Chapter 9
Activity Theoretic Approaches

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9.1 Introduction

Activity theory[1] (AT) is an approach to the study of human practices—any human practice and human practice in itself. It warrants a chapter in this book on tools and mathematics because artefacts/tools[2] are intrinsic to its approach and many mathematics educators use theoretic approaches to study mathematical practices. I first consider this approach in general but then focus on the practices of doing, learning and teaching mathematics, and the light that activity theoretic approaches shed on tool use in these mathematical practices. The roots of AT go back to early Soviet approaches and the section on Vygotsky in Sect. 7.2 serves as an introduction to these roots. This chapter has four sections. Section 9.1 provides an overview of AT. Section 9.2 traces early influences of AT in mathematics education research. Section 9.3 considers foci of a set of mathematics education papers recent at the time of writing. Section 9.4 explores emphases and tensions in papers considered in Sects. 9.2 and 9.3.

[1] As will soon become apparent, there are a number of schools of thought within what is called ‘activity theory’ and I use the term ‘activity theoretic approaches’ as a collective noun for these different approaches.

[2] A note for readers who are reading this as a ‘stand alone chapter’. In Sect. 1.3.1 I stated my distinction between and artefact and a tool as, an artefact becomes a tool when it is used by an agent to do something. I use this distinction in this chapter. For example, a compass as a metal thing which holds a pencil and rests on a desk is an artefact but when it is picked up by someone to draw a circle it is a tool. When its status is ambiguous I use the term ‘artefact/tool’.

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9.2 The Development of Activity Theory

It is important to start with a clarification of the word ‘activity’ as ‘activity’ is an everyday word for ‘doing something’ and it is not always the same as the word ‘activity’ in AT. *Activity* became a focus for Vygotsky in the 1920s in his consideration of consciousness as a *problem* for psychology. Kozulin (1986, pp. xxiii–xxiv) explains:

> The major objection Vygotsky had to the mentalist tradition was that it confined itself to a vicious circle in which states of consciousness are “explained” by the concept of consciousness. Vygotsky argued that if one is to take consciousness as a *subject* of study, then the *explanatory principle* must be sought in some other layer of reality. Vygotsky suggested that socially meaningful activity (Tätigkeit) may play this role and serve as a generator of consciousness.

Activity, going way back into our ancestors’ prehistory, can be conceived as that which continues the species. Hunting, gathering, cooking and schooling are such activities *writ large*. In AT ‘object orientated activity’ is the *unit of analysis*, that which preserves the essence of concrete practice. The ‘object’ here is not the object-thing but the object-*raison d’etre*; indeed if two individuals perform similar actions but have different objects, then it can be said that they are involved in different activities. Although activity theorists all agree that object orientated activity is the unit of analysis, they argue amongst themselves about what constitutes this ‘essence’. The unit of analysis is a means to understand the *Piaget vs Vygotsky* debate (see Monaghan, 2007). The cognitive activity (that Piaget was interested in) of a student engaged in a mathematical activity is, to an activity theorist, only a part of the unit of analysis which includes why the student is doing this mathematics, who s/he is doing it with and what tools s/he is doing it with—and the why/who/what cannot, to an activity theorist, be separated and analysed in themselves.

Such thinking was, though not through this example, present in the original work of Vygotsky and this was continued after his death by Leont’ev who considered individual and collective *actions* (usually with tools) and *operations* (things to be performed or modes of using tools) involved in socially organized *activity* (Leont’ev, 1978). Tool use here can be considered to include the primary, secondary and tertiary tools of Wartofsky (considered in Sect. 7.2.2); tool use is not, by this thinking, an activity in itself though tool use and activity are dialectically related (the activity shapes the tool use and the tool use shapes the activity). Leont’ev emphasised that all activity is motivated (though the motive may not be explicit) and transforming the object into an outcome is essential to the existence of an activity; this has immediate implications for considerations of the role of the mathematics teachers who may be mere facilitators to post-Piagetians but who are central, to activity theorists, in ensuring that students realise the object of learning. The upshot of learning is ‘change’, the student and the object are involved in a dialectical transformation: the object transforms the activity of the student and at the same time the object is transformed by the psychological reflective activity of the student. Parallel with the work of Leont’ev was activity theoretic work in

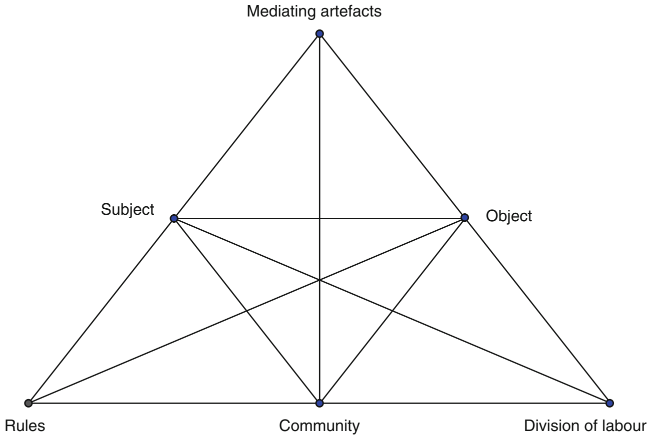

Fig. 9.1 Engeström's expanded mediational triangle

neurology but I do not consider this as it appears, to me, to have had little impact on mathematics education or tool use to date.

The ideas of Leont'ev were barely known outside the Soviet Union until the 1980s. Around the same time Scandinavian work in AT, and that of Yrjö Engeström in particular, began to attract the attention of education researchers. Engeström (1987) extends Vygotsky's focus on mediation through signs and tools to multiple forms of mediation and extends Leont'ev's frame to 'activity systems' to include the community and social rules underlying activity. These ideas are commonly schematised as in Fig. 9.1 below.

Figure 9.1 is designed to show multiple forms of mediation, for example: the top triangle (subject—mediating artefact—object) is the mediational triangle considered in Sect. 7.2; in the lower left triangle (subject—rules—community), social rules (norms and conventions) are mediational means; in the lower right triangle (division of labour—community—object) the division of labour mediates the object-oriented actions of the community. Figure 9.1 as a whole is used, in specific cases, to represent *activity systems* and the subsystems considered in this paragraph should be considered only in relation to the activity system. Activity systems research often examines interactive activity systems such as a hospital and an outpatient clinic with a focus on the objects of activity in the two systems. Engeström (2001) presents five principles for his form of AT: the activity system as a whole as the unit of analysis; *multi-voicedness*, 'multiple points of view, traditions and interests' (Engeström, 2001, p. 136); *historicity*, 'Activity systems take shape and get transformed over lengthy periods of time' (Engeström, 2001); *contradictions*, 'as sources of change and development' (Engeström, 2001, p. 137); and 'the possibility of expansive transformations … A full cycle of expansive transformation may be understood as a collective journey through the *zone of proximal development* of the activity' (Engeström, 2001). It might be thought that tool mediation attracts less attention in activity systems research than in those of

Vygotsky and Leont'ev (and there is some truth in this) but activity systems research emphasises tool use in the context of the whole system; and it is appropriate to take a paragraph to emphasise 'tool use in context' in all the above forms of AT.

AT in Vygotsky, Leont'ev and Engeström's forms is often referred to as cultural–historical activity theory (CHAT). Cole (1996, p. 108) is an eloquent proponent of CHAT and states that the central thesis is that 'the structure and development of human psychological processes emerge through culturally mediated, historically developing, practical activity' and these three components are interrelated. I have addressed practical activity above but I feel a few words on culture and on history are appropriate. 'Culturally mediated' includes tool mediation. The book you are reading is focused on tools and mathematics but CHAT is focused on all tools in activity and language is 'an integral part of the process of cultural mediation' (Cole, 1996). We downplay language as a tool in this book to address our focus but we do not deny its place as the 'tool of tools' (Cole, 1996). There is a sense in which the interrelated set of tools (with no special status given to mathematical tools) used in collective activity is, to a CHAT researcher, the basis for the culture of that collective. With regard to history, we are each born into a culture based on a set of interrelated artefacts/tools, and our immersion in this culture continues in our (mathematical) development/education. We attend school where we have a teacher who was born into a prior form of our culture (who had a teacher . . . who had a teacher . . .). Our teacher looks to our future (including what tools she/he/society feels we need to master) but this vision of our future needs is grounded on valuations of what should be preserved from the past. So, tool use is of fundamental importance to CHAT researchers but this is tool use in the context of cultural–historical activity.

Apart from scholars, such as Michael Cole, who could read Russian texts, AT came to be known by Western scholars after the appearance of Vygotsky (1978), Leont'ev (1978) and Wertsch (1981).[3] The third book has not been mentioned until now. It is a primer on AT edited by James Wertsch which contains a preface by Cole, an introduction by Wertsch and translations of key Soviet AT texts grouped under the headings: theoretical foundations; Vygotsky's influence; the role of sign systems; and empirical studies. AT (in its various forms) is used as a framework in many fields of study. Three fields of study relevant to this book are human–computer interaction (HCI; see Nardi, 1996), ergonomics (see Daniellou & Rabardel, 2005) and education (see Daniels, 2002). Wertsch (1991, 1998) has attracted the attention of mathematics educators interested in tool use because it focuses, amongst other things, on the person-tool dialectic or, as he puts it, 'the *irreducible bond* between agent and mediational means' (1997, p. 27); the bond in, say, a person using a calculator, is irreducible because the act of calculating with a

[3] I will focus on English language texts due to (1) the dominance of the English language in Western academic writing, (2) English is my first language and (3) to keep this chapter to a reasonable length.

calculator cannot be reduced to what the human alone can do or to what the calculator can do, the calculation is done by a human-with-calculator. Wertsch emphasises that "the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of 'individual(s)-acting-with-mediational-means' than to speak simply of 'individual(s)'" (1991, p. 12). I now consider the genesis of activity theoretic influences in mathematics education (mainly in English language writing).

9.3 The Genesis of the Influence of Activity Theory in Mathematics Education Research

In this subsection I trace, to the best of my ability, the early influence of activity theory in Western[4] mathematics education research. I do this via two subsections. In the first I consider two books from around 1990. I then consider the influence of AT in academic journal papers.

9.3.1 Two Activity Theoretic Mathematics Education Books

I believe that the first English language text by a Western mathematics educator was a book on the politics of mathematics by the Norwegian Stieg Mellin-Olsen (1987). This book focuses on the alienation of learners of mathematics and he employs the approaches and constructs of Vygotsky and Leont'ev. Mellin-Olsen considers tools in a broad sense, 'both thinking-tools and communicative tools . . . Their functionality is dependent on whether they are experienced in the process of Activity or not' (Mellin-Olsen, 1987, p. 48) and that language is the basic human thinking tool. Three years after Mellin-Olsen (1987) the National Council of Teachers of Mathematics published a translation of a 1972 book by the Soviet educator Davydov. The aspect of Davydov's work that attracted most attention was his consideration of abstraction and generalisation, as Jeremy Kilpatrick, in the Introduction to Davydov (1990, pp. xv–xvi), wrote:

> Much work on the learning of concepts and principles has assumed that such learning occurs "from the ground up." Students need to see many examples so that they can use induction to form a generalization. The generalization reduces the diversity in the specific examples. Davydov argues that we ought to conceive of learning differently. The specific examples should be seen as carrying the generalizations within them; the generalization process ought to be one of enrichment rather than impoverishment.

[4] This caveat is important as the influence of AT in mathematics education research in (what was known as) 'communist bloc' countries was long standing at the time AT started to influence Western mathematics education research.

Davydov's views on abstraction, his *ascent to the concrete*, which refers to the development of an idea via a dialectical *to and fro* between the concrete and the abstract, was to become the basis for a well-respected framework of 'abstraction in context' which stemmed from Hershkowitz et al. (2001). Fascinating as Davydov's work in this area is and despite his references to tools and social interactions, he says virtually nothing of the place of tools in the formation of abstractions and generalisations.

9.3.2 The Genesis of the Influence of AT in Academic Journal Papers

The remainder of this section considers the early influence of AT in Western mathematics education research journals.[5] In planning this subsection I encountered two problems which I relate for the sake of intellectual transparency. First, how do I overcome the bias of simply considering papers with which I am familiar? My solution to this problem was to adopt a systematic means of considering paper. The second problem is, how do I do this in a reasonably short word length? My solution was to choose one primary source, the highly respected international journal *Educational Studies in Mathematics* (ESM). I searched the ESM web site using the keywords 'activity', 'Vygotsky', 'Leont'ev' and 'Engeström'. I choose the three names to ensure that I considered all the dominant approaches to AT. I stopped my search when I had papers that I considered represented all current approaches to AT employed by scholars in mathematics education research (at the time of writing). The remainder of this subsection provides a summary (with specific regard to tool use) of six papers from the period 1996 to 2003.

Two AT papers appeared in ESM in 1996, Bartolini Bussi (1996) and Crawford (1996). Although Crawford (1996) does report research it is largely an exposition of Vygotskian AT. It asks question such as 'What difference does the use of tools such as computers and calculators make to the quality of human activity?' and states that these, to Vygotsky, 'are cultural artefacts' (Crawford, 1996, p. 57) but the paper does not explore the nature of tools further. Three aspects of this paper in a leading academic journal suggest that AT was not, in 1996, widely known: the fact that the paper has an expository style (such styles are often used when a subject matter is new); there is no reference to Leont'ev or Engeström; the paper references only three works (all books not journal papers) from the field of mathematics education and these three books are only loosely associated with AT in having a social/practice orientation (Lave, 1988; Papert, 1994; Walkerdine, 1988).

Bartolini Bussi (1996) also has an expository style (with regard to the AT of Vygotsky and of Leont'ev) but its main focus is a report on a 3-year primary

[5] I focus on academic journals as I regard them as a dominant media through which ideas are circulated in academia.

mathematics teaching experiment on geometric perspective which was part of a wider project on mathematical discussion. The paper analyses the teaching experiment 'by means of the theoretical construct *semiotic mediation* (Vygotsky, 1978) in an attempt to substantiate its crucial effect on pupils' learning and metalearning' (Vygotsky, 1978, p. 13) and 'The *theory of activity, actions and operations* developed by Leont'ev (1978) is supposed to offer a suitable tool to either differentiate or coordinate the analysis of long term and short term processes' (Leont'ev, 1978, p. 15). The design of the teaching experiment includes tasks, mathematical discussions and 'appropriation of existing cultural artefacts (e.g. devices, texts and so forth' (Leont'ev, 1978, p. 22). The word 'tool' has two uses in the paper: Leont'ev's theory as a tool for analysis (see the quote two above); 'semiotic tools'. The term 'semiotic tool' is actually not defined in the paper but examples of semiotic tools are provided. One such example is a 'two column scheme', 'In the left column there was *reality*, in the right column *representation*' (Leont'ev, 1978, p. 26), which was 'built collectively in a discussion orchestrated by the teacher' (Leont'ev, 1978, p. 33). The two column scheme was created to highlight invariant and non-invariant properties of 3D objects in 2D representations; one column was for 'reality', the other for 'representation'. The two column scheme served to focus students' attention not only on what has changed but on what has not changed (the cultural–mathematical idea of invariance). Once the scheme had been created, it 'acted as a *semiotic tool* in perspective drawing for either producing or reading an image' (Leont'ev, 1978, p. 33).

Bartolini Bussi (1996) and Crawford (1996) show that 'AT had arrived' in mathematics education research in 1996 and Bartolini Bussi (1996) reveals a very specific appropriation of the word 'tool'. In 1998 there were two ESM papers that considered tool use in very different ways to Bartolini Bussi (1996), Chassapis (1998) and Pozzi, Noss, and Hoyles (1998), which I now consider.

Chassapis (1998) focuses on the processes by which children develop a formal mathematical concept of the circle by using various instruments to draw circles. It considers drawing circles: by hand; using circle tracers and templates; and using the compass. The primary theoretical influences are Vygotsky (the similarities and differences between signs and tools in activity; the difference between spontaneous/everyday and cultured/scientific concepts) and Soviet and Western interpreters of Vygotsky, e.g.: Zinchenko, 'tool-mediated action must be considered as the primary unit of analysis for a Vygotskian account of human mental functioning' (Chassapis, 1998, p. 276); and Wertsch, that tools "have been developed in a culture over extended periods of time and have become an integral part of human activity, being 'the 'carriers' of socio-cultural patterns and knowledge'" (Chassapis, 1998, pp. 275–276). Chassapis (1998) stresses that:

> The process of learning to use a tool, for example, an abacus, involves the construction of an experiential reality that is consensual with that of others who know how to use an abacus. As a consequence, when we use an abacus individually or while interacting with others, we participate in a continual regeneration of a consensual reality which both constrains and enables our individual ways of thinking and calculating. (Chassapis, 1998, p. 276)

Chassapis (1998) concludes that children's everyday concepts of a circle are global and static curvature concepts, not that of a set of points equidistance from a fixed point. These everyday concepts are in the realm of perceptual thinking and the use of freehand circle drawing and of circle tracers and templates does not radically change these everyday concepts. The use of the compass, however, 'structures the circle-drawing operation . . . may give rise to concepts constructed in the realm of action-bound practical thinking . . . constituting a potential ground for the development of analytical, more formal mathematical concepts of the circle' (Chassapis, 1998, p. 292).

Pozzi, Noss, and Hoyles (1998) results from research on nursing. As the chapter you are reading is on activity theoretic approaches, it is appropriate to mention that this research is one of several studies by this team where the object of the research activity is to understand mathematical practices in workplaces. The goal of this paper is to address the question 'how do resources enter into professional situations, and how do they mediate the relationship between mathematical tools and professional know–how?' (Pozzi et al., 1998, p. 110).The paper focuses on nurses administering drugs and monitoring fluid balance. The opening paragraph of the paper includes an unambiguous homage to the value of activity theory in such work:

> the entire corpus of work on activity theory, offers compelling evidence that individual and social acts of problem solving are contingent upon structuring resources, including a range of artefacts such as notational systems, physical and computational tools, and work protocols (Gagliardi, 1990). These artefacts are 'crystallised operations' (Leont'ev, 1978), borne out of needs within a given set of social practices, and in turn playing their part in shaping and restructuring future practices: artefacts exhibit an ongoing dialectic of producing and being produced by activity. (Leont'ev, 1978, p. 105)

The paper's conclusion is also framed in activity theoretic terms. Mathematics is bound into nurse's action, especially when there are concerns, for example that the wrong dose of a drug may have been given, but nursing activity is not arithmetic activity. Nursing activity includes mathematical artefacts/tools, such as rules for drug dosages and fluid balance charts, but the use of these tools is but a part of the activity of nursing.

Two years after the papers by Chassapis and Pozzi et al. ESM published an AT paper, Radford (2000), that signalled a new emphasis in mathematics education research, semiotic-cultural analysis. Radford's focus is on the early algebraic thinking (generalisation) which "is considered as a sign-mediated cognitive *praxis*" (Radford, 2000, p. 237) where the term 'sign' includes symbols, words, gestures, indeed anything that signifies. He grounds this conception in the work of Vygotsky, Leont'ev[6] and also Bakhtin (but a consideration of the later would be inappropriate in my brief exposition). I select a long extract which gives a flavour of the radical action/activity regard that Radford has towards signs in mathematics education:

[6] Leont'evs actually, father and son.

> instead of seeing signs as the reflecting mirrors of internal cognitive processes, we consider them as tools or prostheses of the mind to accomplish actions as required by the contextual activities in which the individuals engage. As a result, there is a theoretical shift from what signs *represent* to what they *enable* us to do . . . the signs *with* which the individual acts and *in* which the individual thinks belong to cultural symbolic systems which transcend the individual *qua* individual. Signs hence have a double life. On the one hand, they function as tools allowing the individuals to engage in cognitive praxis. On the other hand, they are part of those systems transcending the individual and through which a social reality is objectified. The sign-tools with which the individual thinks appear then as framed by social meanings and rules of use and provide the individual with social means of semiotic objectification . . . the conceptual and the signifying aspects of signs need to be studied in the activity that the signs mediate in accordance to specific semiotic configurations resulting from, and interwoven with, social meaning-making practices and cultural forms of signification (Radford, 2000, p. 241).

Radford focuses on small groups of Grade 8 students engaged in tasks in which they are to use circular counters ('chips') to generalise from visually presented sequences representing linear algebraic expressions (e.g. $2n - 1$). The role of the teacher is not only for the students to get the answer but to see for themselves the kind of answer they are to get. The analysis of the activity includes discourse analysis as students struggle to express the general through the particular, 'you always add 1 to the bottom, right?'. I cannot summarise the paper in this paragraph but I can point to Radford's focus on the intersection of semiotic means which allow the students to appropriate cultural forms (the use of letters):

Student: How many chips to have vertically . . . you would subtract 1 from how many chips
Teacher: But now you have to say it without using words! Use letters! OK?
Student: You have to do 1 *n* minus . . .

There is a sense in which Radford both continues and breaks with the traditions of cognitive studies in mathematics education, AT and semiotics: cognition is reconceived as social and cultural sign-mediated cognitive *praxis*; Vygotsky's distinction between signs and tools is blurred; the classic semiotic approach where sign, object and signified are regarded in isolation is replaced by an approach which focuses on joint acts of symbolising in context.

I close this section on the genesis of the influence of activity theory in mathematics education research by bringing the work of Engeström (and co-workers) into the picture. The first mention of Engeström in ESM appears in Jaworski (2003). This paper outlines a framework for 'both *insider* and *outsider* research and *co-learning* between teachers and educators in promoting classroom inquiry' (Jaworski, 2003, p. 249). It is not essentially concerned with tool use in doing mathematics though it does consider classroom inquiry as a 'developmental tool' (which may, in AT terms, be taken as 'a mediational means to assist the development of teachers and researchers in pursuit of their educational objectives'). The paper has a brief afterword:

> It seems important to mention the suggestion of one reviewer that discussion of knowledge and learning relating to social and societal significance might be recast in terms of an

activity theory perspective. Subsequent work on these ideas led to my development of a mapping between the framework here and Engeström's "mediational triangle". (Jaworski, 2003, p. 276)

Barbara Jaworski ends the paper with 'I plan to work further on these ideas' which, as we shall see in the next section, she did. There are a number of issues worthy of discussion arising from the papers considered above but I leave this until the final section of this chapter and I now move on to consider activity theoretic approaches in mathematics education in the early part of the twenty-first century with, of course, particular regard to tool use.

9.4 Activity Theoretic Approaches in Mathematics Education in the Twenty-First Century

Since the Jaworski paper, 2003, AT has exerted a strong influence on research in mathematics education and it would be rather foolish of me to attempt a summary of this research. Further to this, in selecting research reports to review I wished, as I stated in the preamble to Sect. 9.2.2, to avoid bias by simply considering work with which I am familiar, so I once again looked for a source. ESM would be a suitable source but 2012–2013 saw the publication of a two-volume special edition of *The International Journal for Technology in Mathematics Education* devoted to *Activity Theoretical Approaches to Mathematics Classroom Practices with the Use of Technology* and this seemed a closed but appropriate set of papers in which to examine current AT research practices as it is likely, with its focus on technology, to raise issues related to tool use.

I first describe the corpus of papers in this Special Issue. Of the 11 papers 5 are more or less 'straight AT': Abboud-Blanchard and Cazes (2012), Chiappini (2012), Jaworski, Robinson, Matthews, and Croft (2012), Ladel and Kortenkamp (2013), and Maracci and Mariotti (2013). Another two, Robert (2012) and Abboud-Blanchard and Vandebrouck (2012), are AT with a specific French interpretation. A further three jointly consider several approaches (called 'networking theories'): Fuglestad (2013) networks AT and the instrumental approach; Kynigos and Psycharis (2013) networks constructionism and the instrumental approach; Lagrange (2013) networks AT and the anthropological theory of didactics. Monaghan (2013) considers a socio-cultural theory, Valsiner's 'zone theory', that shares Vygotskian roots with AT. The diversity of papers illustrate that AT is open to national variation and networking with related theories. I now summarise the purportedly 'straight AT' papers with specific regard to artefacts/tools.

Chiappini (2012) focuses on the teaching and learning of mid-level algebra (equations, functions, inequalities and equivalence associated with expressions such as $x^2 - 2x - 4$) with software, called *Alnuset*, with a visual 'algebraic line' and conventional algebraic notation, to draw students' attention to the culture of mathematics (see Fig. 9.1). Chiappini is a software designer as well as a

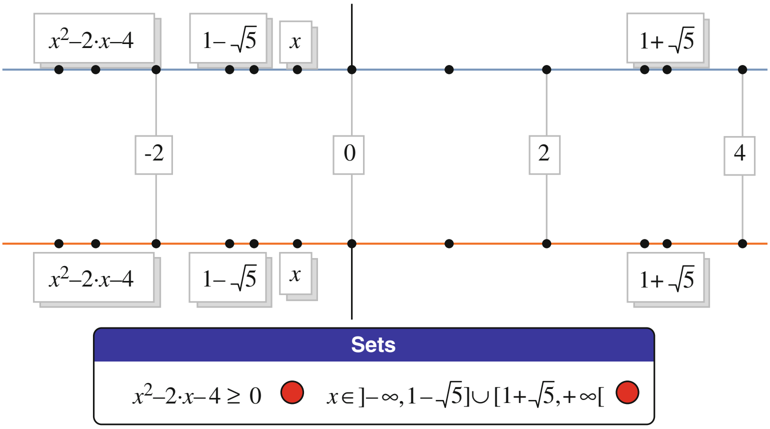

Fig. 9.2 A screen shot from A*lnuset*

mathematics educator and draws on work from Fig. 9.2, *Alnuset*'s algebraic line the HCI strand of AT which employs the Gibsons' construct of *affordances* (considered in Sect. 7.2) with regard to 'usability research both as an heuristic and an ad hoc design principle to describe the potential of a (computer) system with regard to its effectiveness' (Chiappini, 2012, p. 135). There are two interpretations of affordances (of a system for a user) in the HCI community which hinge on whether the affordances are perceived or not; the significance of this difference for HCI work lies in the potential for user actions. This difference leads to a distinction between the usability of a system (how a task can be completed) and its usefulness (how a system responds to user actions). Chiappini regards this distinction and, in particular, the construct 'usefulness' as

> important in educational contexts where students may not focus clearly on the objectives of the task at hand and teachers' goals do not necessarily coincide with those of their students in a didactical activity mediated by a digital artefact. In particular, the notion of usefulness makes it possible to evaluate the affordance provided by the system software: to promote in students the emergence of the objectives for the solution of the task they are engaged in; to support the development of the teacher's cultural goals (development of knowledge, meaning, principles and values of the discipline) that may also transcend those of the task in which students are involved. (Chiappini, 2012, p. 135)

Chiappini seeks to employ this distinction to evaluate student–teacher use of *Alnuset* and, to this end employs an HCI breakdown of affordances: perceived affordances; ergonomic affordances, which allow 'embodied actions involved in solutions of tasks and sub-tasks peculiar to the context' (Chiappini, 2012); and cultural affordances, which concern

> the cultural teaching/learning objectives underlying the system being used. Evaluation of cultural affordances can be carried out through the analysis of how meanings, values and principles underlying the action mediated by the use of the embodied actions, get to be known through the artefact-mediated activity. (Chiappini, 2012).

Chiappini's framework can be viewed as an elaboration of Leont'ev's activity–action–operation triple specifically designed for the evaluation of mathematics education with software systems. The final section of Chiappini (2012) develops a framework to evaluate the cultural affordances of *Alnuset* which uses Engeström's expansive learning cycle in four phases. In terms of tools these are as follows.

1. The students are given a task (an open algebraic problem) and the artefacts/tools embedded into the software provide output to student input, some of this output surprises the student and produces cognitive conflict.
2. Tasks are then:

> designed in order to exploit the visuo-spatial and deictic ergonomic affordance of the algebraic line to allow students to explore the conditions, causes and explicative mechanisms of conflicts . . . the teacher's crucial task consists in the introduction of terms and algebraic notions found in the visuo-spatial and deictic narration of the various problematic situations (Chiappini, 2012, p. 139)

3. The teacher encourages the student to recast their work using *Alnuset*'s algebraic manipulation facilities in order for them to mathematicise surprises encountered in earlier work:

> In this phase the teacher encourages both the establishment of the algebraic axiomatic model in the student's practice and the development of meta-cognitive processes involved in the re-configuration in symbolic terms of the algebraic meanings expressed beforehand in visuo-spatial and deictic terms. (Chiappini, 2012)

4. Students, with a transformed understanding of algebraic activity, and teachers, with a transformed understanding of their students' understandings, engage in teacher-led whole class consolidation of their understandings.

There are strong parallels in this paper to Radford's (2000) paper considered above. Both emphasise the cultural–historical (one might say 'unnatural') objectification of mathematical knowledge. But there are differences too: Chiappini employs Engeström's expansive learning cycle; Radford places greater emphasis on semiotics and does not employ digital technology.

Jaworski et al. (2012) focuses on an undergraduate mathematics module for engineering students that employs inquiry-based tasks and a computer system *GeoGebra*. The teachers had put a lot of effort into designing a module to enhance student engagement in mathematics and the object of their research was to evaluate this design from a learner and a teacher perspective; the paper focuses on the aspect of this evaluation related to the use of *GeoGebra*. An initial evaluation using student surveys revealed some positive comments in terms of better understanding but also 'Just because I understand maths better doesn't mean I'll do better in the exam'. The AT analysis conducted put the *GeoGebra* aspect of the work in perspective, 'It is the whole with which we work and in which we participate'. They analyse the whole from the perspective of both the students and the teachers using separately both Leont'ev's activity–action–operation triple and Engeström's expanded mediational triangle. Both analysis reveal differences between students and teachers:

> Perhaps the most important difference is the *object* of activity (Engeström) or *the motivating force* (Leont'ev) for the two systems. Both are valid, but the fact that they are different means that along with other factors—values placed on forms of understanding (the *rules* of the enterprise) or whether GeoGebra is positively helpful in promoting learning (mediating artefacts)—they result in the tensions observed. (Jaworski et al., 2012, p. 151)

This paper says virtually nothing with regard to tool use. There is no explicit mention of tools in the paper and two instances where the word 'artefact' is used (one in the quote above and one in relation to Engeström's expanded mediational triangle). At one level this is surprising in a paper considering the use of *GeoGebra* in a mathematics module but the paper does take a holistic view of the module (we shall return to this via a consideration of the unit of analysis later in this chapter).

Ladel and Kortenkamp (2013) focuses on the design and use of a multi-touch-table (a large touch-screen artefact that registers input from fingers, not just a finger) to engage young children (5–7 years of age) in meaningful work with whole number operations. The paper notes a feature of the child-technology environment which has similarities to the Gibsons' construct of *affordances*, they note that 'such technology ... enables children to work with virtual manipulatives directly instead of being mediated through another input device' (Ladel & Kortenkamp, 2013, p. 3); and they also note that 'We want to restrict the students' externalizing actions to support the internalization of specific properties of the objects[7] in consideration ... Thus the mediation through the artefact is characterized by restriction and focussing.'

Ladel and Kortenkamp (2013) adapt Engeström's expanded mediational triangle in what they call an 'artefact-centric activity theory' model (ACAT, see Fig. 9.3). They note that:

> the artefact itself does not have agency and is only mediating ...[but] the artefact changes the way children act drastically and in non-obvious ways ...we use Activity Theory not only for analyzing the interaction between subject and object, but in addition for designing the artefact. We adapted the activity system diagram of Engeström ...We believe that *Rules* ... should also affect the design of the artefact, thus we need a new relation between these two nodes. For clarity we omit the division of labor from the diagram. Because our focus lies on the artefact, we are not considering the relations between the rules and subject, object and community in this article, though they are important for a full activity system ... (Ladel & Kortenkamp, 2013, p. 3)

Ladel and Kortenkamp (2013) view students' arithmetic work in the light of Leont'ev's activity–action–operation triple and conclude that 'Through the lens of ACAT that places the artefact in the center of attention we can locate the various areas of didactic and pedagogic design that have to be taken into account' (Ladel & Kortenkamp, 2013, p. 7). In contrast to Ladel and Kortenkamp's account of artefact mediation Maracci & Mariotti (2013) present a very human-centred view of mediation.

[7] Ladel and Kortenkamp (2013) consistently use the word 'object' to mean a 'thing'. This occurs elsewhere in papers in this Special Issue. We consider this interpretation later in the chapter.

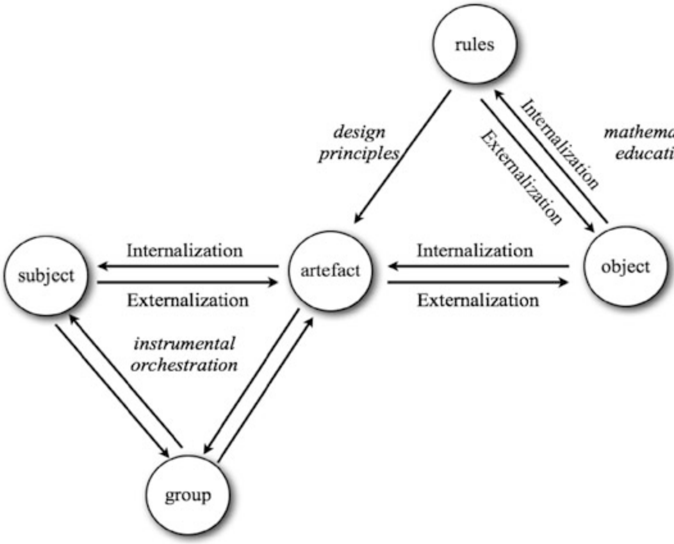

Fig. 9.3 A representation of the ACAT theory

Maracci and Mariotti (2013) outline the Theory of Semiotic Mediation (TSM) with regard to 'the use of artefacts to enhance mathematics teaching and learning, with a particular focus on technological artefacts' (Maracci & Mariotti, 2013, p. 21). This paper continues a long line of papers on semiotic mediation originating from Bartolini Bussi (1996) considered in the previous section. TSM draws on the AT of Vygotsky and of Leont'ev but they are critical of research where 'the mediating function of the artefact is often limited to the study of its role in relation to the accomplishment of tasks' (Bartolini Bussi, 1996). TSM is essentially semiotic in that:

> teaching-learning . . . originates from an intricate interplay of signs. . . individuals have to be involved in semiotic processes leading to the explicit formulation of the meaning they have developed in relation to an activity, in order to become conscious of such meanings . . . mathematical meanings can be crystallized, embedded in artefacts and signs . . . (Bartolini Bussi, 1996)

TSM also draws on the work of the socio-linguist Hasan who distinguishes between the mediator, the thing (which may be a concept) that is mediated, the mediatee and the *circumstances* for mediation. Following Hasan, Maracci and Mariotti (2013), claim that:

> The mediator is not the artefact itself but it is the person who takes the initiative and the responsibility for the use of the artefact to mediate a specific content . . . artefacts are among the constitutive elements of the "circumstances for mediation". In fact, the modalities of use of the artefacts, the tasks to be accomplished, the whole organization of the classroom work, the classroom interactions among students and between them and the teachers are constituents of the "circumstances for mediation". (Bartolini Bussi, 1996, p. 22)

Leont'ev's activity–action–operation triple provides a frame for studying *circumstances* for realising the desired semiotic mediation. To mediate the learning of mathematics the teacher has to design specific circumstances, a didactical cycle, aimed at fostering specific semiotic mediation processes: accomplishing a task with the artefact; producing signs related to the artefact use; and classroom discussion. A central aim of the didactical cycle is the '*unfolding of the semiotic potential* of the artefact' which I interpret as having similarities to what Radford (above) calls 'objectification': students sitting in a mathematics classroom at the beginning of a (sequence of) lesson(s) are there to learn mathematics and do not know what they are to learn; the central aim of the teacher is that the students appropriate cultural (scientific) meanings. It is crucial that teachers design tasks which 'lead students to develop personal meanings related to the artefact use having the potential to evolve towards mathematical meanings' (Bartolini Bussi, 1996, p. 23). All three parts of the didactical cycle are essential for personal meanings to become shared meaning and for the teacher to shape these shared meanings into public scientific meanings. Artefacts are an essential part of this cycle but they are not mediating agents in the TSM.

Abboud-Blanchard and Cazes (2012) interprets research on Electronic-Exercise-Bases (EEB), digitised mathematical exercises. The research was carried out over 3 years with 30 teachers with a focus on three phases of teachers' use of EEBs, 'the preparation of the lesson, its progress and the reflexive return that the teacher makes on this lesson' (Abboud-Blanchard & Cazes, 2012, p. 142). The research questions that the paper addresses are, 'Why and how do teachers use EEB? What effect does this use have on their teaching activity?' (Abboud-Blanchard & Cazes, 2012, p. 141). The paper uses Engeström's expanded mediational triangle (unamended) to interpret the data (teacher interviews and classroom observations). Like Ladel and Kortenkamp (2013), the authors sometimes appear to use the word 'object' to mean a 'thing', e.g. AT 'studies a subject acting on an object to produce a result' (Abboud-Blanchard & Cazes, 2012, p. 142). The paper does not use the word 'artefact' but does use the words 'tool' and 'instrument', for example in explaining the terms of Engeström's expanded triangle they write 'The tool allows the subject to exercise her/his activity. It is a set of tools or of instruments. The essential instrument in this study is the EEB' (Abboud-Blanchard & Cazes, 2012).

Abboud-Blanchard and Cazes are French researchers but, apart from the use of the word 'instrument', there is nothing particularly French about Abboud-Blanchard and Cazes (2012). In contrast, the final two papers I consider (Abboud-Blanchard & Vandebrouck, 2012; Robert, 2012) do present 'a French take' on AT. I would like to add here that I do not regard one's nationality as determining one's theoretical framework: Chiappini, Mariotti and Maracci are all Italian but their papers present differing foci within AT. But there is a specific line of inquiry within French mathematics education research that takes its cue from Leplat (1997). Leplat focused on the psychology of the workplace, viewing the characteristics of the tasks and the characteristics of the workers in two dialectical feedback loops with 'activity' in the middle, the 'double approach': a production loop in which activity is object-oriented to the task(s) at hand; a construction loop in which activity is subject-oriented to the development or well-being of the workers. Rogalski (2013, p.7) summarises this thus:

> The situation is a determining factor of the activity, and is simultaneously itself modified by the activity. This modification primarily affects the object of the activity, but can also include modification of resources and constraints. Subjects, too, both determine the activity and are modified in turn by their own activity.

Some (not all) double approach researchers also 'network' this approach with Rabardel's *instrumentation* theory and Chevallard's *anthropological theory of didactics* (considered in Chap. 10). I now consider the papers of Robert (2012) and Abboud-Blanchard and Vandebrouck (2012).

Robert (2012) outlines the 'double approach' with regard to 10 years of research on students' and teachers' activities in and out of mathematics classrooms. She stresses that this work addresses AT 'from a cognitive individual perspective, not as a whole system . . . [not] the socio-cultural contexts of students and teachers' (Robert, 2012, p. 151). The main foci of this work has been on students solving exercises and teachers' monitoring of student work (this is consistent with Leplat's characteristics of the tasks and of the workers). The focus on student work continues a French cognitive strand and Robert references Douady and Vergnaud and stresses 'knowledge': old knowledge, new knowledge, knowledge to be used, states of knowledge, reorganisation of knowledge, recognition of knowledge, lack of knowledge, knowledge to be adapted, . . . Further to this:

> student learning is tied to the quality of the so-called "scenario," but it is also tied to the precise way the students work on the corresponding tasks. So, the better we describe the offered (proposed) tasks, the better we succeed in understanding students' actual activities (Robert, 2012, p. 155)

The focus on the teacher in this body of research is at a local and a global level. At the local level this involves studying the ways that teachers interact with students and their mathematical work and (with an implicit references to Leplat) distinguishes between procedural help (directed at the task completion) and constructive help (with a focus on the students' interpretations of the task). The global level considers the management of student activities with respect to the craft knowledge of the teacher-in-context. The local and global level are interrelated. Robert also stresses (again with an implicit reference to Leplat) the interrelated productive (students completing tasks) and affective dimensions of teaching. Robert (2012) is a general introduction to the double approach and does not focus on the artefact/tools used in mathematics classroom.

Abboud-Blanchard and Vandebrouck (2012) is written as a follow up to Robert (2012) with a focus on teachers' practices in technology-based lessons with particular regard to Leplat's production and construction loops. The genesis of teachers' practices with technology are considered to have 'external aspects which correspond to the evolutions of the teachers' productive activity throughout technology-based lessons, but also have internal aspects related to the constructive activity which accompanies these evolutions' (Abboud-Blanchard & Vandebrouck, 2012, p. 159). It is assumed that teachers' practices are stable and the evolution of these practices involves three levels: the micro level, which has similarities to Leont'ev's 'operations'; the local level, which refers teachers' goals and actions; and the global level, which refers teachers' motives. Abboud-Blanchard and Vandebrouck (2012) explanation of teachers' technological geneses puts forward a two-stage process in which these levels interact.

In the first stage the local level is regulated by the micro level. When a teacher first uses a new tool in the classroom the 'the automatic regulation of teaching practice at the micro level allows the teacher to cope with difficulties emerging during the technology session at the local level' (Abboud-Blanchard & Vandebrouck, 2012, p. 160) but this is usually short-lived and 'some teachers feel the need to build new specific practices with technology, while others will tend to reduce the role of technology within their teaching' (Abboud-Blanchard & Vandebrouck, 2012). This stage concerns Leplat's 'production loop' but 'it generates constructive activity at the medium and long-term' (Abboud-Blanchard & Vandebrouck, 2012, p. 161) and this leads to the second stage.

The second stage has two parts. The first part concerns the movement from the local to the global level and includes an evolution of the production loop and the development of the construction loop, 'There is a new balance between traditional sessions and technology sessions, between collective work and individual phases of students' activity or between old and new mathematical knowledge in students' activity' (Abboud-Blanchard & Vandebrouck, 2012). The second part concerns the movement from the local level to the micro level (the refinement of the teachers' understanding of the artefacts/tools they are using in their classrooms). This part develops over time as a teacher goes from 'tinkering' with an artefact, to using it as a tool for personal mathematics, to 'tinkering' with an artefact in the classroom, to assisting the technomathematical development of their students' use of a new tool for doing mathematics.

I now consider emphases and tensions in these papers together with the approaches considered in the previous section.

9.5 Emphases and Tensions in Mathematics Education Activity Theoretic Approaches

There are similarities in the approaches in the mathematics education papers considered in Sects. 9.2 and 9.3. Every paper: pays homage to Vygotsky by mentioning his works directly or indirectly (via Leont'ev or Engeström); places a positive valuation on considering 'practice' (though what 'practice' involves varies); attempts to describe (rather than prescribe) a practice (bar, possibly, Chiappini, 2012). But there are also differences which I shall consider briefly under the following interrelated categories: sign and tool; unit of analysis; cognition; the cultural–historical dimension; mediation.

I have mentioned (in Sects. 7.2 and 9.1) Vygotsky's observation on the similarities and differences between signs and tools. Vygotsky's view of these similarities and differences is present in the early papers (Chassapis, 1998; Crawford, 1996; Pozzi et al., 1998), is implicit in Bartolini Bussi (1996) and, as mentioned above, Vygotsky's view is extended and, to some extent blurred, in Radford (2000). But when we consider the more recent IJTME papers there are differences and omissions. Neither Jaworski et al. (2012) or Ladel and Kortenkamp (2013) consider signs or tools in the body of the text, though the latter does place emphasis on artefacts. Maracci and Mariotti (2013) make much of signs and artefacts but only mentions tools once in a quote. In the three papers by French authors 'sign' is only mentioned (twice) in Robert (2012). These differences, I feel, go beyond terminology used and reflect differences in the basic fabric of scholastic mathematical activity.

I shall consider differences with regard to the unit of analysis and cognition together as it seems important, to me, for mathematics education research, whether cognition is an explicit part of the unit of analysis. Despite the importance of the unit of analysis for AT research, not all papers explicitly consider the unit of analysis. Considering the early ESM papers it is explicitly mentioned by Chassapis (1998, p. 276), 'tool-mediated action must be considered as the appropriate primary unit of analysis' and Radford (2000, p. 244) who used it to guide his data analysis, '*situated discourse analysis* whose elementary unit (i.e. the unit of analysis) was constituted by the refined (i.e. contextualised and cadenced) identified salient segments'. Cognition, mathematical thinking with signs/tools, is central in both of these papers. Neither Bartolini Bussi (1996), Crawford (1996) or Pozzi et al. (1998) explicitly mention the 'unit of analysis' but (1) Bartolini Bussi (1996) clearly considers the long-term teaching and learning process as the unit of analysis, and (2) in Crawford (1996) and in Pozzi et al. (1998) cognition is viewed in a wider context where there is bi-directional 'shaping': 'mathematical knowledge increasingly shapes and is shaped by human activity' (Crawford, 1996, p. 46);

> In the past, the issue tended to be seen in purely cognitive terms . . . Now investigations tend to focus on how activities are shaped by the social practices and goals of the working culture, and to examine how this shaping informs our understanding of mathematical behaviour and learning. Pozzi et al. (1998, p. 105).

There is no mention of 'unit of analysis' in any of the IJTME papers but my reading of the seven papers puts them into four camps with regard to what this unit might be and the place of cognition in this unit. The first is a 'systems approach' (Engeström's model with reference to Leont'ev's triple) where the implicit unit of analysis is the activity system and, in which, cognition is an implicit part of this system; I put the papers by Abboud-Blanchard and Cazes (2012) and Jaworski et al. (2012) in this camp. The second camp is reflected in the papers by Robert (2012) and Abboud-Blanchard and Vandebrouck (2012) which consider 'Activity Theory from a cognitive individual perspective, not as a whole system. It does not address a more general point of view, involving the socio-cultural context of students and teachers' (Robert, 2012, p. 153). The third is the papers by Chiappini (2012) and Ladel and Kortenkamp (2013). Both of these papers use the Engeström model (adapted in the case of the second paper) where the model is the implicit unit of analysis but, in which, individual cognition (with artefacts) is an intrinsic component. The fourth camp is a singleton, the paper by Maracci and Mariotti

(2013) where the implicit unit of analysis is the teacher-mediated didactical cycle and cognition is an intrinsic component:

> A didactical cycle, or an iteration of didactical cycles, can be seen as an activity whose motive is to promote the generation of students' personal signs related to the accomplishment of a task through an artefact and their evolution towards desired mathematical signs. (Maracci & Mariotti, 2013, p. 23)

With regard to the cultural–historical dimension, the papers considered, in my opinion, fall into two camps: those that embrace this dimension (Bartolini Bussi, 1996; Chassapis, 1998; Chiappini, 2012; Crawford, 1996; Ladel & Kortenkamp, 2013; Maracci & Mariotti, 2013; Pozzi et al., 1998; Radford, 2000); and those that appear to ignore this dimension (Abboud-Blanchard & Cazes, 2012; Abboud-Blanchard & Vandebrouck, 2012; Jaworski et al., 2012; Robert, 2012). Those in the first camp do not view mathematical activity as a 'natural' unfolding of psychological development. Mathematics has a culture steeped in a history and, in workplace mathematics:

> Fluid balance charts, like many informational resources in the workplace, are not products designed for the benefit of individuals; they are cultural products, in constant use by members of a working community. (Pozzi et al., 1998, p. 115)

Radford's (2000, p. 240) provides a non-ambiguous statement of the importance of the cultural–historical dimension:

> as long as the relation subject/object is seen as a non-culturally-mediated, direct one, meaning construction appears to be the result of the relation that the isolated subject entertains with the *a*historical object

I do not claim that those in the second camp view mathematical activity as a 'natural' unfolding of psychological development but they do not say that it is not this.

My final consideration of differences in the approaches in the mathematics education papers considered in Sects. 9.2 and 9.3 concerns mediation. Mediation, by people and/or language and/or sign/artefacts/tools, is a central concept in the majority of papers considered except in Robert (2012), which does not mention 'mediation', and in Abboud-Blanchard and Cazes (2012) and Jaworski et al. (2012), where consideration of 'mediation' is mainly restricted to mentioning its importance in the theoretical frameworks of Leont'ev and Engeström. But behind the 'and/or's in the previous sentence are different emphases with regard to mediator. These emphases are most clearly marked in the papers by Ladel and Kortenkamp (2013) and Maracci and Mariotti (2013). To Ladel and Kortenkamp (2013) artefact mediation is in the centre of their model, which goes by the name of 'artefact-centric activity theory' but to Maracci and Mariotti (2013, p. 22), 'The mediator is not the artefact itself but it is the person who takes the initiative and the responsibility for the use of the artefact to mediate a specific content'. I suspect that behind 'theoretic statements' on mediation, there are the phenomena that interest us as researchers. Ladel and Kortenkamp are clearly interested in the potential of their artefacts, multi-touch-tables, to improve learning. Maracci and Mariotti, as noted above, continues a line of papers on semiotic mediation that can be traced back to Bartolini Bussi (1996) who focused on mathematical discussion, where human mediation is a central consideration. Further to this, they state that they follow Hasan in formulating their interpretation of mediation. Hasan, as noted above, is a socio-linguist who has based her academic career on the study of everyday discourse, for example, between mothers and daughters. Hasan is interested in such things as daughters' appropriation of the language of their mothers and Maracci and Mariotti appear to have appropriated Hasan's focus for mediation.

I feel that the differences considered above show that AT is a loose collection of approaches (at least in mathematics education) and is not a unified theory. In closing this section I would like to bring in my interests in tools and mathematics and consider tensions in activity theoretic approaches with regard to Leont'ev's activity–action–operation triple. Artefacts/tools are important in each element of the triple but the focus on artefacts/tools is different in each element. In the *operation* element we may focus on the details of manipulating an artefact/tool. Such a focus is likely to interest a mathematics educator who is convinced that a calculator or a dynamic geometry system (DGS) or whatever can help students do/learn mathematics by establishing relationships between mathematical objects but that certain configurations (e.g. modes of dragging in a DGS) of the artefact/tool are important to optimise learning. In the *action* element the mediating qualities of an artefact/tool become paramount and mathematics educators may focus on the transformation of actions by different artefacts/tools; the differences, for example, in drawing the graph of a specific function using pencil, ruler and graph paper compared to drawing the graph of the same function using *GeoGebra*. The human part of this focus on action may be an individual or a group of individuals but when we consider the *activity* element it is always a group. The analysis *activity* element includes the *operation* and *action* elements but its consideration of mediation goes beyond artefact or person mediation to 'include the institutional contexts and history of the systems of activities' (Cole, 1996, p. 333) under investigation.

Although Leont'ev's activity–action–operation triple is not supposed to be ripped asunder, it can be difficult to combine the elements. Cole (1996, pp. 332–334) considers similar matters in relation to Wertsch's focus on mediated action and Engeström's focus on activity systems, he concludes:

> Mediated action and its activity context are two moments of a single process, and whatever we want to specify as psychological processes is but a moment of their combined properties. It is possible to argue how best to parse their contributions in individual cases, *in practice*, but attempting such a parsing "in general" results in empty abstractions, unconstrained by the circumstances to which they are appropriate. (Cole, 1996, p. 334)

LaCroix (2009) goes further than Cole (though not with regard to Wertsch) and argues, in the context of an individual case, that Engeström's approach and Radford's approach 'do not sit well together'. The case concerns adult students (pre-apprentices in the pipe-trades) learning to read fractions-of-an-inch on a measuring tape (an essential trade-skill) in a course. LaCroix was a participant observer and collected data from multiple sources over the 8 week course. He analysed the data in two separate stages using Engeström's approach and then Radford's approach and notes that the analysis from the point of view of Radford's approach sometimes required 'frame-by-frame analysis of videotape to assess the role and co-ordination of spoken language with the use of artifacts and gestures' (LaCroix, 2009, p. 856). Both analyses produced interesting results but:

> [Engeström's] foci, while useful for research in many contexts, run counter to mathematics educators' practical interests in teaching and learning activity, that is, individual students' mathematical enculturation on a day-to-day, if not minute-to-minute basis . . . [Radford] provides a way of defining and positioning mathematics as a cultural practice within particular forms of activity . . . [Engeström] theorizes learning within activity theory as change in the activity itself, Radford focuses on the learning of individuals as they come to be part of an existing historical-cultural activity. (LaCroix, 2009, p. 859)

This statement is similar to my statement above on the phenomena that interest researchers and is linked to my statement, early in this chapter, that activity theorists argue amongst themselves about the appropriate unit of analysis. I think that AT has contributed much to our understanding of tool use in mathematics but it offers us nuanced understandings.

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