representations reflect some of this thinking as indicated by sociocultural studies (Adler, 2002; Barton, 2008; Gerdes, 1999; Owens & Kaleva, 2008a, 2008b).

Cultural Ways of Thinking Reflected in Language

Rivera and Rossi Becker (2007) queried that culture might not necessarily provide a reason for diverging from western school mathematics because there are still individual differences in mathematical thinking in a specific cultural group. However, culturally shared language provides evidence of a widely taken-as-shared way of mathematical thinking that does show a cultural difference in mathematics that should be taken into account in understanding mathematics. Barton (2008) shows from a study of languages that mathematics is much more relative and dependent on human experience than is usually accepted. Mathematics expressed in school with English words is only one mathematical experience. There are many languages that express mathematical thinking quite differently. Furthermore, Barton shows that it is worthwhile pursuing mathematics as it is expressed in different languages. The world needs to nurture difference for its rich possibilities and creativity as well as providing culturally responsive mathematics education (Averill et al., 2009). As Rivera and Rossi Becker suggested, taking cultural ways of thinking together with school mathematical pursuits means the learner can identify culturally and mathematically, and develop his/her own ways of thinking mathematically. "A more powerful ethnomathematics program in contemporary times involves understanding the structure of complexity of cultures in ways that explain how members in such cultures are able to preserve valuable mathematical practices and might overcome those that constrain them from fully participating globally" (Rivera & Rossi Becker, 2007, p. 222). Anthropology has led the way to contested spaces where negotiation, institutionalisation, and internalisation of practice might be resisted. Thus it is important to appreciate the ecocultural reasons behind certain Indigenous practices and how they are shared, constrained, and pursued and to assist in reconciling different conceptual and practical variances.

Meaney, Trinick, and Fairhall's (2012) study pursued similar issues. They showed (a) how creative, language- and culture-rich, place-based mathematics education can develop in the unavoidable political climate; (b) how cultural background can be used to resource students; and (c) how to meet the challenges of context—community, teacher knowledge and ability, and professional development. Furthermore, Meaney et al. provide a comprehensive coverage of how one large Indigenous language group in New Zealand *Aotearoa* nationally developed its mathematics register and met the challenges of implementing education in *te reo Māori*. They provide guidelines and salutary messages for others, including smaller language groups with greater language changes as in PNG, to establish similar projects successfully. Furthermore, they support the importance of every mathematics educator recognising the importance of ethnomathematics in current school education. The language permits and encourages teachers to draw students' attention to important points through linguistic markers. Furthermore, the way that mathematical terms in *te reo Māori* were developed has resulted in assisting students to develop deep meaning. In addition, logical connectives within the syntax of the language clarify logical relationships. Their study shows the strengths of using culture and language in learning mathematics, not only on national testing regimes but also in terms of how the language and culture resource mathematical thinking and learning.

Most researchers on mathematics education in bi/multilingual classrooms have argued for the use of the learners' home languages as resources for learning and teaching mathematics (Adendorff, 1993; Adler, 2002; Moschkovich, 2002; Ncedo, Pieres, & Morar, 2002). Further, the active involvement of parents in the children's homework, often in the children's first language, can be a source of confusion if different notations and algorithms are used. If conflict between home and school exists, a suitable balance needs to be worked out. The family comes to understand the value of different approaches to learning and the implications of this in what the children do in mathematics lessons while the school acknowledges and builds on family reasoning through activity or discussion.

Children who speak a language other than the language of instruction confront a substantial barrier to learning. In the crucial early grades when children are trying to acquire basic literacy as well as adjust to the demands of the school setting, not speaking the language of instruction can make the difference between succeeding and failing in school, and between remaining in school and dropping out (Lockheed & Verspoor, 1991). The first language of a child is inseparable from his/her cultural heritage and as such deserves our recognition and support. If a child does switch to his/her mother tongue in an effort to solve a mathematical problem that child should not be discouraged (Then & Ting, 2011). Then and Ting suggested that a better policy is to recognise the value of the mother tongue, encourage its full development beyond surface fluency, but at the same time allow English or main language of instruction to become a naturally preferred medium for mathematical thought. Children taught mathematics by their parents in first language find themselves switching languages in class more frequently. Many multilingual primary-level pupils have great difficulty with reading and writing mathematics. They have poor mathematical vocabulary and can read only the simplest mathematical test, according to Then and Ting (2011).

Among the challenges for developing the *te reo Māori* schooling was that of writing mathematics in *te reo Māori* given it was predominantly an oral language (Meaney et al., 2012). In their professional development, teachers learnt to use different genres and explicitly develop field, tenor, and mode for reflection, recording mathematical activity, and conventional interaction with others. Meaney et al. provided considerable data to justify the point that not only is writing important but also the expected quality of the writing is important and possible in *te reo Māori*. The importance of revitalising the language strengthens the mathematical learning. Another issue is how to balance the need to use other terms for explaining and at the same time developing the approved mathematical register. It is important to decipher those words that easily connect from other uses to the mathematical use and those that need further translation/explanation.

Another study that particularly looked at the issue of language and geometry was carried out by Jawahir (Jawahir, Owens, Sukon, & Sunhaloo, 2011; Jawahir, 2013) in Mauritius. In his intervention comparative study, he chose not to look at written Creole but he used oral Creole for discussion and teaching (see Chap. 8 for more detail). In particular, Jawahir noted that researchers like van Hiele (1986) pointed out the importance of language in developing levels of geometric reasoning so that holistic reasoning was a foundation for later analysis, relational connections between shapes and later justifications and explanations. For example, in analysis, students may describe the properties of the shapes informally and imprecisely or they may describe explicitly and exclusively using formal geometric concepts and language to describe and conceptualise shapes in a way that attends to a sufficient set of properties to specify the shapes. Thus Jawahir considered verbal skills along with visual, drawing, logical, and applied skills. Furthermore, he noted that gestures and actions were significant in communication especially with manipulatives. Students who used Creole particularly with investigative processes of learning performed better than those using English which was a second language.

Locating, Visuospatial Reasoning, and Communicating

Structural features of language have significant implications for cognition and learning in visuospatial reasoning within mathematics particularly in terms of location or spatial understanding and processing. François et al. (2013) support this issue stating that some Indigenous languages like Athapaskan and Cherokee (Native American languages) and some PNG languages are basically viewing reality as events with substantially no noun categories.

> The structure of the Indo-European languages distinguishes between verb and noun forms. With this distinction corresponds a differentiation between things/states and operations/processes in the conceptualization of reality. Intuitively, mathematical thinking sophisticates these deep structural linguistic and cultural differentiations. Hence, the emphasis on geometric figures (with a thing-character) and their constitutive forms, on sets and their elements, on operations (of multiplication and so on) [that is] performed on entities (a number, a series, etc.). The point we want to make is that formal thinking elaborates the intuitive world view which is given in language and in folk knowledge (Atran, 1990). … Actions in Navajo begin, stop, change or transform. The cosmos can be understood as a universe of events, rather than a universe of things. In such a view no part-whole logic of 'beings' or 'objects' and their elements [exists] … (Pinxten et al., 1983). In Academic Mathematics, on the other hand, the very basis for formal reasoning is a part-whole logic: the world of experience is split in parts. For example, in geometry a line is defined as a set of points, or a plane is said to be a set of lines. (François et al., 2013, p. 30, pp. 29–30)

Thus language does reflect but also impacts on visuospatial reasoning and needs to be taken into account when we argue that visuospatial reasoning is impacted by culture. For example, for the Navajo, with a dominance of verbs rather than nouns, and the use of movement to describe position, reasoning is different to English descriptions that make use of prepositions to express relationships. Pinxten et al. (1983) also show that ecology is taken into account in geometry. For example, in expressing position, Pinxten et al. define this as actions related to certain landmarks evident on land that might seem to western eyes as fairly devoid of objects. Thus the description of position is unique and within a totally different system to the Euclidean western system but nevertheless rich not only as a system but also as part of a worldview of objects.

From a perspective based on western languages we might consider that space is initially referred to in terms of the planes associated with the body. These are the central vertical planes providing (a) left and right and (b) front and back. The third plane may be at our feet as the plane of the ground providing a height dimension. Such a way of referring to space is consistent with a three-dimensional orthogonal Euclidean approach that provides for pathways, areas, and volumes. The natural symmetry of the left–right plane and the expectation that one is standing in a vertical position underlie these expectations. The speaker's position and orientation are important referentials (Senft, 1997). In fact,

> One of the main features of natural language is its 'contextuality'—and it is in this context-boundness that language, perception, and cognition meet. … Space, our perception of space, and our orientation in space are basic for human action and interaction in a number of domains- Konrad Lorenz even regards our spatial cognitive capacities as one of the roots for human thinking. (Senft, 1997, p. 2)

In many western referencing systems the speaker or the listener is considered but it is also possible to locate in terms of a third object. However, there are times when the context actually provides the meaning. For example, a ball is in front of the tree usually means the ball is between the speaker and the tree but if the tree is in the front yard of the house, it may mean in front of the tree in alignment with the generally accepted front of the house and the speaker can be anywhere. In addition, static configurations may use the way one faces but a dynamic configuration may be more about alignment or parallel relationships. Furthermore, the metaphoric and extended use of words can be linked by visuospatial reasoning (Lakoff, 1987). For example, “over” is used in a number of ways associated with position and action on a hill. Words and oppositional concepts such as “here” and “there” are very much determined by sociocultural experiences. English also changes when it is reported, so “it is cold here” is reported as ‘it is cold there’ (Ehrich, 1991, cited in Senft, 1997). Finally the words may also be associated with a symbolic use rather than a descriptive relationship to the object. “Over the top” refers to a person’s expression that is unreasonably exaggerated. Furthermore, some words also have an emphatic purpose like the *su* in Turkish which is something that the addressee should take into account (Ozyürek, 1998, cited in Senft, 2004a). Similar emphatics are evident in Papuan languages (Tupper, 2007) and te reo Māori.

Language Patterns in Papua New Guinea

The role of language in an ecocultural pedagogy for visuospatial reasoning in space and geometry is further explained by considering the rich diversity of languages in PNG. Some PNG Indigenous languages have

> a greatly complicated verbal system, but pay little attention to the noun, lacking perhaps any system of classification or giving very little attention to distinctions of number and relationships of case to other parts of the utterance. … (Others have very elaborate gender or noun-class systems) often involving grammatical concord with all conceptually connected parts of the utterance (Capell, 1969, p. 13).

In verb-oriented systems like the non-Austronesian language (NAN) of Kâte, the emphasis lies apparently in what happened, when it happened, and how it happened, rather than in the people or object involved or the place of the occurrence. The verb with post-, pre-, and infixes might take six English sentences to convey the same message. On the other hand, in noun-oriented systems such as Baining, a NAN language, East New Britain Province, an utterance gives attention to the persons and objects such that the action words are allowed to take care of themselves (Capell, 1969). A rarer type of language classified as numeral dominated is the Kiwai language spoken by the Indigenous people who live around the mouth of the Fly River in Western Province. As noted by Capell (1969), in Kiwai language, there is “prefixal indication of the manner of the actions—one action only, one action repeated, a number of actions together or in sequence needed to carry out the task in hand” (p. 15). Nearby are languages with “many” being determined by repeated action or increasing number of actors, so there is little counting per se and more classification of “many” or “few”. These are some of the structural features that illustrate a staggering and complex linguistic and cultural diversity found among the

Indigenous languages of PNG. These structural features have significant implications for cognition and learning in mathematics education.

One early PNG study of directionals to indicate a spatial relation of person, place, or thing to another person, place, or thing was in a Narak speaking village in Jimi valley (Cook, 1967). A few examples are provided here together with metaphoric uses. Cook lists 19 directionals. *pla* refers to a high relative position on the vertical axis but also in the case of a pig, it means large and fat while it can refer to where God is thought to reside in heaven. It is also used to refer to up over a person, the direction of climbing, looking away from the person, and a long or short distance. *kalA* is vertically straight down in direction, so God came down or a place down a ridge but it can also refer to the poor condition of a small, skinny pig. It has an opposite sense to *pla*. *paNo* refers to the middle of something while *kora* means on the same level as something else. These examples show the metaphorical extensions and complex meanings associated with just a couple of words.

Frames of Reference for Space and Place

Position depends on frames of reference and different cultural descriptions. In general space is referred to by local and directional prepositions or postpositions ("at", "on", "in"; "in front of"; "behind"), locatives—local or place adverbs ("here", "there"), dimensional or spatial adjectives ("high", "low", "wide"), demonstratives ("this", "that"), static and dynamic motion verbs ("to stand", "to come", "to go", "to bring", "to take"), directionals (e.g. "to", "into"), and presentatives ("there is") (Senft, 1997, p. 8). These terms form deictic systems and there is a large variety of these systems across languages. In addition, languages have gestures such as pointing or raising the eyebrows to indicate position. However, the number of terms used in any one language may vary. Senft (1997) presented an argument made by others that the more the man-made spaces in a society, the smaller the size of the spatial deictic system. He gives as examples the fact that English has two terms for position ("here" and "there") but Yup'ik has 30 terms and East Eskimo has 88 terms. This is partly attributed to the man-made function given to the object associated with the position. For example, "the key is in the door" or "the satellite is in space". The locative markers of a language impose an implicit classification on spatial configurations. Indo-European categories are topological relationships (e.g. proximity, inclusion, surface contact), Euclidean notions, and functional notions concerning typical uses.

The Inuit have four different suffixes, roughly corresponding to "at", "from", "via", and "to", but they also indicate (a) if the event is at the beginning, middle, or end of the sequence; (b) expand on the position further away with an idea of "up", "down", "in", "out", and "same plane"; (c) perspective of speaker, the addressee, or some other reference point; and (d) the relative size/shape of the place partially determined by the speed and nature of the motion involved (Ascher, 1994).

Typical of studies that look at language and frames of reference (Gerdes, 1999) is that carried out by Edmonds-Wathen (2012a) using cards that have pictures with a tree and a man in different orientations (facing different ways and on different sides of the tree). A person describes the position of the man when compared to the tree. In general, frames of reference fall into three categories although the language may have a word that could cover more than one of these categories. These categories are usually described as being intrinsic, that is, the words link the items of the representation (picture or drawing); relative, that is, the words link the items to the describer's position looking at the drawing; or absolute, that is, terms such as north, south, east, and west are used. For Iwaidja (Edmonds-Wathen, 2011) on Croker Island, northern Australia, there was a second absolute frame of reference being "deep sea" (west) and "mainland" (east) but if the describer was facing west, this could also be used in a relative sense if the describer noticed that the man was behind the tree in an English description. Other languages use landmarks to denote position. Local landmarks and environmental features are also used to denote places and the position of objects. One example is the use of "west-sea-down" and "east-land-up" relevant to the geography and ecology of the Iaai on Uvea, an island in New Calendonia (Ozanne-Rivierre, 2004). (See in Chap. 5 a classroom use mentioned by Muke and in Chap. 6 the Mayan use of seas.)

Prepositions or postpositions generally provide a connection that is obvious by context or by the expected relationship between objects, e.g. the book on the table. For this reason, prepositions are frequently minimal and may or may not impact on word order. Alekano (Eastern Highlands Province, PNG) has up to 15 slots or positions for different types of words and relationships between words in a sentence. Wiradjuri in NSW, Australia, has three positional suffixes—one for being next to a person or on or in an object, another for coming to a person, and another for going away from a person which vary with the class of noun (Grant & Rudder, 2010). For example, the suffix -*gu* is added to the noun for movement towards the person or thing, that is, the person or thing is the purpose of the action (but also -*gu* is added to all the nouns for two or more objects owned by a person, rather than using "and"). Table 4.1 also shows how suffixes are used for movement away illustrating variance over type of word. In some languages, words vary with addressor and/or addressee or a third person or object as reference point and the number of people involved may also modify the words to be used (e.g. Samoan as presented by Mosel, 2004).

Table 4.1 Examples of added suffixes for movement in Wiradjuri

Word ending	Suffix	Wiradjuri base word	With suffix	English
"*ang*"[a]	-*dhi*	*ngurang*	*ngurandhi*	From the camp/home
"*i*" or "*ny*"	-*dyi*	*mirri*	*mirridyi*	From the dog
"*aa*"	-*ri*	*yinaa*	*yinaari*	From the woman
"*ang*"	-*ga*	*ngurang*	*nguranggа*	In/by/at the camp/home
"*i*" or "*ny*"	-*dya*	*wiiny*	*wiinydya*	At the fire
"*a*", "*ir*", "*n*"	-*dha*	*dhaagun*	*dhaangundha*	In the dirt
"*l*", "*r*", "*rr*"	-*a*	*gibir*	*gibira*	By the man
"*ng*"	-*gu*	*galing*	*galinggu*	To the water

Note: [a]for "*ang*" ending, "*g*" is dropped

Sometimes words vary with the use of gestures (e.g. Saliba, Milne Bay, PNG; Margetts, 2004).

Stokes (1982) also notes that for the Anindilyakwa from Groote Island in Northern Territory, Australia, the word for "come" and "go" is the same generic word requiring context for meaning, that the suffix *-manja* is used for "at", "in", "on", and "by" requiring the context to provide meaning, and that the question "where" can be asked by an adverbial for a person but of an object by an adjective.

Codrington (1885) much earlier noted that Melanesians and Polynesians have a habit of continually introducing positional suffixes, "adverbs of place and of direction such as 'up' and 'down', 'hither' and 'hence', 'seaward' and 'landward'". One deictic system that can be found in a number of Austronesian and non-Austronesian Papuan languages in PNG, in languages of the Pacific, and in Australian Indigenous languages includes varying words which refer to a place quite distant, ones that encode medial distance, and ones that imply proximity with visibility impacting on choice of words (Senft, 1997). Wiradjuri has three suffixes for "here" *-nha*, "there", *-nhana*, and "way over there", *-nhanala*. Senft's edited books (Senft 1997, 2004b) provide many linguistic examples of deictic differences from the work of linguists among Austronesian and non-Austronesian Papuan languages of Oceania, PNG, and Melanesian areas to the west (such as West Papua and Suluwase). These alternatives also affect the way people measure (see comments on Kilivila later in this chapter on measurement).

It is common for "behind" and "front" to be used for denoting persons but in different ways in diverse languages. The diversity of frames of reference in PNG and Australia illustrates particularly the difficulties of using the metaphorical "before" and "after" with numbers on the number line for various reasons. For example, when Matang (2008) carried out his study (personal observation) and used the Tok Pisin terms, he was using the words in the opposite way to that used in English. Similarly, the Iwaidja, Australia (Edmonds-Wathen, 2012b), and Walpiri in Central Australia (Graham, 1988) both used "before" and "after" in ways different to that of English. "After" in some languages can be a word used only in a relative sense so that it can also be translated into English as "before", "previously", or "after" depending on the context. "Before" in Walpiri is also used for "larger" because it has an associated time factor linked to growth. That is, a child who is "before" (that is born before) will be larger than the other child. This leads to confusion with the metaphor of the number line but it can also be easily overcome by teachers who only refer to the "number one less than" rather than "before". However, there is also a spatial orientation difficulty. In Iwaidja, a person could say a man is before the tree when the man has his back to the tree. This intrinsic approach linking the items on a diagram would be different to the English description if the tree was (partially) covering or in front of the man from the position of the describer. It could also be the case from the position of the describer of the drawing that the man was to the left or right of the tree with his back to the tree. So the word for "in front of" which has a stronger directional component in terms of the items of the diagram may be better than a term which has a different meaning in Iwaidja like "before" (Edmonds-Wathen, 2012a).

Anindilyakwa use words like “below” for a canoe on the sea but not on the land, and it is the same as the word used for “inside” the shelter in that there was a cover only “over” rather than surrounding the person (Worsley, 1997) and separating from the outside (Fig. 4.1). “Outside” is outside the jungle in a clear space and could be distant and an area explained as “from here to there” (Stokes, 1982). *Angwurn.dikirra* is for a space like a strait or between objects, usually narrow, while a confined space is a verb stem for enclosing. So the connection between lines and area is not expressed in Anindilyakwa as lines are generally associated with aspects of an object such as a spear (Stokes, 1982) and can apply to both horizontal and vertical lines which can introduce straight lines often associated with “becoming straight”. Other words like “horizontal”, “oblique”, “vertical”, “corner” would be distinguished by verbs for “lying”, “leaning”, and “standing upright” and the adjective for “crooked”, respectively. There are “flat” objects. Shapes like rectangles are only recognised in terms of objects such as a bark painting or rectangular sail (Stokes, 1982). However, there are several ways of noting round and specifically to denote the sphere shape of the turtle egg from the other ovoid-shaped eggs (Fig. 4.2).

Compass points or cardinal points may be denoted (Harris, 1989). In addition, dimensional axes, usually in reference to the body, are used but in some cases, the position of the axes can be moved. For example, Ralph Lawton (personal communication, 2010) noted that measurement varied depending on whether the plane was on the ground or at the arms for Kilivila, Milne Bay, PNG (an Oceanic Austronesian language). Such diversity merely hints at the diverse ways of representing space verbally but also the effect of the ecocultural influence on language and visuospatial reasoning.

Fig. 4.2 Shelters in the camp, Yalata, South Australia (Owens, 1966)

Language in Comparing and Measuring space

Each language has its own way of referring to sizes. Some have comparative words such as “longer” or “more distant”. For the Anindilyakwa from Groote Island, northern Australia, size is covered by three adjectives, “small”, “big”, and “huge” (Worsley, 1997) although Stokes (1982) notes a diversity of words for thick or fat and thin for people, animals, and things but additional ones for things. There is remarkable flexibility with the use of qualifiers such as “more” or “very” (Stokes, 1982; Worsley, 1997). There was a word for “short” usually used with a noun and interestingly the word for “foot” was also used. In Tok Pisin in PNG *centimetre* is used for the short unit. Worsley also pointed out that other taken-as-shared meanings in western education take on different meanings in Anindilyakwa language. “Empty” has a spatial sense but it implies what might be expected and while comparisons of “more than” and “less than” were measured, the approximation was adequate (Worsley, 1997). Distance is seen as the time taken to reach a place and the words for “soon” and “near” are interchangeable. However the words for “to another place” can be used for “far away” and the prefix for “rather” and suffix for emphasis are used for comparison as well as the intensifier for “more” further away. The verbs of motion have certain features to express a great distance or length of time (Stokes, 1982).

Providing a sense of size often requires narratives. In PNG, if I wanted to know how long it would take me to walk to a village, I would ask how many hours it would take the speaker (a villager) and how long it would take me (a white woman), then I would take the average. Usually it turned out to be a good estimate! Time was often used to indicate length of the track from village to village. However, the idea of an hour was not well sensed. This did not mean that they were unfamiliar with time bodily and mentally. When I was staying in a village in the mountains behind Lae, Morobe Province, women who had gathered greens from the garden would get up early enough to wrap the bundles and walk to the road head in time for the truck for the 1-h journey to sell at the market by 7.30 am. In a village in Oro Province, people without watches needed to be at the nearest airport at 8 am. They rose in the dark at the right time to prepare, knowing how long it would take to sail and paddle, negotiate the winds and swells, and arrive in time. After 4 h, we arrived at 8 am. En route, the two men worked in unison with hardly a word especially as they added paddling to the sail to round a particularly difficult point, watching the rolling waves carefully as the canoe was heavily loaded. There was clearly taken-as-shared visuospatial reasoning.

Jones (1974) argued that it was not always easy for languages in PNG to express certain concepts related to measurement. Ways of reasoning about size and making decisions in activities involving measurement in PNG are covered in Chap. 5 and implications for education in Chap. 8. Here we will discuss language patterns related to space and measurement. A study by Owens and Kaleva (2008b) in PNG revealed that vernacular words for size and other related ideas are complex. Data came from questionnaires completed by tertiary students, village observations and discussions, and linguistic records for 360 of the 850 languages of PNG. Most languages appear to be able to refer to volume, mass, area, and length but a verb may be involved implying an action on size, e.g. making bigger. There may be a limited number of comparative adjectives or very general concepts like *size*. Participants talk about size but often thought for some time before completing the word lists as there is often not an exact match between English words and their own language for meaning nor necessarily the same kind of speech pattern. For example, the word may not be an attribute word or adjective. Mussau in New Ireland has an auxiliary clause (van den Berg, SIL, personal communication, 2006). They also have the word "mother" to refer to large things. It is similar to the Huon Peninsular *awara*. In a similar way, Korafe speakers in Oro Province use metaphors for size. For example, a child is a chunk of the father or a smaller version of the father. However, they use suffixes for "bigger" and "biggest" but they will also use reduplication (repeating morphemes) which is found for descriptive words (larger and/or smaller) (Farr (deceased), SIL, personal communication, 2006).

Reduplication is common in PNG languages and is used for similarity or other purposes (e.g. continuing verb, plurals, groups like two by two, emphasis). Dobu speakers further along the coast in Milne Bay Province use the word *kaprika* for "pumpkin" which changes to *kapukapurika* for "small pumpkin" (Capell, 1943). This may indicate a different type as well as different size. Manam on Karkar Island Madang Province and Tinatatuna (Tolai or Kuanua) on East New Britain are spoken many hundreds of kilometres away but they also use reduplication. A Manam speaker gave *dadaka*, *memekei*, and *kanabibia* for "big" and *kengekenge*, *sikisiki*, *mukumuku*, *seisei*, and *bisibisi* for "little" (see Table 4.2, from the measurement study in PNG).

"Very" is expressed in consistent ways in most PNG Austronesian languages. It might be translated "enormous" and has equivalents in Tok Pisin of "mama" or "tripela". In non-Austronesian languages this term might not exist. "Larger" might be expressed by comparison of two objects with a comparison word between (order is important). Sometimes one word like "long" was negated (usually within the verb structure) to suggest "short" and a language with this structure is likely to use other paired opposites in a similar way. Other languages use a range of diminutives (linguist focus group, personal communication, 2006).

Table 4.2 also shows a number of different words in some of the languages for the same concept in English for either different objects or purposes.[1] This variation is found in Manam for several concepts, in Tinatatuna (Tolai or Kuanua) for "compare", and in Kewapi words for "little". Some variations can be explained in terms of the influence of other languages. For example, for Tinatatuna the word "to measure" includes *mak*(*ai*) which is similar to the Tok Pisin (PNG's lingua franca) word and the overlap of words in the three highland languages (Huli and Kewapi in

[1] The variations in the table may be due to different sources of data (for example, different participants writing down an oral language or the date on which the data was collected, e.g. the 1900s first contact or later records and recently collected data) but in other cases, the same participant provided the multiple number of words confirmed by others from the language group (e.g. Manam).

Table 4.2 Words related to size from a small selection of different languages in PNG

English	Fore	Manam	Tinatatuna	Enga	Huli	Kewapi
Compare	Koviga	Tongaka	Varvadaina, vandadauane, valarue, varagopina	Makande, kapakap	Manda, mandapia	Anamealapa, manda-mamea
Measure	Amakaga	Tongaka	Mak(ai), angewe, valarue	Makande, makade	Manda pia, kimagi, kemagi	Mandamea
Size (volume)	Mangawa'e	Ilo	Ngala	Kapkap, makande	Arane, timbuni, luni, taliga	Ekei-yapa andai
Big	Tave	Taila, dadaka, memekei, kanabibia, ilaba	Ngala (variations)	Andaik, yale	Timbuni	Andai
Little	Amanagando	Kengekenge, sikisiki, mukumuku, seisei, bisibisi	Kapakapana	Kolam, kokilyam, yaalam	Emene (various spellings)	Egepusi, ekei, ekesi, ogesi
Short	Alo	Tupeka	Gur tutuk, ngu ngu, tutukana	Moui, muu	Tomaki	Rundu rundai, rundsi
Long	He'elo	Salagabuli, salagauta,	Lolovina	Londe londe, londakai	Luni	Andalu (with suffixes)
Full	Puma,e	Ikauri	Buka	Tubelam, tubabpah, tuimbilam, perpertta	Toho laya, caralipa, catlope, to	Manda, rege-pe-lea, rayo
Heavy	Gunda	Moatubu	Mamat	Kend, kenda, kendepi, kendaping	Kend, kenda, kendepi	Kend, kenda, kendepi
Light	Ewasa	Malama	Papa, papanga	Yapalume, kende napinge	Hale, je-pe-pi, emene, jepeapi, yapipi, atatapi	Yapa-pea, pa

Southern Highlands Province and Enga) suggests interlanguage influence. Other words building on the same morpheme indicate relationships between the words. One particular Huli speaker noted that volume was a combination of length, width, and height (Piru, 2005). The impact of western education might be evident in this comment but only one student from the hundreds who completed questionnaires from the Southern Highlands Province noted that her family (unlike others) discussed area in terms of area units.

Some languages have an adjective for a measurable attribute that is used for all objects and others have words and suffixes for specific classifications, e.g. round objects, flat objects, people, and food (classifiers). In other cases, the attribute word varies depending on the item being discussed. For example, in Korafe *big* for fish is different to *big* for people (Farr (deceased), SIL linguist, personal communication, 2007). Other languages have words, suffixes or prefixes, or action words for different types of objects. In other cases, only certain kinds of objects may be compared (e.g. volume of stone is not compared to volume of water).

Two further linguistic features, ways of indicating emphasis and order of words, can impact on discussions about measurement activities (Tupper, 2007). Emphasis is used to draw attention to a particular point and the point could be the size of the object although there may not be a particular word used for size. Emphasis was seen as a strength of *te reo Māori* as mentioned above (Meaney et al., 2012). In the second case, the smaller object is placed before the word for the larger object in the sentence but there is no comparative morpheme, so the context of the sentence indicates that size is being considered.

Although this is only a small selection of the data from around 360 PNG languages, it does indicate that concepts of size exist and that there is a diversity of ways of expressing attributes related to size and difference in specificity. However, just the vocabulary fails to provide the more complex ways of discussing size that are embedded in sentence structure and in other ways of representing the attributes. Complicating this issue is that measurement and size may not be the only considerations taken into account in reasoning. For example, when comparing the area of two gardens people would take into account fertility, distance from the village, the number of people needing to use the land, closeness to water, cleared or currently bush or fallow, and crops. Thus size of area is only a part of the visuospatial reasoning. The visuospatial image may be much richer in terms of other attributes and features relevant to the decision making.

A further consideration in discussing locating and communicating visuospatial reasoning is the way in which a group might reach a decision about a place. In other words, it may not be a single or small number of words that locate or describe an object or person but it might be part of a larger discussion about the position or object. It is the discussion itself that can be significant to the speakers (cf. Salzmann, 2006, on disease in Mindanao, Philippines).

Implications of Language for Visuospatial Reasoning

In the previous chapter, I referred to the study of Hutchins (1995) on the navy team's use of tools including a Mercator map that required additional information to improve navigation. These tools had been developed over many hundreds of years to assist navigation and the team with specific roles required information from different people and certain commonly held and understood rules of thumb such as the better choice of visual sites for accuracy with position or the expected amount of travel distance at certain speeds.

Other historical situations illustrate the difficulties of referring to space by a spatial frame with the speaker as the centre and two orthogonal (perpendicular) axes denoted by the directions—north, south, east, and west. This static frame was an issue by the sixth century BC for the Greeks viewing the world as a sphere, so they divided the heavens into zones and the earth into five latitudinal zones (Tuan, 1977). Furthermore, reference to space also had place-based and cultural connotations. For example, European folklore linked people to their environment; for example, the north were hardy, the south easy-going (Tuan, 1977) whereas in some Arabic dialects the word for "south", where the once flourishing Yemen lay, was also used for "right", and "plenty" (Senft, 2004b). Valuing place and position is often encapsulated in the language of the cultural group. For the Chinese the four sides of the rectangle were represented by animals. Ancient Greece used planetary gods—east denoted light, white, sky, and up while west was darkness, earth, and down. For Europe, zodiac star signs were linked with patterns of farm work like the coming of rain, breeding flock, harvest, mowing, and raking (Tuan, 1977). Thus reference terms were associated with other aspects of life that required decision making.

The analysis presented in this chapter illustrates how the ecocultural context is significant in the language of spatial referencing. The language directs the visuospatial reasoning in both the metaphorical use and the significance of position/location in the culture where ideas of movement and relationships of people in a context are more important than the diagrammatic representation to be described by a viewer. This is complicated by other cultural views such as the position of the man and the woman when walking. In some cultures, the man should go before the woman or vice versa whereas in others the expected position is side by side. Similarly, women are never to be above a man such as stepping over his legs. I also suggest that a diagram which is more abstract than the man and the tree would also present some difficulty because it does not have the directional relationships clearly presented as illustrated by the Wiradjuri words (Table 4.1). This brings us to the use of maps in cultural contexts.

Maps as Representations of Visuospatial Reasoning

Location is usually considered in terms of an orthogonal coordinate system (Uttal, Fisher, & Taylor, 2006). Students may begin with locating on one dimension before using two and three orthogonal dimensions. The use of x and y axes with later developments for positions described in terms of negative to positive numbers is familiar and a basis for coordinate geometry or the visual representations of algebraic statements or relationships. Higher levels might provide polar coordinate referencing of position such as angle from north clockwise and distance from the origin or reference point. A similar idea of amount of turn and distance travelled is used in Logo geometry (see Chap. 9). Affordable programmable toys have encouraged practice and recent research shows how effective these toys are in developing spatial and pattern thinking in early educational settings (Highfield, Mulligan, & Hedberg, 2008).

Early childhood experiences in western schooling emphasise the use of prepositions like "in", "on", "inside" and words like "left" and "right". Some studies of mapping have discussed developments in primary schools indicating that responses tend to move from more pictorial representations with some indication of direction to those showing greater accuracy in terms of angles formed by non-orthogonal roads, and relative lengths (Owens, 2000a). Mapping is introduced as a plan view and in mathematics little attention is usually paid to the common use of contour lines on maps. The use of landmarks in big space is also noted by researchers (Liben, 2006) as an everyday way of giving spatial positions. There is an accepted discontinuity about descriptions of spaces that can be within a person's immediate view such as on a piece of paper and descriptions of big space.

My research with school children (Owens, 2000a; Owens & Geoghegan, 1998) showed that children can map their way from home to school from 4 or 5 years of age. Initially they note landmarks and indicate turns generally in the correct direction. As their mapping develops, they provide more detail in terms of less obvious landmarks and provide turns with right angles. Later they are able to indicate other angles, and later the proportion of parts of the land is better represented. According to Clements and Sarama (2007a) consciously self-regulated map reading behaviour through strategic map referral increased 4- to 6-year-olds' competence with reading route maps.

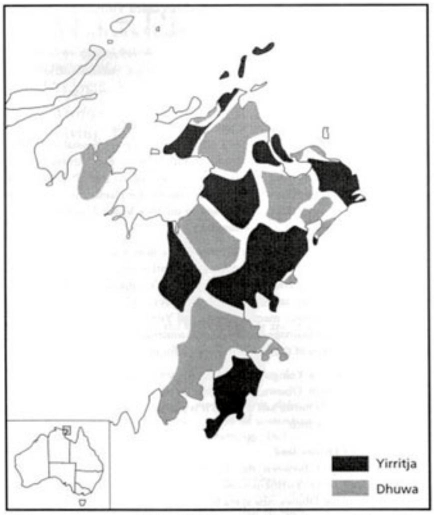
At a primary school in Goroka, PNG, I was observing a good teacher with her class. She had explained mapping clearly and demonstrated with the whole class participating in mapping the classroom. They had also mapped their houses. She said that some had not put an outer wall on the house but just made a map of objects relative to themselves in the house. I wondered if the "walls" of some self-help houses were not considered as part of the map of the internal space of the house and not part of the place they called home. This was the second lesson on mapping the school buildings and playground. Some had not considered the map in terms of north and some had drawn a mirror image of the school. Then the children shared their maps and discussed what was good and what could be improved with the maps (Fig. 4.3). This was a multilingual school and a mixed socioeconomic status school with a good proportion of well-educated parents. I also wondered if some of the difficulties resulted from the lack of maps in the culture and the different structure of language for location from English. I recalled the earlier work by Bishop and Lean (Bishop, 1979) in which they noted that tertiary students initially had difficulty interpreting the position of photographs of a visible structure taken from different

Fig. 4.3 Mapping the school (multilingual town school, Goroka, PNG) (Owens 2001a, 2001b)

perspectives but that they quickly learnt to read these two representations (solid model and 2D picture) and that it was indeed a lack of pictures (photographs, films, and books with illustrations) in their ecocultural context. How different might ideas be about mapping in different cultures in different places?

For the Yolngu of northern Australia, every person and every other thing is either *Yirritja* or *Dhuwa*, the two clan groupings. The division of land is dependent on the sacred sites (Fig. 4.4a). The creation of the sites comes from the dreaming creatures who created the clans who are now responsible for the sacred sites and who maintain the power by observing appropriate ceremonies and by painting, dance, and song (Thornton & Watson-Verran, 1996). "There is a metaphorical force essential for their way of life and sustaining their world" (Watson-Verran & Turnbull, 1995). In discussing and drawing the various places, a clan Elder represented the connectivity of the water flow; thus, a line is not a Cartesian mapping but a topological mapping in western mathematical terms. However, such a description (topological mapping) does not present the fullness of the representation (Fig. 4.4c). Each place has connections with activities carried out by ancestors such as a place for camping when visiting, or a place for washing cycad nuts to remove poisonous chemicals. A walking track was the Elder's responsibility and he would maintain it in songlines,[2] ceremonies, and practices. The land is represented by areas around sacred sites being either Yirritja or Dhuwa like a patchwork of nondescript shapes with the areas between being grey and not clearly delineated as the distance from the sacred site diminishes (Thornton & Watson-Verran, 1996). This referencing of space is not

[2] While walking/maintaining a track, an Elder would sing the song associated with the land and clan referencing the parts and points of the track as he went.

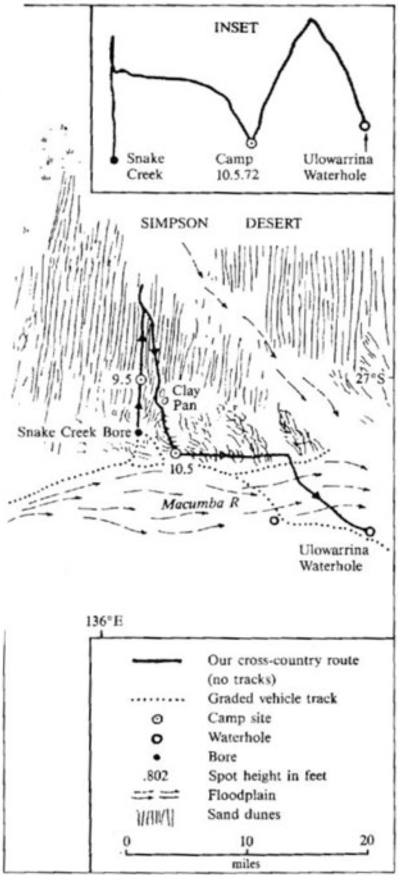


a Yolngu map of their land, Northern Territory, Australia. (Thornton, 1996, p. 10).

b Half-man story of Kaveve, Eastern Highlands, PNG 2007

a

c Shield with map by Peter showing connections of places of significance, Central Australia, 1972.



d Wintinna's diagram inset with map of travel with Lewis, 1976, p. 266, cited in Harris, 1989, p. 30).

Fig. 4.4 Non-orthogonal maps indicating visuospatial reasoning about place by an orthogonal grid. Furthermore, to solve problems of space as place, people cooperate as some knowledge is known by Yirritja and some by Dhuwa.

The place where negotiations occur is roughly the shape of a stingray which buries its tail in the sand just as one of the Elders and the ancestors before him buried their spears in the sand when negotiating a peaceful solution for revenge. This is near the stingray-shaped lagoon. Thus the shape and the place are metaphors and powerful images for complex ideas. Activity is set in kin relations, land rights and responsibilities, and sacred understandings. The land is constituted by living it. The conventions of the map that the Elders drew are representations interpreted in terms of systematic relationships. The Yolngu system of spatial knowledge (which they call *Djalkiri*) is detailed and provides a means by which a person can find his/her way anywhere across the land. The structures of the various forms of representation in ceremonies, everyday living, and in the land itself locate space appropriately in the footsteps of the ancestors.

One aspect of the visuospatial reasoning for Aboriginal Australians is that the spatiotemporal entities are not as paramount as the relationship entities (Watson-Verran & Turnbull, 1995). Similarly, the travels of the ancestors in creating the landscape constitute tracks or song-lines that traverse the whole country. For the Yolngu, *gurrutu* the recursive relationships and *djalkiri* the location and their overlap form a strong mathematical structure representing ecocultural living.

People represent the position of places using dance and song. "Indigenous dance isn't just Indigenous dance—it's a map in itself, a directory of the culture behind the dance" (E. Johnston, nd, on Northern Territory languages in particular). Songs are used in many activities while traversing land and sea and for various reasons usually associated with spirituality or for rhythm of movement (personal experiences in PNG). A community project in the Blue Mountains (on the outskirts of Sydney) involves maps, pathways, and "song-lines" (Cameron, 2003). People use song-lines to maintain the connection with the route that is taken when traversing their land.

Time becomes evident in many map representations of space and place. Wassmann (1997) noted that with the descriptions and even more the map drawing (both of which are not generally required in everyday communication except with people from outside) some sense of walking the route was involved. For example, a slightly longer line represented a difficult time-consuming stretch of the track for a Yupna man, Morobe Province. Similarly, Harris (1989) provided an example of the direction and nature of walking in an Aboriginal map and discussed the meaning of maps that could be described as topological (connecting places) but deeply embedded in relationships of place and people (Fig. 4.4d).

To walk a trek in Kaveve village (Eastern Highlands Province, PNG) is to walk the story of the half-man who lived in that place (personal experience, Fig. 4.4). The story connected the place across time. In PNG, songs communicating with the spirits are used when traversing the land or remembering people who traversed the land (Rumsey & Weiner, 2001; personal experience). Hence I am vividly reminded that representations of land whether in words, diagrams, actions, or land formation embed relationships between people and between people and the land and hence reasoning about relationships.

Language is a major clue to the limitations of describing position in terms of two orthogonal reference lines or by distance and direction from a reference point. Pinxten's (Pinxten, 1997; Pinxten et al., 1983) study of Navajo concepts highlights even more the difficulties of concepts and language that are not easily connected. For example, a word like space for the Navajo referred to a saucer-like grand container. Thus the notion of infinite needs to be established in another way. With the issue of static object, the idea of a "snapshot" of motion is helpful along with the recognition that the western mathematics wants to emphasise the object that in Navajo mathematics was a spot in passing in the dynamic moving approach to place. Surface could be seen as where two volumes come together. What is quite distinct is the emphasis in western mathematics of part-wholes, of the hierarchical logic of point, line, plane, and three dimensions or of the distinction between distance and time.

There were examples in PNG mathematics of similar difficulties. However, the westernisation of curriculum meant that many teachers in their projects (see Chap. 8 for examples) were unable to focus on the mathematical thinking of their Elders and family but rather saw objects as a static object to link to western mathematics. Thus they emphasised their material culture especially built objects and final design features if they were seen as mathematics, e.g. symmetry and shapes. At best, these teachers described "deciding by eye" or "in their head" but did not have the mathematical language or connection between school and home mathematics to discuss the use of ratio or rate estimates, to note complete groups or encompassing sizes, or spiritual values of design or design making. Simply to explain as thirdspace thinking is one way forward (Soja, 2009) but Pinxten and François (2012b) outlined the depth by which Indigenous mathematical systems are more complex requiring their own unique ways of seeing western mathematics as a part of the larger mathematical complex. Furthermore, Indigenous mathematical systems are not only important but should also not be lost to the world.

Shapes

One of the issues in school mathematics is the use of labels and names for shapes as we mentioned in Chap. 2. It is important for children to describe and classify shapes and for them to recognise the generally accepted concept behind a shape name. I recall a conversation with Usiskin (personal communication, Utrecht, 2001) about the myriad of definitions for common shapes like "rectangle" and "trapezium" ("trapezoid") across the USA. Some countries have rhombus, others do not; some have oblongs, others do not. No doubt definitions are culturally determined and often without the same ways of thinking as mathematicians might claim. I have had conversations with experienced teachers about terms such as "regular", "diagonal", and "pattern" since they seemed to be using the terms differently to me. It is no wonder that Battista (2007b) noted how two boys were coming to an agreement about the use of a rectangle maker to make a particular shape in a Microworld.

> Matters were further complicated because ‘slant’ meant not-perpendicular for Matt but tilted from the vertical for Tom. … In school geometry, shapes are described by referring to *relationships between their parts*. … Common-language use of the word *slant* … refer(s) to the relationship of lines and segments to the *up-down or vertical frame of reference*. Thus, Matt’s use of the word slant (to refer to the angle of the shape) was evoking a totally different, common-language meaning for Tom. (Battista, 2007b, p. 71)

The teacher’s discussions with the children continued to indicate that they noticed different parts of shapes and their position differently and language continued to be an issue. When the teacher drew a rectangle with no horizontal sides next to a parallelogram with short sides almost horizontal, the children focused on the fact that these sides were not on top of each other like in the rectangle, meaning vertically above each other on the screen. However, with further manipulation and having their attention drawn by the teacher to the measures also on the screen, the students realised that the rectangle’s angles remained at 90° whereas the other shape’s angles varied.

The episodes reminded me of watching children with elastic loops to make shapes. One small boy made a triangle and to convince his partner, he turned it as it was a right-angled triangle, so there were horizontal and vertical sides as if that was the only way of having a right-angled triangle. “Activities assist students to start with holistic reasoning, they constantly encourage and support students’ development of ever-sophisticated knowledge of the properties of shapes” (Battista, 2007b, p. 78). However, the most significant issue in regard to shape naming is the use of words for categories. In Chap. 5, I discuss further how categories of objects when counting may denote shapes in different PNG and other languages—there are different counting words for different categories.

Such contextual references are common place in visuospatial reasoning whether they limit or extend thinking. In a study on language in a collaborative classroom in PNG, Muke (2012) found that a teacher chose to assist children to know left and right by reference to places located at some distance from them in the school on their left and right. This might of course not be helpful for the children if they were in a different orientation or place but for the moment it was a reference point for them. (See above for discussions about cardinal points and left and right). It was common practice in PNG, for people to think about east and west with little thought for north and south due to their familiarity with the rising and setting sun that does not vary much throughout the year close to the equator. In temperate zones, the sun’s position in the north and south is far more important in terms of its heat inside a dwelling.

Similar problems arise with terms like “straight”, and “right angle” often perceived as straight ahead or vertical on the page. “Diamond” often prevents children from realising a square is a square in any orientation or it confuses the rhombus and kite names. Thus the term in Tok Pisin is particularly difficult. However, words for Euclidean shapes are not necessarily part of traditional PNG cultures and languages. The ways in which shapes are made are not necessarily linked to the Euclidean property approach to shapes. “Diamonds” are more likely to be made than squares in weaving and bilums (see Figs. 5.14 and 5.16). Shapes are associated with changes to number patterns rather than changing properties. Parallelograms are distinguished from rectangles by the diagonals being equal although every attempt is made to get angles at right angles in house building, for example. Students will provide shape names in some cases but one teacher identified "starry" as a common shape which he related to pentagon, hexagon, and octagon, all of which people knew how to make through open cane or bamboo weaving, bilum-making, and tattoos. The angles of weaving and the over–under routes of the bamboo are significant (see Chaps. 5 and 8). However, other people were not familiar with these names for the shapes as found with the shape on a bilum called naming of the *fifti toea* (a coin that is not round but not a hexagon which is the shape on the bilum, Fig. 5.16g).

It is evident that language can either hinder or be used to develop concepts about shapes or other spatial concepts. If students learn through talking mathematics, then it is important for teachers to spend time on assisting children and parents to explain and justify their thinking especially in multilingual classrooms. If there are language confusions or lack of language words then it is important to spend time on the constructing of meaning and explaining around activities that are culturally relevant (see Chap. 8 for one project attempting to do this).

Moving Forward

This chapter shows that language about space and measurement is not just associated with representations mentally as suggested by information processing psychologists nor physically as suggested by mathematicians who ignore the ecocultural context of mathematics. Language reflects an ecocultural perspective on space, place, and visuospatial reasoning not only for communicating purposes but also for visuospatial reasoning associated with place. Language gives insight into the ways of reasoning in comparing size or determining position. In the examples provided, language in words or visually is associated with communities living in the places and with communities' relationships with those places.

The differences between various frames of reference are indicative that visuospatial reasoning and decision making are reflected in language and that language and reasoning are closely interwoven and supportive of each other. With visual and verbal representations, visuospatial reasoning is extended. For example, the time needed for walking a track is more clearly portrayed in maps that reflect hard time-consuming sections of the track. However, both verbal and visual representations could be misinterpreted if the cultural and linguistic context is removed.

I return now to the diagram presented in Chap. 1 on identity as a mathematical thinker. Language is a tool for expressing visuospatial reasoning in a cultural way. Thus cultural identity and valuing that identity will be expressed in responsive social interactions and in clearly presenting meaningful relationships. This expression of culture about the land and ecology of the person, the place of the person, promotes cognitive, affective responsiveness in solving problems in a visuospatial way. Language assists the person to structure the appreciation of the environment and to use words and diagrams as tools for problem solving. As a result, a person develops his/her identity, not only as a mathematical thinker but also as one thinking ecoculturally. Visuospatial reasoning reflected in mathematical literacy is a critical part of that identity.

Visuospatial reasoning is expressed in language but are there other ways in which ecocultural perspectives are portrayed? To explore this, I will discuss visuospatial reasoning in practices of people in PNG in the next chapter and from other countries, particularly with Indigenous communities in the following chapter.