Chapter 2
Visuospatial Reasoning in Twentieth Century Psychology-Based Studies

Ensuring that knowledge and skills are meaningful requires engaging the imagination in the process of learning.

(Egan, 1992)

I would say that all discovery requires imagination.

(Donald Coxeter, 1907–2003, cited in Hagen (2003))

The Challenge

From early in the twentieth century, there was interest by psychologists and educators about visual and spatial abilities along with other abilities perceived as valuable for learning. The scientific approach to research dominated the scene. Visual perception and spatial abilities were the main areas of interest for educational psychologists. Both constructivism and information processing theories were important drivers of research on visuospatial reasoning (or at least spatial abilities and visual imagery) in the twentieth century. Many mathematics educators emphasised that concepts are not passively received but are actively constructed as the learner uses existing schema to interpret information and draw inferences from this information (for example, Lohman, Pellegrino, Alderton, & Regian, 1987; Skemp, 1989; Steffe, 1991). In this learning, visuospatial reasoning plays a part when "the stored memories and information processing strategies of the brain interact with the sensory information received from the environment to actively select and attend to the information and to actively construct meaning" (Osborne & Wittrock, 1983, p. 4). The immediate context of the student was seen as relevant and it was accepted that memory was influenced by external prior experiences in a broader context. What was the legacy of the twentieth century from studies on visual imagery and spatial abilities? The influence of psychology on mathematics education was significant in this area of visuospatial reasoning but what impact could it have in the

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classroom? For some educators, Krutetskii's (1976) idea of visual and verbal reasoning was sidetracked into multiple intelligences or was there more to be learnt about visuospatial reasoning for mathematics education and in particular space, geometry, and measurement? In this chapter, I set out to research these questions, firstly through an extensive critical literature review and then via a number of empirical studies. Much of the work on visual imagery and spatial abilities was carried out in the 1970s and 1980s, so much of the foundation work for our understanding of visuospatial reasoning comes from that literature. A generative model of learning (Osborne & Wittrock, 1983) assisted to bridge the gap between information processing theories and constructivist learning theories. Other areas of research on visuospatial reasoning have been prompted by how children with disabilities learn visuospatial knowledge. Age-related studies are critiqued especially in terms of diversity of tasks in which visuospatial reasoning occurs and can be affected by the task. Then I explore in my studies how children are using visuospatial reasoning in school. I develop this research to show how students' attention and responsiveness are critical to their learning. However, it is salient at first to note the complexity of terminology generated by theorists and researchers in developmental psychology, factor analysis, and information processing studies on visual imagery, visualisation, and spatial abilities (Eliot, 1987).

Visuospatial Reasoning and Studies on Spatial Abilities

Terminology in these studies varied. For example, the word *visualisation* may refer to internal (mental) representations or external representations (Goldin, 1998), or to a specific spatial ability which was described and assessed by different kinds of testing items by different authors. It is worthwhile explaining this at the start of this chapter because it also gives the reader a greater appreciation of what is meant by visuospatial reasoning, a term that I say encompasses all these areas. The term *visual imagery* was usually used as an alternate to other forms of information processing or mental skills such as verbal processing. Spatial abilities were seen as a more stable intellectual quality than using visual imagery (Bishop, 1983) although training studies and age or maturation studies have shown spatial abilities can improve and change over time and with experience (Cox, 1978; Eliot, 1987; Lean, 1984). Problem-solving studies suggested some people preferred to process visuospatially while others preferred processing verbally (Krutetskii, 1976; Moses, 1977; Quinn, 1984; Suwarsono, 1982). This chapter teases out some of this complexity and then synthesises it drawing out important points for geometry education.

Visualisation or visual synthesis is contrasted with verbal reasoning in some intelligence tests but visualisation in other studies refers to one of the spatial skills—the mental rotation of a representation (visual image) of an object—in contrast to orientation in which the person considers the view of the object from another perspective (Eliot, 1987; McGee, 1979; Michael, Guilford, Fruchter, & Zimmerman 1957). Tartre (1990a) argued that the idea of limiting visualisation to mental rotation

Table 2.1 Visualisation and orientation

Tartre's categories	Descriptions and similar tests	Comments
Visualisation	"Mentally moving"	• Manipulation (Eliot & McFarlane-Smith, 1983) except alternative perspectives
Mental rotation		
• Rotating 2D shapes • Rotating 3D shapes		• More than rotation especially of 3D given it was often done by analysis
Transformation		
• 2D to 2D • 2D to 3D • 3D to 3D • 3D to 2D	• Form board tasks, integration of detail, tessellations, tangrams • Surface development tasks • 3D tessellations • Unfolding tasks	• "Integration of detail" (Pellegrino & Hunt, 1991) and "spatial relations" (Johnson & Meade, 1985; Thurstone & Thurstone, 1941) except related to orientation—completing figures and fitting parts together
Orientation		
Multiple representations		See comment above
Re-seeing		
• Reorganisation of the whole	• Alternative perspectives	• Thurstone's spatial relations possibly
• Part of field	• Completing figures	• Pellegrino & Hunt's "adding detail", "deleting detail"
• Ambiguous figures	• Find part or fit part	• Lohman et al. (1987) have flexibility of closure (disembedding) as separate factor
• Hidden figures	• Figure-ground perception (Del Grande, 1990) • Also called "disembedding" • Recognition (Eliot & McFarlane-Smith, 1983)	• Eliot & McFarlane's – visual memory – copying – maze tests are not included in Tartre's examples

alone, especially of three-dimensional objects, is too limiting. She included all forms of transformation under visualisation and expanded orientation to include other forms of re-seeing shapes as shown in Table 2.1 which shows how various terms are used for similar spatial abilities based on examples of test items used by the various authors.

The term spatial relations is also used in different ways. Pellegrino and Hunt (1991) used it to refer to mental rotation tasks because, in terms of information processing, it is likely that, in fact, parts are rotated and checked in relation to other parts in sequence. Although "adding detail" and "deleting detail" were not classified by Pellegrino and Hunt (1991) with surface development and the integration of detail tasks, they do appear to be the same as Tartre's "part of field". Examples of items for assessing and investigating these spatial abilities can be found in my two tests: *Thinking About 2D Shapes* (Appendix B, see also Owens, 1992a, 1993) and *Thinking About 3D Shapes* (Owens, 2001a) discussed later in this chapter. These tests were for young children (5–10 years) and more like school experiences than most tests.

Visuospatial Reasoning from an Information Processing Perspective

While some information processing theorists' perspectives were incorporated into the discussion above on spatial abilities, they emphasised perceptual speed and the effects of speed and accuracy in spatial abilities. Poltrock and Brown (1984) suggested that individual differences were particularly due to the visual buffer (short-term memory of the image) and speed of processing. Measures of the processing for particular tasks depend on their complexity, speededness, and susceptibility to more than one solution strategy, so spatial abilities are reliant on creating structures which are abstract and relation-preserving and on which transformations can be easily and successfully performed (Lohman et al., 1987). Time is also significant for processing not only static spatial relations but also dynamic spatial relations which involve a time order and are generally studied by a series of computer images (Aust, 1989; Pellegrino & Hunt, 1991). New contextual areas requiring visuospatial reasoning include dynamic information presented, for example, in representing past and future weather patterns, and graphing data with traces.

Within the information processing theories, there are different emphases pertaining to visual imagery as a processing/storage medium. First, Paivio's dual-coding theory (Paivio, 1971, 1986) states that there is a non-verbal as well as a verbal symbolic modality for processing physical objects, scenes, environmental sounds and images, and general images. Kosslyn's surface representation theory (Kosslyn, 1981; Kosslyn & Pomerantz, 1977) suggests that during perception, units are abstracted, interpreted, and stored in long-term memory. In Pylyshyn's abstract transformational model (1979) the verbal and non-verbal information can also be transferred between modes by a set of propositions. A visual representation, sometimes accompanied by a verbal one, is generated by this proposition (Kieras, 1978). Imagery and propositions together with other memory structures interpret and are used for testing perceptions in short-term memory during learning (Gagné & White, 1978). Pictorial images then are not original photographic images but "quasi-pictorial representations that are supported by a medium that mimics a coordinate space" (Kosslyn, 1981, p. 46) explaining Bruner's (1964) notion of concrete, pictorial, and abstract representations. Support for visual images being processed, based on reaction time, in a way that is similar to manipulation of physical objects showed a linear relationship between the degree of rotation or number of transformations and the time taken to respond to the task (Cooper & Shepard, 1973; Shepard, 1971, 1975). However Shepard also noted that analysis rather than rotational methods could account for reaction time.

No matter how the storage of imagery occurs, the need to generate spatial representations is an initial stage in the processing of a spatial problem, according to the flowchart models of Egan (1979) and Carpenter and Just (1986). The emphasis is on part-whole relationships. For orientation, the model requires comparisons on dimensions one at a time while visualisation tasks require a search followed by a looping of transformation and checking. The particular task will affect the processing (Carpenter & Just, 1986; Paivio, 1971). Images can be generated by encoding a physical stimulus, retrieving a previously constructed representation, constructing a new representation from non-iconic (verbal) descriptions, or by some combination of these processes. Visuospatial reasoning is affected by the adequacy, efficiency, and accuracy of the encoding and the retaining of detail during transformations or comparisons. Some tasks do not require transformations but only assessment. Choice of frame of reference for encoding, consideration of size and proportion, and interpreting perceptual distortion are three aspects affecting processing and would be related to the ability of interpreting figural information which Bishop (1983) contrasted to the ability of visual processing.

Carpenter and Just (1986) based much of their work on detailed analyses involving retrospection and eye fixation, but a study by Poltrock and Agnoli (1986) further describes the importance of efficient imagery and what is entailed in it. They used structural equation modelling and found that a range of tests of spatial abilities required a number of visual imagery processes. The resultant model was used to relate the imagery-cognitive components as determined in laboratory tests to spatial-test performance by a linear regression analysis and then to a factor analysis of the spatial tests. Efficient image rotation and efficient image integration contributed to performance on all the spatial tests, but image generation time did not. This last factor was correlated with image memory performance. Adding detail and image scanning were two further imagery components suggested by Kosslyn (1983) and others (Brunn, Cave, & Wallach, 1983, cited in Poltrock & Agnoli, 1986; Poltrock & Brown, 1984). Visual memory and vividness of imagery did not correlate with spatial ability (Lohman et al., 1987) and Burden and Coulson (1981) also found that students used a variety of approaches to visual processing and that these processes could not be restricted to the processing methods suggested by Egan (1979) and discussed above.

Lohman et al. (1987) concluded that visualisation is the most general spatial-ability factor. The tests that load on this factor were quite diverse: tests of rotation, reflection, folding of complex figures, combining figures, multiple transformations, or no transformations. They listed another nine spatial factors: spatial orientation, flexibility of closure (embedded figures test), spatial relations, spatial scanning, perceptual speed, serial integration, closure speed, visual memory, and kinaesthetic memory. This list is not a complete list of spatial abilities and, indeed, Guilford's structure of the intellect was a model schematising a multifaceted intellect involving three dimensions—content, product, and operations—and it encompassed many cells with figural content that could be related to spatial abilities and visual processing (Magoon & Garrison, 1976). Lohman et al. (1987) stated that tasks which were complex tended to load only on the one factor called visualisation, but simple tasks, generally involving time, tended to involve more specific factors. While they summarised basic categories of processes as pattern matching, image construction, storage, retrieval, comparison, and transformation, Kosslyn (1981) listed other processes (rotate, scan, pan, zoom, and translate images, inspect and classify patterns). Among others, Carpenter and Just (1986) emphasised the use of analysis and checking in both orientation and visualisation procedures and this might explain the conflict between Tartre's classification and others. If this is the case, then visuospatial reasoning is not just a skill but it involves the understanding of concepts because analysis and checking are limited when images are not conceptualised; a point that is generally not mentioned in the literature but which is taken up in discussing types of visual imagery later in the chapter.

The question remains whether visuospatial reasoning is a spatial ability or a higher order ability encompassing spatial ability. Visuospatial reasoning can be used in non-spatial problem solving (Deregowski, 1980; Krutetskii, 1976; Owens, 2002c). The terms "imagistic processing" or "imagining" capture the creative use of mental visuospatial reasoning in solving problems (Goldin, 1987). The extent of visuospatial reasoning is reflected in the following statement:

> producing or using geometrical or graphical representations of mathematical concepts, principles or problems, whether hand drawn or computer drawn … that is, the use of mathematical visualisation is intended to be a mental process but also to produce a drawing to assist in understanding or problem-solving. (Zimmermann & Cunningham, 1991, p. 1)

Visuospatial reasoning also incorporates "the ability to represent, transform, generate, communicate, document, and reflect on visual information" (Hershkowitz, 1990, p. 75) and to relate certain concepts to physical embodiment, pictorial or concrete through which each person would develop certain conceptualisations (Bauersfeld, 1991). Visuospatial reasoning then is a mental process that may come from, create, or manipulate physical representations (see also the discussions reported by Goldin, 1998). Visuospatial reasoning encompasses spatial abilities but goes well beyond these skills.

Studies of Learners with Disabilities

Another area that assists us to know about visuospatial reasoning is the studies with people with disabilities. Witelson and Swallow (1988) suggested that both hemispheres support spatial performance with maturation points at age 5 years and at puberty. Damage to the left hemisphere of the brain (often seen as dominant in language acquisition) reduces this performance. Landau (1988) noted that basic principles of spatial cognition of students who confronted their environment mainly by hand were the same systems as those of sighted children. By contrast, Stiles-Davis, Kritchevsky, and Bellugi (1988) showed right hemisphere-damaged infants display normal ability to identify class relations so long as these were not spatial relations. Furthermore, for spatial groups and relations, they were impaired compared to others whereas the left hemisphere-damaged children did not show the same difficulties. In the cases of a child with a disability that reduces spatial thinking, then appropriate language development seems to provide alternative pathways (Mandler, 1988). Visuoperception can be adequate for tasks like recognition of unfamiliar faces, perception of form, and closure for children with Williams Syndrome. However, visuospatial thinking is limited for these children as evident by their focussing on irrelevant features like height reduction to conserve quantities, by not showing connectivity of parts of perceived objects, and by not recognising transformed shapes indicating deficits in the visuospatial skills of drawing, spatial construction, line orientation, spatial transformations, and spatial memory (Bellugi, Sabo, & Vaid, 1988). This study in particular indicated a distinction between visuoperceptual skills and visuospatial reasoning.

Lillo-Martin and Tallal (1988, p. 437) also note that "while the well-known left- and right-hemisphere distinctions are upheld, some degree of plasticity, transferability, and compensatory change are indicated [by the studies reviewed by Stiles-Davis et al., (1988)]". In the area of attention, studies of subjects who were deaf and hearing who knew or did not know American Sign Language (ASL) provide further information. Deaf subjects showed compensatory mechanisms with occipital activity in both hemispheres while the hearing group with ASL (deaf parents who signed) had increased left temporal-parietal activity compared to the hearing group without ASL showing functional reallocation (Neville, 1988). In a further study (Poizner & Tallal, cited in Lillo-Martin & Tallal, 1988) there was no compensatory performance and Lillo-Martin and Tallal (1988) suggest this was due to the critical flicker frequency, lack of verbal labels, and the positioning of the visual stimuli on the eye. These last-mentioned researchers suggest that a spatial language still uses the left hemisphere although some brain reorganisation takes place. They conclude that "function rather than form dictates cerebral organization, at least for language and spatial cognition" (Lillo-Martin & Tallal, 1988, p. 438). While the acquisition of language and visuoperceptual functions are innate in certain parts of the brain, a limitation on that area may limit performance in early childhood but will lead to changes in brain organisation and limited plasticity. It may be that children without brain dysfunction or limitation may process differently and there is no implication for adult performance from these studies. In addition, timing might also affect performance on tasks related to motion and localisation in space (Anderson, 1978; Neville, 1988; Shepard, 1988). "Interactions of spatial processing with other, related areas, such as temporal processing, is an integral part of understanding spatial cognition" (Lillo-Martin & Tallal, 1988, p. 440). However, the studies suggest that context and social experiences in early childhood will dramatically affect development in the area of visuospatial reasoning.

Processes include spatial perception, object location, line orientation, spatial synthesis, spatial memory, spatial attention, spatial mental operations like rotation, and spatial construction (Kritchevsky, 1988). Spatial attention seems to be influenced by both sides of the brain and so does construction with one part particularly requiring more thought than the other to draw, for example, an image with adequate angles and detail. Objects can be located using both visual and verbal information. Importantly training that involves areas of the brain other than the perceptual, visual memory section assists in spatial construction needed for basic tasks designed to improve spatial attention, memory, and construction.

Healy and colleagues (Healy & Fernandes, 2011; Healy & Powell, 2013) have also studied learning of blind students. In a unit on symmetry they particularly noted

> There were differences between approaches to symmetry adopted by the two students. For example, the student who had never had access to the visual field tended to treat geometrical objects as dynamic trajectories and attempted to look for invariance relationships among the sets of points which defined the trajectories; the second student attempted to characterize the objects he was feeling in terms of objects he remembered from before he lost his sight. Nevertheless there were also similarities. Notably, both students tended to move their hands or corresponding fingers from each hand in a symmetrical manner over the materials they were exploring. (Healy & Powell, 2013, p. 78)

Reisman and Kauffman (1980) provided a range of visuospatial issues for consideration in this regard from work with disabilities. Visuoperceptual disorders underlie difficulties in spatial orientation, recognising position, discriminating figure from ground, and distinguishing near–far relationships together with sequential memory, visual spatial memory, or constancy of form difficulties. These difficulties impact on arithmetic skills and understandings as well as spatial-geometry understanding. Similarly Farnham-Diggory (1967) showed that alternative ways of reading using pictographs are possible although disability may slow progress. These studies on learners with alternative abilities indicate that visuospatial reasoning occurs using different pathways.

Age and Visuospatial Reasoning

While I argue later that strategies for visuospatial reasoning are found across ages, it is important to consider earlier studies and to build on them but at the same time show how modifications to assessing provide evidence to critique stage and age-related limitations. Piaget and Inhelder (1956, 1971) claimed that children who had not yet reached the concrete operational stage could not solve problems requiring mental rotation of images because this task required conservation skills. Visuospatial reasoning was linked to maturation and considered available only to those who had developed certain levels of thinking. However, Rosser, Lane, and Mazzeo (1988) who considered age as a predicting variable contributing to level of development actually found that young children could solve rotation problems which were not difficult (such young children may not be conserving). The children reproduced the simple models of two rods, which formed a T or an L, and a circle placed at the end of a rod or in the right angle. Most children aged 4 and 6 could reproduce a model present in front of them and when it was shown and then hidden while 8-year-olds could also memorise and represent an anticipated rotation (which was indicated by hiding and rotating a model), and represent another perspective by moving a model. Owens (1992a) developed an innovative paper-and-pencil test that used cardboard cut-outs in explaining the items and stickers for some responses. On an item inspired by Rosser et al. (1988), she found that these items were relatively easy (on a Rasch analysis) for children aged 7 and 9 years. The test incorporated items that linked to spatial abilities (Eliot & McFarlane-Smith, 1983) but more closely linked to typical classroom activities. It was developed in two equivalent versions, a copy of one is available in Appendix B.

Invariance of parts of a shape was more complex than that required by the Piagetian conservation of length task (the staggered lines test involving two equal horizontal sticks with non-vertical starting points). Kidder (1978) found that only a small percentage of conservers could choose the correct length of a side of a transformed triangle, and Thomas (1978) found that non-conservers (determined by the Piagetian task), irrespective of grade (1, 3, or 6), were less likely to be correct in assessing invariance of length of the side of a triangle under rotations, translations, and reflections than conservers in that grade. The older students considered the vertices as well as the sides of the triangle. This result suggests that conservation may not have been the most important determinant of the results of this study but some other factor such as the strategy used to make the decision or some features of the task.

van Hiele (1986) suggested that concepts in geometry such as equality of angles develop through the following stages and depend very much on experience. Students do not tend to reason about properties, although they may about parts, without first apprehending (attending and noticing) and reasoning visually. According to van Hiele, the stages are the following:

1. The student reasons about basic geometric concepts … primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components. …
2. The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established…
3. The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept. (Martlew & Connolly, 1996, p. 31)

Students with less developed approaches to concepts such as equality of angles may be operating in the earlier two stages. Several later studies suggested that development through these stages was concept specific (see summary in Owens & Outhred, 2006).

From a study of 2- to 5-year-old children's constructions and drawings of geometric shapes, Fuson and Murray (1978) reported that the verbal descriptions given by children were holistic and that, if an attribute was mentioned, it was in the context of describing a whole shape, for example, "the pointy one". The study showed that children could construct each of the shapes before they could draw it or analyse it suggesting that there were at least two prerequisites for drawing shapes:

1. The ability to discriminate the parts of the shape
2. The ability to operate on a mental image of a shape so that
   (a) The parts of the shape can be related in a sequential order.
   (b) The part(s) of the shape already drawn on the paper can be coordinated with the mental image of the whole shape that is projected onto the paper. (p. 80)

Support for an interaction between the visuospatial reasoning and the external actions (verbal and visual) as critical to our understanding of visuospatial reasoning comes from two interview and observational studies by Mansfield and Scott (1990) and Wheatley and Cobb (1990). Instead of determining the kinds of transformations that students could carry out by giving them test items in which students had to recognise transformed shapes, Mansfield and Scott's (1990) study observed 23 pre-school to grade[1] 1 children selecting shapes to cover other shapes which were either marked with suitable divisions or not. (For example, a square could be covered by two right-angled isosceles triangles or two rectangles.) Although older children in this study tended to be able to solve more problems than younger students, this was mainly the result of their persistence rather than their more efficient or varied strategies. Covering shapes which did not have divisions was more difficult for children than covering those with divisions. Recognising shapes which would not lead to a solution and re-positioning pieces increased success. Rotating shapes and turning the pieces over were more advanced strategies. Children tended to use the same strategies in two interviews over time since persistence meant that a poor strategy could gain success eventually (Owens & students, 2007).

In Wheatley and Cobb's (1990) study, 24 children from first and second grade were given five pieces in the shapes of a right isosceles triangle, a parallelogram, and a square, and two smaller similar triangles which could be joined to form the other three shapes. The children were briefly shown a square with lines drawn to indicate that it could be covered by the three triangles. They were then asked to cover a blank square with the pieces. Wheatley and Cobb determined that the overt actions of the children represented images and conceptual structures. Students seemed to be using the following aspects of imagery and structures:

1. The divisions of the square could be thought of as being made up of two-dimensional space rather than just lines.
2. The size of shapes could be compared with imagined shapes.
3. Mental rotations could be used to anticipate how the space might be filled.
4. The whole is made up of parts in specific positions.

Wheatley and Cobb described the children's behaviour in terms of several levels: (a) imagining two-dimensional shapes as linear objects (matching shapes using their lengths); (b) covering the shapes globally (covering with overlaps or gaps

[1] This study and my own were undertaken in Australia where in fact grades are called Year 1, Year 2, etc. but grade is used here for consistency with other countries.

without aligning sides); (c) structuring an unfilled space as a shape (after positioning some pieces, children perceived the remaining space as a shape); (d) partially constructing images (mental images tended to involve only one aspect of the whole); and (e) constructing relational images (parts and properties were noticed as parts of the image). Such a description of students' visuospatial reasoning suggests a growing alignment between conceptual understanding and visuospatial development.

In attempting to provide a summary of children's uses of shapes, Clements, Wilson, and Sarama (2004) also suggested levels such as precomposer, piece assembler, picture maker, shape composer, and substitution composer but these appear to be a guide but not definitive levels in terms of students' behaviour or ways of thinking. Especially in terms of some consideration of the difficulties and nature of tasks is needed (Wilson, 2007). While these studies provide evidence of visuospatial reasoning, it is clear that trying to bring levels to these ways of reasoning is restrictive of students' diversity of thinking for any puzzle. However, experience, visuospatial reasoning, and decision-making are evident. One puzzle was to cover a bone shape by five regular hexagons. One student who had placed four isosceles trapezia on the shape but not as a hexagon was not immediately sure of covering it nor could he imagine where each trapezium would be placed. The visual and his train of thought may have prevented recall of other facts that he knew such as trapezia make a hexagon. His mental imagery was sophisticated already as he had begun a puzzle which required placing, imagining, and mentally counting trapezia in quite a difficult way compared to the tasks in my own studies (NSW Department of Education and Training Curriculum Support and Development, 2000; Owens, 1993).

Visuospatial Reasoning on Different Tasks

There is a further caution raised in comparing research using different tasks. Task features are significant factors in tests of spatial abilities. For example, students in all grades (up to 11) found it was very difficult to visualise the rotation of letters which had rotational symmetry (the S and N) and the horizontal reflection of the non-symmetric J. The half-turn clockwise also yielded greater differences between the grades than the two reflections or the counterclockwise rotation. Vurpillot (1976) explained that the use of a horizontal reference line in spatial perception tasks encourages subjective preference for distinguishing a "top" and a "bottom" of a shape while a vertical reference line encourages preference for homogeneity of perception favouring recognition of symmetry.

The need to consider variations in the type of transformation as well as the type of figure involved in the task was taken up by Schultz (1978). She varied the type of transformation, the mode (horizontal or diagonal), the lengths between positions before and after the transformation, the size of the configuration, and the type (meaningful, that is, the sailing-boat configuration, or not). The configurations were made of three coloured parts. She found the following: (a) lack of familiarity and unexpected sizes of shapes interfered with comprehension but not as much as type of transformation and features of the transformation itself; (b) "meaningful configurations apparently facilitated the operational comprehension of a task" (p. 205) and large shapes were preferred; (c) translations were far more "do-able" than reflections and rotations by 7-, 8-, and 9-year-old children; (d) rotations and diagonal reflections increased error rate or were found to be not "do-able"; (e) diagonal translations often resulted in re-orientation of the shape in the same direction; and (f) the distance of a displacement was a significant variable. However, the study did not give the significance of the differences in the percentages of different categories. Horizontal and vertical displacements in translation and rotation tasks were significantly easier than diagonal-displacement tasks for first graders but orientation of the figure made it even harder (see also Owens & Outhred, 1997). First graders' scores on subtests on the recognition of shapes and left–right orientation were relatively high but were low on subtests on perspective, figure-folding, and reasoning. After instruction the experimental group only improved significantly on the perspective subtest. Moyer (1978) found that explicit knowledge of the physical motion associated with a transformation did not necessarily help the child's ability to perform the transformation task.

Lehrer, Jenkins, and Osana (1998) considered children's reasoning for choosing two out of three shapes they considered alike. They suggested there were nine types of visuospatial reasoning with one kind of reasoning using properties and two kinds based on class of shape. The visuospatial reasoning was seen to vary with immediate context. For example, a skinny rectangle placed in an oblique orientation was considered similar to a skinny parallelogram with oblique small sides by a large proportion of children but when the parallelogram was enlarged, there was not the same degree of error in terms of definitions of shapes. Visuospatial reasoning was influenced by context within the page but also by children's schooling about what makes shapes the same. In other words, the school culture and the degree to which they had been enculturated into this Euclidean, definition-based system of shapes influenced their decision-making.

However, studies of children's intuitive behaviour yield other findings in terms of symmetry. Children from a very early age experience symmetry because it is an aspect of our bodies, of nature, and of many person-made constructions. Booth (1994) studied pre-school students' art and showed a natural tendency to paint symmetrically such as matching coloured lines on opposite sides of a central vertical line of symmetry and in patterns such as rows of coloured dots. Nevertheless, other ideas influence their paintings such as a desire to fill the whole page with paint. More formal, paper-and-pencil studies around 1990 showed children's difficulties with symmetry as illustrated by an analysis of grade 6 students' responses in New South Wales (NSW) on Basic Skills Tests. Two questions on symmetry involving mirror reflections were poorly answered by grade 6 students: 69 % were correct on a question involving a grid and a vertical reflection line, but only 20 % coloured in parts of a reflected face correctly. By comparison, over 80 % of students in grade 3 and grade 6 were correct on questions involving folding (Owens, 1997a). It seems that recognition of transformed shapes depends on the nurturing of natural symmetrical experiences.

The influence of visual skills, and diversity of means by which students can answer a simple angle-matching task should not be underestimated. Spatial skills such as disembedding or re-seeing were noted as helpful in using imagery and in solving spatial problems. Tasks themselves, especially the directions given to students, may encourage use of different kinds of reasoning; for example, novel tasks and tasks which relate to physical objects may encourage visuospatial reasoning (Paivio, 1971). Krutetskii (1976) pointed out that some students preferred visual methods, others analytical or verbal methods while other students preferred to use both methods. Lowrie (1992) found that students chose visual or verbal methods depending on the nature of the problem and how difficult they found it. Many studies (see, for example, Burden & Coulson, 1981; Lohman, 1979; McGee, 1979; Poltrock & Agnoli, 1986; Shepard, 1975) indicate that different people use different strategies for doing the same spatial tasks. For example, on tasks in which the subject has to decide if the object has been rotated, some subjects have rotated the visual image to the new orientation, others have considered the object from a different perspective, and others recognised features and used more *analytic* strategies. Studies by Egan (1979) and by Carpenter and Just (1986) have shown that part-whole analysis can be used in both "orientation" (other perspective) and "visualisation" (transformation) tasks. The skill of being able to disembed shapes and parts of shapes seems to be a different skill from those requiring mental manipulation of images (see Table 2.1; Eliot, 1987; Tartre, 1990a) but the tasks which seem to require this skill may still be completed by analytic procedures.

If this is indeed the case, then visualisation (used in the broad sense of all visual imagery) is a skill which can involve analysis and checking and hence concepts (Clements, 1983; Krutetskii, 1976). This point was not recognised in the earlier factor analysis literature on spatial abilities. Despite their differences both Pylyshyn (1981) and Kosslyn (1983) would agree that both verbal (analytic) and visual information can be processed, and that there is a means of mental storage which can be used either verbally or visually as needed in the working mind. Individuals vary in their preference for mode of mental representation whether by verbal, visual, or both mediums. Hence I incorporate these mental activities into visuospatial reasoning, avoiding conflict of terminology and emphasising these are using reasoning.

Personal Approaches to Visuospatial Reasoning

As Lohman et al. (1987) have suggested, visuospatial reasoning depends on a range of spatial abilities, visuospatial memory, and image integration and manipulation. In Poltrock and Agnoli's (1986) study, efficient image rotation, image integration, adding detail, and image scanning contributed to performance on spatial tests but image generation time did not. Numerous studies have assessed the impact of visual skills and choice of visual or analytical methods on problem solving. In the narrow area of spatial tasks, Barratt (1953) found that the choice to use imagery was important on tests with high loadings on a spatial-manipulation factor but less important on tests loading on a reasoning factor. Carpenter and Just (1986) found that those who solved tasks sequentially tended to have lower scores than those who rotated shapes holistically. However, Sheckels and Eliot (1983) found students who performed well on visual rotation tasks and processed visual materials analytically performed well on visual and combined visual/verbal mathematical problems.

One study of a personal characteristic, namely the preference for visual processing, that is, visuality, was carried out by Suwarsono (1982). His Mathematical Processing Instrument (MPI) consisted of a Mathematical Processing Test (30 verbal problems) and a questionnaire that asked subjects to choose between a visual and a verbal solution as similar to their own solution method. From the questionnaire a mathematical visuality score was obtained. He considered the effect of training in verbal and visual methods on performance and the use of visuality in mathematical problem solving. Suwarsono found that spatial ability and picture-completion ability were not related to mathematical visuality. This was also found by Lean and Clements (1981) with tertiary students in Papua New Guinea.

Suwarsono (1982) found that visuality did not assist or hinder mathematical problem solving. However, Lean and Clements (1981) found that students who used analytic–verbal processes tended to perform better than those preferring visual processes. As the MPI was designed for seventh-grade Australian students, it may have been too easy for the tertiary students of Lean and Clements' study (the mean test score in Lean and Clements' study was 11.1 out of 15 as opposed to 17.3 out of 30 for Suwarsono's sample). Furthermore, Tartre (1990b) found in a problem, in which the area of an irregular figure was to be estimated and calculated, that spatial-orientation ability (picture-completion test) was related to each of the following: the quality of the estimate, changing unproductive mind set, adding marks to show relationships, mentally moving or assessing size and shape of part of a figure, getting the correct answer without hints, and relating to previous knowledge structures. Barratt (1953) asked students to indicate the extent to which they used visual imagery. He claimed that those who used it extensively did well on tests with high loadings on a spatial-manipulation factor but no better than others on tests with high loadings on a reasoning factor. Thus there is no simple explanation for achievement but rather an indication of the complexity of visuospatial reasoning.

Students who have high spatial ability can still choose to use verbal methods of solving problems. In several studies, scoring on the test of verbal reasoning was the only variable explaining variance on post-training mathematical problem-solving performance except pre-training performance (other variables included pre-training mathematical visuality, spatial ability, and picture-completion ability) (Lean & Clements, 1981; Quinn, 1984; Suwarsono, 1982). The importance of verbal reasoning, at least on problems presented verbally, could be explained by better abstract thinking (as Lean and Clements have suggested) or by the nature and familiarity of the problem (as Paivio has suggested). Further support for the value of analytical thinking despite high visual processing ability comes from Sheckels and Eliot (1983) who found that, as only two visual variables—rotation and embedding—were related, the choice to use visual imagery (visuality) was unrelated to the ability to rotate visual material or to the preferred visual processing of material.

By contrast, Webb (1979) found that, besides mathematical achievement and verbal reasoning, only pictorial representation out of 13 variables accounted for a significant amount of variance. Moses (1977) also found that there were correlations for scores on the problem-solving inventory, measures of spatial ability, reasoning, and degree of visuality which were all significantly different from zero. However, she analysed students' written responses to the problem-solving tasks to determine degree of visuality but this procedure has doubtful validity, especially when it is considered that the problem-solving inventories were too difficult for most students. However Hegarty and Kozhevnikov (1999) have found that there are two types of visualisers: concrete imagery and abstract imagery affecting performance especially on items that did not require a high verbal skill. Why might this be the case? The key study described in this chapter helps provide an answer and explains the role verbal skills play together with visual imagery in problem solving.

A number of the above studies have used spatial-ability tests which could be high on reasoning factors rather than visual imagery. The type of task and level of difficulty make it problematic to conclude whether there is value in using visual approaches to solve problems. In order to overcome this uncertainty, training studies were used to assess the situation. This approach, together with exploratory qualitative studies of students involved in problem solving, has provided alternative methods of exploring visuospatial reasoning.

Training

Kyllonen, Lohman, and Snow (1984) found that short strategy training and performance feedback improved performance on a spatial-visualisation (3D rotation) task and a surface development transfer task but visualisation training was otherwise ineffectual. In general they found verbal–analytic training assisted more difficult paper-folding problems and for low visual–low verbal subjects a combination of enactive practice and feedback with visualisation strategy training helped. Higher aptitude students especially in verbal reasoning were already proficient in analytic strategies in the same way as Fennema (1984) found with the strategies of "encoding and classifying folds, rehearsing the sequence of folds, and deducing the solution using the rules provided by the analytic treatment" (p. 143). General spatial activities were as effective as short general training according to Baenninger and Newcombe's (1989) meta-analysis of correlational students. However, a three-week training programme did increase spatial visualisation for students in all grades 5–8 in Ben-Chaim, Lappan, and Houang's (1988) study. Lean (1984) comprehensively summarised studies on training in 3D visuospatial reasoning and concluded that general geometry courses are less likely to improve the skill of interpreting figural information (a term used by Bishop, 1983) than specific training courses. Furthermore, he noted that there is less conclusive evidence for being able to train visual processing. Lean (1984) warned that two major features could lead to misinterpretation of the value of training: (a) the training or testing may be indicative of skill in interpreting figural information or in some analytic skills rather than a visualisation skill (see also Deregowski, 1980), and (b) any improvement may merely be from practice rather than from a real improvement in visual skills as indicated by retention and transfer of skills to other tasks. (The latter argument was expounded by Piaget, Inhelder, & Szeminska (1960).) Cultural factors will also influence development of spatial skills (Bishop, 1983, 1988).

Nevertheless, kindergarten children showed an improvement on a perspective task after eight training sessions (Miller, 1977, cited in Lean, 1984), but in Cox's (1978) study with 20 individual training sessions, there was no transfer to a matrix task, prediction of a cross section, or the prediction of the water level in a tilted jar, and he concluded that the basic requirement for learning and achieving on the spatial tasks was not just operational thinking but spatial skills specific to the task. Retention scores (after 7 months) on the tasks which were similar to those in their training were also significantly different from the control group. Moses (1977) carried out a problem-solving training study in which grade 5 children improved their scores on spatial-ability tests as well as reasoning and problem-solving tasks as a result of the training (see also Lean & Clements, 1981; Quinn, 1984).

There have been a few articles outlining programmes developed to improve geometric and visual skills in younger children (Abe & Del Grande, 1983; Flores, 1995; Frostig & Horne, 1964; Kurina, 1992) but a carefully evaluated programme by Del Grande (1992) found that a course involving transformation of shapes did in fact improve the spatial visualisation (perception) of grade 2 students. The activities involved concrete shapes, geoboards, other common classroom aids, and pencil-and-paper activities. Similarly, Perham (1978) found that instruction in flips, slides, and turns (using activities involving tracing paper, geoboards, and free drawing as well as class and group discussion) assisted performance on tasks involving slides, flips, and reflections except those involving diagonal transformations, and some of those involving turns (see also Genkins, 1975).

Other training studies have involved older students. Although Lean (1984) concluded that general geometry studies tended not to show improvements in spatial abilities, a study by Bishop (1973) provided evidence that active participation in a geometry course did positively affect spatial abilities. A significant feature of this course was the use of manipulatives. Bishop's result lends support to the van Hiele's (1986) theory that recognition should precede analysis in geometry and that manipulatives and everyday experiences have an important part to play in this. Saunderson (1973) is another to make use of concrete activities at the post-secondary level in Papua New Guinea. His training programme involved both three-dimensional and two-dimensional activities and his tests also covered both areas. He used informal activities including three two-dimensional activities—tangrams, pentominoes, and enlarging tile shapes. Both the use of form board tests and the nature of his activities suggested that the improvement in spatial skills after training was linked to improvement in analytical skills. Rowe's (1982) training study considered the effects of different types of spatial programmes. The study involved grade 7 students, with one group undertaking training of spatial skills for transforming two-dimensional shapes, another group undertaking training on three-dimensional shapes, and a third group acting as a control. The group involved in the two-dimensional programme improved statistically significantly more than those involved in the three-dimensional programme but only on the test items involving two-dimensional shapes and easier spatial skills. Wearne (cited in Lean, 1984) found that the greatest improvement in scores for secondary students was associated with an increased number of analytic solution strategies. Caution is needed in applying studies and the van Hiele theory applicable for older students to younger students. The studies described later in this chapter address these concerns and provide a less structuralist approach to learning and using visuospatial reasoning.

Key Study on Children's Visuospatial Reasoning

In order to overcome this problem, I undertook a classroom study with children in grades 2 and 4 (Owens, 1993; Owens & Clements, 1998). The children came from three different schools in low socioeconomic areas of Sydney with most children having English as a second language. Within each class, based on their pretest scores, children were matched and randomly allocated to one of the teaching groups: geometry investigations working individually, geometry investigations in groups of three or four children, or number investigations. Children in the geometry groups participated in 10[2] one-hour investigative tasks requiring visuospatial reasoning over 5 weeks involving pattern blocks, tangrams, matchstick puzzles, and pentominoes while the control group undertook number problems. Children were also learning about shapes and angles by comparing them. The lessons are detailed below to indicate the kind of learning plans used to provide appropriate investigations for visuospatial reasoning:

1. Explore similarities and differences in the seven tangram pieces.[3]
   (a) Compare the pieces and decide what is similar about the pieces. What is different? What is the same about the square, parallelogram, and middle-sized triangle?
   (b) Notice what shapes you can make by joining two or three pieces together in different ways. Draw them.
   (c) Estimate how many small triangles are needed to make each of the other shapes, for example, the large triangle. Check it.
   (d) How many different ways can you make the large triangle with the smaller pieces? Draw them. When you wanted to make the shape, how did you move the pieces?
   (e) Extension: Make squares out of the pieces.

[2] All children participated in an introductory lesson, so the kind of interactive behaviour expected in investigations was established and children and I came to know each other. The class teacher taught the other half of the class and then we swapped.

[3] Tangram sets were made from cardboard with three sizes of right-angled isosceles triangles (two large, two small, and one medium), a parallelogram, and a square which combine to make a square. This is a well-known puzzle that can be used to make many shapes and pictures and the shapes have special relationships, e.g. the square, parallelogram, and medium triangle can all be made from two small triangles.

2. Explore the variety of pentomino shapes you can make with five squares.[4]
   (a) Take five square breadclips. Put them together so that the side of one joins exactly onto the side of another. When you make a shape, leave it. Take five more breadclips and make another shape. Keep making new shapes.
   (b) Check there are no two shapes which are the same although they are turned over or around another way.
   (c) How did you decide two shapes were the same?
   (d) How did you try to make new shapes?
   (e) What is the same about all the shapes in space?
3. Explore how squares have to be arranged to make more and more squares from the same number of matches.
   (a) Take 12 matches. Make one square. Now try to make two squares of the same size. Try to make three, then four squares. One of these number of squares can't be done. Which one?
   (b) Draw your answers.
   (c) Now take 24 matches and make one, then two, then three … up to nine squares of the same size. Which one can't be done?
   (d) Why did you decide to arrange the squares in a certain way?
   (e) Why does it help to join the squares?
   (f) When did you use a similar arrangement?
   (g) Extension: How did you know something won't work?
4. Explore ways of making each pattern block shape larger. How do you know the shape is the same but larger?[5]
   (a) Take one of each kind of pattern block. Next to it make the same shape but larger using a number of the same pattern block. Record or draw how you did it.
   (b) How do you know the shapes are the same?
   (c) Extensions: Is there another way of making the same shape but using different blocks?
   What can you say about the area of the bigger shape?
   Can you make the shape even larger? How many blocks do you think you will need?
5. Explore how to make angles using other angles of the tangram pieces.[6]
   (a) Which angles are the same, larger, and smaller? Which angles are the largest? Draw each in your book.

[4] Grade 2 started with four squares; square breadclips were used.

[5] Foam sets were used consisting of an equilateral triangle, an isosceles trapezium (equal to three triangles), a square, two sizes of rhombus, one of which is equal to two triangles, and a regular hexagon (equal to six triangles), a readily available set.

[6] Angles of shapes were marked by the thumb and forefinger to show size. The forefinger is rotated to line along the other arm of the angle.

(b) Join the angles of the pieces together to make the angles.
(c) How many of the smallest angle are needed to make each of the other angles?
(d) Can you make them another way? Try making bigger angles.
(e) Use an angle to draw angles in different ways on paper.
(f) Extensions: Draw shapes which are different but have one of the angles the same.
(g) Is there another way of making the same shape but larger?
What do you notice about the shapes you used to make the large shape?

6. Explore how to make angles using other angles of the pattern blocks.

(a) Compare the angles of the pattern blocks. Which are the same? Which are bigger than a right angle (angle on the square)?
(b) Draw each angle in order of size.
(c) How can we make each angle out of other angles?
(d) How many of the smallest angle are needed for each of them? Write it down on your drawing.
(e) Extension. Draw some shapes which have these angles but are different to look at.

7. See shapes in three different designs made with matches.

(a) Two squares were joined at a vertex on the workcard.

- Make the design.
- Add two matches to make three squares.
- Return to the first design, add four matches to make three squares.
- Return to the first design, add four matches to make four squares.

(b) A hexagon from equilateral triangles was on the workcard.

- Make the design.
- Remove three matches to get three equal shapes with four sides.
- Return to the original design, remove four matches to leave two of this four-sided shape.
- Return to the original design, remove four matches and leave two equal shapes with four sides but another kind.
- Return to the original design, remove three matches and leave three triangles.

(c) A square made from four squares was on the workcard. Make the design. Return to the original design each time.

- Remove two matches to leave three squares.
- Remove four matches to make two squares.
- Remove two and leave two squares.
- Move three matches to make three squares.

(d) Extension: Try your own ideas.

8. Explore more about shapes by making their outlines (tangram pieces and pattern blocks).
   (a) Take one of each shape. Next to each shape, make the outline of the shape with matches. When you need different lengths, use the sticks.
   (b) Draw each shape without tracing.
   (c) Which shapes have sides of the same length?
   (d) What is the same and what is different about any two shapes?
   (e) What is the same and what is different about the triangles?
   (f) Extension: Join two shapes and make the outlines of the new shapes.

9. Explore lines of symmetry and other types of symmetry for the pentomino shapes.[7]
   (a) Guess where a shape can be folded in two so that the two sides lie on top of each other. Try it. Draw over the lines that you find make two symmetrical halves.
   (b) How can you explain the two halves match?
   (c) Are there any shapes which look symmetrical but don't fold so the two sides lie on top of each other? How can you move the piece so it lies on top of itself?
   (d) Extensions: Use pattern blocks to make designs with symmetry.
   Add a square to the pentominoes to make symmetrical shapes.

10. Explore why some pentomino shapes tessellate and why others do not[8]
   (a) Try to arrange the tiles of the same shape so there are no gaps. Will the same pattern go on in all directions?
   (b) Why do they fit together? Why don't they fit together?
   (c) Extension: Join two kinds of shapes so there are no gaps.
   Join one of each pentomino shape together to make rectangles.

A test (Owens, 1992, 1993; see Appendix B) was developed specifically for the study. Items that fitted well for an underlying trait on visuospatial reasoning based on a Rasch analysis were used for analysis. This test was deliberately designed to cover the range of areas discussed previously in reviewing the literature on visual imagery and spatial abilities but relevant and interesting to young school children. It was coloured and involved coloured stickers. It was introduced wtih carboard cutouts to match practice examples. The results of the test showed that grade 4 students reached a higher level of visuospatial reasoning than grade 2 as shown in Table 2.2.

An analysis of covariance with pretest scores and factors of gender, year level at school and different learning groups indicated a significant difference in scores for the groups in the delayed posttest ($F= 5.072$, $p = 0.026$). Furthermore, the confidence intervals of the means of the differences between delayed posttest scores of two-dimensional thinking and pretest scores showed that the mean gain scores of the students involved in spatial learning experiences were significantly greater than for students

[7] Each shape was printed on paper.

[8] Each shape was made from cardboard and a number given in each packet. Packets were swapped between children.

Table 2.2 Comparison of grade 2 and grade 4 students' scores

	PRE2D			PRE3D			POST2D			POST3D			Delayed Posttest 2D			Delayed Posttest 3D		
	N	MN	SD	*N*	MN	SD	*N*	MN	SD	*N*	MN	SD	*N*	MN	SD	*N*	MN	SD
Year 2	86	36.34	11.47	86	3.21	2.4	83	43.29	13.7	83	3.36	2.61	85	46.26	13.48	85	3.94	3.16
Year 4	99	45.56	10.38	99	4.11	2.5	96	52.77	9.68	96	4.25	2.92	93	54.89	10.27	92	5.40	2.96
t values	6.32**			2.51**			5.28**			2.13*			4.77**			3.17**		

Note. Maximum score for the 2D test was 75; maximum score for the 3D test was 11
N = the number in the group; MN = the mean score for the group; SD = the standard deviation for the group
*t values are significant, $p < 0.05$, two tailed test
**t values are significant, $p < 0.01$, two tailed test

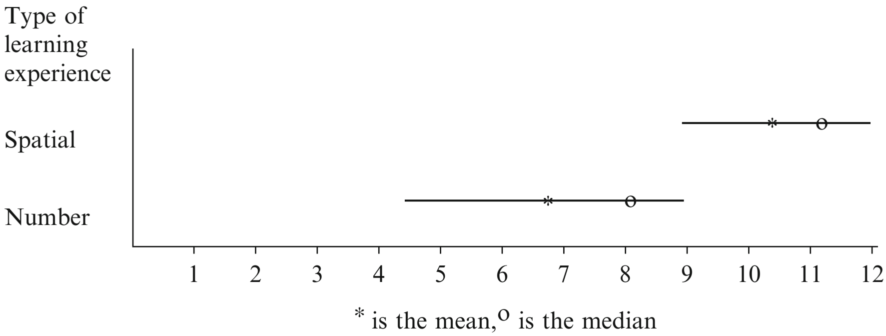

Fig. 2.1 Confidence intervals for means of the difference between scores on two-dimension delayed posttest and pretest for spatial versus number groups

participating in number learning experiences (Fig. 2.1). The learning experiences had a significant effect on children's visuospatial reasoning as assessed by this test.

Visuospatial Reasoning—Getting Inside Children's Heads

However, I also carried out a grounded theory study to explore how children were thinking during the investigations. Initially, I gave the problems to teacher education students and then to five individual children from pre-kindergarten to grade 5 in order to get spoken comments on visuospatial reasoning. Besides teaching in the three schools mentioned above, I also explored whether the findings were evident in a fourth school in another part of Sydney from a slightly higher low socioeconomic area and in a school in PNG. While some children worked individually on the visuo-spatial geometry lessons, others worked in small groups of three (or occasionally four). Groups in classrooms were videotaped but I also observed and videotaped[9] 12 groups of three children (from each year group, there was a group working individually although they could talk to each other and another working as a cooperative group sharing materials and findings). Following on from the problem-solving lesson each day, I used stimulated recall interviews in order to "get inside children's heads" and add to the observed behaviour and conversations. The use of materials meant that their reasoning was "out there on the table" (Richard Skemp in *Twice Five Plus the Wings of a Bird*) (Campbell-Jones, 1996; Skemp, 1989).

All incidents were replayed and analysed based on the children's descriptions and actions. A constant comparative method was used to make assessments of the nature of thinking. For example, if certain movements with materials were associated

[9] John Conroy, a retired mathematics educator from Macquarie University assisted with videotaping. All children were taught by myself. Lapel microphones were attached to children. To avoid class disruptions half the class working individually were taught followed by half the class working in groups on number or space problems.

with visuospatial reasoning explained in the stimulated recall of children being interviewed, then it was assumed that similar movements by another child were of this nature. The results indicated that there was frequent use of visuospatial reasoning of different kinds, some more than others.

Coding of over 1,800 incidents[10] (identified sections of actions or interactions with people or material) from all the videoclips indicated that visuospatial reasoning was involved in 540 cases and that three fifths involved holistic recognition and/or memory of visuospatial procedures but a half involved other types of visuospatial reasoning (Table 2.3). It should be noted that an incident could involve more than one kind of reasoning.

The study found imagery was important in reasoning, in creating new concepts, and more generally in directing the actions of children. The results supported the perspectives of Lakoff (1987) and Johnson (1987) who argued that imagination was a complex, embodied basis for making meaning about concepts and propositional judgements. Such a view suggests that visual imagery plays a pivotal role in conceptual development (Shepard, 1971; Tartre, 1990b).

Kaufmann (1979) has suggested that visuospatial reasoning occurs with parallel mental transformations enhancing problem solving more than sequential verbal processing. According to Kaufmann (1979) verbal processing is too bound to convention to allow for new ideas whereas visuospatial reasoning is

> more idiosyncratic, varied and flexible as to rules, and this fact makes it potentially more adaptable as a representational system for the transformational activity needed in solving tasks which possess a high degree of novelty. … [This is not the] traditional Gestalt view of problem-solving as consisting of an immediate restructuring of the perceptual field. On the basis of our findings, we hold the view that the solution to a problem is obtained by building an analogous situation from other areas of visual experience. This process we regard as mediated by transformational activity effected through the visual symbolic system. (p. 79)

This kind of interpretation of problem solving provides support for the conclusion, which is suggested by the data in the present study, that the role of visuospatial reasoning is crucial in the problem-solving process. Dreyfus (1991) is another to argue that visuospatial reasoning plays a significant role in higher levels of thinking. According to Dreyfus,

> visual reasoning is not meant only to support the discovery of new results and of ways of proving them, but should be developed into a fully acceptable and accepted manner of reasoning. (p. 40)

This study illustrated the variation within visuospatial reasoning and how visuospatial reasoning develops and assists learning. While simpler names were used for in-school programmes based on this research, descriptions of different kinds of imagery were later confirmed as a useful tool for teaching and assessing (see later in this chapter).

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[10] The videorecorded actions and interactions were described and spoken words recorded. An incident was a small self-contained segment of learning that could be described. After analysis, these tended to be a small cycle (context, context providing input, child or children's thinking, response affecting context), many of which formed a cycle within learning. (See Fig. 2.17 on responsiveness in problem solving towards end of this study.)

Table 2.3 Percentage of incidents involving different types of visual imagery

	Total number of incidents	Percent with visuospatial reasoning	Type of reasoning as % of incidents with visuospatial reasoning					
			Holistic	Dynamic	Action	Episodic	Pattern	Procedural
Tangram shapes	293	60	27	3	12	3	9	46
Pentominoes	218	47	35	8	11	1	14	31
Enlarging	274	76	27	6	19	1	26	21
Squares from matchsticks	260	30	33	9	4	3	34	17
Tangram angles	141	45	42	16	6	5	6	25
Pattern block angles	120	37	22	16	0	4	9	49
Outlines	135	81	45	2	6	5	2	40
Matchstick designs	85	52	36	0	11	0	18	36
Symmetry	198	45	34	4	1	3	9	49
Tessellations	109	64	16	0	28	0	41	16
Total	1,833	537	317	64	98	25	168	330

Holistic, Concrete, Pictorial Imagery

Students using concrete pictorial imagery (as named by Presmeg, 1986) tended to recognise the whole shape but some would make a shape but not hold an image in mind, and some would not recognise the configuration until it was completed. In the pictorial form, the image was often given a name that corresponded to a real-life object. For example, Michael,[11] in kindergarten, frequently named the pentomino shapes, "That's a cup" (for the C shape), while Sam in grade 2 named a configuration of tangram pieces as a sailing boat. This natural tendency helps to place the names of shapes into a wider ontological perspective. Indeed the pentomino activity especially helped children to realise that there were two-dimensional shapes without names or symmetry. This was a significant step in conceptualising the meaning of the word "shape". When Sam was making outlines of the trapezium and the parallelogram, he was pushing the pieces as if he were trying to get the pieces into place so the configuration matched his image. Holistic imagery generally did not enable students to recognise a lack of proportionality when they were making a trapezium or a parallelogram that was not similar to the given shape.

When students had made one large square with 24 matches and then had to make two or more squares with the matches, it was clear that often they made decisions on the basis of visual stimuli, with no counting or calculating being used. They seemed to use visuospatial intuition as a basis for predicting whether the required number of squares could be made with the matches that were left. Similarly, in the tangram activity several of the students, who had made the large triangle in two or three ways, responded very quickly to the question on its area by saying that four of the small triangles were needed to make a large triangle; it was only later that they began to reason verbally from their image. This visuospatial intuition is raised again in Chap. 5 where I discuss visuospatial reasoning in PNG.

James had a clear conception of the lengths of sides and this was strengthened by actually comparing sides. Later James and Victor made shape outlines for the tangram and pattern block pieces when Victor explained that James had not made a right-angled triangle, as James had thought, but that he had just made an equilateral triangle in another orientation. Victor himself had made the right-angled isosceles triangle with the long side horizontal and he checked it with the tangram piece which he put on top ("a lid", he called it). This discussion between James and Victor helped James to perceive the right-angled triangle in both orientations.

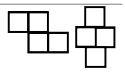
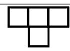
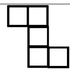
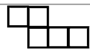
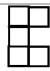
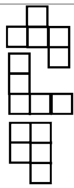
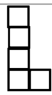
One developing visuospatial reasoning skill was the ability to recognise shapes in different orientations, including the more uncommon pentomino shapes and the right-angled triangle in unusual orientations (see Kathy, para. 4.03; James and Victor, previous paragraph). The problems themselves encouraged the use of this skill. For example, once students realised that two pentominoes were the same, they more readily avoided or recognised another pair of congruent shapes, either because the meaning of the problem was clearer or because they had developed that

[11] All names are pseudonyms.

visuospatial reasoning skill. Those students who had already imagined actions on shapes in their minds, especially the flipping of shapes, tended to manipulate materials more fruitfully.

Although James was part of a cooperative group, he began the second spatial activity somewhat competitively. He was thoroughly involved in making new shapes from four square breadclips and then in making pentominoes. He also enjoyed commenting and in other ways expressing his achievements and feelings of pleasure. (“Names” have been used for each pentomino shape and illustrated.)

Excerpt 1

1.01	James continues to count how many he has made, comparing his number with his friend’s number.	
1.02	Using four squares, he makes a “Z”, checks that it is all right, and then makes a “cross” avoiding repeating the Z.	
1.03	His friend points out “it is half her”, so he changes it to a “T”.	
1.04	He begins with five squares deliberately positioning the pieces to make a Z. Then he makes a “lineZ”.	
1.05	He notes his friend’s shape saying “yours has three columns. Mine has two; she copied me”. (Each made the lineZ in different orientations.)	
1.06	Despite the teacher suggesting that they work together, he keeps making shapes quickly and happily, commenting on how well he is going. He uses a tactic of beginning a new shape with “three in a row”. He counts his shapes and says “I’m beating her”. He knows what he is making before he completes the shape, showing joy before he finishes making the shape. He places three in a row and claps as he makes a “C”.	
1.07	He cannot recognise the “odd” shape in different orientations despite moving his body to assist orientation. He changes the shapes to make the easily recognised shapes “L3” and the “square-like shape”, comparing the incomplete shapes with his short-term memory images of those he has made (that is, he is not physically glancing at his shapes).	
1.08	He changes his tactic from starting with three in a row to beginning with four in a row. He makes the “L4”.	
1.09	He quickly grabs the last five breadclips so that he can make another shape.	
1.10	He wants to make a car but ends up with lineZ, globally deciding it is different and says “Oh, I can’t make any more”. His activity wanes when the teacher asks if they can find any shapes that are repeated in the group’s work.	
1.11	He recognises the repeated lineZ and L4.	

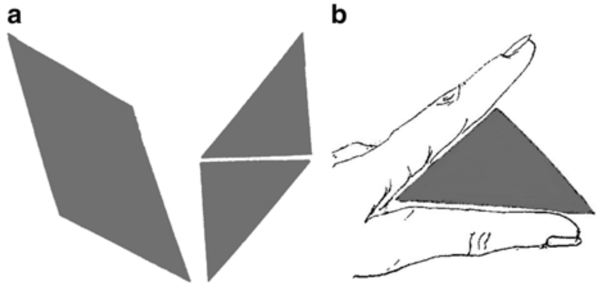


Fig. 2.2 Children attending to parts of shapes. (**a**) Michael attends to the sides of the shape. (**b**) Children attend to the angle by marking and turning their finger away from their thumb

Similarly children marked angles with thumb and forefinger. James' affective responsiveness, visuospatial reasoning, and tactics are evident.

Michael in kindergarten was joining tangram shapes to make another shape. He had the two small triangles joined to make the parallelogram but he was concerned that they were not the same size when he put them next to each other although he could see they were the same when placed on top. He was pointing to the sides and saying, "no, they aren't, see" as he points to the two unequal sides near each other (Fig. 2.2a). In the next few moves he matched sides of the different shapes, disembedding the side from the shape and realising sides could have the same length although the overall shape and area were the same and a shape could have the sides with different lengths. Similarly children marked angles with thumb and forefinger (Fig. 2.2b).

Holistic concrete imagery assists students to learn about concepts such as a shape does not have to have a name or be symmetrical, or a side of a shape is not the size of a shape. In addition, holistic concrete images can play a part in visuospatial reasoning especially as a basis for size estimates and checks. The parts of an image may be recognised as parts of an everyday object or picture but they may be the geometric features such as lines and angles. However, it is soon enhanced by dynamic visuospatial reasoning and concepts as illustrated in the example below of Victor recognising equal angles on shapes that are turned. The important skill of disembedding parts from the shape and imaging concrete objects or pictures in two or more ways depends on past experiences, the current problem, and on which aspects of the objects or pictures are taken account of and which are ignored (Thomas, 1978).

Dynamic Visuospatial Reasoning

Dynamic visuospatial reasoning[12] is significant in problem solving. In the past, teachers have often regarded dynamic imagery descriptions, for example, "a rhombus is a pushed-over square", as inadequate and unhelpful. By contrast, this study indicates how students have made connections between images and associated concepts through dynamic imagery. A common example of a verbal description arising

[12] Presmeg (1986) referred to dynamic imagery as involving movement in remembering formulae such as moving letters in expanding a product of two binomials.

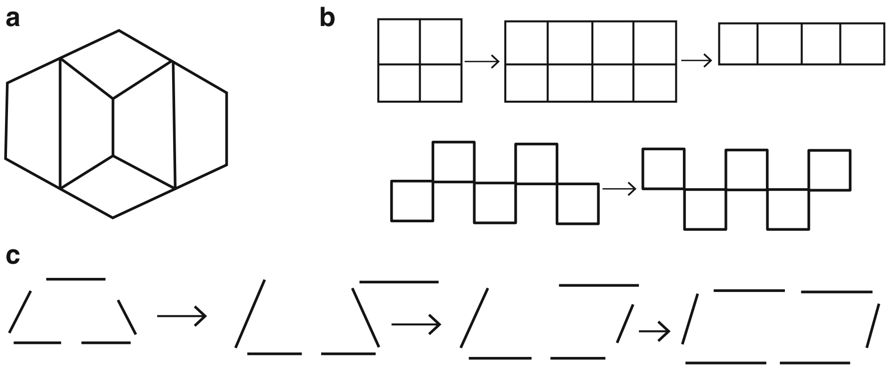


Fig. 2.3 Examples of dynamic visuospatial reasoning. (**a**) Sam's hexagon "like a square". (**b**) Michael's rectangles, and M to W. (**c**) Peter's trapezium to parallelogram

from visuospatial reasoning is the use of the phrase "a pushed-over square" for a rhombus. In this study, many students thought of shapes as being modifications of other shapes; for example, Sam described an irregular, symmetrical hexagon as "It's bigger. … It looks a bit like a square" (Fig. 2.3a).

Michael in kindergarten has already established dynamic visuospatial reasoning. Before making something, he stops and thinks and later he says the shape that was in his mind.

Excerpt 2

2.01	Michael is asked to make shapes using four squares. He makes a square and when he is given four more squares, he adds them on, saying as he starts "It's a rectangle".
2.02	He then proceeds to use another four squares and says "I know. I could make a longer triangle, I mean rectangle". As he makes another shape he smiles and says "a rectangle. I made a skinny rectangle" (Fig. 2.3b).
2.03	Next he makes a shape and says "an icecream cone" and scoffs that others would call it a diamond. When I challenge with "but that is a square", he says "we made the square" (now modified as a rectangle). At first, he decides that the diamond is not a square but then concedes, commenting that names can be confusing.
2.04	He is given four more squares. "I know". He fiddles with them under the table and, having decided to make an M, asks for another square. He makes an M with five squares on the table. "It's a bit upside down for you". He modifies the M so that I can see the M on the other side of the table. "An M for you, a W for me" (Fig. 2.3b).

Michael's extension of the square to a rectangle (para. 2.01) and his use of symmetry to change the W into an M (para. 2.04) are examples of dynamic visuospatial reasoning. He found no difficulty in regarding both a thin and a fat rectangle as examples of a rectangle, and he appears to have decided on making the thin rectangle as a result of his visuospatial reasoning of the larger one becoming thin (para. 2.02). Nonetheless, a degree of analysis of a shape is needed with this form of imagery if one is to be fully successful in making and describing shapes.

Peter, in grade 4, also made use of dynamic imagery when he transformed a trapezium to a parallelogram by sliding one parallel side along and swinging the lateral match across to meet it (see Fig. 2.3c).

Dynamic visuospatial reasoning is an important step in extending prototypical images and concepts. For example, visuospatial reasoning provides for a diversity of triangles if one imagines moving the vertices of an equilateral triangle to other positions (see James and Victor's discussion about triangles above). Some properties of a square are invariant within a rhombus, but not all of them (Sam's efforts above). Similarly, the extension of a square to form a "rectangle" maintains some properties (for example, right angles and parallel sides) but deliberately changes others (see Michael's description and action above).

Emphasis has been given to the use of dynamic visual reasoning in many computer-based geometry experiences. In dynamic geometry software, for example, dynamic changes can be made to shapes, and students can see that the changes which occur in some parts of the shapes affect some properties while other properties remain constant. In addition, the basic notion of partial inclusion in visual reasoning permits connections to be made between shapes (Owens & Reddacliff, 2002; Tartre, 1990b). Furthermore, dynamic transformations can lead to property recognition; from this perspective computer-software microworlds which enable transformations to be carried out easily can be useful. However, one of the advantages of equipment like that used in the present study is that shapes can be changed physically. There is the disadvantage, though, that with static shapes there are no "in-between" positions. This disadvantage can be overcome to a certain extent with cutting, folding, uncovering, superimposing, and using a movable perimeter to produce "in-between" states. My favourite piece of equipment is a loop of thin elastic which each pair of children can use to make different triangles or different quadrilaterals. The use of a loop of string makes a thought-provoking comparison (see activities later in the chapter).

Visuospatial Action Reasoning

Presmeg (1986) classifies imagery, which has a strong emphasis on muscular activity, as "kinaesthetic imagery" but Wickens and Prevett (1995) suggested this is spatial imagery, rather than visual imagery. In fact, Kim, Roth, and Thom (2011, p. 207) have noted the following:

(a) Gestures support children's thinking and knowing
(b) Gestures co-emerge with peers' gestures in interactive situations
(c) Gestures cope with the abstractness of concepts
(d) Children's bodies exhibit geometrical knowledge

However, in the cases described now, the visual imagery holds spatial actions and whether or not there is spatial imagery, there is visuospatial reasoning.

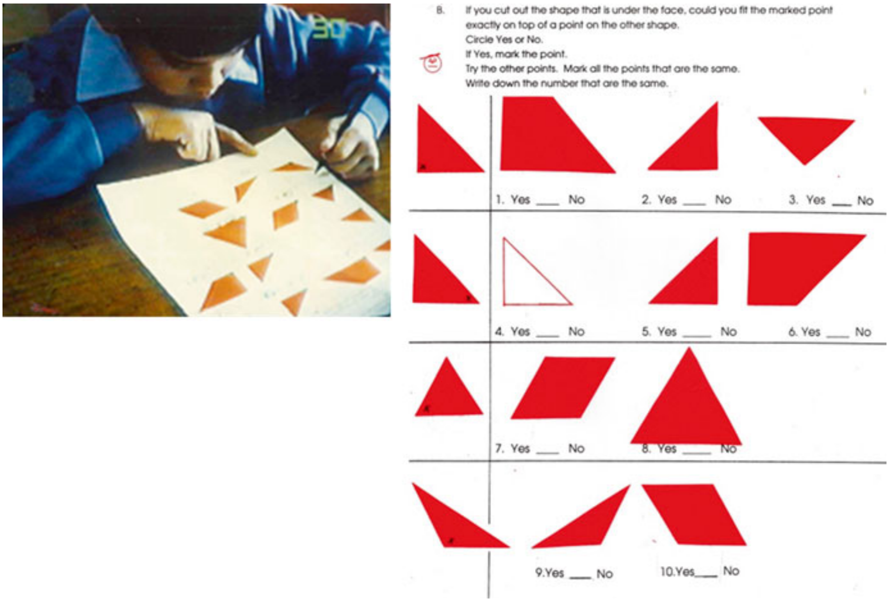


Fig. 2.4 Victor, grade 2, recognising equal angles on different shapes and on shapes in different orientations

Fig. 2.5 Peter and Victor mentally folding the pentomino shapes to form an open cube

Action imagery is also important in children recognising parts of shapes in different orientations. When children were doing the test, it was clear they were thinking about the angles in different orientations as shown in Fig. 2.4 where Victor is concentrating on the angles of the shapes. Later when asked how he first did not recognise the right angle on the triangle in a different orientation but then self-corrected, he said, “because at first, I didn’t recognise it as the same shape” and he turned the paper to indicate how he recognised it in his head.

When children were doing the test item that asked whether the pentomino shape on paper could be made into an open box, you could see them thinking and doing slight hand movements like Peter and Victor in Fig. 2.5.

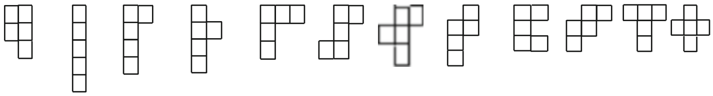
In viewing the playback of her work with pentominoes, Sally who had just finished grade 4 commented, "In my mind, I pictured my hand moving the pieces around the shape". This clearly matched my observation of her efforts to obtain new pentomino shapes—she was reasonably systematic in moving the pieces around partly made shapes searching for new pentominoes. While solving the problems, she said that she was using "ideas in her mind" but she did not give the imagery names as she might have done if she had evoked whole shape images. In fact, this was commonly noticed with children making pentomino shapes and a comparison between adults and children raises some interesting points about experience on forming pentominoes (Owens, 1990).

In this study, I asked whether certain pentomino shapes prompt the making of other shapes? If so, was this due to the relative strength of a shape in visuospatial memory, the modelling of shapes with names or symmetry, the grid analysis of the shape, or the simplicity of the shape? The task was completed by 52 adults and 12 children. To investigate the differences for adults and children, each shape was given a value based on the order in which it was made (1–12 with a value of 13 if it was not made). The median scores for each group—adults and children—were calculated and correlation coefficients between shapes were calculated (see Fig. 2.6). The square-like shape was made early by the children but they often discarded it initially because "it really wasn't a square". Once accepted as a shape, it was frequently remade by children, often in different orientations, but they would recognise it as the same and change it. This shape correlated highly with the + shape as children were deciding what was acceptable as a shape, the T shape was made early, often first. Children tended to begin with three in a column (see left diagram in Fig. 2.6b) while adults often started with four in a column and hence the line (five in a column) and L with four were made early by the adults and often quite late by the children. The other common starting point was the three with two in one column and one in the next (see top of Fig. 2.6b). From both starting points shapes such as a T were made. Children often started the same way each time and when they seemed to have exhausted ideas, they would switch to this shape with either three or four in a column. The high correlations between the T and LineZ, W and LineZ, L4 and C, and W and C suggest that the movement of one square from the previous configuration to make a new one was common. From the L3 many shapes were made; the C gave some pleasure as a recognisable shape.

It was evident that certain tactics were used as illustrated in Fig. 2.6 as well as in the key study. For example, Jodie in grade 2 began with three squares and made shapes from this base until she could make no more. Then she tried another starting combination. Besides the figural similarities between shapes, the comparison of adult and children's data would suggest that imagery, short-term memory, strategy, and propositions (e.g. what constitutes a shape) influence order of appearance of different shapes. Thus from the observational data and the above analysis, it seems that experience influences children and adults' decision-making indicating an eco-cultural perspective to visuospatial reasoning is appropriate.

Students knew that pieces were to be joined or moved in a particular way even though they did not know the entire procedure to make a required shape. Once Kathy, in grade 4, was comfortable with the problem, she made deliberate moves to

a



Shapes	sq	line	L4	Slip L4	L3	Z	Odd	Line Z	C	W	T	+
Children	1.5	8.5	7	8	8.5	9	5	10	5	10.5	4.5	4
Adults	1	3	8	8	10	5	8	10	4	12	6	7.5

b

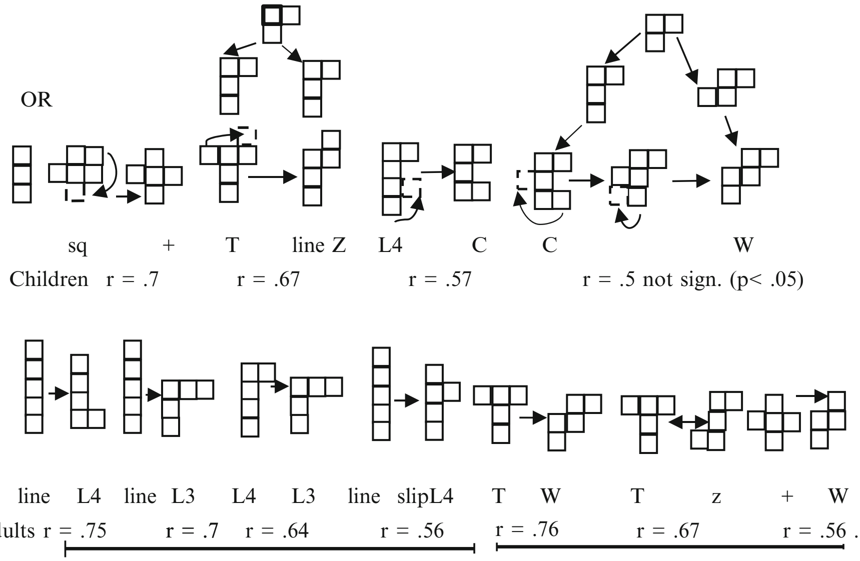


Fig. 2.6 Mean order of pentominoes being made and correlated data for adults and children. (**a**) Mean order of shapes being made by children and adults. (**b**) Correlations in order of appearance of shapes

Fig. 2.7 Kathy's large triangle and rectangle from the tangram pieces

create various shapes. In viewing the part of the video which recorded her making the large triangle with smaller pieces of the tangram set (Fig. 2.7), she commented, "I was sort of moving them around in my brain. … Like I was just seeing the triangle in my brain moving and me putting the square there so I got it". In fact when

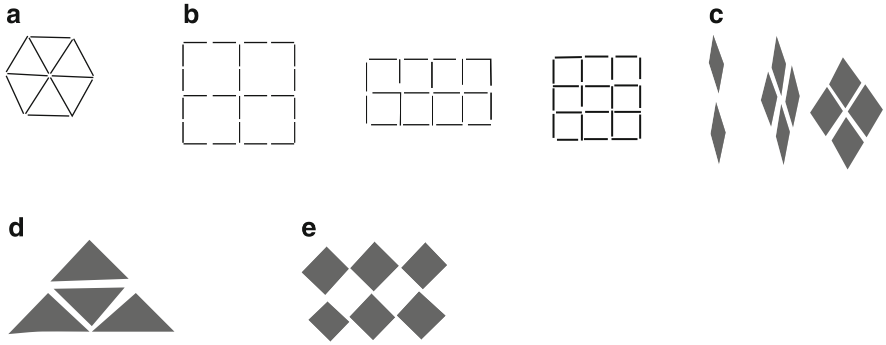


Fig. 2.8 Examples of pattern visuospatial reasoning. (**a**) Peter's hexagon. (**b**) Sally's 4, 8, and 9 squares with 24 matches. (**c**) Pattern repeated for both types of rhombus. (**d**) Right-angled triangle pattern that was also used for equilateral triangles by Sally. (**e**) Lena's pattern of squares

she was fitting in the last little triangle, I stopped her messing it all up with a "no, no" which gave her time to imagine it turned and placed correctly in place.

Action-based visuospatial reasoning occurred more frequently once students began to develop and implement tactics for solving problems. This reasoning was supported by concepts relating to the effects of operations or transformations on pieces and linked to dynamic or pattern imagery. Action imagery was a common means by which students solved the physical problems in this study. Action imagery is closely related to physical manipulations and to operational concepts. As action images were combined, procedural imagery was likely to develop. At this point, one of the paradoxes of learning occurred. While procedures are being developed, a high level of thinking takes place, but once they become automatised, and are reduced to algorithms, the level of thinking is reduced.

Pattern Visuospatial Reasoning

Pattern imagery[13] was evident when Sally carefully counted as she made a rectangular array of eight squares with the matches (Fig. 2.8b), and during the video playback stated "Like the picture was in my brain but it didn't work". In fact, she had interpreted the problem as meaning that the squares had to be in a square or a rectangle; she recalled "you couldn't have odd shapes like that—they had to be square or rectangle". Interestingly, she quickly succeeded in making both four and nine squares (Fig. 2.8b). In fact, students in PNG, who were less experienced

[13] Presmeg (1986) used the term "pattern imagery" and illustrated it with symbolic and numeral patterns.

with structured materials, frequently made the nine squares first and then tried to make the four squares. Perhaps the strength of the image of tessellated squares resulted from familiarity with squares (or diamonds) used in their pandanus weaving and string bag designs, providing support for an ecocultural perspective on visuospatial reasoning (see Chap. 5 for more details on PNG designs and coloured pictures).

Sally also used pattern visuospatial reasoning. Sally chats as she manipulates the tangram pieces. She gives a clear description of how she remembers about the triangle pattern (Fig. 2.8d). “Last year we made a Christmas tree out of triangles. That is how I know you put one up and one down. Four of these small triangles would make the larger one like this”. This comment represents the use of imagery associated with a specific experience, and relates to what Gagné and White (1978) called episodic memory. Clearly there was a pattern involved as well as actions such as slides and rotations of triangles.

Similarly, Victor in grade 2 explained how he knew that three triangles made up the trapezium in the retention test, by referring to his making of the shape earlier with the pattern blocks (a similar pattern as the right-angled triangles). When Jodie, in grade 2, was asked why she had been able to make the triangle with pattern blocks so quickly, she said that she had remembered that there was a similar task before (in the pretest, an equilateral triangle was illustrated with appropriate lines for folding into a triangular pyramid). Jodie, Jonah, and other students called the pentomino cross “a box”, relating it back to the net for an open box given in the pretest practice item. Pattern visuospatial reasoning was used by Peter when he was having difficulty making the hexagon outline with matches (Fig. 2.8a). He commented, “I know, I’ll make it like the other day”. And he proceeds to add one triangle next to the other as he had done with the pattern blocks and designs-with-matches problem.

In the tangram problem, students remembered patterns such as the configuration of the square and two triangles for making the large triangle, and they relied on this pattern when they rebuilt the large triangle for the class. Pattern visuospatial reasoning became important for students making the enlargement of the second rhombus. Generally they positioned the pieces to repeat the pattern of the previous enlargement rather than trying other possibilities. The following excerpt illustrates Sam’s use of pattern visuospatial reasoning (Fig. 2.8c) especially when he explains to his friend but he also generalises about how pattern block shape enlargements can be made using pattern knowledge by using four similar pattern blocks to make an enlarged shape. It should be noted that Sam and his collaborative group have English as a second language; his language is the same as one of the other group members but he does not use this language in class.

Excerpt 3

3.01	Sam discusses whether they can make a square or not with trapezia and suggests that you can only make a square with squares. He makes a 3×3 square and claps.
3.02	He says "I'm going to make a triangle", and he collects triangles. He nearly has the triangle and knows how to complete it. When asked how he did it, he says "because it is a triangle and you make a triangle with little triangles and the corners are sharp so you can make it like that".
3.03	He turns to the blue rhombi (with 60° and 120° angles) and says, "You can make a square with these. Oh, no, you can make a diamond".
3.04	He takes two rhombi and touches points symmetrically, but misses seeing the enlarged rhombus and joins the sides.
3.05	He listens to the teacher talking to his friend and then concentrates on his own work and quickly puts pieces together to make the rhombus. He is happy, and the teacher praises him and asks if it is the first time he has made it and he says, "Yes".
3.06	He then describes to his friend how to do it "You put this here and this here" (touching the points of the rhombi). He goes on to describe how to make the triangle, "Up and down, up and down" to help his friend make her triangle. He is pleased with himself.
3.07	He now selects four trapezia and places three together but cannot get the fourth one in correctly. After a while he leaves it as a five-sided shape saying, "It looks like a trapezium", clearly knowing that it is not one yet.
3.08	He then moves both end pieces, leaves the symmetrical "butterfly", and then makes two joined hexagons… and then places triangles on the sides to make a long hexagon. The teacher asks him what is different about his hexagon and the yellow one. "It's bigger".
3.09	The teacher runs her fingers along the sides and asks about them. He says "It looks a bit like a square" [he is referring to the angles of a square]. When the teacher asks about his friend's parallelogram, made from joining six rhombi, he says "It is bigger and kind of like a square". Returning to the hexagon, he says, "It is unstraight". When the teacher asks about the sides of the parallelogram, he says "not pointy".
3.10	He places two trapezia together and asks his friend what it is called and is told "a hexagon".
3.11	The teacher asks him if he can make the brown shape (rhombus with angles of 15° and 135°). He says he has made it, pointing to the blue one but she says, "No, a skinny one". So he collects the narrow brown rhombi and quickly follows the same pattern to make it. "I'm the best in the world", he laughs.
3.12	He remakes the squares and then he remembers he still has to make the red one (the trapezium) but he thinks no one can, if he can't think how to make it. He sees that his friend is making the red one with rhombi and triangles, so he tries the same and makes a trapezium but not of the correct proportion. He realises and tries to adjust it without success.

Sam described or demonstrated several patterns and used pattern visuospatial reasoning to make the second rhombus (Fig. 2.8c), and this was supported by his notion that larger shapes were made by joining four similar smaller shapes. He also noted when a shape had some features like but not the same as another shape. For him, the angles of the shapes were more dominant than the lengths of sides, a feature noted in other students' work.

A group of children in PNG also seemed to make considerable use of pattern visuospatial reasoning. This was their first experience with pattern blocks (although the school had some dusty attribute blocks which contained a parallelogram, square, oblong, triangle, and circle).

Excerpt 4

4.01	Lena puts squares in a pattern, touching corners but not joining sides (Fig. 2.8e). Susan puts the brown rhombi with sides together and then touches the middle points and puts the third in place before disarraying quickly. The teacher says, "Close. You nearly had it. Do it again". They do and Nora places in the fourth rhombus.
4.02	Meanwhile Lena collects the blue rhombi, joins two with the 120° angles at the top, and then joins the third and completes the enlarged rhombus with the fourth. But then she spaces them apart and puts the rhombus the other way as if she didn't recognise the rhombus. … (Fig. 2.8c).
4.03	The teacher (not knowing what they have already made) asks if they have made "this diamond" (blue rhombus) yet. There is no reply but they don't make it and instead Susan collects up the narrow rhombi and puts four of them together but as an arrow. Susan looks up, satisfied and perhaps baffled. …

The initial pattern with squares was reminiscent of bilum (string bag) patterns (see Chap. 5 for examples). It is interesting to note that Susan did not really establish a firm pattern image of the rhombus after making it the first time, and that Lena did not recognise her first rhombus. Lena tried to have three obtuse angles on the shape (perhaps because of the usual positioning of the "diamond" with the obtuse angles to the sides). Nevertheless, she did finally succeed in making the patterns.

A specific type of pattern visuospatial reasoning is the image of a grid. Sally, like other students, used this form of imagery in the pentomino problem when she systematically searched for new shapes by imagining where the pieces would be on the grid in order to make a new shape (see Fig. 2.6). Recognition of the structure of the pattern is important for imagining the covering of a rectangle with tiles and understanding the ideas behind area especially linking the equal rows to calculating areas (Outhred, 1993). Simultaneously students were showing similar diversity of approaches with some evidence of becoming more efficient in the paper-and-pencil test used in my study (Fig. 2.9) (Owens & Outhred, 1997, 1998).

An analysis of the responses to the items on the test on covering areas revealed some interesting features about drawing and visuospatial reasoning. Figure 2.9 gives the items of the test referring to covering areas and a picture of a child completing the test. Children who drew on their worksheets were considered for analysis. Table 2.4 provides the percentages of responses indicating that covering with triangular tiles (Items 3, 4, and 5) was more difficult than with rectangular (including square) tiles (Items 2 and 7). The difficulty was particularly marked when the shape to be covered was the unfamiliar trapezium shapes (Items 4 and 5). On the first testing more than half the students thought the trapezia could not be made by tessellating the given triangles, and less than a third of them could give the correct

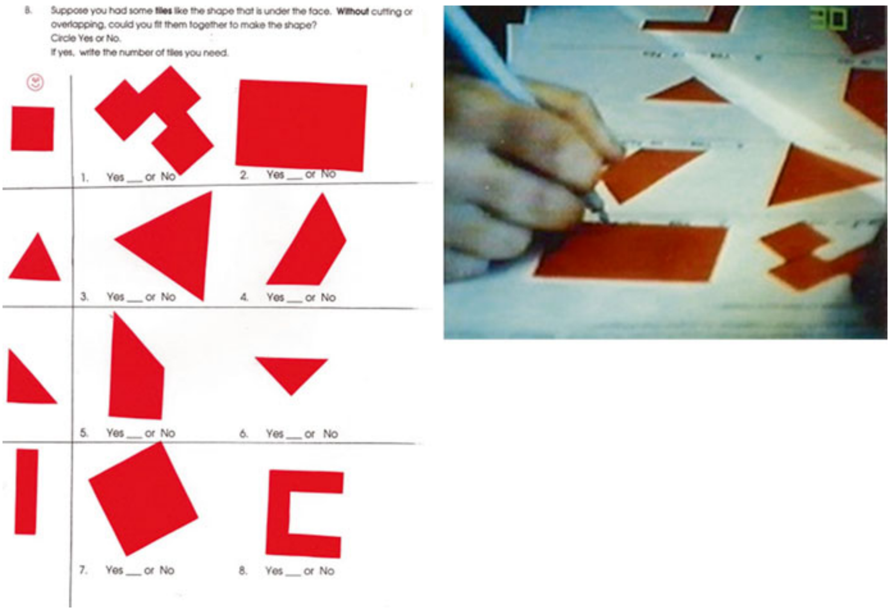


Fig. 2.9 Worksheet for tiling shapes (Form S) and grade 2 child completing the items

Table 2.4 Percentage of responses of children on pretest and on delayed posttest

Item	"No" response		"Yes" response, wrong number		"Yes" response, correct number	
	Pretest	Delayed posttest	Pretest	Delayed posttest	Pretest	Delayed posttest
2	30	21	42	39	29	40
3	45	35	27	32	28	32
4	52	36	19	16	28	48
5	57	53	15	17	27	30
7	33	26	28	25	39	48

Note. All these items (2–7) could be covered. Items 1 and 6 had no drawings so are not included in the table and 8 was a "no" answer

number of triangles. After teaching, students performed better. Although many students seemed to realise that the square (Item 7), the rectangle (Item 2), and the equilateral triangle (Item 3) could be made by tessellating the given tile, they were unable to visualise the tessellations to work out how many tiles would fit. For both the equilateral triangle (Item 3) and the square (Item 7), many students wrote 3 or 5 tiles as their answer. For the rectangle (Item 2), common answers were 8 and 9, but larger answers were also given which suggests that some students disregarded the size of the tile. Students' drawings frequently indicated difficulties with size estimates.

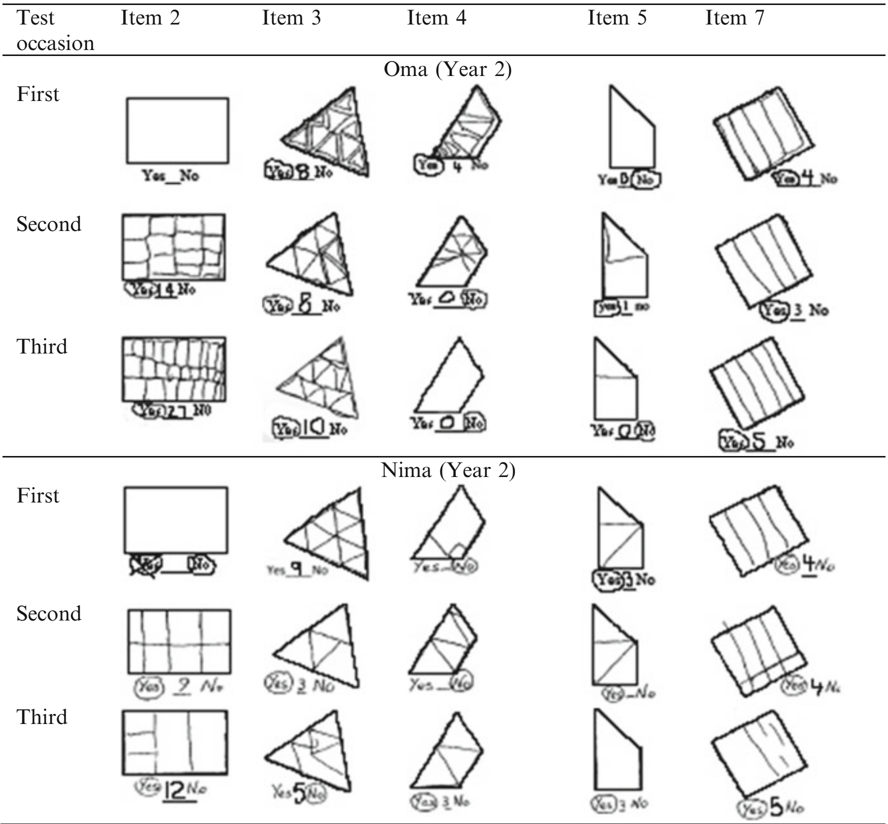


Fig. 2.10 Examples of children's responses to test items on covering areas with tiles

Figure 2.10 illustrates some of the responses from children in PNG. Students' responses were influenced by their cognisance of the following: (a) maintaining tile size, (b) covering without gaps or overlaps, (c) aligning tiles, (d) matching features of tiles such as angles, sides, and the triangular parts of the trapezia, and (e) relating the diagrams to various activities encountered during the learning experiences. Students needed to imagine or draw the relevant tessellation and to be aware of its structure. The triangle and trapezia tessellations were more difficult than the rectangular case as students had to consider the orientation of the tiling unit. Finally, students needed to be aware of the limitations of their own drawings. Students first considered covering a region with tiles by filling in from the sides and corners. Gradually they became more systematic, aligning tiles accurately and attending to features such as size and shape. Participating in activities or doing the test appeared to help students but not necessarily if they were remembering the visual image only. They might have been visualising as a picture rather than as a grid. In the interviews, several students who had been involved in the activities with concrete materials spontaneously remarked that they had made the isosceles trapezium from equilateral triangles in class and with their overall improvement from pretest to delayed post-test, it seems they had episodic imagery (Gagné & White, 1978).

The development of the grid structure for covering the rectangle with square tiles is particularly worthy of discussion. As an emerging strategy, a child might draw one to a few squares somewhere inside the rectangle, and then might align a square with an edge or corner of the rectangle. Children then tend to draw the squares one at a time in a row, row after row. There is a tendency for the rows to slant and narrow and for squares to get smaller. Children often recognise that there are too many squares in the drawing and may or may not discount the drawing as giving the possible answer. In some cases, children chose not to draw but could give the correct answer. Size and perpendicularity of lines improve as children continue to draw individual tiles until they attempt to draw a grid structure, partially or fully (Owens & Outhred, 1998). Occasionally, it is evident that the child's thinking is not fully reflected in the paper-and-pencil test score. This is evident in Oma's triangles on trapezium and her attempts for the rectangular tiles. It is also clear that she put more weight on her diagram when reasoning than what might have otherwise made sense.

By contrast, Nima discounted the small parts of square tiles on the rectangle in her second attempt. Her attempt to enlarge the triangles resulted in a less favourable structure than on her first attempt. Nevertheless, there is clear development in her attempts across all the diagrams based on various reasonings. For example, the tiles must not overlap (rectangle tiles) or my diagrams are not good enough to decide as she used a good sense of size and pattern in her mental imagery. However, she seemed to have difficulties in counting. Children were not required to draw and this part of the study used only examples where children attempted to draw but there was alignment with the simultaneous study by Outhred resulting in the findings presented in our joint papers and discussed above (Owens & Outhred, 1996, 1997).

Procedural Visuospatial Reasoning

The use of procedural visuospatial reasoning can be associated with parts being deliberately placed in relationship to each other. For example, in making the large triangle with the tangram pieces, some students deliberately turned the square so that the right angle of the square matched the right angle of the large triangle and having done that they then placed the small triangles. In comparison, less experienced students tended to place the square on the large triangle so the bases were together. Other less experienced students made many trials of the possible positions of pieces. This was partly due to the dominance of the features such as similarity and horizontal lines (see Fig. 2.11).

Students also considered overall size of area. Unlike Wheatley and Cobb's (1990) claim in their study with early covering, overall length was not dominating but rather overall size (area). Kathy, for example, stated during video playback of her covering the large triangle of the tangram with the smaller pieces, "I chose it (the square) because the others would not fit, they were too big [sic]". Kathy meant the other pieces would not cover enough and she went on to position the square so

Fig. 2.11 Dominance of similarity and horizontal lines in initial trials before visuospatial reasoning improved

that the triangles also fitted. Visual analysis often began with students joining pieces together by matching angles and equal sides. This led to further analysis and subsequently to successful completion of the task. (See Fig. 2.7 which is the large triangle that Kathy made.)

Through their actions, children often had imagery of procedures in their minds, and this procedural image they used as they deliberately positioned pieces to remake shapes. Remaking the large triangle with different pieces (three only) was easy for most students. When a shape was made from a larger number of pieces, the students often remember part of the procedure. This was common with the square made from five pieces. Colin, in grade 2, remembered how to position two pieces but then had to re-image the rest; on the other hand, Tess and her group, in grade 4, remembered how all the pieces needed to be positioned. In fact, Tess' making of the square, with some help from her friends, indicated how action imagery developed into procedural imagery.

Excerpt 5

5.01	Natalie makes the large square (Fig. 2.12a) and she, Tess, and Damien decide to cover it. Tess puts the square into the corner and proceeds, saying "Wait, wait". (She completes the stages shown in Fig. 2.12b–d.) However, both Damien and Natalie also see the places for the triangles. She then shifts the square across (Fig. 2.12b) and suggests that they need another square, but they only have the medium triangle.
5.02	Damien removes the pieces and begins with the medium triangle in the corner, places the parallelogram next to it, but then the square won't fit, so he removes the square and parallelogram, and then the medium triangle.
5.03	Natalie suggests they put the parallelogram on the side, so Tess picks it up but returns to putting the square into the corner (Fig. 2.12e), and they remake the first configuration in another orientation.
5.04	Tess places the parallelogram against the medium triangle and then slides it across to the corner with the triangle (Fig. 2.12f–h). She flips the triangle as she moves it away. She continues to reposition the parallelogram on the large square.
5.05	Natalie says "perhaps if you turn them over". Tess places the parallelogram into the corner as she had before (Fig. 2.12i), seeing the various spaces that are left. Natalie gets bored and wants to use the book.
5.06	Tess picks up the square and fills in the top section carefully (Fig. 2.12j). She then collects the small triangles and restrains Damien from placing the medium triangle correctly, taking it from him. Although she is thinking she is unsure of herself, relying on action imagery rather than a completed pattern picture.
5.07	She flips the triangle into position (Fig. 2.12k) so that the point completes the right angle with the parallelogram. "Yeah, yeah", says Natalie. They all see where the small triangles will fit. Tess sits back, content.

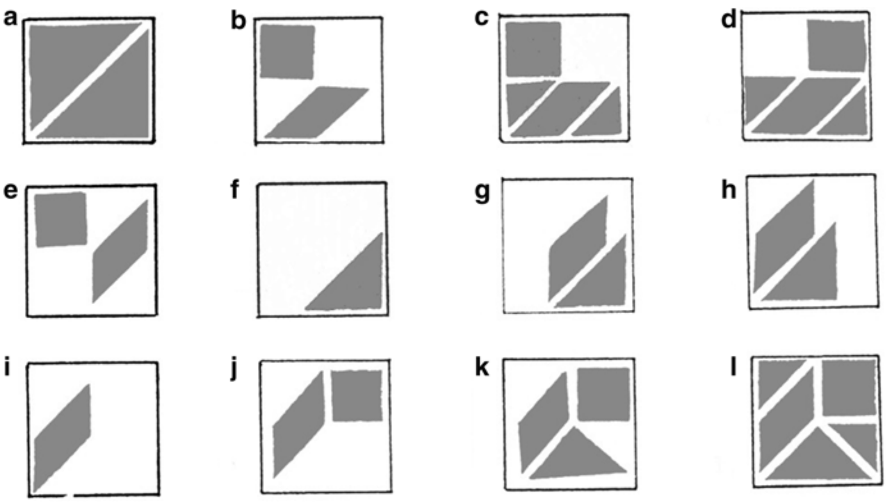


Fig. 2.12 Using action imagery to develop procedural imagery

Most of the positioning was done by Tess but Natalie, who made the initial square with the big triangles, also made some comments. Towards the end Damien began to manipulate the medium triangle, but Tess took it from him and positioned it herself. Certain procedures were more common than others. Initially, there was a tendency to match the right angles and to choose pieces to cover areas, but then came the realisation that two angles could be used to construct the right angle (para. 5.04, Fig. 2.12f). The juxtaposition of the two pieces and the observation of the ways that areas could be filled by the shapes assisted later problem-solving attempts. When the design was spoilt, the students had no trouble in reassembling the design, even though it was slightly rotated. The deliberate positioning of pieces into corners or against sides, and the checking of spaces that were left gave rise to the use of procedural imagery by all three students, whether they were watching or doing most of the manipulating. This kind of imagery is reminiscent of the procedures suggested by Carpenter and Just (1986) for recognising shapes in rotated positions.

Visuospatial Reasoning in Concept Development

Hershkowitz (1989) involved preservice and experienced primary teachers in her investigation into changes in the use of visual images which support concepts arising in the course of an activity. She investigated the use of concept images for a right-angled triangle, an isosceles triangle, an altitude of a triangle, a quadrilateral, and two shapes defined specifically for the investigation. She concluded that there are three levels of use of concept images:

1. The prototypical example is used as the frame of reference and visual judgement is applied to other instances. This seems to be the most common behaviour in the identification of right-angled triangles, where subjects failed to identify examples which contradict the vertical–horizontal prototype, and in the altitude task, where subjects failed to draw altitudes which contradict their concept image of an internal altitude.
2. The prototypical example is used as the frame of reference but subjects base their judgements on the prototype self-attributes and try to impose them on other concept examples. When this does not work, they do not accept the figure as a concept example.
3. The critical attributes are used as a frame of reference in the formation of geometrical concepts. In this case there is a chance that the individual will form concept images that are less (or not at all) visually biased (p. 74).

Hershkowitz pointed out the importance of ensuring that students see various examples of concepts. This procedure should prevent some students from imposing a visual bias on concept images.

In looking at the development of the angle concept in one of the collaborative groups in the key study described above, classroom interactions and use of manipulatives were predominant over the series of lessons (Owens, 1996b). There were a number of activities in which students were required to notice and begin to develop their concept of an angle. In particular, a case study of a grade 2 cooperative group provides a good example. Jodie, James, and Victor were asked to find small, middle-sized, and large angles on the tangram pieces. Immediately they checked the points of the pieces by overlaying them as the teacher had demonstrated in introducing the pretest where they were to mark angles on different shapes equal to the marked angle on a shape. Jodie called the 45°, the sharp angle, and associated it with “big”. When the teacher called it the small angle and illustrated with the thumb and forefinger that they were only turned a small amount to lie along the arms of the angle (see Fig. 2.2b), she quickly readjusted her language pairing “sharp” with “small” (she was a bright child with English as a second language, so was used to learning new English words). The third member of their group, Victor, was absent from the previous lesson, so he was self-regulating and still doing the previous activity of making the large triangle with the other pieces of the tangram set as well as thinking about the angles. The discussion indicated how Victor, who seemed to know what was meant by the size of points (the word generally used by these children to refer to angle), temporarily considered that he should be comparing the size of the sides of the shapes. The interaction between students helped Victor to clarify what was meant by “the point of the same size”. James, who had been able to match points in the previous activity, began this later session by choosing the wrong points, largely because he was choosing the small or middle-sized triangles. He established the meaning by listening to the teacher and to Jodie and by checking points with the drawings in their book. Thus the perceptual size of triangles dominated but the interaction with those around him and the visual representations assisted him to establish the meaning of angle size.

In their next activity, the children were able to order the angles in the pattern block set and draw them in order. They were encouraged to give them size values of the unit equal to the smallest angle (the 15° angle of the narrow rhombus of the set). In a later activity, the group made shape outlines and Victor explained that James had not made a right-angled triangle as James had thought but that he had just made an equilateral triangle in another orientation. Victor himself had made the right-angled isosceles triangle with the long side horizontal and he checked it with the tangram piece which he put on top ("a lid", he called it). The teacher asks what was meant by bigger angle. Jodie replied "more spread out" and picked up the tangram right-angled triangle and the pattern block equilateral triangle, put one on top of the other and said, "see it is bigger".

Interestingly, the children in different groups often noticed and recognised angles and they were perceptually more noticeable than length, or the starting and ending of a side, or straightness of a side. However, they had more difficulty to describe angles and without the use of the fingers would have struggled to show their understanding. Communication encouraged identification and representation of the angle-problem situations, and development of cognitive processes for solving the angle problems. Interactions with others and internal representations assist analysis which is an important aspect of concept development and problem solving (Krutetskii, 1976; Lean & Clements, 1981).

In a further study (Owens, 1998a), adults were required to identify equal angles in complex figures. Different conditions—visual, physical, aural, or spoken cueing—did not make a statistically significant difference on students' ability to solve the tasks. The reasoning provided by the adults suggested that prior experiences at school, often with negative feelings, and their view of themselves performing mathematically impacted on the their performances. Few of the adults felt comfortable about just perceiving the angles to be equal without "proof", so many drew on remembered knowledge about vertically opposite angles and angles in isosceles triangles. Other school-based knowledge such as angles related to transversals across parallel lines or exterior angles of polygons were not recalled. The adults who were given information in training audibly as well as visually had significantly less variation in scores on the test suggesting selective attention resulting from the additional auditory cueing played a part in their visuospatial reasoning.

These studies have shown that prior experiences and informal experiences can help students to establish visualisations. A unit on similarity for grade 5 prepared by Woodward, Gibbs, and Shoulders (1992) was to provide informal experiences for students to gain a good foundation for concepts about ratio and proportion. Such experiences included comparisons of angles and sides of similar figures. In fact, van Hiele (1986) maintained that the level of development in reasoning is more dependent on instruction or informal experiences (incidental learning) than on age.

Visuospatial Reasoning in Problem Solving

As the students in the key study described in this chapter continued trying to solve a problem, they seemed to use less pictorial/global imagery and more dynamic and action imagery, and finally more pattern and procedural imagery. Imagery, judging from the students' actions, involved recognition of different parts of pieces. Active involvement in the problems clearly increased students' use of imagery and their skills with images. Generally the improvement was associated with the following:

1. Students began to relate concepts to their visuospatial reasoning. Concepts associated with manipulations occurred mostly when action visuospatial reasoning was used. However, other conceptualisations related to size, angles, shapes, patterns, and symmetry were used. Concepts supported the imagery that guided tactics and manipulations, rather than vice versa. The meanings of verbalisations were not always clear suggesting that only limited conceptualisation had occurred.
2. Students, upon settling into the tactical stage of the problem-solving process, generally used reasoning other than concrete visuospatial reasoning.
3. Some problems encouraged students to manipulate visual images mentally while others encouraged the use of disembedding skills and yet others pattern or action visuospatial reasoning. Any one student may use a variety of types of imagery (see, for example, Sam and Tess in excerpts 3 and 5 above, respectively). Some children completed some activities more easily than others such as Sam enlarging pattern block shapes whereas he struggled with other activities.
4. Dynamic and action visuospatial reasoning developed into other forms such as pattern and procedural visuospatial reasoning. However, there is no hierarchy of types of visuospatial reasoning.
5. Pattern visuospatial reasoning especially provided the necessary connection between a visual image and an abstract conceptualisation, possibly because the processes of looking for, recognising, and describing patterns are basic forms of mathematical thinking.
6. Visuospatial learning experiences can assist in developing these mathematical thinking skills and structured materials, like those used in this study, can encourage recognition and use of patterns (Owens & Outhred, 1998).
7. The structured nature of the types of visuospatial reasoning described in this study not only reflected images of the physical embodiments which were used but also served as a way by which imagery was structured and used for reasoning.

Although imagery is necessarily individualistic, in the sense that an image "resides" in a particular person's mind, it makes sense to say that different people can have more or less common images and visuospatial reasoning in the same way that we say people have a shared understanding of a concept. Shared visuospatial reasoning particularly develops as a result of shared physical phenomena, problems to solve, body movements, and social interactions.

Visual representations in mathematics are not simply personal or disassociated images but they convey explicit knowledge structures that are constructed and negotiated in a context of visual representations that operate within shared rules, habits of seeing, and cultural practices (Voigt, 1994). Further, the different kinds of visuals that are generated depend on signs that are taken-as-shared but personally created (perhaps limited or enhanced by experience) although tools and hegemony (e.g. the common two arcs for a bird, the equilateral triangle on its base) may determine their nature. Visuals may be learned (e.g. the name of a type of triangle and a representative diagram), associated with a relevant experience (e.g. manipulation of string to form a triangle), or established through relational structural similarities (e.g. drawing a square as a rectangle with all sides equal).

Some visual representations require patterns that may lead to structures. Figural patterns often lead to a description or algebraic representation (Outhred, 1993; Owens & Outhred, 1998). Early generalisations are often additive but then multiplicative thinking occurs, at least in children from European backgrounds but in other cultures there may be a more multiplicative approach (see later reference to research on enlarging houses and counting groups in PNG, Chap. 5). Figurative patterns need a high Gestalt effect such as children picturing a bag of lollies for a multiplicative pattern rather than a series of dots although arrays can be physically created. In line with Dörfler's (2004) arguments, diagrams are valuable for visuospatial reasoning if they are structural and relational and the arrangement expresses the relationship. They need to possess internal meaning or rules for transforming the diagram. They have an external referential meaning, inside or outside mathematics. They need to be generic or visually general and transformed in a perceivable way. A visual template such as the circling of parts of each stage of a figurative pattern that is growing according to the pattern encourages pattern recognition and apprehension of the pattern (Rivera, 2011). The role of directing attention and then the self-developed selective attention are part of the visual reasoning process associated with diagrams.

Visuospatial Reasoning in Learning

Pirie and Kieren (1991) suggested the beginning of problem solving for new learning is "primitive knowing" and this links to intuition which is discussed later. Learners then make and hold an image to which they "fold back" (revisit) in order to go forward again with their learning through noticing properties of the imagery, and then "formalising", "observing", "structuring", and "inventising". Presmeg (2006) suggested that imagery was also a way of reifying conceptual understanding and could be considered part of formalising on which observations can be made, structured, and developed or used creatively. Perhaps Hegarty and Kozhevnikov's (1999) concrete and abstract visualisers are better understood as those who select one type of visualising more than the other. Visualising about everyday objects, concepts, and processes is different from mental objects, concepts, and processes

(Rivera, 2011). The latter can be associated with the higher levels of Pirie and Kieren's model when mentally formal definitions and structural complexities are recognised. The dynamic and pattern imagery associated with concepts, metaphors and metonymies reifies the concept ascertained from various, often concrete and practical, sources (Presmeg, 1997). This imagery develops through problem solving in which actions of a learner (e.g. to interpret or to construct by way of predicting, classifying, translating, or scaling), situations (e.g. abstract or contextualised), variables (e.g. the data type and whether concrete or abstract), and focus (i.e. the location of attention) are identifiable (cf. Leinhardt, Zaslavsky, & Stein, 1990; Owens, 1993; Rivera, 2011). Thus visualisation can vary in each circumstance.

Giaquinto (2011) analysed a number of diagrams to tease out visuospatial reasoning. He particularly noted that first there was perceptual recognition of a concept such as a square that depended particularly on symmetry in the horizontal and vertical plane but also in recognising the equality of angles. He noted that people have a tendency to attend to the vertical with the influence of gravity or external reference frames such as the page, table, or body position more than other positions and orientations (see earlier work, e.g. Vurpillot, 1976 and inexperienced responses in Fig. 2.11). If this experience was repeated it would become an acceptance of a square in a geometric sense. In my own study with adults (Owens, 1998a), perceiving equal angles was generally achieved but the context such as the complexity of the diagram or orientation of the angles or an exterior angle of a figure made it harder for equal angles to be perceived. In these cases, adults would recall school mathematical information to assist in identifying equal angles such as vertically opposite angles, angles of an isosceles triangle, or those associated with transversals of parallel lines, for some adults. Sides of triangles were more readily perceived as equal.

The adults used visualising and imagining to assist in decisions about angles. Similarly Giaquinto (2011) found visualising and imagining assisted to reinforce the dispositions or beliefs about the square. He also noted that dispositions could be given different degrees of support. For example, directly perceiving or remembering an experience that could be easily judged by memory, e.g. countable objects, may be easier than imagining a length compared to a remembered length. However, some explanation to support the comparison would be supportive of the disposition or belief. This might rely on a past experience or an intention to carry out a visual imagination. The intention focuses the attention and noticing of certain aspects such as the visual comparison of line lengths or the more holistic shape being translated or rotated or reflected. Squares can be both recognised when partially obscured drawing on visual memory or representations in the mind and imagined (see also Rivera, 2011). However, while physical proof and imagined proof require similar brain functioning, the imagined proof is often more convincing rather than perception of a physical representation because it is not likely to hold the imperfections of a physical representation. There can be difficulties such as imagining an object in an unusual orientation and not actually perceiving the imagined image correctly. It might be that another figure is in the mind distracting the visual imagination. There is also the possibility that the imagination is too complex or has too many steps or parts to be carried out easily and thus repeated in the mind consistently (see the story of the two girls re-enacting the string figure to recall the next step in Vandendriessche, 2007). It is also possible that a person imagines incorrectly because of a lack of conceptual knowledge.

Visual imagination can provide sufficient discovery and in some circumstances justification for a belief or proof in geometry. This visualising provides stronger reasoning than seeing because aspects of the diagram or object might not be generalised sufficiently and may require text to establish the justification. Giaquinto (2011) concluded that there is not a dichotomy between geometric and algebraic thinking but rather spatial thinking is used in conjunction with symbolic arguments. He used the four proofs of the sum of numbers as a culminating example to illustrate. While one algebraic induction proof was symbolic, the Gaussian proof of ordering the numbers in reverse order below and seeing that horizontally there are so many numbers and vertically the sum is $n+1$ requires spatial thinking as well as symbolic understanding. In the other proofs, the forming of a mat of dots by joining two triangular stairs of the numbers or by looking at the area of squares is basically spatial with the square one being more geometrical in nature. Nevertheless, spatial thinking is key in a proof requiring both vertical and horizontal reading approaches or when proofs involving arrays require both their horizontal and vertical size to be noted. This is also the case for rectangular area arrays (Owens & Outhred, 1996).

Although visuospatial reasoning is often usefully employed in problem-solving situations, such reasoning is not always recognised or regarded as legitimate. Often the person who evokes an image does not necessarily appreciate its richness, and the use of external representation to communicate its meaning to another person may not be successful (Dreyfus, 1991). Therefore visuospatial reasoning may have been undervalued in a number of twentieth century geometry studies.

Visuospatial Reasoning, Metaphor, and Metonomy

Johnson (1987) and Lakoff (1987) refer to the possibility of visuospatial reasoning in communication when they discuss the use of metaphors in thinking. Through metaphor, connections among existing image schemata are made and extended. Metaphorically (through new contexts) and metonymically (through partial representations) imagery develops mathematical thinking. While Vurpillot (1976) noted that young children in her study tended to categorise items on partial equivalence and indicated that this was a limitation in their conceptualisation, the excerpts and examples in this chapter suggest that metonymically part-whole connections of image schemata assist children to develop more abstract thinking. The act of problem solving, in itself, and interactions with others, tended to facilitate the formulation of alternative perceptions of concrete materials. Learning from problem solving is more than just associating conceptual knowledge to visuals. This view is suggested by Clements, Battista, and Sarama's (1998) careful analysis of young students' verbal and visual responses. The details of their report reflect those in my own study. Thus it is evident that imagery is likely to promote flexibility in thinking and creativity in problem solving. Nevertheless, inferences need to go beyond the imagery of the physical (Giaquinto, 2011).

In the key study described above, there were many examples in which ideas were creatively reconnected. Jonah, a grade 2 student, made two pentomino crosses in different orientations and said, “This is a box and this is a robot”. Tess (excerpt 5) remade a right angle from two pieces despite the pieces having non-matching sides and Sally (discussed under pattern visuospatial reasoning) linked her image of her Christmas tree to the triangle enlargement. In each case, imagery did not appear to be primarily structured in terms of propositions; rather the proposition (such as Sally’s description of the pattern) supported the image.

Metaphor and metonomy are often the genesis of connections. The connections enable features, for example, a pattern, associated with one configuration to be applied in a related situation. The so-called “concept images” (Fuson & Murray, 1978) can act metonymically for a concept, emphasising certain characteristics of the concept. Images need to be embedded in various visual and conceptual schemata if they are to provide a dynamic influence on a person’s approaches to a problem-solving task. Without this, the concept image can limit conceptualising and creative thinking. Furthermore, the dynamic moving of images of shapes into related shapes can assist the development of conceptual relationships.

The continuous manipulation of materials meant that students were able to see where shapes could be added or taken away and this experience encouraged their visualising of results before trying the manipulations. The tangram tasks, in particular, involved children in a great deal of turning around and over pieces, and of matching angles in order to fit shapes together. The making of shapes, the comparing of angles, and the finding of shapes in designs improve students’ visuospatial reasoning in that students were encouraged to disembed shapes and parts from more complex shapes and to imagine where other shapes could be (cf. Tartre’s classification of re-seeing, Table 2.1). Students were using both their short-term and long-term visual memories in order to achieve greater problem-solving efficiency.

Based on Goldin’s (1987) model of problem solving there are five interconnected language systems (the word “language” suggests “re-presentation” or processing of information). The categories are related here specifically to the processing of visuospatial problems which are likely to be met in early childhood:

1. Verbal/syntactic processing has input which can be verbal (as it is in word problems) but it can also be non-verbal in visuospatial problems. For example, students learn whether a diagram of a parallelogram is representative of all parallelograms or whether it is intended as a precise drawing such as a scale drawing of an area of land. The output can be imagistic processing or formal mathematical notation.
2. Formal notational processing usually refers to arithmetic or algebraic statements or to statements in geometric proofs such as $AE \parallel BD$. In the spatial area, another example would be the categorising of shapes and the schematising of these categories in a tree diagram or Venn diagram or drawing a triangle to represent all

triangles. Goldin (1987) points out the dangers of direct translation into this system without involving the imagistic configuration.

3. Imagistic processing involves the "feel" for the problem. In addition to visuospatial reasoning, this representation can include pattern recognition, and the matching of non-verbal sensory inputs to previously encoded information. The task content and context are incorporated into the processing.
4. Planning and executive control language include heuristic processes such as plans, strategies, tactics, and self-assessments. This category involves a recursive capability so that the processes can act as control not only on the other domains but also on itself. It incorporates the notion of metacognition described by Flavell (1987), Lester (1983), Mildren (1990), and others.
5. Each of these four processes is affected by and influences the affective system of representation. Affects include feelings, attitudes, beliefs, and values.

Context and Visuospatial Reasoning About 3D Shapes

Following on the analysis of how important noticing and imagining were in the development of children's angle concept over several sessions as described above (Owens, 1996b) and the impact of audible cueing in the adults' angle study (Owens, 2004a) described above, I began to explore how children make images and notice parts of three-dimensional shapes and how they might consider properties (Owens, 2004a). Here I report on the section of the study involving testing children in grade 3 in three different ecocultural areas: a girls' private school in an Australian city, a school in a lower socioeconomic, multicultural area in an Australian city, and a multicultural city school in PNG with children from a diversity of family backgrounds and with a greater range of ages. All classes had teachers who taught mathematics well. The study also involved individual interviews of six children immediately after they had completed each page of the test on 3D thinking (they came from the lower SES school in Australia). Prior to this study, test items were developed for different categories of visuospatial reasoning and those that had the best validity from a Rasch analysis were selected (Owens, 2001a). The test *Thinking About 3D Shapes* covered the following: recognising 3D shapes within shapes, joining two 3D shapes together to make a third shape (a rectangular prism) or an illustrated shape, tessellating a block to make a given shape, viewing objects from different positions, imagining folding, marking, and unfolding paper, and imagining folding a 2D shape to make a variety of 3D shapes or objects. The test was introduced by showing 3D shapes, ensuring the children knew the names, showing the children how to draw them on the board in isometric form, and illustrating the folding of paper, marking, and unfolding, and folding a rectangle and a net to form 3D shapes.

Some results are presented here. In the following tables, the lowest percentage and the highest percentage from the three classes are given. In this study, I made no attempt to link ecocultural background with results other than to note difference. The results showed that for recognising shapes in other shapes some degree of

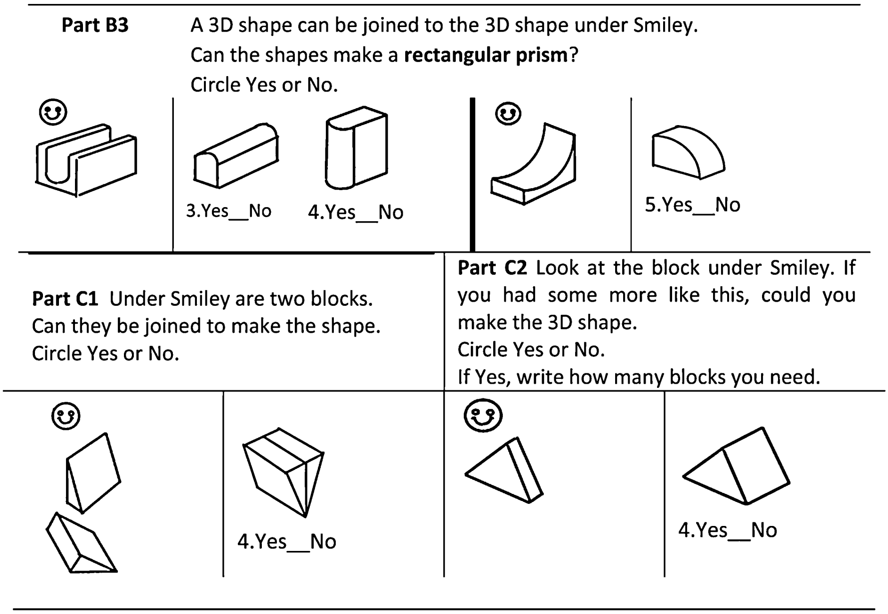


Fig. 2.13 Selection of test items: joining 3D shapes together to make other shapes

sophistication of reading diagrams was necessary because many children ignored the absence of a line in one diagram. For spatial relations or joining blocks to form a rectangular prism or illustrated block, children could explain how they were thinking (see Fig. 2.13 showing some of the items from the test, reduced in size). Ahmed at first could not see how these curved shapes would make a rectangular prism but then he referred to putting in the piece to make it smooth.

Ahmed: (B3, 5) No because it is too long and can't make it, because it would be pointy at the top, so if turn over would be pointy at top?
Interviewer: Yes, what makes you turn it over,
Ahmed: So it is like this pointy.

It was interesting that the students considered the flat and in some cases rectangular surface that would be made by joining blocks together. Students did the items by analysis and mental rotation.

Ian: If put that triangle [sic] same as that but facing other way, make a flat surface just like that (points to triangular face). (C1, 4)

Ian is aware that two blocks will join to give a flat surface together. Other students also expressed similar approaches. Visuospatial reasoning is important for tessellating blocks to fill a 3D shape (test, C2) in order to understand and calculate volume. Students generally found it easy to select whether blocks of the type illustrated will tessellate but are unable to keep the size stable in considering the number required, generally imagining many more blocks. Like the 2D tiling activities (see above, Owens & Outhred, 1997, 1998), students find it difficult to see how blocks

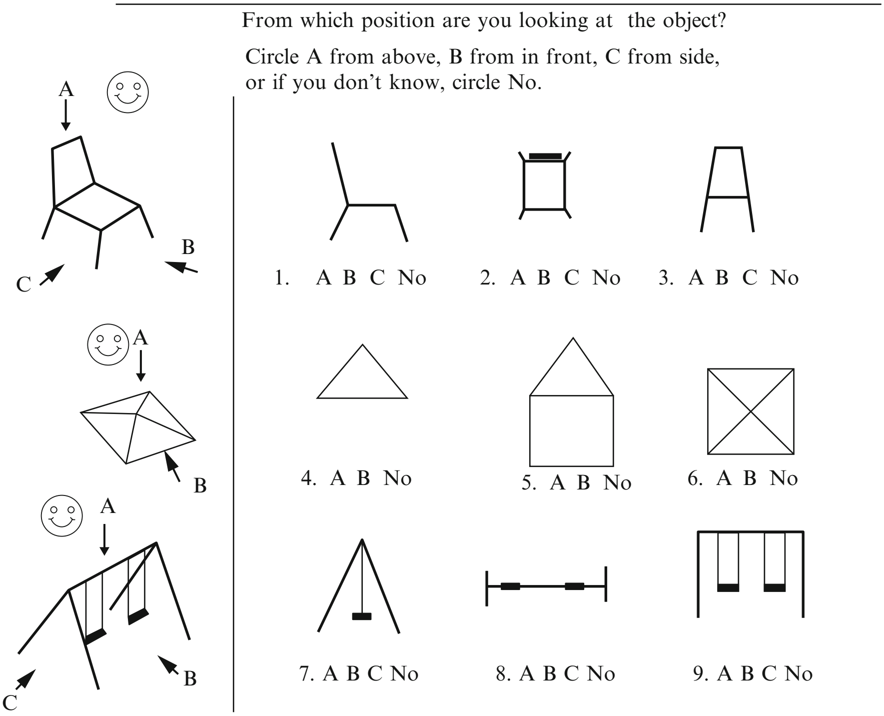


Fig. 2.14 Items for recognising shapes from other perspectives

join to give a distinctly different shape. For Item C2, 4, student difficulty was in seeing how the square face can be formed from the thin triangular prisms. Some students also find it difficult to consider size; and two interviewed students counted many rectangular prisms as making up the cube in another item whereas two rectangular blocks, one sitting vertically on and perpendicular to the other, had quite low percentages (36–53 %).

Ahmed: (Points to C2, 4) If joined together, it will make that
Interviewer: What was going on in your mind?
Ahmed: A square
Interviewer: You seemed to be counting? Can you explain what you were doing,
Ahmed: It's like a book?
Interviewer: Can you explain,
Ahmed: (Counts) one two three four.

On the other hand, Joe says "but won't fit into square" still seeing the triangle fitted onto the square face of the larger triangular prism. He counts but is unable to draw to explain what he was thinking.

For Part C1 there was little difference between the schools. For Part C2, two schools fluctuated from item to item between the middle and lower ranking suggesting ecocultural context was influencing items differently.

Part D1 "Can you see it another way?" (Fig. 2.14) involved students in recognising shapes from different perspectives.

Table 2.5 Percentages of students who were correct on completing prisms and recognising shapes from different perspectives

B3	3	4	5	C1	4	C2	4 Yes	4 number = 4	
Lowest percentage	22	38	28		85		52	24	
Highest percentage	87	73	73		95		73	53	
D1	1	2	3	4	5	6	7	8	9
Lowest percentage	71	47	14	42	54	54	62	43	48
Highest percentage	87	63	47	67	90	67	93	67	80

Most items were around the middle of the difficulty range but some were quite difficult (confirmed by a Rasch analysis). Percentages in different schools are given in Table 2.5. The chair viewed from the front (D1, 3) was the most difficult one to recognise especially for children from the school with less experience with reading books and pictures but they did not have a difficulty recognising that D1, 5 was not a representative of the square pyramid (they were not the lowest scoring school on this item). Recognising the small parts in Item D1, 2 as legs, and the seats of the swing in D1, 7 and D1, 8 was a key difference between those getting these items correct and those that could not, reflecting the issue of the importance of lines in the earlier questions and Bishop's (1983) interpretation of figural information.

The last two parts of the test required visuospatial reasoning in mentally folding, punching a hole, and unfolding and then in making various 3D shapes (not necessarily closed) from a net or a rectangle. Lack of experience seemed to affect the results for the hole punching and opening questions since percentages for the two items for the different classes ranged from 29 to 75 %. Similarly for the items in Fig. 2.15, typical mistakes were not seeing the triangular prism as hollow prisms despite being told suggesting prior experience affected thinking in line with an eco-cultural perspective.

Although most students drew the lines for the "table" (E2, 2) correctly, in other cases students drew lines horizontally. It seemed that the move from the isometric view to a flat paper was particularly difficult. Interestingly, only one interviewed student could imagine the rectangular paper being rolled to form a cylinder (E2, 3) reflecting the overall lower percentages for this question (25–67 %). Many students put lines on the rectangle, usually curved at the ends to illustrate rolling. The successful student, who knew there were no fold lines, was asked if he had done this in class but he could not remember and the current teacher confirmed this but they had used solid cylinders in building with blocks.

The items for folding open cubes were generally difficult. The order of difficulty was similar to that found on a previous test *Thinking About 2D Shapes* that had incorporated these items (Owens, 1992a). If students began to use a square that was not going to be the base, then they often struggled to imagine an alternative starting point for folding or to turn their shape to decide the base. The net requiring folding up and two sides to be turned (E3, 3) was the hardest (percentages for correctly

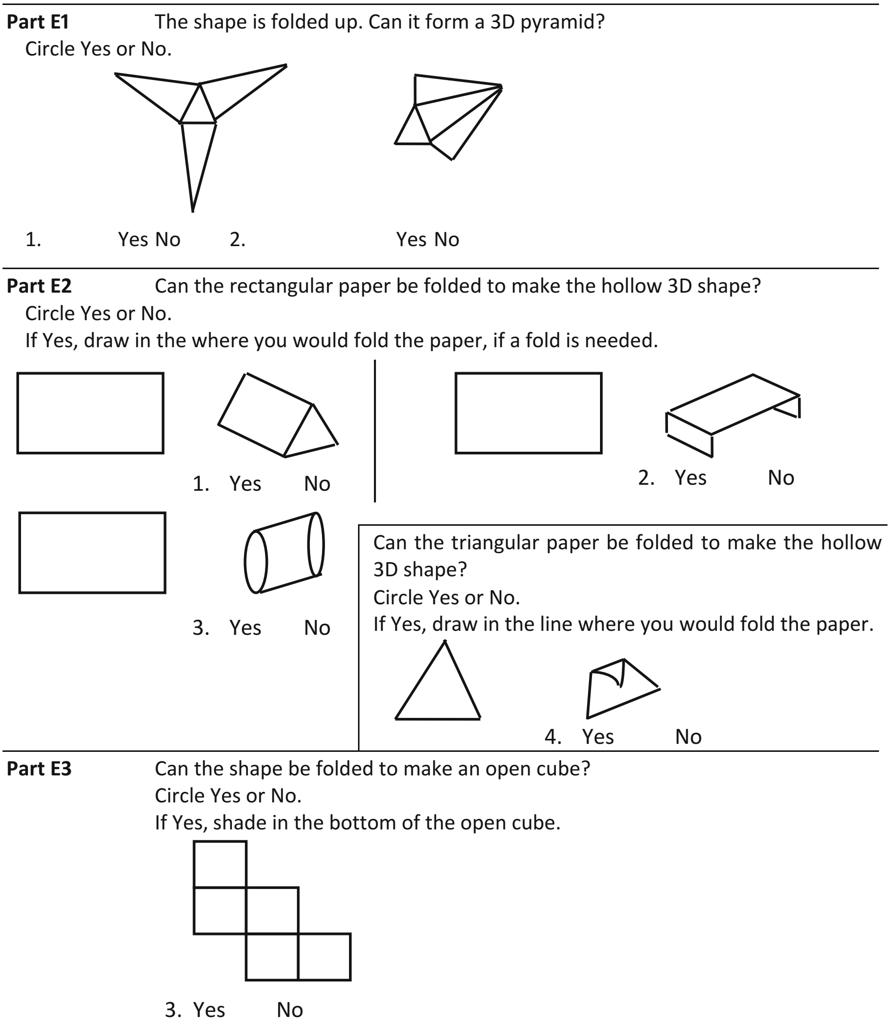


Fig. 2.15 Selected items from test: imagining folding 2D shape paper to make a 3D shape

selecting it ranged from 38 to 48 % with the correct base shaded being 33–43 %). These percentages were similar to the unusual net for the triangular pyramid, E1, 2. The shape (an L shape) that could not be folded to make an open cube was not consistent with the rest of the items but it provided a negative response to at least one of these items. Many students selected this item as a net but failed to imagine that some faces would be doubled with others left open.

The wide variety of items involved students in a range of visuospatial reasoning skills including fitting objects together, mental rotation of objects, viewing from another perspective, and mentally folding. The interviews indicated that students were mentally manipulating objects or parts of the figures that were perceptually accessible in the form of the diagram and they commonly analysed the shapes and considered parts. The test was not timed and so speed of image making and that of image scanning were not teased out as separate processes.

Re-seeing shapes and recognising shapes in different orientations, and noticing and imagining parts of shapes were skills being developed by this age group. Students appreciated parts of figures were hidden in some cases but could not imagine the piece that is hidden. It seems that details of figures are not always noted. This is evident when lengths, points of intersection, and size are not distinguished but shape is more important. Responses from students who were not strong (e.g. Ahmed) supported Tartre's (1990b) suggestion that low scoring problem solvers did not integrate analytical and spatial skills well.

Some recent studies have considered the impact of symmetry of objects (both familiar toys and abstract arrangements of blocks) from different perspectives. One recent European study (author unknown) suggested that symmetry might in fact assist students to concentrate on other features for determining perspective whereas asymmetry was less of a problem with a familiar toy than an abstract block arrangement. Children's reasoning for front/back perspectives was predominantly related to features of the object or the alignment of features within the object for both symmetric and asymmetric objects (animals and blocks) rather than an extrinsic alignment. However, for side views of symmetric animals, students in lower elementary school struggled to provide a reason or struggled to use a description whereas most students could describe characteristic differences for asymmetric animals. For side views of block arrangements, there were more descriptions of symmetric arrangements as well as asymmetric and front/back perspectives. It is suggested teachers should develop front/back perspectives using both symmetric and asymmetric objects but for side views it is worthwhile beginning with asymmetric objects. The left–right relation could also be stressed as what can be seen is not always successful. Importantly, the world around the child in terms of complex living creatures seems to provide more clues for reasoning intuitively.

School Learning Experiences and 3D Visuospatial Reasoning

The test was later used to show the effectiveness of a series of lessons from the Count Me Into Space project in NSW, Australia, on visuospatial reasoning of grade 2 students from five schools across three districts of the city of Sydney, Australia, matched with schools from the same districts (Owens, 2004b). While there was a significant difference between students who undertook the activities on orientation and motion (see below) including work on 2D to 3D shapes on the immediate post-test, there was no significant difference after 6 months. The confidence interval of the mean of scores for the groups overlapped. However, when the students were broken into three groups according to their pre-intervention scores, there was virtually no overlap for those students in the lowest group—intervention had confidence intervals for the mean of 38 ± 3.5 and non-intervention 31 ± 3.5 (see Fig. 2.16).

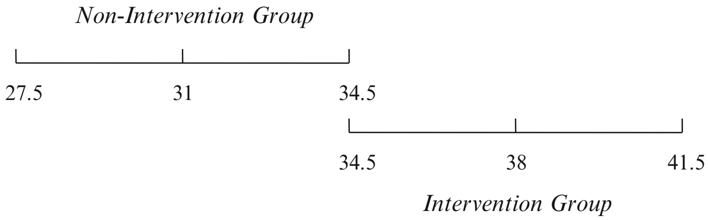


Fig. 2.16 Performance of low attainers from both groups on three-dimensional shape test

This confirms reports by teachers that the weaker students gained considerably from the classroom experiences. The group work, discussion, and hands-on experiences encouraged a sense of ownership of their work and helped these students to improve. The programme captured the essence of the research especially in developing imagery for (a) recognition of 2D symmetry and 2D and 3D shapes in different orientations, (b) modifying shapes that keep certain properties (dynamic changes), (c) perceiving parts of 3D shapes, and (d) imagining 2D nets of 3D shapes.

Given the differences in results from the three schools (grades 2–4) in different ecocultural contexts and the impact of school learning experiences in the studies discussed above, it seems important to pursue the influence of context on visuospatial reasoning.

Visuospatial Reasoning in Context

Kaufmann (1979) claimed that visual imagery did not necessarily lead to flexibility in problem solving, and that this might have been the result of limitations brought about by socially induced gender differences (see my discussion of many studies on gender and visuospatial reasoning, especially the meta-analysis by Linn and Hyde (1989) in my thesis (Owens, 1993)). Perceived rules of the classroom also impact on using visuospatial reasoning as well as children's interactions. From my key (1993) study as described above, this was evident from both the competitive approach of James in his group with making pentominoes (excerpt 2) and later Victor's discussion with him about the right-angled triangle with a horizontal hypotenuse. It was also evident in the classrooms when children moved the shapes too quickly to allow all the groups to think about the shapes. Tess was doing this initially in making a square from the tangram pieces (excerpt 5). Susan (in grade 2 in PNG, excerpt 4) tended to be a dominant figure within her group and her quick pulling apart of trial configurations may have prevented the students seeing shapes within shapes. Interestingly, none of the students in her group saw the nearly completed trapezium in enlarging pattern block shapes, and the complete trapezium was never made by the group despite several more attempts. (Most groups in all grades and schools were unable to make the trapezium and many were unable to complete it even when three pieces were correctly positioned.) In one class in PNG, a girl made one shape and sat with finger on her lips and hand up waiting for the teacher to give her praise and further instruction (until she realised she was to make many shapes if she could) (Voigt, 1985).

In my study (Owens, 1993), I found the classroom context had to be considered to fully appreciate the results. Classroom learning environments should provide not only receptive-language opportunities when students process another person's communication by listening, reading, interpreting diagrams, pictures, and actions but also expressive-language opportunities for speaking, writing, drawing, performing, and imagining (Del Campo & Clements, 1990). If this is the case, then students who manipulate and speak about their angle-matching tasks are more likely to perform better on angle-matching tasks in future. I decided to see if preservice teachers could put this into practice. (Later I will discuss a widely used programme Count Me Into Space which encouraged visuospatial reasoning.) The preservice teachers planned learning experiences after learning about the different types of visuospatial reasoning and the importance of substantive communication in the classroom. They used Wood's (2003) model involving strategy reporting and inquiry to prepare the learning experiences. The following extract indicates children, perhaps for the first time using visuospatial reasoning, to respond to teacher's questions. The following transcript from the pentomino lesson shows how she encouraged students to interact and give their opinions. (T stands for teacher.)

T: Is that the same shape or a different one
D: Same
T: How come it's the same?
S: It's been rotated
E: It's different
T: Why do you think its different E?
E: Because the square we're looking at is in the top row not the bottom row
T: Someone else
V: They're the same because if you rotate it's on the right side not the left side
T: What happens, yep someone else
J: If you flip it over and rotate it once.

In a later lesson the children were drawing examples of shapes, first in small groups and then discussing whether some given descriptors fitted the shape. In the process they tried to draw a 40°, 40°, and 100° triangle to visualise the obtuse-angled isosceles triangle.

T: So everyone got the isosceles triangle.
J: You know how you call it an acute isosceles triangle, doesn't it have to be acute?
T: What does everyone else think? Do you think you can have obtuse angled isosceles triangle?
R: No then it would be scalene. (Other students comment in the background.)
T: Then it would turn into a scalene
D: If both angles (pause) in the corners, it would go out like that (shows with hands)

Students continued to discuss other shapes on the paper deciding on whether they were irregular or not. In this extract and later in the lesson, students were initiating conversation. So conversations in the classroom can direct students' attention to features of shapes encouraging visuospatial reasoning to make decisions and develop concepts. I now consider attention in more detail. It is not just external attention.

Attention

Flavell (1977) commented that attentional processes become increasingly interwoven with other cognitive processes such as memory, learning, and intelligence. Attention is attracted by perceptually outstanding features such as nearness, isolation, size, special form, colour (Gell, 1998), number of items, and the inherent interest of the items (Bishop, 1973). As a result, people attend to certain features of a visual stimulus.

Selective attention is the result of focusing on both external and internal stimuli (Flavell, 1977). Selective attention can be affected by the visual ability of making ground-figure changes. For example, a student can change focus from a part of a shape to the whole shape. Less experienced students may focus on partial features to decide equivalence and may not be logical or recognise relevant orders such as size (Vurpillot, 1976). Selective attention can be improved by repetition and the recognition of a relationship which can be employed to solve a problem (Vurpillot, 1976). If students consciously or unconsciously assess information as incoherent, then they do not attend to the input (Egan, 1992; Lévi-Strauss, 1968; Mason, 2003). Such restrictions may reduce the effectiveness of selective attention in developing conceptual links but students' attention can be influenced by others through looking and listening to others as noted above with adults and children.

When students respond to problems that require visualisation skills such as those required in spatial problem-solving tasks with manipulatives or computer assistance, there can be an interference effect. Some researchers have contributed the difficulties to cognitive overload (e.g. English, 1994). English argues that the equipment can make excessive demands on the individuals' working memory and this cognitive overload interferes with the learning of desired concepts. Studies suggested that chunking material, practice, and reducing redundant and irrelevant material especially if it splits attention in the same perceptual mode can assist selective attention or learning (Sweller & Chandler, 1991). The nature of the material and its familiarity, difficulty, uncertainty, and modality of presentation also influence attention (Baddeley, 1992; Kahneman, 1973; Liu & Wickens, 1992). Disputes about selective attention were about the effect of early and late selection and about limited and unlimited capacity. However, Johnston and Heinz (1978) demonstrated that selection can be either early (based on physical characteristics) or late (based on semantic analysis) depending on the nature of the task, the instructions, and so on. Attention is assisted by ecocultural contexts that encourage observation, repetition, interest, and chunking material together from a holistic perspective.

Unlimited Capacity Model of Attention for Action

Then a more flexible view of attention was developed. Rather than identifying rigid upper limits, studies have demonstrated that our capacity to attend and use information is influenced. Allport (1987) argued that early selection is really about "the relative efficiency of *selective cueing* (which) is simply irrelevant to questions about the level of processing accorded to the 'unselected information'" (p. 409). Processing of both cued and non-cued information proceeds at least to categorical levels of analysis. Allport argued that unlimited capacity for perceptual attention for action explains results of experiments. He referred to

> crosstalk interference between parallel processes. … Whenever the task-specified inputs are not the single most compatible among concurrently available inputs for the task-specified actions, (inputs need to) be actively decoupled from the control of particular actions. … It is a radically different conception, however, from the earlier notion of a central, limited capacity, or even from that of multiple limited 'resources' (Allport, 1987, p. 411).

Selective attention has been described as like a spotlight on possible inputs and as a filter of sensory information. However, van der Heijden (1992), based on his experimental findings with short exposures, disagreed with both these metaphors for selective attention which imply limitation and loss of sensory information. Supporting Allport, he provided a model to avoid limitations and loss. The model involved the separation of location and identity for stimulus inputs and the importance of a feedback loop during processing from the location to the inputs. Thus attention can shift mentally to notice other information. Different sensory features of objects are coded automatically and spatially in parallel and are located in appropriate maps (Treisman, 1988; van der Heijden, 1992). Uncued information may take longer to locate but combinations of features specify objects through a master map of where features are located by neuronal activity selectively enhancing (not inhibiting or attenuating) processing (van der Heijden, 1992). Higher order centres involving past experiences and conceptualisations improve the locating. These centres involve expectations and intentions which influence selective attention. For example, if persons expect only to see an angle without a line dissecting it, they will not attend to angles that are dissected. Expectation influences the location (a) directly with verbal cueing, (b) via another module with attribute cueing, and (c) with a link from identity to the higher centre and then to location if symbolic cues are used. This theoretical position suggests a dependence on prior experience. The end result, though, is action (Allport, 1987).

Clements and Sarama (2007a) noted that mental maps are not like paper maps. They distinguish between the areas of the brain that note what an object is, "spatial visualisation" (its identity in van der Heijden and Allport's term), and the way upon which it is perceived "spatial orientation". While this may be a helpful distinction, it is not clear cut in that interaction with objects, their contexts, and people influences both skills. Part of visualisation and location involve recognition of objects. Furthermore language plays a role in such mental maps.

The above summary of van der Heijden and Allport's work provides a way of understanding selective attention in classroom settings as well as perception experiments. Various aspects of the classroom environment—words from the teacher or fellow students, the position of concrete materials, the expectation associated with a routine of classroom activities, and the task description—may influence selective attention. The student identifies, processes through higher centre schema, to give a location that leads to attending selectively to the inputs, with further loops as needed. Selective attention is influenced by expectation and intention as well as perceptual inputs and internal feedback through the higher centres. Expectations and intention are part of the inner visual system and alter internal and external feedback. Classroom and other social interactions form part of past experiences, and they frequently influence expectations and intentions. For example, the prior knowledge and feelings associated with the angle-matching tasks in the adult study on angles (this chapter) influenced students in the computer environment. Thus contexts and ecology of learning become important influences on learning.

Ecology and Visual Perception

One of the earliest theorists to discuss visual perception and ecology was Gibson (1979). He discussed the affordances that the ecology provided in perception. In particular, he noted the position of the head, the body, and the way in which the eyes were looking relative to the head in perceiving but he also noted the texture, curvature, and blocks to vision that the ecology produced that impacted on visual perception. Motion was integral to visuospatial perception and "ambient light" resulting from the environment impacted on visual perception. Thus like van der Heijden's processing model, further connection between context and perception results in visuospatial reasoning at a relatively basic physical level of the brain and nervous system. Ecology, however, impacts on the higher processes almost immediately as seen in studies with children crawling and viewing their surroundings (Cheng, Huttenlocher, & Newcombe, 2013).

Visuospatial reasoning can be involved not only in tasks with objects or drawings which are smaller than a person but also in tasks in which the person is part of his or her surroundings requiring spatial ability or visualisation in the larger spatial arena (Clements, 1983; Werner, 1964). Near spaces are first identified with recognition of where and what is there with developing discernment and discrimination (Newcombe & Huttenlocher, 2000). Thus we note that external contexts can feature in the development of visuospatial reasoning from an early age. Learmonth, Newcombe, Sheridan, and Jones (2008) showed when children were placed in a rectangular space, they were able to use geometric features such as the lengths of sides of the rectangular walls at around 18 months while they use the landmarks such as colour at around 5–6 years suggesting language was essential at this stage. Nevertheless, 18-month-old children can make decisions if the space they are using is small rather than large and so movement is available to them. A more definitive and comparative study suggested that children by 3 are able to use geometric features and by 4 they are able to use landmark information. In this study, walls of a rectangle with one wall being red were used but the child was not able to move outside the smaller rectangle so that nearness and distance were separated. As a result, they found that 3-year-olds were able to use previous experiences (four preliminary trials in which they were able to move to the outer walls) to successfully notice and use the features in a second set of four trials. This study suggests that experience assists students to make decisions. By comparing the various conditions of their experiment and three earlier studies, Learmonth et al. showed that effort was not an effect for the children not being able to make a correct decision in the larger room but the age of the children. The age at which students could make decisions based on features was between 3 and 4. "Spatial language may be one of these factors but not the necessary and sufficient condition for developmental change" (p. 424). The difference is not due to verbal versus visual strength. Instead an adaptive combination view suggests that both geometry and features affect decisions in which movement can assist attention to the spatial framework. Furthermore, young children aged 5–9 years have an ability to reason about nonlinear relationships.

Clements and Sarama (2007a, 2007b) noted that mental structures develop with what they call Euclidean or horizontal/vertical organisations associated with large and small objects. Clements and Sarama supported the point that interactions influence development but ecocultural perspectives best support a diverse range of findings that go beyond the more narrow studies of children influenced by western languages, perspectives, and built environments. Whatever the mental mapping, it seems that younger children (3.5 years) need to move through the space to show their visuospatial mapping. These are interesting results since later chapters (e.g. Chaps. 4, 6, and 7) establish the influence of Indigenous families moving with young children around their lands on the knowledge of space that these children bring to school (e.g. Pinxten & François, 2011; Pinxten, van Dooren, & Harvey, 1983).

Attention and Responsiveness

Attention for action (Allport, 1987) is an apt concept that links well with the model of problem solving that developed from my earlier studies (Owens, 1993; Owens & Clements, 1998). This is illustrated in Fig. 2.17 which suggests that responsiveness or action results from the complex interaction of cognitive processing. The term *responsiveness* implies a degree of understanding of the situation, involvement, and interest in the activity. The analysis of data indicated that cognitive processing embraced selectively attending, perceiving (e.g. listening, looking), visual imagining, conceptualising, intuitive thinking, and heuristic processing (such as establishing the meaning of the problem, developing tactics, self-monitoring, and checking). Responsiveness has an underlying affective aspect. With the changes in imagery, selective attention, and understanding, there is active progress in problem solving.

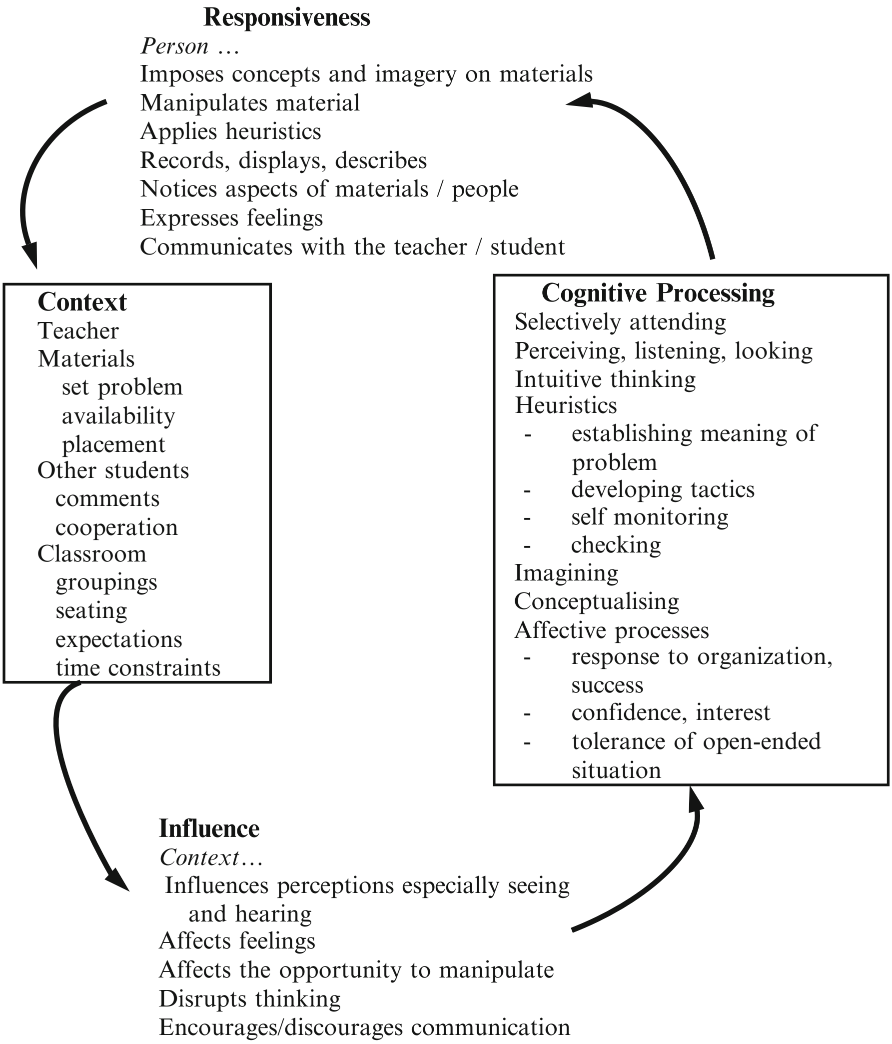


Fig. 2.17 Aspects of problem solving

Individual responsiveness also impacted on students' learning. There are several points to note about James' responsiveness in excerpt 1. First, a friendly competition existed between the students and this motivated them to participate and achieve (para. 1.01 and 1.06). Certain affective characteristics are evident in his behaviour—his responses to his successes (para. 1.01 and 1.06), his competitiveness (para. 1.01, 1.06, and 1.09), his desire to make shapes (para. 1.09), and his loss of interest at the end (para. 1.10). James' use of visuospatial reasoning influenced his responsiveness—not only his manipulation of materials (para. 1.03, 1.04, 1.07, and 1.10) but also his comment to his friend (para. 1.05) and his self-assessments (para. 1.06, 1.10, and 1.11) which tend to keep him on task. His visuospatial reasoning helps him to stay on task (para. 1.06 and 1.10). Third, he assessed or monitored his own progress on the task and this, too, influenced his responsiveness. He showed his monitoring by expressing how he was progressing (para. 1.01 and 1.06) and by changing his tactic in an appropriate way (para. 1.08 and 1.10). Finally, he expressed his understanding and knowledge (para. 1.03, 1.04, 1.05, and 1.11). The changes in his responses (para. 1.03 and 1.10/11) were precipitated by comments to him by his friend and by the teacher. Thus we see how his responsiveness was affected by (a) his understanding of the problem, (b) his use of visual imagery associated with comments by other students and the teacher, (c) his self-monitoring, and (d) his attitudes. At the same time, we can see how his visuospatial reasoning and tactics improved and influenced his responsiveness.

Materials or words spoken by others are important in students selectively attending and hence using concepts and images actively to solve problems (Owens, 2004a). While imagery has a role in generating intuitive responses, in inducing selection of and reflection on concepts, and in precipitating the direction of actions, the verbalisation of concepts often assists in interpreting perceptions and actions. In this way, conceptualising and verbalising are important in assisting meaning and later attention where analysis of the imagery is possible.

An example (already discussed above in the study of pentominoes from both adults and children) might clarify the role of selective attention and show why spatial concepts are constructed largely by idiosyncratic means. Students joined five squares to form different (pentomino) shapes. At first, some students made only symmetrical shapes or shapes with names. When they realised they were required to make more shapes, they realised that symmetry and a common name were not essential for defining "a shape". Their expectation influenced selective attention and initial schema location. Intentional and conceptual changes also occurred when they considered shapes in different orientations and the students developed their understanding of what constituted sameness and difference for that problem situation. (See Owens & Clements, 1998, for other examples.)

A subsequent study corroborated the findings this time in the context of adult students learning mathematics through interactive construction of concepts. An analysis of critical incidents revealed that interactions, affect, and responsiveness were important features of learning in a problem-solving classroom setting (Owens, Perry, Conroy, Geoghegan, & Howe, 1998). With further research, discussed later in this book, this model was modified to the diagram on identity in Fig. 1.2 taking even greater note of context.

Giaquinto also supported the argument that visuospatial reasoning requires an aspect shift (Allport, 1987; Owens & Clements, 1998). This particularly relates to disembedding and embedding as discussed earlier and motion as depicted often by arrows in diagrams although motion may be implied in other kinds of diagrams imaged with paper-based symbols and on computer screens (Giaquinto, 2011). Dynamic imagery entails motion in imagination (Laborde, Kynoigos, Hollebrands, & Strässer, 2006; Presmeg, 1986).

Shifts within an ecocultural perspective are evident in the thinking of Indigenous communities. For example, in Malalamai when asking about the relationship of houses to measurement and I pictured the square units tessellating depicted by squares with corners at the posts of the houses, the participant researcher Sorongke Sondo noticed half as much again as the size of the house which provided the additional floor space on which people sat, lay, and built rooms. In school mathematics, this perspective relates to ratio of areas rather than area units. When discussing the planting of crops, both Malalamai and Yupno people referred to the two equal lengths used for spacing plants at the points of equilateral triangles. However, comments were about the beautiful tessellating pattern of equilateral triangles represented by this planting. They had an overview of the shapes and the pattern but little way of connecting the geometry associated with the equal lengths to these shapes or patterns. The intention of the person was also influencing visuospatial reasoning. The villagers and myself attended to different aspects because our intentions in terms of cultural and school mathematics dominated our attentions and perspectives. Chapter 5 will provide other examples in which disposition, metonomy, motion, intention, and visuospatial reasoning impact on activity.

Visuospatial reasoning with number size and number lines is also cultural. While in western society most people recognise small numbers, it is less likely that one can immediately estimate larger collections as many Indigenous people do. In Chap. 5, I discuss the work of Paraide that shows that cultural context influences not only arithmetic knowledge but also the imagination. A similar result was found by Willis (2000) and by Treacy and Frid (2008) in Australia but not necessarily by others working with traditional representations on testing cards (Warren, Cole, & Devries, 2009). Furthermore, the cultural symbolism of a society impacts on visualising size of number (Giaquinto, 2011). For example, western societies are more likely to note the size of 0.45 than the binary 101101. This might not be the case in an oral society with a two-cycle system as found in PNG.

Furthermore, there is a tendency to have a left–right orientation of size for the number line in western societies. By summarising results from a number of studies, Giaquinto (2011) noted that participants' reaction times for deciding whether a number was greater or smaller than a given number varied when the smaller number buzzer was in the left hand compared to if it was in the right hand. The reverse was the case for Arabic monoliterates who read from right to left and reaction times were less strong for bilingual persons. It is interesting to note that societies with body-part tally systems such as the Oksapmin have strong visualisation of number (Saxe, 2012) but unlike the western number line, it might be considered that they have less of a sense of infinity since the last number tends to end at the point symmetrically opposite the first number such as on the little finger of the other hand from where the counting system starts (Owens, 2001c). While some PNG and Australian groups would want to complete counting at the end of the cycle, others considered counting people or reiterating the numbers since the notion of cyclic repetition was also important in the cultures and in those ways establishing an infinity in number.

However, image scanning, zooming-in, and extrapolating are tools available to be used on number lines when numbers out of current range are required (Giaquinto, 2011). While visual imagery, number sense, and the desire to illustrate concepts by drawing might be innate, the number line is based on cultural conventions. Non-written-symbolic cultures and young children will use a variety of representations of number, not necessarily a number line (Thomas, Mulligan, & Goldin, 2002). Thomas' study showed children's imagination with numbers written in a spiral but also school experiences such as a line of numbers and contextual experiences such as watching calculator screens changing with the constant addition of one. The whole recent movement on number learning (e.g. NSW Department of Education and Training, 1998), however, has emphasised the importance of figurative or visuospatial reasoning in the mind and much teaching and research is supporting this visuospatial aspect of learning arithmetic.

Developing a Theoretical Framework of Visuospatial Reasoning

Reviewing the earlier studies resulted in the development of a theoretical framework that could be used to inform teachers of young students' early visuospatial reasoning in geometry. The framework was also designed to build on ideas developed by The Count Me in Too project for arithmetic (NSW Department of Education and Training) through which teachers became familiar with such terms as emergent, perceptual, and figurative (imagery) stages. The success of emphasising both investigating and visualising together with describing and classifying for both part-whole and orientation and motion aspects of geometry is given in several papers (Owens, 2002a, 2002c, 2004b; Owens & Reddacliff, 2002). The framework is summarised in Table 2.6.

The actual activities (NSW Department of Education and Training Curriculum Support and Development, 2000) consisted of ten lessons where students make triangles, explore symmetry, build with blocks, and draw.

Table 2.6 A framework for geometry based on visuospatial reasoning

	Investigating and visualising	Describing and classifying
Part-whole relationships		
	The student:	The student:
Emerging strategies	Attempts to put pieces together to see what is obtained	Matches shapes with everyday words, e.g. ball for a circle
Perceptual strategies	Recognises whole shapes used to build a shape or picture	Describes similarities and differences and processes of change as they use materials
Pictorial imagery strategies	Disembeds parts of shapes from the whole shape Matches parts of different shapes Completes a partially represented shape or simple design	Discusses shapes, their parts, and actions when the shape is not present
Pattern and dynamic imagery strategies	Develops and uses a pattern of shapes or relationship between parts of shapes Plans and dynamically modifies a shape to illustrate similarities between different representations of the same concept	Discusses patterns and movements associated with combinations of shapes and relationships between shapes
Efficient strategies	Assesses images and plan the effective use of properties of shapes and composite units to generate shapes	Describes effective use of properties of shapes to generate new shapes
Orientation and motion		
	The student:	The student:
Emerging strategies	Recognises shapes that match the child's fixed image(s)	Uses a shape word for a fixed image
Perceptual strategies	Recognises shapes in different orientations and proportions; checking by physical manipulation	Describes similarities and differences and processes of change as they use materials
Pictorial imagery strategies	Generates a series of static images of shapes in a variety of orientations and with different features	Discusses shapes, their parts, and simple actions when the 2D and 3D shapes are not present but recently seen
Pattern and dynamic imagery strategies	Predicts changes by mentally modifying shapes and their attributes using motion or pattern analysis Represent patterns and relationships of change by modelling or drawing	Describes a number of changes that will occur with one or more actions Discusses patterns and movements associated with combinations of shapes and relationships between shapes
Efficient strategies	Selects effective strategies to make changes needed to achieve a planned product	Describes effective use of properties of shapes to generate new shapes

Assessment Tasks

Teachers were also provided with assessment tasks for individual interview. These were also used to evaluate the programme. A number of carefully established cardboard cut-outs, drawings, sticks, and string are required and all tasks are presented so that students can show if they are using mental visuospatial reasoning before they are allowed to use materials in using perceptual strategies. Figure 2.18 provides some items from the test to illustrate how the task is presented with probe questions for extension or simplification. The first task is about recognising shapes (represented by cardboard cut-outs) in the environment.

The tasks did provide a range of strategies to be observed by different students. While students did not necessarily show the same type of strategy across all questions, there was a tendency for this to happen. Table 2.7 shows how one task could be used to decide what strategies were being used.

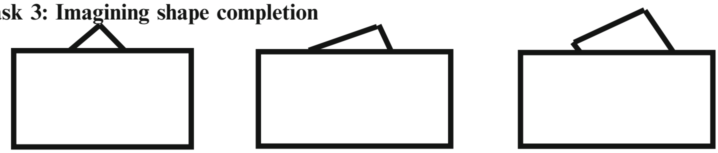
Task 4 (Fig. 2.18) shows how the skill of re-seeing parts is manifested in visuospatial reasoning while responses to the orientation and motion Task 2 (Fig. 2.18) indicated the development of orientation skills and noticing angles. Results for Task 6 (Fig. 2.18 and Table 2.7) on making triangles show how a carefully designed task can illustrate a full range of strategies. It was a particularly novel task for consultants and teachers.

Table 2.8 gives the results of assessment on the tasks (Owens, 2002b). These results indicate the effectiveness of the framework, series of activities, and teachers' professional development. The number of students who improved on each item and overall in the classes whose teachers undertook professional development and taught with the activities was significantly higher than those without the geometry lessons that emphasised visuospatial reasoning. This was the case whether professional development was through a consultant or a school facilitator. An attitude question also indicated that more students felt they were good at mathematics most of the time, more decided this because of self-assessment, and more recalled specific activities. Teachers confirmed that students enjoyed the lessons and remembered content well. Thus the framework implemented by teachers, the tasks, and assessments were valuable in increasing visuospatial reasoning but also in establishing self-regulation and positive attitudes leading to evidence of the development of a mathematical identity.

The tasks can be used for individual assessment or for the basis of activities for the class (see Owens, 2006a). The questions and probes can be used by the teacher to assist in students' learning and assessment during class experiences. The technology may be as simple as card cut-outs but computer-generated tasks could extend learning from previous activities with concrete materials. It is worth noting that Lehrer et al. (1998) had taken several different geometric and measurement tasks and had used probe questions and described different levels of assistance on each item, moving from more abstract to concrete to demonstrated responses. One of his questions related to transformation of a core square made up of four smaller different squares. Repeating transformations made a strip for a quilt.

Part-Whole Relationships

Task 3: Imagining shape completion



A square is gradually revealed. Each time, the student is asked what it might be and to trace where it might be. They are encouraged to give more than one answer.

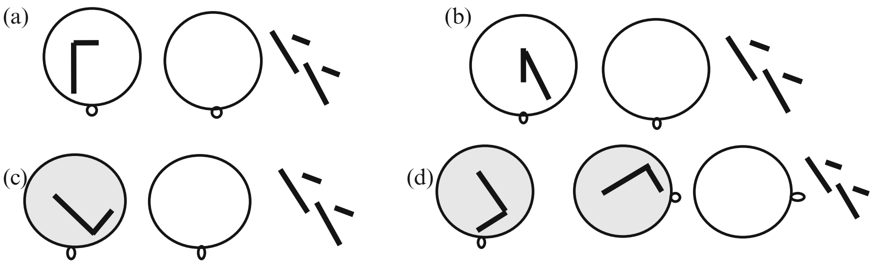
Task 4: Reseeing shapes

Students use sticks of the same lengthto form 2 squares joined together along a side and then 2 triangles joined along a side. They are asked to draw the 2 triangles, while covered and are asked "If I take the middle stick away, what shape would I have?"

Orientation and Motion

Task 2: Angle recognition, visual memory, and rotation skills

Make the following diagrams on a circle using long and short sticks, point out the tab, let the student make the same diagram on their circle with tab mark aligned with yours. The first two are uncovered, the third is covered before the student starts, and the fourth is shown to the student, covered, and turned before the student starts.



Task 6: Dynamic imagery

Use 40 cm string, joined to form a loop; a firm stick.

Place the loop of string on the table and hold two points firm, about 12cm apart.

Provide the student with the stick.

"Use this stick to pull the string tight and make a triangle."

How would you describe the triangle you have made?

Make other triangles?

How would they change?

Probe: If the student cannot explain, let them use the stick to demonstrate and tell about the triangles they are making.

Point to one of the sides of the triangle.

Tell me what you would have to do to make this side shorter.

Point to the other side.

As the first side is made shorter, what will happen to this side?

Fig. 2.18 Tasks for assessment in Count Me Into Space

Table 2.7 Examples of different visuospatial reasoning strategies for a task

Visuospatial reasoning strategies	Indicators of investigating and visualising	Indicators of describing and classifying
Part-whole relations: *task 3—imagining shape completion by tracing possible hidden shapes*		
Emerging	Traces an edge	Says any shape name
Perceptual	Attempts to trace hidden shape or traces visible triangle	Says triangle
Pictorial	Traces for a triangle or square or rectangle or two of the same kind	Says triangle or square or "diamond" or rectangle
Pattern and dynamic	Traces possible shapes of varying sizes	Explains how the shapes change by lengthening or shortening the sides
Efficient	Indicates tracings and various changes	Readily explains how different shapes could be underneath
Orientation and motion: *task 6—dynamic imagery using a stick to move a loop of string*		
Emerging	Moves stick but does not make or recognise a triangle	
Perceptual	Makes a triangle	
Pictorial	Makes two or more triangles, e.g. right angle, isosceles	Knows names and properties of different types of triangles
Pattern and dynamic	Automatically slides stick to make different triangles Makes both acute and obtuse-angled triangles	Comments on changes to triangles and gives names of different types of triangles
Efficient	Shows an arc of points to shorten side	Explains why continuous range of triangles can be made in general

During implementation of the tasks, a grade 2 girl, who mostly showed emerging strategies and seemed to have most difficulty with describing and classifying, was able to show perceptual or pictorial imagery strategies in other tasks such as Task 6. These were novel questions for her and may have been less associated with her general struggle with learning shape labels in English. The assessment provided the basis to plan suitable activities for her. For example, she needed experiences in sorting and grouping many different kinds of triangles, squares, and rectangles (two kinds at a time); talking about the reasons for grouping, e.g. four sides or four corners; seeing shapes within shapes in matchstick type puzzles; doing more jigsaws; and making geometric shape like squares with tangram pieces.

On the other hand, a boy in the same class generally showed pattern and dynamic imagery strategies and an ability to see shapes within shapes assisted by good general language. However, his recognition of diversity when referring to a shape like "a triangle" still needed extension. He needed activities like matching parts of different shapes in order to notice similarities and differences, and to develop properties. He also needed more language to describe the parts and types of shapes. Interestingly, in Task 6, he showed some hesitation in positioning the stick to mark the vertex of the triangle to shorten its side, trying to indicate that it would be further

Table 2.8 Student improvement on assessment tasks

Task	Number (%) who improved with school-based facilitator		Number (%) who improved with consultant	Number (%) who improved without programme	χ^2 value comparing consultant and non-intervention group
	Group 1	Group 2			
Part-whole relationships	$N=135$	$N=193$	$N=140$	$N=75$	
Task 1	89 (66)	129 (67)	87 (62)	31 (41)	8.54*
Task 2	63 (47)	130 (67)	85 (61)	25 (33)	14.65**
Task 3	74 (55)	113 (59)	72 (51)	18 (24)	15.10**
Task 4A	95 (70)	131 (68)	74 (53)	22 (29)	10.94**
Task 4B	73 (54)	122 (64)	84 (60)	27 (36)	11.26**
Three or more tasks	79 (64)	141 (73)	77 (55)	20 (27)	15.83**
All tasks	17 (14)	40 (21)	19 (14)	0 (0)	**,†
Orientation and motion	$N=136$	$N=160$	$N=73$	$N=34$	
Task 1A	57 (43)	73 (46)	33 (42)	9 (26)	4.48*
Task 1B	63 (49)	98 (61)	Not included	Not included	
Task 2	41 (31)	69 (43)	43 (59)	13 (38)	3.97*
Task 3	73 (54)	94 (59)	42 (58)	9 (26)	8.97*
Task 4	72 (53)	81 (51)	44 (60)	12 (35)	5.80*
Task 5	66 (49)	80 (50)	38 (52)	8 (24)	7.70*
Three or more tasks	70 (53)	103 (66)	37 (51)	9 (26)	5.55*
All tasks	16 (12)	19 (12)	8 (11)	0 (0)	**,†

*Difference assessed by chi-square analysis is significant at $p<0.05$ level
**Difference assessed by chi-square analysis is significant at $p<0.01$
†No chi-squared value calculated because $n=0$ in one cell of the table

away than the line of the string (tending towards efficient strategies). He was ready to use properties to establish that squares are rectangles and that the same names apply when the shapes are in turned positions (a problem that can be exacerbated by the use of words like diamonds), and to use words like rhombus, trapezium, quadrilateral, or four-sided shape.

Moving Forward

This chapter has synthesised psychological literature around spatial abilities and around visual imagery especially from the last century. These were heavily influenced by psychological studies and experimental designs. Visual imagery research was particularly common from the information processing theories of psychology but several theorists have linked it to perceptual and contextual aspects of learning.

In my research, I attempted to draw all these psychological literacies together to discuss children's thinking when problem solving. I drew on qualitative research in order to get inside children's heads to see how they were visuospatially reasoning. Visuospatial reasoning relies on four skills (Wessels & Van Niekerk, 1998) that I elaborate as follows:

- Visual skills especially seeing and re-seeing aspects of the environment, objects, and shapes
- Verbal skills that support comparisons and decisions with words, and encourage interactions about the visuospatial reasoning
- Tactile skills such as cutting, joining, and folding that support or provide affordances in the visuospatial reasoning
- Mental skills especially mentally manipulating spatial images

Encouraging these skills together strengthens measurement and geometry education. These skills come together through pattern and dynamic imagery used in visuospatial reasoning supporting the learning of processes and concepts in measurement and geometry and expressed in conjecture, explanation, argument, and proof.

Visuospatial reasoning emphasises reasoning associated with and dependent on visual and spatial imagery but also expressed, developed, and argued spatially. Visuospatial reasoning is the important part of reasoning with visual and spatial imagery or imagination. It is a mental process linked to physically seeing and doing in a spatial world that has spatial relations. Geometry is about spatial relations. We reason not just in verbal written proofs often associated with high school geometry such as congruent triangles and trigonometry or circle theorems but with perceiving and interpreting diagrams. In primary school that reasoning relates to shapes, both two-dimensional and three-dimensional, their interrelationship, and lines; and to transformations and symmetries. It also relates to interpreting drawings. A drawing may be used as a metonymical representation of a class of shapes thus "knowing what a triangle is, is more than being able to label an equilateral triangle sitting on its base as a triangle" (D. McPhail, Count Me Into Space videos). Initially we know that students cannot always verbalise why a shape is, for example, a triangle—they seem to have a global understanding much as they do that a chair is a chair. On the other hand, a young student may just focus on the pointiness without seeing the whole or noticing other important properties. Students may also have a fixed image that needs to be developed by experiences. For example, one young boy making a triangle with a loop of elastic thinks that a right-angled triangle must be placed with horizontal and vertical sides. Students will realise that a variety of examples of a shape can be categorised as one particular shape. Students will begin to associate more and more properties or parts as necessary for that shape. They will also begin to decide what is not necessary for a shape to belong to a particular category. None of this is restricted to the school mathematics shapes. These comments could be noted in other ecocultural environments.

It is often thought that children need to develop words first but they in fact develop a visual image of a shape before they have the language to talk about it.

When children talk about their images, their explanations help them to clarify what is in their images and to develop their concepts. Children often say about triangles that they have properties like having "three sides" by rote (note how often children leave out that they are straight and intersect) but children need to be able to perceive these sides separate from the whole shape and to reason visually often by running their finger down each side as they count. A good example of physically representing visuospatial reasoning through dynamic imagery is that of pulling a vertex of a triangle formed on a computer screen or a piece of thin elastic. There are an unlimited number of triangles. Prior to reasoning in that way, children might only recognise a couple of images of triangles or think they are the shapes with "pointy bits and not corners". Without extending children's imagery of triangles they may have a prototypical first image and procept (Gray & Tall, 2007) or beginning conceptual understanding.

Visuospatial reasoning occurs when a child seeing part of a hidden shape says, "it can't be a triangle because it has two corners" (pointing to the right angles of the partially revealed shape) (Count Me Into Space video) or when the same child in seeing one "corner" and a triangular section of the shape can show that "it could be a larger triangle or an even larger triangle underneath or even a rectangle or a larger rectangle or a square underneath". Every time the child told us what shape it might be, she traced with her finger where the shape might be. In the research on this hidden shape task, one child from grade 2 said "it could be any shape". When asked what he meant, he called it by an imaginary name and traced out a zigzag line at the end of the imagined extended sides. (Being an English-as-a-second-language learner, this child had learnt to "play" with words and this strengthened his visuospatial reasoning.) Children can mentally slide, rotate, and turn over shapes or reflect them. By talking and pointing, students indicate that they notice parts and visualise their relationships. These are skills required in visuospatial reasoning.

Students learn to attend to the more important aspects of images, overcome initial static perceptions in favour of pattern and dynamic ones, and acquire appropriate mathematical conventions in developing and conceptualising visuospatial reasoning (Hegarty & Kozhevnikov, 1999; Owens & Outhred, 2006). Episodic and illustrative visuospatial reasoning is important in transforming visuospatial images to new situations as shown by the above examples such as Sally's tangram and pentomino problem solving. Diagrams need to be manipulative whether mentally and/or virtually and then visuospatial reasoning can be applied through the use of structures and propositions to new situations (Dörfler, 2004) as illustrated in the examples in this chapter. However, it depends on the valuing of the visuospatial representations and reasoning whether these remain significant in memory and purpose (Rivera, 2011). Gestures in cultural practices are mathematical representations in use and constitute the interface between embodied and cultural aspects of knowing and learning geometry (Kim et al., 2011). Significant are the manifestations of visuospatial reasoning, especially through actions, when two communities of practice merge whether they be western and Indigenous or community and school as the chapters that follow develop (Civil & Andrade, 2002; Gutstein, 2006; Téllez, Moschkovich, & Civil, 2011).

Visuospatial reasoning and a move away from stereotypical images and practices is important for visuospatial reasoning to be evident in all areas of mathematical problem solving. The following example illustrates this well as it links to limitations in both geometry and number as a result of teaching practices that fail to encourage students' use of visuospatial reasoning. M. Clements (whose work has been discussed earlier in this chapter) reported on a study by Zhang with grade 5 children (2012, p. 14). The teacher used a textbook that used area-model representations of fractions (circle and rectangle) following a Standards curriculum. At the end of the teaching, the children, and the teacher to a lesser extent, could only represent fractions and not use fraction concepts to solve a simple problem, "find a third of the way around an equilateral triangle". He noted that "These students thought about simple fractions in terms of parts of a circle, and many of them knew of nothing else" because of the overuse of one kind of "visual algorithm". A similar limitation has been found with base 10 block representations of fractions. However, to correct this through a verbal, especially symbolic representation, would be worse and curricula that encourage multiple visual representations should not be crowded so they result in visuospatial reasoning in only one context or medium. Rather there needs to be a visuospatial reasoning approach in which problems that require some visualising are set but then students are encouraged to act through heuristics such as to draw, compare with other representations of a third, compare with other fractions of this representation, and represent with another model. This argument also applies in geometry and measurement education.

The complexity of visuospatial reasoning and the way it relates visual imagery, spatial abilities, and other forms of thinking is important. Nevertheless, the case is established that the context, both within the classroom and in the community and indeed the school with its curriculum and teachers and government policies, is impacting on visuospatial reasoning. In fact, attempts to recognise visuospatial reasoning in the geometry area of mathematics, at least in Australia, have met with structuralist theories of development, rigid thinking of two categories of 2D and 3D separately, poor teacher content knowledge or pedagogical content knowledge, paper-and-pencil testing, and the view that visuospatial reasoning cannot be assessed by such testing without even realising its role in the test (Lowrie, Logan, & Scriven, 2012).

Some of the authors cited in this chapter made reference to the importance of context, in terms of perception in small and large spaces, in terms of development and reasoning, and in terms of classroom routines and expectations. My own research in classrooms indicated a strong influence of teachers, peers, and materials on children's ways of thinking and learning but also the role of expectations in learning. Learners rely on "deep, personal, and situated structures" (Goldenberg & Mason, 2008, p. 183) to provide a possible variety and range of examples of a concept but at the same time their attention needs to be drawn to the generality whether intuitively or by interaction with an external source.

However, what are the possible impacts of family and community's shared knowledge, values about aspects of education, and ways of teaching on visuospatial reasoning? In the next chapter, we will establish a case for considering visuospatial reasoning from an ecocultural perspective. Examples of culture and ecology and theories of education related to an ecocultural perspective will be developed.