Chapter 3
Changing Perspective: Sociocultural Elaboration

A paradigm shift occurs when a question is asked inside the current paradigm that can only be answered from outside it.

(Goldberg 1997)

… what young children learn and remember are things that arise as a "natural" and often incidental consequence of their activities … Setting out deliberately to commit a body of information to memory is quite a different affair from such examples of natural or spontaneous remembering, where what is subsequently recalled is something one literally handled, attended to or in some way had to take cognisance of in the course of doing a practical activity.

(Dave Hewitt, 2001)

The Challenge

In the previous chapter, I illustrated that earlier research on cognitive psychology was limited in explaining how we learn about and use visuospatial reasoning. While it helps to establish how the brain may be processing information of this nature and how it is attending to aspects of visuospatial information, there remain questions about how the world outside our minds is indeed interacting with our mind. An exploration of intuitive, tacit knowledge shows the importance of situated learning. Much of this learning occurs in activity. Questions arise about bodily involvement in visuospatial reasoning and how practice—participation in activity—influences this reasoning.

There are also questions about how different ecologies, environments, societies, and cultures impact on visuospatial reasoning. There has been an argument for a sociocultural perspective in education for some time and these aspects need to be considered for visuospatial reasoning. I wanted to understand the impact of place and

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culture on reasoning. Since place has a spatial aspect, it is expected that visuospatial reasoning should be understood in terms of place and culture. First we need to appreciate how a critical approach might impact on our thinking especially about space and place. To do this we draw particularly on elaborations of a critical pedagogy of place, that is, how a questioning approach to the relevance of curricula and classroom practice might influence our view on learning and subsequent social justice issues (Gruenewald, 2008; Gruenewald & Smith, 2007; Somerville, 2007). Furthermore, people with a sociocultural perspective also espouse quality education as the child belonging, being, and becoming as expressed in Australian early childhood education (Department of Education formerly DEEWR, 2009) and no doubt elsewhere (Radford, 2006). How is identity perceived in these terms? What does this mean in terms of visuospatial reasoning?

Much of the work in this area results from anthropological, cultural, and semiotic research in cognitive psychology as an explanation of meaning and learning. Has it remained too focused on the psychological aspects rather than the sociocultural perspectives that might link, for example, intentions and self-regulative approaches to learning? (Macmillan, 1998). Part of our thinking in this area is also challenged by the genetic developmental and anthropological psychologists. These psychologists investigate by asking specific questions to different cohorts in order to look at the sense of meaning both for different age groups and also within the diversity of the cultural group at any one time. These approaches were used by Wassmann and Dasen (1994a, 1994b) and Saxe (Saxe 1985; Saxe & Esmonde, 2005) in exploring classification and number in remote communities of PNG. Saxe's (2012) theory of change over time but with recognition at any one time of diversity within a cultural group informs an ecocultural perspective of visuospatial reasoning.

However, there are questions about how there is change over time in our visuospatial reasoning in terms of both age and experience of fairly common activities such as schooling and what might change in terms of new types of experiences. The question can be addressed from a sociocultural perspective particularly by considering cultures where rapid change is now occurring. However, there is a need to explore how this ecocultural perspective can be integrated theoretically with the psychological research.

Local Context

One issue that van Hiele (1986) grappled with in his book was that of intuition. He felt that intuition was often incidental learning that occurs in everyday activities. A parent who uses the term "cylinder" to talk with a child when a particularly noisy tanker goes past encourages the child to note the shape and often the word. The parent is likely to draw a circle in the air by hand and describe it as round reinforcing the nature of the shape. The noise attracts the child's attention and the significant person, the parent, provides words and representations of the shape. Thus a reinterpretation of van Hiele's work illustrates that the ecocultural context of learning is

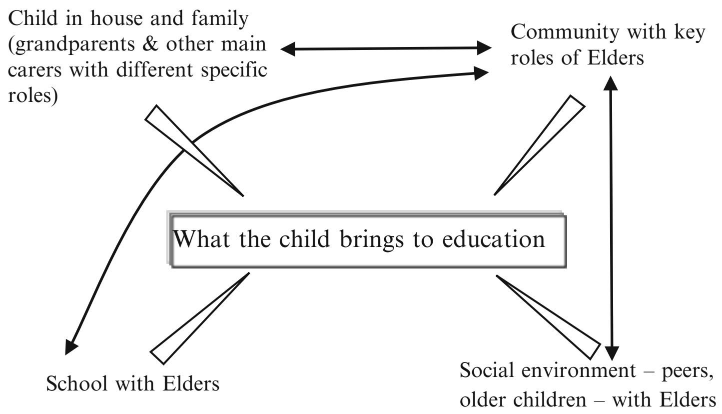

Fig. 3.1 The role of Elders in the education of the child. *Source*: Owens et al. (2012)

critical for geometric understanding. However, explicating significant knowledge is important as illustrated by the tanker example (Aikenhead, 2010).

Similar to other researchers' changing perspective (for example, Bishop, Barton, Gerdes, & Saxe), experiences with Indigenous people, their worldview, and their metaknowledge have resulted in my change in perspective about visuospatial reasoning from that of an internal way of thinking within an individual to that of a sociocultural phenomenon. A similar account of how people develop their local identity as one born and bred in a place, but not Indigenous, required Garbutt (2011) to be confronted by what it personally meant including resolving the problem of Indigenous ownership. Spatial links to an area as well as specific ways of thinking, kinship relations within the area, and a history in the area were all relevant. However, from my experiences in PNG, the term "local" referred to Papua New Guineans, generally born and bred in the place. Local was used to avoid the term "native", then regarded as derogative. Hence, I see spatial knowledge that is local as primarily Indigenous.

However, I was to be further confronted. In a forum with local Indigenous community Elders and others in Dubbo (where I now live), the group moved away from the Brofenbrenner's (Brofenbrenner & Ceci, 1994) ecological perspective. They gave prominence to the role of Elders in all spheres of life that supported a child. The child was not in the centre surrounded by the systems of school and family as Brofenbrenner had illustrated but the child in the family was supported by the family with the Elders' influence, the school with the Elders' influence, and the peers with the Elders' influence (Owens et al., 2012, see Fig. 3.1).

The child's knowledge was not independent of the sociocultural context. The child's learning was not just represented in the child's mind but the child in the family. The child did not just internalise knowledge shared by others but the knowledge was part of relationships within the family and with the Elders. Local takes on a meaning that incorporates the family and Elders' knowledge and their worldview and interpretation of space in terms of place. Thus space designations are part of relationships with people. The close connection between culture and place and people is identified. Local, however, can extend in boundaries and in time. Hence, contemporary culture and not just historical culture are part of the sociocultural context. The historical perspective is not forgotten in Brofenbrenner's model or in Indigenous or community's local knowledge, whatever the community may be. The historical perspective is embedded in memory and language and frequently embodied in routines, tools, play, practice, and activity.

Furthermore, border crossing or generative changes in spatial concepts and visuospatial reasoning occur. These changes result from the reasoning or even dogmatic or hegemonic representations of more powerful individuals such as teachers and curriculum writers (Aikenhead, 2010). Indigenous cultures have developed meaning and reasoning in mathematics and these ways of thinking are not necessarily found in western or schooled societies. However, individuals cross cultural boundaries in everyday activities and in school requiring education to be aware of the social injustices and the loss of world knowledge that occurs with dominance and lack of continuity of learning for individuals.

An Example of an Ecocultural Practice

By taking an ecocultural perspective, it is possible to make sense of what seems like discrepancies in PNG counting systems from the same language group. From my investigation into Lean's (Lean, 1992) thesis on counting systems (Owens, 2000b) and my own data, such as the sets of Alekano (Gahuku-Asaro) counting words given by different speakers (Eastern Highlands Province, PNG), there are not universal sets but a diversity of ways of conceiving certain numbers. Table 3.1 provides some numbers with associated embodiments. The discrepancies do not reduce the fact that there is mathematics but a more important ecocultural and individual identity is at play. Both men could understand the other and I as an outsider could understand and accept both based on my broad knowledge of PNG counting systems.

"*Logosigi* squared means 2 plus 2" but "*logosigi* to power of 4 means 2 plus 2 plus 2 plus 2". Since reduplication is common, many words are phrased as squared, e.g. *kau*^{2} for *kaukau*, sweet potato. This is part of PNG humour even if it might be a visual representation that is confusing for school mathematics. It is likely that the variation in Table 3.1 is influenced by a neighbouring language (Bena Bena). Nevertheless across Gahuku-Asaro area, in different villages, slightly different counting words were given. Other words of interest are those words for large numbers. In Gavehumito village (2004), the teacher noted "100 *asasi ligizani luga luga* (stick or 10 hand finished finished); 200 *go*' *hamo* (*bilum* one); 1,000 *mulisi* (*hip* = heap)".[1]

More common were the various ways of combining twos and ones for 6 and 7 especially in languages with 1 and 2 as the frame words for the counting system or there is more than one way to say 4. In Wiradjuri, NSW, Australia, 4 can be expressed *bula bula*, or *bungu*, or *magu* (Grant & Rudder, 2010). However, there are more complex systems. For example, Paraide (2010) explains how the complex ways of measuring and counting in Tinatatuna, the Tolai language (East New Britain, PNG),

[1] *Bilum* and *hip* are words in Tok Pisin, the lingua franca of PNG.

Table 3.1 Alternative numbers in Gahuku

	First man	Second man		First man	Second man
1	Hamo	Hamako	8	Nigizani hamo asu o'oko makotoka logsive hamo ol omalago	Logosi^4 means logosi is repeated 4 times "means 2 plus 2 plus 2 plus 2 plus 2"
2	Logosita	Logosi	9	Nigizani hamo a su o'ko logosive logosive oli'o malago	Luguhagi luguhagi luguhagi
3	Logidigi hamoki	Luguha (logosigi moka)	10	Nigizani logosi asu igo (nagahuni hamo)	Golaha
4	Logosivi logosive	Logosigi^2 "Logosigi squared meaning 2 plus 2"	11	Nigizani logosi asu oko ligisaloka hamo oli'o malago (2 hands finished and one on the leg)	Golohaki hamakoki
5	Logosigi logosi hamo or nigizani hamo asu igo (one hand finished)	Logosigi luguhagi	15	Nagahuni makoki logosi logosi hamo or nigizani logosi asu 'olo ligisa hamo asuigo (2 hands finished–one leg finished)	Golohaki luguhagi logosigi
6	Luguha luguha	Luguha logosi	20	Nagahuni logosi or nigizani logosi aso'oko nigisa logosi asu igo (2 hands finish, 2 legs finish)	
7	Luguha luguha hamoki	Luguha logosigi makoki (segininaga)			

builds on both 10s and 12s (3 groups of 4). For example, 12 coconuts are called a *pakaruati* (one lot) and then 10 or 20 of these lots are named. These forms of counting are associated with practical activities and with the visual representations of the sets. For Hagen and related languages, there is both a (4, 8 cycle) system and a 10 system. Gestures are associated with both systems and different morpheme combinations are used for numbers depending on what is being counted, the ceremony, or purpose. From these various examples, even simple mathematical activities like counting are a way of expressing identity and the individual identity within the rich identity of the cultural group.

Values in Education

There is often an emphasis on the cultural conflicts that occur in schooling where colonial or western perspectives dominate. Furthermore, acculturation of mathematics allows school mathematics and those in position of power within the school system to dominate. However, I contend that there are important strengths of cultures that must be highlighted before acculturation or multicultural stances are taken. I am not sure that any of the choices of strategy generally espoused for intercultural relationships, that is, integration, assimilation, separation, or marginalisation, satisfactorily achieve the ecocultural perspective that is contended in this book. Like the Elders in my local Wiradjuri community and others whom I have met from other Indigenous communities (in Australia, Sweden, New Zealand, PNG), relationships between people are dominant. Many Pacific people are relational beings legitimised by

> Sacred relationships built on the values of tapu (sacred bonds), alofa (love and compassion), tautua (reciprocal service), fa'aloalo (respect and deference), fa'amaualalo (humility), and aiga (family); to them, culture is the core of their very existence, both individually and collectively. Nevertheless, being cognisant of the negative consequences of colonisation and forced acculturation among Pacific cultures is critical for working towards balanced intercultural relationships that can lead to positive outcomes for people of the Pacific. … [Educational developments should align] with Pacific cultural values of shared responsibility, reciprocity, and interdependence. (Vakalahi, 2011, p. 87)

Thus I suggest that a strong recognition, extension, and valuing of cultural mathematics in a school setting are important for effective and efficient learning of other mathematics through an effective transition approach.

Signification, Meaning, and Becoming

One of the challenges for cognitive psychologists has been that representations of objects in the mind have not remained constant over time or place or people or indeed within a person. Objectification is indeed a probabilistic determinant or decision of best practice of people's responses to a stimulus. In other words, the debate between subjectivity and objectivity around meaning and interpersonal communication is around the certainty of words or diagrams signifying objects without dispute. Objective knowledge was challenged by radical constructivists and social constructivists (Davis, Maher, & Noddings, 1990; von Glasersfeld, 1991) through either an emphasis on personal, radical, individualistic construction of meaning or the social impact of others on meaning-making. One approach to finding a way forward was to talk of taken-as-shared meaning (Lerman, 2001). In that situation, cultural and social influences were recognised in discussions on mathematical concepts that might have been taken as a universal mental possibility for all persons even though some might favour visual or verbal or other ways of solving problems and knowing.

"Concepts of meaning indeed are based on presuppositions concerning the relationship between the cognizing subject and the object of knowledge" (Radford, 2006, p. 40). Referencing systems are related to the relationship between the person and the object and its position in space. Generalisations of patterns found in experiments (Peirce, 1998), no matter if embedded in everyday activity, are often associated with numbers but are established in terms of the sociocultural contexts and what they might mean for future communities. Intentions then closely monitor the meaning that is established for the person individually and the person within the cultural group. Thus, for example, the mathematics involved in building a village house in PNG requires consideration of many construction principles but also reciprocity, recognising relationships in terms of assistance and gifts. In a collaborative group task in a classroom, the value placed on collaboration, purpose, relevance, sharing, and appropriate communication will influence the resulting mathematising from the task.

However, by building on and modifying the constructs of Peirce (1998) and Radford (2006), I suggest nature (the environment) might indeed provide possibilities and restrictions on the experimentation, abstraction, and interpretation within mathematics. An ecocultural perspective is more comprehensive than either a psychological or sociocultural perspective. Furthermore, it is possible to extend these generalities and concepts by relating meanings and by making logical possibilities. Thus intention in learning is controlled by more than sensing and perceiving as the psychologists suggested in generative learning theories (Osborne & Wittrock, 1983) though these information processing theories provided a good theoretical background for earlier work on visuospatial reasoning. Furthermore, according to Radford (2006, p. 42), "ideas and mathematical objects … are conceptual forms of historically, socially, and culturally embodied reflective, mediated activity". In that way, language plays a part in the establishment of meaning and representation and reasoning. Habits and patterns of language and activity provide a mediated realm for signifying objects and their position in space. This point is taken up further in Chap. 4.

Furthermore, it is possible to have an intuitive sense of relationships based on our intentions, according to Radford (2006) in expanding on Husserl's notion of praxis. Thus reflective practice increases the likelihood of learning and making meaning. For example, geometric generalisations are ideals that develop from the more specific examples that are generated through reflection from one to another. Such generalisations occur through visuospatial reasoning about the examples and how they can be modified to provide a different example but with certain properties maintained and represented in both words and diagrams. For example, a triangle made from a loop of thin elastic can be modified by pulling points or moving their position in a dynamic way as occurs if the loop is held by three fingers. A similar occurrence occurs in dynamic geometry software. The person may generate some triangles by chance but the construct is best appreciated when the person compares the examples and deliberately tries to make specific examples. Thus through praxis, conceptual meanings are generated. Through intention, we attend to certain aspects of the phenomenon or generalisation in order to create meaning and conceptual understanding. The intending and referring that occur in practice are creating meaning not only for the individual but also for others. It is the historicity of culture that provides the meaning for problem solving and learning and inter-subjectivities. "A set of morphological instruments, syntactic and lexical systems, literary genres, figures of speech, forms of representation of events, etc. that are part of our cultural inheritance" anchor the referring and signifying of the ideal of the object through interaction and negotiation (Radford 2006, p. 53). This set of representations provides our way of referring to ideas embedded in culture. In other words, “mathematical objects are conceptual forms of historically, socially, and culturally embodied, reflective, mediated activity” (Radford 2006, p. 59).

However, alternative perspectives, different ways of attending to the object or concept, and different people can reach different but probably related ideals. It may be that the spatial textures of the object or environment are what draw the attention or affect the intention of the person making meaning. This spatial aspect is encompassed in an ecocultural perspective.

Taking this critical perspective further, Soja (2009) suggests the meanings of human spatiality and related concepts such as place, location, locality, environment, and geography are best understood in terms of thirdspace, a metaphor to keep an open perspective on geographical imagination to incorporate multiple postmodern views as a space for race, gender, and class without privileging. Following Lefebvre (1991), Soja suggests social production of social space as a spatio-analysis. He envisaged multilevels of the right to difference (from body to nation). Firstspace—perceivable spatial dimensions and direct connections such as measuring; secondspace as spatial representations, cognitive processes, and symbolic meaning in the dualistic geographical imagination but now thirdspace as a critical discourse that problematises spatiality and avoids reductionism in interpretation. Thus an ecocultural perspective on space perceives and works with the overlapping perspectives of the various descriptions of a place. There are different ways (western and traditional) of describing a route that is also part of a mythological story of how the place was formed.[2] These multiple spaces intersect to provide new mathematical knowledge such as noticing unusual flat stones, the movement of water, the surrounding vegetation and the provision of food from the plants, distances embodied in walking the track, and the ownership and sharing of the land.

This open perspective on spatial knowledge provides for radical, creative mental spaces and relationships. Mental spaces consist of perceived spaces of practices and social relationships; conceived spaces that are representations; and lived representational spaces that embody clandestine meanings. The thirdspace is a social space that transcends the first two dimensions (physical and mental)—all three are “real and imagined, concrete and abstract, material and metaphorical” (Soja, 2009, p. 52). Soja presents “being” as having spatial, social, and historical aspects that are balanced in the three spaces in spatial imagination. This trialectic ontological assertion of space, time, and being-in-the-world impacts on epistemology and empirical analysis. It provides for a recognition of othering and an emancipatory change which is never fixed but permits lifespaces and life views in epistemologically derived situations such as education. Spatiality as a term prevents the dualism of physical and mental and becomes the perceived, conceived, and lived spatiality as a practical sense of the spatiality of social life. It is incorporated into our sense of visuospatial reasoning.

[2] An example are the song-lines found in Australian Aboriginal and PNG stories such as the half-man story, their river and valley told by men of Kaveve village, Eastern Highlands Province, or the songs of tracks associated with particular Foi hunters, Southern Highland Province.

Thirdspace provides resistance to hegemonic power. A new spatial awareness emerges as a product of spatial imagination that combines the physical and mental in a lived space, or location of culture (Bhabha, 1995). The ecocultural perspective as a thirdspace provides a critique of the dominant culture's liberal permission of cultural diversity. Thus the Yolngu women[3] wash cycad nuts in the same place at the same time of the year every generation, so the place has more meaning and more mathematical generalisations than just a western description of location. Lossau (2009) links thirdspace to borderlines and border crossings (Jegede & Aikenhead, 1999) as a strength rather than a limitation or negative compromise in learning and she suggests it provides an outward and expanding position, even as a hybrid. Thirdspace implies the myriad of circulating crossing of boundaries and movements from centre to periphery. However, it is important to keep multiple perspectives rather than reduce them to a common form. Lossau explains that the spatial metaphor permits ordering of ideals, connections, and relationships in the knowledge. It permits difference indicated by distance between ideals but spatial imagination can only be imagined by a hidden reification. However, I am concerned that difference is, like any representation, reducing difference to dimensions and positioning in all its implied meanings. To that I would bring into the metaphor the notion of topology rather than dimensions as a way of understanding difference in the thirdspace. Topology emphasises relationships rather than metered distances on various dimensions and thus reducing the ordering of different perspectives.

Furthermore the ecologies of the classroom itself, especially during cooperative group work, may also be important in terms of the positioning of students including the various mathematical and social identities of the student (Esmonde, 2009). The relationships between people in the classroom have variation, not only as a result of gender and language and prior experiences but also in terms of the contextual ecologies of the students. As Macmillan (1998) showed in early childhood settings, discussions were assisted by mathematical language and other aspects of game or role positioning within the classroom but language proficiency also positioned students in the social interactions. Teachers extended the students' play and thinking, thus permitting "identity formation by encouraging tolerant and accepting co-participation, responsible self-regulation and clear and negotiable access to resources, activity and meanings" (Macmillan, 1998, p. 122). The mathematical activities and discourses then lead to individual motivations of curiosity, challenge, choice, and imagination resulting in interpersonal motivations and socio-regulative interactions.

A Genetic Approach to Ecocultural Perspectives

A genetic approach considers how form and function of signs (language, objects, and symbols) change over time. In some ways, in a simplistic understanding of a genetic approach, I think it is a sad perspective in that it loses difference and

[3] Northern Territory, Australia.

multiplicity of visuospatial reasoning. It indicates a hegemony of language and practice and an acculturation or enculturation often into school mathematics rather than an enhancement of school mathematics. It is often marked by a loss of language and the role of perceived power (see Saxe's (2012) example of the teacher's authority in giving the changed counting words for Oksapmin). However, it does indicate that cultural identity is fluid just as change is inevitable. Language becomes an important part of change, or at least it is a visible sign of change. The use of language and signs does not necessarily precede the development of thought. Similarly, practice may or may not determine thought and skills. However, both language and practice facilitate communication and may facilitate thinking. However, we explore Saxe's genetic development model in more detail as it illustrates the impact of ecocultural change on valuing and reasoning.

Saxe (2012) does not think the amplification metaphor, that suggests signs and systems assist in the extension of thought, is appropriate for explaining change because the circumstances and the systems may be quite varied. It may not be the result of the sophistication of a system or representation or its function. Each system in practice develops its own complexities. To keep culture and cognition separate and to suggest that one influences the other fails to draw adequate analysis from the sociohistorical cultural and cognitive developmental processes. Saxe maintains that both culture and cognition participate in constituting the other. Both are processes. One thing to note is that there is variation in culture. Descriptions of cultural practice frequently ignore the diversity within the society but Saxe's explanation is similar to de Abreu's (2002) explanation about the common valuing of a practice in a particular ecological circumstance. Culture has representations, practices, and artefacts but diversity is also a property of culture and boundaries. As Saxe puts it, these properties of culture are osmotic in many ways resulting in variance and change. Cognition is a process and is not internal as opposed to external as in representations. Thus the representation is part of the cognitive process and it has cultural forms and cognitive functions for both individual and group activity. Form–function relations as processes in motion are constituted and shifted in the processes of microgenesis, sociogenesis, and ontogenesis.

Saxe has written clearly, engaging the reader in the first chapter with his personal journey into a little known realm of the world, the Oksapmin of the highland reaches of the west Sepik area of PNG. He follows this with his theoretical journey referring to various theories that approach psychology from an anthropological or cognitive (constructivist) perspective to present his genetic model that incorporates microgenetic development at the personal level and how this relates to the sociogenetic level as individuals in the society influence each other and over time to an ontogenetic level that leads to changes also in the individual and at the social level. This criss-cross influence is illustrated diagrammatically in Fig. 3.2.

Furthermore, the interplay between positional identities and the elaboration of mathematical goals may be explored. At the microgenetic level, change is not a mere transmission from individual to individual as in one person suggesting a new use for a particular idea but change for the individual is the result of development in society of the construction and use of ideas and their representations and

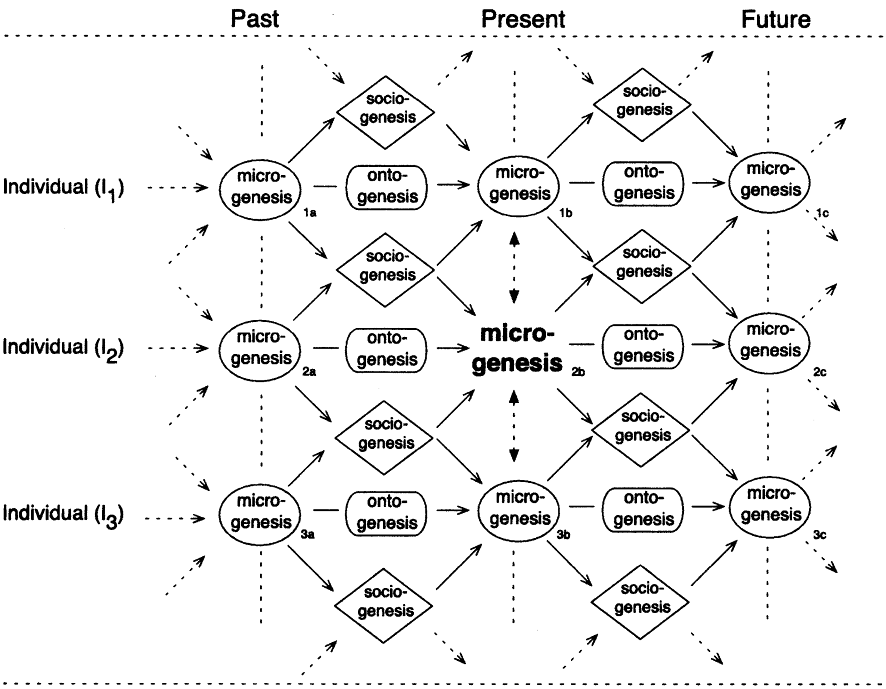

Fig. 3.2 The interplay of genetic aspects in cultural contexts. *Source*: Saxe (2012, p. 33)

modifications. This is the sociogenetic level. These of course limit the individual possibilities and provide an understanding of the form–function relations (ideas, objects, and signifiers and their uses) over time. Ontogenesis is the individual development over time. A blind person who uses a cane to navigate makes adaptive changes to spatial thinking through the physical manipulations and responses of the stick on objects during practice and in new ecologies. Practice in various situations permits adaptation to new situations but there may need to be sound or other guidance for the adaptations to be effective and efficient.

Saxe used an interesting research procedure. Initially in the field he observed everyday activities but also asked people to specifically explain, for example, how they counted which was quite visually demonstrated. They showed their system of tallying objects against parts of their body in a fixed order starting at the thumb (which is not a common starting point for body tally systems in PNG) of one hand to the small finger to the wrist, lower arm, inner elbow, upper arm, shoulder, ear, eye, nose, and then across and down the other side but going from the thumb to the small finger (not a mirror image which is also unusual among the body-part tally systems) (Lean, 1992; Owens, 2001c). Sometimes they stopped at 20, the elbow of the other arm, because that matched with 20 shillings equalling a pound (during the former Australian administration) and then adjusted that to $2 and finally to K2 (two kina note—all amounts below this are coins). Others went back up the arm to the elbow to 30. The visuospatial connection between shillings and 10 toea (the coins looked the same in size and colour[4]) were also used as a standard value for a pile of food in the market. In this study, there was already variation among the people. Saxe then selected specific groups of people to support the view that certain experiences were likely to be associated with certain approaches but to also show that it was a relatively common approach or valorisation within a specific age group.

Having come from an empirical tradition of psychology, Saxe describes his methodologies for a range of studies which he grouped into two areas. The first related to the influence of the outside money economy found in trade stores and used by others who had worked outside the community and to a lesser degree by those who sold garden produce and paid school fees only. The second area related to the two-way influence of societal and school ways. On his visits, 20 years apart, he selected certain divergent groups to interview that illustrated different degrees of contact. This also related to different visits across time providing a check on the genetic explanation of change. However, he did not lose the serendipitous opportunities and the opportunity to review videotapes. Furthermore, he has shared his visual research on the web (Saxe, nd).

I select here to evaluate the findings from his first study on the money economy. The number of interviewees was large, nearly 80 in four categories requiring walking a considerable distance between trade stores and villages. The addition and subtraction problems with or without 10 toea (10t) coins (still called shillings by some) were carefully counterbalanced. Each oral word problem was presented in terms of the trade-store buying and each number indicated on the body-part equivalent. The visuospatial reasoning using body parts and the trade-store experience are evident. Saxe's graphs present interesting differences in results but a good proportion of all groups with coins present correctly answering most questions, with more coaching of older people and of schooled young adults for subtraction (quite a few had been systematic but gave the wrong addition answer). Saxe noted a common understanding of the story problem without actually designating whether the problem was a difference or sum when coins were not present. The visuospatial relevance of coins and the oral communications were evident. It seems that the body-part tally system is indeed a built-in ruler-type representation for numbers that is easily used for problems to which in school we would apply arithmetic concepts of addition or subtraction. This method was used mostly by older adults and to a lesser extent by young adults and plantation returnees but not often by trade-store owners whose practice with costs allowed them, I contend, to visualise or recall in ready-reckoner form the ratio of values for different amounts of goods or for commonly added amounts.

A more intriguing method of responding to problems was the use of double-enumeration or matching of parts obtaining a result by one-to-one correspondences of body parts. Some people used words rather than body parts to match providing a more efficient response. For these people, it was clear that the body parts were a

[4] The coin is sometimes called a shilling and it is not unusual for a younger man to give the kina and toea values in pounds and shillings to the Elders, without any apparent effort in calculating.

number system. This method was used by a significant number of people across groups except the older adults. The body-tallies led to an interesting compensation method where both arms were used with the shoulder (10) being a critical part in the designation and transfer of parts (e.g. 16 might be 10 on one arm and 6 on the other). This method was used by a few of the trade-store owners and by very few others (mainly plantation returnees). In addition, by providing scores for advanced strategies that used body parts as a system to provide accuracy, Saxe was able to show a significant difference between groups. An analysis of variance, supported graphically, showed greater use by trade-store owners, with less use by plantation returnees and young adults with no advanced strategies or use of the body parts in a numerical system for older adults with problems without coins being present. The question remains whether this was lack of practice or indeed genetic development from social experiences. For either explanation, the visuospatial representation and reasoning are evident.

Saxe indicated that hegemony and position of people such as trade-store owners, teachers, missionaries, patrol officers, and returning employees can have a stronger impact on the development of practice than other people. Thus Tok Pisin methods of referring to currency were universal unlike the diversity of ways of developments in naming and valuing currency. The example of K5 is particularly indicative of people "playing" with words to develop a phrase to represent the K5. Unlike the restricted variety indicating ontogenesis for K2 or K20 resulting from the established 20 parts to the whole, K5 was referred to as a combination of K2, K1, and 50t, often with doubles such as two K2 and two 50t. While he might suggest how this illustrates both personal developments that become new forms of knowledge over time (ontogenesis) influenced by sociocultural developments, there are also important values in the play of words when children are learning about money and values. Similarly the various numeric equivalents for *fu*, "a complete group of plenty", are associated at different times and by different people with differing amounts in a consistent way. However, *fu* is also associated with doubling. There is no explanation about how this developed although doubling occurs to turn K1–K2, K5–K10, and K10–K20, each of which is marked by coins or notes. Doubling is notified by "married". The grouping of 20 is common practice among various language groups of PNG and for many travelled participants 20 became the *fu* rather than the original 27 in the body tally system. Visuospatial representations and words were linked through the double arms. With activity and interactions between people, there are changes both of signification and meaning but in each case the meanings between variant ways of naming the currency are in themselves important in reasoning.

Part of the reasoning or "play" with words for different amounts subsumes the visuospatial reasoning. For example, the use of the name "hole flat" for the K1 coin (which has a hole) or the word "leaf" for the green two kina note assists people to know these denominations and use these to make K5. It would be instructive for teachers to make use of these various forms of describing K5 to encourage learning rather than to just replace with English or Tok Pisin (very similar counting systems based on base 10).

In a similar way, when children are first learning to make and remember additions, use of the body-part tally system as a number line (equal spacing is not important because it is signifying counting numbers) in conjunction with English can provide a good visuospatial reasoning platform. The children may learn the order of the words because they are also body parts. They can see them. They can count to a number like six and count on eight by matching corresponding parts of the other hand and/or orally counting up another eight. It is worthwhile noting that it passes the 10 (shoulder) which takes on special significance now that there is a regularly used base 10 counting system in place. I have regularly noticed across different language groups in PNG young adults (whose schooling was in English and not their own language) using their traditional counting systems to make words (string of words or morphemes) for larger numbers by using different combinations of morphemes or words that represent smaller numbers (see discussion earlier in this chapter). In this current period of change, various systems are being used. So this "play" with counting words seems to be strengthening the arithmetic. If one person in Gahuku-Asaro says 18 is three groups of 5 and 3 and another says it is 2 groups of 5 and then 5 and then 3, both are correct and providing different arithmetic expressions. Both have visuospatial representations behind their reasoning (Gahuku-Asaro has groupings of five in the counting system). In forming larger numbers once the pair of hands has been used, people will "borrow" the hands of others standing around in the group with a nod at each person keeping track of their pairs (two hands) of fives. Thus we find the ecologies of people are influencing their construction of visuospatial reasoning behind their number combinations and descriptions and physical representations.

Saxe's theory does allow for the differences within a society and the changes that occur within a society. This was an area that Montiel and Managal (2011) were concerned about in looking at cultural identity. The degree to which there is continuity in change can be significant for the integrity of a person's identity of self-worth by which they make effective decisions. However, in terms of mathematical identity as outlined in the first chapter, self-regulation and the ongoing sociocultural interaction embedded in sociogenesis will impact on mathematical identity in terms of practice, values, satisfaction, and effectiveness. If practices are valued by the person and society in their ecocultural context, then mathematical identity will develop and the loss of cultural identity and ecocultural mathematics will be avoided.

Visuospatial Reasoning in a Navigating Team's Sociocultural Knowledge

In PNG, we had occasion to travel on sailing single-outrigger canoes along the coast or across large rivers as well as canoes in inland rivers and lakes. At other times we travelled on dinghies with outboard motors and larger coastal boats. But it was the use of sail, wave, and rowing around particularly difficult points that amazed us most. These canoes are not easy to steer, let alone balance and manage waves. The recognition of wave, current, and place even in the half light of the early morning and the automatic responses of the sailors and rowers as a group illustrated visuospatial reasoning that was shared among the men and embodied in their strong paddling and steering actions.

The breaking down of boundaries of inside/outside (social/mental) in cognition was a critical premise of Hutchins (1995) in his account of the cognition of a team of navigators on a US navy ship. His study of the team and their cognition showed that the calculations of position and the various knowledge and roles were partially represented by the breakdown in the tasks to be done in the social order of the ship's crew and relationships as well as in the navigation tools that were used. He provided several examples of the breakdown in the flow of information that illustrated well this inside/outside view of cognition. One was of the need of those having access to the chart, assisting the position bearer to find a required landmark. Access to experience and the chart permitted one crew member to give an approximate bearing by which the attention of the bearer was narrowed sufficiently for him to locate the required landmark. In other incidences, the reasoning prompted by a possible mishearing of a position or the inability to know what precise point was required for a bearing was overcome by an interaction of people through a written and oral exchange of information and through a physical gesture as well as a description. The reasoning was completed by a team working on the spatial problem together. In another incident, it was obvious from plots on a chart of bearing lines that three close beam landmarks were not as good as using a beam and another landmark closer to the line of travel. The chart facilitated one crew member pointing out this situation to another. To complete discussion of the team's visuospatial reasoning, he described how the team created their own shortcut or modularisation for speeding up calculations, albeit with an error initially under the pressure of a large moving ship that had lost power. In this critical incident, the team developed and shared a new mathematical approach which was not, unlike many other activities and tools, part of the legacy of navigation techniques. Nevertheless, it would be easy for another person or team to develop this method again.

One of the influences on Hutchins in interpreting the results of his study was his study of Micronesian navigators. He outlines how different cultures establish their representations of position, direction, and distance and I will return to this in Chap. 6. Firstly, in western society, familiarity with small maps representing large space presumes a position will be fixed by orthogonal axes. It is accepted that the Mercator representation of the earth is visually incorrect for representing distances and areas the further one leaves the equator for higher latitudes. Hence radio-beacon, the shortest distance between two places on the spherical globe along great circles, rhum directions which is by line-of-site, or straight-line reckoning are used, often simultaneously, to decide position. Over a relatively small area, these are virtually the same. Navigation charts become a computational tool on which lines and circles of distance are represented and by which computation and decisions can be made about time to reach other destinations at certain expected speeds. These charts and the visuospatial thinking of navigators who use them come from a history of western mathematics. Tools also include previously recorded information, tables for allowing for magnetic north effects on the compass, and routines for sharing tasks.

By contrast, the navigators of the Pacific, specifically those of Micronesia, use alternative but equally effective methods to navigate to unseen places without an extensive array of equipment found on large ships and yachts. There are in both navigation systems certain constraints. There are representations and computations in both systems. This is discussed in Chap. 6.

Visuospatial Reasoning in Measurement

Area measurement is a particularly problematic issue for children if visuospatial reasoning from ecocultural contexts is not incorporated into the learning. Rahaman (2012) showed that students' greater reliance on formal strategies to undertake problems related to area in other contexts reduced the use of their own strategies. She suggested the importance of incorporating contextual and visual experiences into the curriculum. One issue is that people can have two different visual images of a rectangle, one as a border and one as a filled-in rectangle associated with a two-dimensional space having area, an image required for measurement of area. Doig, Cheeseman, and Lindsay (1995) also found these alternatives occurred with children. They reported that more children placed given tiles around the rectangle as if a door frame but if they were given a drawn square, then more attempted to fill in the rectangle. Martin and Schwartz (2005) showed that students tend to realise the grid arrangement and its link to multiplication of areas with square units on small rectangles but not for larger areas when students resorted to addition rather than multiplication. Furthermore, the similarity of a rectangular unit also led to alternate physical and mental processes. It could be argued that ideas are not so stable in different environments requiring learning in different contexts and adaptable processes. In Chap. 5, I will discuss the ecocultural issues associated with visuospatial reasoning used by PNG people in comparing areas.

Visuospatial reasoning is particularly important in estimating and draws on ecocultural experiences. How important is estimation? A study by Adams and Harrell (2003) asked 17 people in different occupations:

1. For what kinds of tasks do you frequently engage in estimating?
2. Why do you choose to estimate instead of using a tool to obtain a measurement?
3. Why do you choose to use a tool to obtain a measurement instead of estimating? (p. 229)

The reasons for estimation are mainly to save time but they also do it to verify the validity of the measuring tools and methods. It might also be that precise measures are not possible or relevant or a quotation for a customer is needed or just because it is enjoyable. However, at other times, it is not appropriate just to estimate such as a product has to be consistent, risks are too high, it is inside the body or it is a new task. However, this form of visuospatial reasoning can be developed through practice, using multiple senses, encouraging it as a step in the procedure that requires prior knowledge, and for making decisions. These kinds of estimations are happening for "employees, customers, consumers, and participants in recreation … estimation is really at work!" (pp. 243–244). Ecocultural context is a vitally relevant part of the visuospatial reasoning of estimating.

Embodiment of Spatial Knowledge

Dialling a telephone, using a calculator, and touch-typing are all examples of embodiment of knowledge of the position of numbers and letters on these tools. Pilots of planes also have spatial imagery as well as visual imagery on which they make split-second decisions (Wickens & Prevett, 1995). From Hutchins' writing and that of others who have sat with the Pacific navigators, there are embodied memories as well as mental memories assisting with the visuospatial reasoning and decision-making. In the Caroline Islands, the tilting of the head to 45° provides a kinaesthetic means of selecting the angle of inclination to view the star constellations (Worsley, 1997). This embodiment of direction is also evident when sailors can feel the state of the swell under the canoe. This may assist in positioning the canoe in reference to the island (leeward, side, or front of the island from the direction of the swell) (Bryan, 1938). In the Marshall Islands, the sailors have sea roads that are taken regularly and that take account of the swells and currents. They physically attend to the forces of nature in determining the extent of travel in a given time, so they take account of fast moving waters or winds appropriately.

PNG Indigenous peoples know of trading and cultural partners far away as observed by their long-distant trading circles such as the kula trade around the Papuan Islands, the Hiri Motu trading and winds along the south coast of PNG, the Rai Coast-Madang trade, and the highlands to coast trade. Australian Indigenous peoples also travelled long distances for trade, relationship building, seasonal adaptation, and knowing the land. The time of year, the winds, and the cycles in food production all influence these navigations together with trading goods such as food, pots, stone, oil, shell money, knowledge, and salt. Distances may be associated bodily with time taken to cover the distance. The crew of the sailing canoe respond to the boat load, swells, winds, and position of reefs (and land if visible) to adjust paddling and sail position (personal experiences). Similar skills are then applied to small "banana" (fibreglass) boats with outboard motors. These tools of movement are then part of the cultural identity and response to space and place.

Early Childhood Experiences

Nativist perspectives might suggest hard wiring like language modularity and sequencing in the brain (Butterworth, 1999; Dehaene, Izard, Pica, & Spelke, 2006). However, this nativist view of spatial sense is only part of the story about visuospatial reasoning. Yakimanskaya's (1991) social theory suggests spatial thinking is based on one's needs, urges, and motivations and that advanced thinking engenders frames of reference requiring active and dynamic manipulation of objects or activity. However, these frames of reference are culturally bound as suggested by Pinxten, van Dooren, and Harvey's (1983) anthropological theory of universalism. They gave three types of space divisions: physical, sociogeographical, and cosmological spaces. They noted all three spatial semantic categorises of near, separate, and contiguous (3 of 118 terms for the Navajo) are used by different cultural groups. Thus we need to explore the bridge between ecocultural and psychological perspectives of visuospatial reasoning.

In Chap. 2, visuospatial reasoning in early schooling was illustrated and examples of different responses were given. The chapter shows the limitations of psychological theories especially that of Piaget for which topological thinking and conservation were considered necessary before visuospatial reasoning could begin in the areas of classification, orientation, motion, and part–whole relationships within shapes. It was also established that in classrooms, students' attention resulted from both internal thinking and classroom context. Hence we began to see that a broader perspective was necessary for understanding early childhood visuospatial reasoning. Ness and Farenga (2007) provided a strong argument for visuospatial reasoning in the everyday context of block play. In spatial development, mental constructions of space are developed after the activity and are culturally bound. By analysing videotapes of children in block play, they developed a theory of how children learn to think spatially and scientifically. They observed patterns of behaviour and development of process skills and cognitive abilities that showed how children begin to learn about space and architectural relationships. As a result they presented a new, alternative way to measure cognitive abilities and development in children noting that topological thinking is not opposed to Euclidean relationships but rather the lack of some Euclidean relationships like parallel lines has not yet been established. They also counter the applicability of Piaget's theory based on cross-cultural and socioeconomic concerns (see also Dasen & de Ribaupierre 1987; Opper, 1977). Socioeconomic status, culture, and family background also impact on mathematical achievement (Ginsburg, Lin, Ness, & Seo, 2003; Pappas, Ginsburg, & Jiang, 2003; Sukon & Jawahir, 2005). Gerdes (1998) has consistently shown the rich mathematics of African groups and illustrated how they link with school mathematics, for example, in the reflections and symmetries of the women's *latima* drawings.

In Chap. 2, intention was noted as an important part of visuospatial reasoning directing attention and responsiveness. Ness and Farenga (2007) expanded on the role of intention as a search component of visuospatial reasoning. Intention can lengthen the time spent observing and attending to aspects of the environment (Baillargeon, 2004) showing that young children may have greater cognitive appreciation than Piaget contended. The link between action in say Logo programming and abstract geometric concepts is evident in school children.

> Spatially structuring an object determines its nature or shape by identifying its components, combining components into spatial composites, and establishing interrelationships between and among components and composites. (Clements, Battista, & Sarama, 1998, pp. 503–504)

Studies with small children illustrate how spatial as well as visual experiences are important in visuospatial reasoning (Greenspan, 2007; Learmonth et al., 2008). Since visuospatial reasoning has both a spatial and visual aspect, Greenspan (2007) suggests that a child who has difficulty in remembering and problem solving in a spatial environment needs more practice in problem solving in highly motivating visual–spatial situations. In some cases, the tasks need to be simpler and small, so success is achieved in navigating spaces. For example, block play provides opportunities for visuospatial reasoning through rotating and joining objects to create the playful idea. Some of the play indicates early measurement concepts such as larger and smaller in area, volume, or length. Furthermore, geometric and architectural concepts are also used in block play in making enclosed and connected spaces, straight surfaces, right angles, edges, and balancing blocks (Clements et al., 2004; Ness & Farenga, 2007).

However, Learmonth et al.'s (2008) experiments go further in suggesting that visuospatial reasoning is not merely the linking of spatial features with language. The ecology and size of a space require not only movement experience but also other spatial features to solve problems of orientation. When features in the space are closer or the space itself is smaller, young children are more able to orientate themselves than in larger spaces or with features being more distant to them. This applies to children much younger than the 6 years that Piagetian studies may have indicated. Landau, Gleitman, and Spelke (1981) noted that a child blind from a few hours after birth and blindfolded adults can be shown points in space twice which they touch and then they are able to move between them along any line, albeit not straight but as if determining gradually the position. This would suggest an embodied spatial awareness. However, Cheng, Huttenlocher, and Newcombe (2013) have reviewed the literature on navigation and reorientation in a relatively small space and suggested there are more factors involved than at first realised. There is the recognition of geometric features found in spaces with corners and surfaces meeting (with less adaptability for animals raised in circular environments). There are the landmark features and horizon features especially relevant in larger spaces. There are a range of cues and values from the environment that direct attention and encourage navigation. It seems that even unconscious visuospatial thinking is affected by the environment.

Mathematics should build on the child's sense of place from the beginning. An important role for formal education is to ensure that children explore their space and be given the tools by which to explore this space.

> Children, developing at their own individual rates learn through their active response to the experiences that come to them; through constructive play, experiment and discussion … (to) become aware of relationships and develop mental structures which are mathematical in form … about … spatial aspects of objects and activities which … (they) encounter. (The Mathematical Association 1956 Report cited in National Research Council Committee on Geography, 2006, pp. v, vi)

However, this link to home and culture may not be recognised by mathematical curricula and school systems especially for Indigenous communities.

When we look at the processes of education and the theoretical underpinnings for the educational process, there is much to be learnt from studying different cultures. There may be ideas about concepts that are foundational for our understandings but need to be made more explicit for school and early childhood education (Ness & Farenga, 2007). Furthermore, there may be different constructions of concepts that differ from those commonly pronounced in western developmental psychology that states the child constructs relative (projective) spatial concepts, those in relationship to his or her own body (in front, behind, left, right), before developing absolute (geocentric) concepts. In a small study in Bali, Indonesia, Wassman and Dasen (2007) found indications that in some cultural and linguistic contexts, this sequence could be reversed. Using two tasks they found that young children in Bali used a completely absolute (geocentric) encoding of spatial arrays; older children and adults, while also showing a preference for the absolute encoding system (coherent with the culturally sanctioned orientation system), were also able to use a relative encoding. If confirmed by further research, this would be the first demonstration of a reversal in stages of cognitive development that dominates western and hence current early educational theory worldwide, and an argument in favour of (moderate) linguistic relativity. However, Dasen and de Ribaupierre (1987) summarised a number of studies and showed that Indigenous cultures in both Africa and Australia despite a degree of acculturation favoured the spatial area and showed equivalent reasoning capacity to western neo-Piagetian testing but not necessarily in quantification suggesting child-rearing practices or ecocultural experiences strengthened their visuospatial reasoning. They also suggested that to give levels and order in development was not possible without similar ecocultural backgrounds (see also Fischer & Silvern, 1985). Thus Piagetian and neo-Piagetian perspectives and views of levels and indeed mathematics education as set out in curricula often limited our understanding of children's visuospatial reasoning.

There are other examples to be found in the literature where there is variation from the dominant western view. Harris (1989) noted that small children responded to north–south, east–west descriptions in remote desert Australian Indigenous groups including with north referencing the direction that the speaker was looking. Spencer and Darvizeh (1983) compared the route descriptions of British and Iranian preschool children. The latter group gave more vivid and fuller accounts of sites along a route, but less directional information than the former. By 3 years of age, the two groups of children were found to communicate spatial information to others in the manner of adults in their culture, suggesting that communicative competence in the spatial domain involves the acquisition of culturally patterned skills for describing space. This linguistic feature is further expanded in Chap. 4 on language.

Cultural artefacts like maps and diagrams also facilitate an individual's visuospatial reasoning. Even preschool children can acquire a sense of large scale space from maps (Clements & Sarama 2007a). This visuospatial knowledge may be restricted to certain cultural groups as indicated by studies reviewed later in this book.

Critical Perspective of Place

We often need a stimulus to think differently about mathematics whether it be linking cultural experiences with technology (Eglash, 2007), recognising non-European mathematics (Joseph, 2000), or emphasising a social justice stance supported by ethnomathematics (Civil & Andrade, 2002; D'Ambrosio, 2006; Gomes & D'Ambrosio, 2006; Tutak, Bondy, & Adams, 2011). In this chapter, I have been developing a critical approach emphasising the importance of sociocultural and, to a lesser extent, ecological aspects that impact on spatial relations for reasoning. In this section, I will expand on the critical approach and show that ecologies need to be considered as well as sociocultural aspects. Thus I develop an ecocultural perspective. This perspective is centred in the students' understanding of space from their experiences in the place (Gruenewald & Smith, 2007; Somerville, Power, & de Carteret, 2009) and their culture's systematic ways of thinking about space. In this way, space understanding is place based.

While some critical theorists note the importance of recognising the hegemony of the more dominant in society (Freire, 1992; Giroux, 1997), locally, nationally, or globally, and that this hegemony may result in disempowerment based on race or class or locality, place-based education emphasises that the ecology of place itself impacts on the learner and on education (Bush, 2005; Gruenewald, 2008). Thus ecology of place for establishing funds of knowledge with the community and family (Civil & Andrade, 2002; González, Moll, & Amanti, 2005) and the impact of those funds on the thinking of the teacher who involves the community and family in education are critical influences on pedagogy.

Ferrare and Apple (2012) suggest that the tools of visuospatial reasoning and the spatiality of ecology apply to education at different levels, that is, the size of the space. It may be the space covered by a curriculum at the policy level and how that is perceived to be enacted. It may be the issues of home-schooling, rural and urban schooling, or schooling and neighbourhoods. At the micro-level, there are the spaces within classrooms, spatial positioning and spatial arrangements for students in classrooms and the "messages" that they contain, and facilities and other aspects related to schooling, e.g. for the child with disabilities. It is also about the place of schooling. In a study in Brazil, learning was in the park for the street children while Sweden and Alaska talk of outdoor education and PNG's cultural mathematics encourages teachers to go out to the gardens and places where mathematics is carried out in activities. When working with Indigenous students in NSW, motivation was gained by looking at shapes of parts of plants and leaf arrangements, uses of the plants, and their positions in the outdoor environment. A number of studies in this region have involved mapping community areas, planting gardens with plants of significance to their Indigenous cultures, and other activities outside in the community but all with their Indigenous Elders.

Ferrare and Apple (2012) also discuss the value of having some spatial analysis tools to investigate and apply spatial theory to education. For example, social analyses and mapping tools are important to ethnomathematics and the placement of schools and the population attending the school in terms of cultural diversity. So in PNG, the urban school may use the lingua franca but a village school might be on the grounds of a particular language group and use that language. Other language groups may or may not utilise their Tok Ples language and there are varying influences of neighbouring languages, proximity of languages, and the social networks, e.g. through marriage with other language groups. As we saw in Saxe's study, access to towns or work created other linguistic changes for their own language and its maintenance. In addition, spatiality influences the connections made between a developing economy and the position (value and location—space and place) of local lands.

Furthermore,

> Bowers (2001) advocates "eco-justice" as a critical framework for educational theory and practice. Eco-justice has four main focuses: (a) understanding the relationships between ecological and cultural systems, specifically, between the domination of nature and the domination of oppressed groups; (b) addressing environmental racism, including the geographical dimension of social injustice and environmental pollution; (c) revitalizing the non-commodified traditions of different racial and ethnic groups and communities, especially those traditions that support ecological sustainability; and (d) reconceiving and adapting our lifestyles in ways that will not jeopardize the environment for future generations. Like critical pedagogy, eco-justice is centrally concerned with the links between racial and economic oppression. (Gruenewald, 2008, p. 310)

For example, Gutstein (2007) involved his class by considering the issue of a block of land that could be a play area for the youth and children of the barrio in a US city. The project involved the class in mathematics, visuospatial reasoning with walking to the place (embodying knowledge), maps, and measurement. They used their mathematics as part of their argument for the play area. Thus the sociopolitical activity gave value and meaning to mathematics (Mellin-Olsen, 1987). There is a connection between culture and place that visuospatial reasoning can enhance. Furthermore, place also links with relationships between people and the environment. Thus visuospatial reasoning is also critical for the environment and its relationship to people.

> Woodhouse and Knapp (2000) describe several distinctive characteristics to this developing field of practice: (a) it emerges from the particular attributes of place, (b) it is inherently multidisciplinary, (c) it is inherently experiential, (d) it is reflective of an educational philosophy that is broader than "learning to earn", and (e) it connects place with self and community. … Current educational discourses seek to standardize the experience of students from diverse geographical and cultural places so that they may compete in the global economy. Such a goal essentially dismisses the idea of place as a primary experiential or educational context, displaces it with traditional disciplinary content and technological skills, and abandons places to the workings of the global market. (Gruenewald, 2008, p. 314)

Engagement with experiences in space results in valuing place (Tuan, 1977). Sobel (2008) emphasises that children will develop a sense of place through different stages but the beginning is being in the place and being able to act within that place. It is our childhood experiences that give us a love of place and begin our literacy of space and place (Somerville, 2007). A sense of space as place develops through movement (crawling, bike-riding, exploring, and walking) in the place, and visual, aural, and olfactory images of places that arouse a pleasant emotion (Sobel, 2008; Tuan, 1977). The mind discerns a network of places or objects in a spatial relation and distinguishes the unique characteristics of shapes and objects. Furthermore, these experiences occur within a culture and family so that the values, relationships, and ways of thinking about the place are embedded in culture and associated language.

An ecocultural perspective is more than just engaging students; although that is important, it is also impacting on how they reason. It is this argument that the following chapters address. While classrooms and state curricula limit experience and hence reasoning, place-based education shows how visuospatial reasoning can develop when "socioecological places" are a part of "exploration and action" and developing relationships (Gruenewald, 2008, p. 318). For Indigenous communities, there is a decolonising and reinhabitation of thinking. Reinhabitation also connects and provides continuities with living in a place and utilising visuospatial reasoning in the process. It provides for globalisation and transformation of cultural perspectives as a matter of course but in emphasising visuospatial reasoning, it is possible for adequate decision-making relevant to a community and culture in the geographical space in which they live, supporting their sense of being and belonging. Decolonising thinking requires a return to cultural ways of visuospatial reasoning that maintain places and relationships and ensure that with school mathematics education, these cultural ways strengthen and extend beyond the limitations of intended and implemented curricula and high-stakes examinations. Embedded in this view is diversity of visuospatial reasoning that may come about by diverse ecocultural contexts. One way of exploring this diversity is to study cultural and natural diversity affecting visuospatial reasoning and community decision-making resulting from the reasoning.

This ecocultural approach to teaching and learning requires intellectual inquiry, practical inquiry, and emotional inquiry by the teacher and the student (Barnett, 2010; Department of Education formerly DEEWR, 2009; Pinxten et al., 1983). Ecocultural knowledge inquiry is centred in place, family, and community and is frequently undertaken in practical activity but it has a strong connection in terms of identity and meaning (emotional inquiry). Activities relevant to the community may strengthen the conceptual development of spatial concepts and indeed strengthen relationships between the mathematics of the community, students' understanding of mathematics, and other aspects of life. Indeed, this ethnomathematics is richer than school mathematics which may restrict the mathematical thinking of the user or learner. For example, the referencing of space may link to activity and associated visual imagery providing not only a symbolic and verbal representation for learning but also a visual, kinaesthetic, and episodic connection. The ecocultural mathematical identity of the student and the contextualisation of the mathematics are strengthened. The specificity to place may be closely linked with linguistic structures and the purpose for the mathematics. It is not, however, necessarily restricted or limited in its system of mathematical relationships since the principles can be applied to new spaces and shapes. Furthermore, they can be adapted to new representational systems as occurs with the nexus of school mathematics and communication with outsiders to a community. Such inquiry results from appreciating space and geometry within a global transcultural community.

Moving Forward

Rather than seeing mathematics as a body of knowledge to be learned through fixed levels of presentation as structuralists like Piaget and van Hiele suggested or curricula often suggest, ecocultural perspectives on visuospatial reasoning imply that there is not a dualism between the mind and objects but that learning is an active involvement of being and becoming enshrined in one's identity. Mathematics is not just a set of procedures or tools to be used but it is enacted through history and society by people's responsiveness to an ever changing world. Furthermore, individual and collective knowledge are "in dynamic, co-specifying and ecological unity. … Systems of mathematics can be as divergent as each of our histories" (Davis, 1999, p. 331). How then do individuals communicate this knowledge? How might ecology, society, and culture impact on visuospatial reasoning? How might ecology impact on teaching? We will consider the first question in the next chapter, the second question will permeate the rest of the book as we consider different societies, and the third question is addressed specifically in Chap. 7.