Chapter 19
Connectivity in Mathematics Education: Drawing Some Lessons from the Current Experiences and Questioning the Future of the Concept

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19.1 Introduction

'Connecting' is certainly the best representative verb of the Internet era. It gave matter to a lot of constructs, some grounded in research and some purely speculative, 'digital natives' being an example of the later (Helsper & Eynon, 2010). The noun coming from this verb, connectivity, is now used in a number of contexts. The wiki based open content dictionary Wiktionary proposed three meanings: the state of being connected, the ability to make a connection between two or more points in a network in a graph, and a measure of concatenated adjacency (the number of ways that points are connected to each other).

In this Chapter, I focus down on mathematics education issues grounded, as far as possible (because we have not always a sufficient hindsight) in research. In the mathematics education community, the 17th ICMI study (already evoked in Chap. 12) evidenced a strong emergence of the notion of connectivity, constituting the focus of a panel of the conference (Hoyles et al., 2010). The subject Index of this ICMI study proceedings (Hoyles & Lagrange, 2010, p. 486) reveals a number of occurrences, with different meanings: the first one is a *technological* one (the potential, for a given artefact, or an environment, for connecting people to people and/or to Internet); the second one is a *social* one (the state for people, of being connected vs. the ability to make connections) to other people and/or to Internet; the third one is a *cognitive* one (the state vs. the ability, for an individual, of connecting different mathematical representations and meanings); the fourth one is a *theoretical* one (the state, for theoretical frameworks, of being connected vs. the ability to make connections to other theoretical frameworks).

Such a dispersion of meanings is a feature of an emerging concept, and of its potential. As stated by Hoyles et al. (2010), p. 440: '[. . .] if and how connectivity, in whatever form, transforms mathematical practices in school is a matter of future investigation'. I will conceive this chapter with respect to the emerging situation of this concept, looking at connectivity in the thread of my own experience as a

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teacher and researcher in mathematics education, over the last (at least) 30 years. It seems to me that.

The first section looks at connectivity through the evolution of students' connections to other students, and to mathematics, over their classroom activities. The second section considers connectivity through the evolution of teachers' connections to other teachers, and to mathematics resources, over their *documentation* work (Chap. 15). The third section, back to the ICMI study connectivity panel, questions the notion of cognitive connectivity. The conclusion discusses the dynamics of the concept of connectivity itself.

19.2 Connecting Students and Mathematics Through Digital Artefacts

I present in this section three environments I successively work with, bringing out the strong evolution of students activity according to the available *connecting tools*, keeping in mind that an environment for mathematics learning is not only constituted by sole tools, but also by mathematical problems and teacher' *instrumental orchestrations* (Sect. 15.2.5).

19.2.1 The Sherpa Student Configuration

I could say that the environment based on the sherpa-student configuration (Fig. 19.1) has grounded my reflection about the teacher's role in computerised environments. As a teacher (around 1990), confronted to the usages by students of more and more powerful calculators (see Chap. 13), I wanted to make communicate the small screens, i.e. to go against a spontaneous tendency of each student to keep for him what he was doing, and thinking, with his own calculator. The opportunity for doing that was offered by the calculator manufacturers, providing teachers with a 'view-screen', that is: a tablet with a transparent screen, and a *short cable* connecting it to a calculator (see Fig. 19.1). This device, posed on an overhead projector, allows the calculator screen to be displayed on the classroom whiteboard, or a screen, and then to be visible by the whole classroom. I underline the expression 'short cable', because it was quite obvious, for the manufacturer, that the calculator at stake was the teacher's one: this device was intended to allow the teacher to project *his own calculator* on the screen (it was then in this way that the advertising pictures demonstrates its use). My idea was, instead of connecting to the view-screen *my* calculator, to connect one of my students' calculators.[1] I name

[1] To be noticed, shortly after its first appearance, the cable at stake became longer, allowing a wider use of the view-screen: the material evolves for fitting the usages...

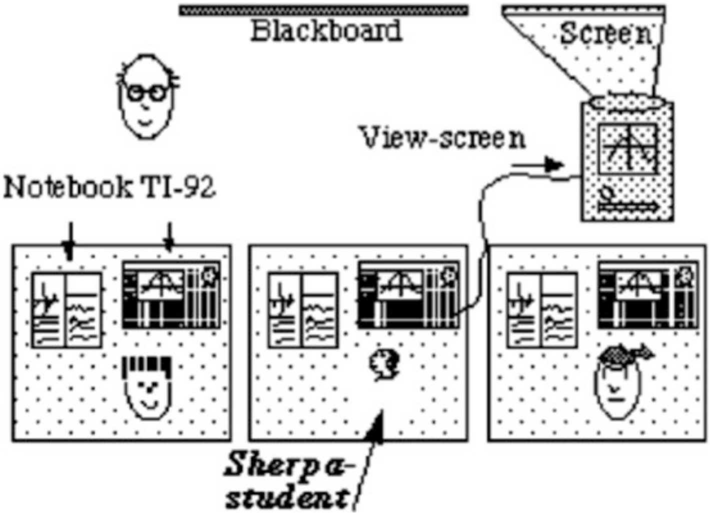

Fig. 19.1 The didactical configuration of the Sherpa-student (Trouche, 2004, p. 298)

him/her the sherpa-student, referring to the person who guides and who carries the load during expeditions in the Himalaya Mountains. My intention was then to underline the *responsibility* of this student helping the class to find its way towards the solution of a given problem, and the *difficulty* of the task.

I presented (Trouche, 2004, p. 299) various exploitation modes of such a configuration, and their possible consequences, in terms of *instrumentation* and *instrumentalisation* processes (Sect. 10.4):

- Sometimes calculators are turned off (and so is the overhead projector): it is then a matter of paper/pencil environment work.
- Sometimes both calculators and overhead projector are turned on and work is strictly guided by the sherpa-student under the supervision of the teacher (students are supposed to have exactly the same thing on their calculator screens as is on the projector screen). Instrumentation and instrumentalisation processes are then strongly constrained.
- Sometimes calculators are on as well as the overhead projector and work is free for a given time. Instrumentation and instrumentalisation processes are then relatively constrained (by the type of activities and by referring to the sherpa student's calculator which remains visible on the big screen).
- Sometimes calculators are on and the projector is off. Instrumentation and instrumentalisation processes are then only weakly constrained.

> These various modes seems to illustrate what Healy (2002) termed filling out and filling in, in the course of classroom social interaction:—when the sherpa-student's initiative is free, it is possible for mathematically significant issues to arise out of the student's own constructive efforts (this is a filling out approach);—when the sherpa-student is guided by the teacher, it is possible for mathematically significant issues to become appropriated during the student's own constructive efforts (filling in approach).

Finally, the usage of such a configuration (Trouche, 2004) evidences that the sole connection of one student's calculator to the common classroom screen, monitored by the teacher, contributes to foster the interactions between the students' instruments and their mathematical thinking. It was, for me, a first occasion of analysing the strong potential of connectivity, understood actually in the three first meanings of this word (see Introduction below): opening opportunities of connections between one student and the whole class through a technological device; opening opportunities of connections between students through the common classroom screen; and opening opportunities of connections between different mathematical representations and meanings, each student having to combine what appears on her calculators screen, and what appears on the classroom screen.

19.2.2 The Calculators Network Configuration

The second occasion for meeting connectivity happens 10 years after (around 2000), when, as a researcher, I analysed the potential of a new device, TI-Navigator, providing wireless communications between students' graphic calculators and the teacher's personal computer (Fig. 19.2). This device consists in an amplification of the view-screen potential, as it allows the teacher to see, on her screen, all the students' screens; she can then decide to connect a given calculator (or some of them), throughout her own computer, to the classroom screen.[2] It leads to a new organisation of the classroom workspace. A manufacturer advertising (Fig. 19.2, left) proposed a configuration attached to a technical constraint: the wireless connection works between hubs, each of them linking four calculators, and the teacher's computer. Then, a natural decision is to split the class into groups of four students. The team of teachers I observed (Hoyles et al., 2010, p. 449) decided

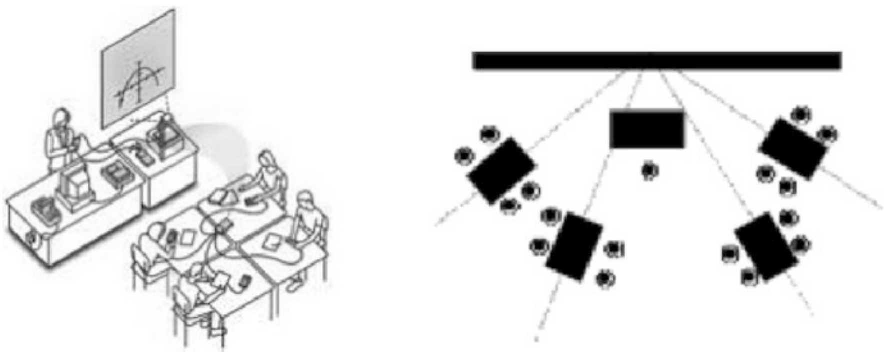

Fig. 19.2 New supports for connecting students' calculators and classroom screen (Hoyles et al., 2010, p. 449)

[2] Actually, in the context of the sherpa-student configuration, I used also to change, during a given mathematical activity, the student playing this role, but, for doing this, I had to plug the cable in another calculator, or to exchange the places occupied by two students. Not so easy to do on the fly.

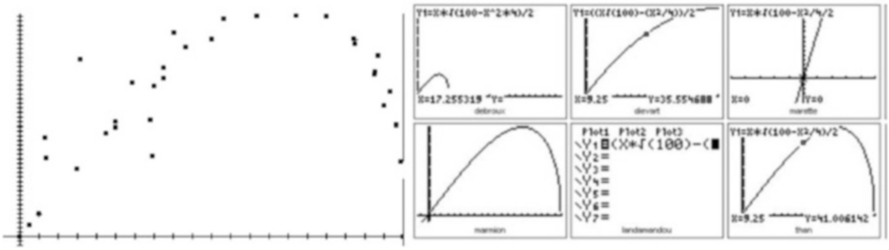

Fig. 19.3 Two main configurations for the TI-Navigator configuration, examples (Hoyles et al., 2010, pp. 447–448)

to slightly adapt this organisation (Fig. 19.2, right), aiming to structure the discussion not only between each students group and the teacher, but between the different groups through the common screen; it appears that the mathematical activity was very sensitive to such an adaptation, that fostered the mathematical discussions in the whole classroom. Actually, the drawing (Fig. 19.1, right) looks like the spatial organisation of an orchestra, evidencing the role of the teacher orchestrating the mathematical situation at stake, taking care of all the students' instruments.

As for the view-screen (Sect. 19.2.1), the technical device does not give matter for a sole teacher's mode of use (the spatial organisation of students, above, constitutes already a strong didactical choice). Actually, Ti-Navigator allows the teacher to use two main configurations: the *common coordinate system* configuration: displaying all of the pupils' data, for example, points or curves, in a single coordinate system (Fig. 19.3, left); the *screen mosaic* configuration: displaying, on the class screen, all (or some) of the pupils' calculator screens in quasi-real time (Fig. 19.3, right).[3]

These two configurations have the common property of connecting all (or some of) the students' calculators to a *common workspace*, situated on the class screen. The orchestration of such a device remains then in the teacher's hands, having to choose the relevant configuration corresponding to her didactical choice, and to select students' calculators to be 'published'.

The mathematical problem giving matter to these screens was (Hoyles et al., 2010, pp. 447–448): an isosceles triangle ABC has two sides AB and AC measuring 4 cm. What is its area? The students tried various values for the third slide BC, drawing the corresponding triangles, measuring their height, and computing their area. Then they send, via their calculators and the hub, the couples (length of BC; area of ABC) to the common screen, obtaining, as a collective result, a cloud of points (Fig. 19.3, left). Then they tried to model this phenomenon with a function,

[3] This application comes actually from the development, by Uri Wilenski, of the HubNet module, see Sect. 19.4.1.

obtaining different curves that the teacher decided to display on the common screen (Fig. 19.3, right).

In Hoyles et al. (2010), p. 447, I draw some lessons of a long term use of TI-Navigator by experienced teachers:

> The work with the TI Navigator was found to foster an emergent real *community of practice* (Wenger, 1998) in the classroom in which we could distinguish three fundamental aspects, *participation*, *reification*, and the existence of shared resources, whose major elements are summarised below:
>
> – Participation with the engagement of students in the mathematical activity and debate.
> – Reification with the collaborative creation of mathematical objects (a good example being the collective creation of the graph of a function that gradually becomes an easily identifiable object) (Fig. 19.3, right).
> – Shared resources most notably the public shared board, which is a place where every student can show her/his mathematical creation. Each student is confronted to her/his production and those of other pupils.
> – In traditional classrooms, speech or writing (when asked or allowed by the teacher) directly on the board are the ways students can express themselves and share with others. With TI Navigator, the situation is very different, for two main reasons:
> – A new interactivity was fostered between the artefact and the student, and between students themselves: students conveys their messages through the artefact; the artefact acts on the students enabling them to extract themselves from their productions thus freeing themselves to become more easily involved in peer exchanges. Thus the common space became a space of debate and exchange that aimed to elaborate a social 'mathematical truth'.
> – Each student becomes detached from his/her production as a distance is created between student and the expression of her/his creation; this distance seemed to improve the reflection on practice. The student became involved in the class activity in a different way as the tool maintained this distance between a student and the results s/he proposed to the class and to the teacher.

As in the case of the sherpa-student configuration, we can notice here the three aspects of connectivity (technological, social and cognitive), with, clearly, didactical difficulties added for orchestrating mathematical activities in such environments: the teacher has to simultaneously manage *all the students calculator screens*, and to take relevant decisions on the fly. She has then to have a deep understanding of the didactical variables of the situation, in order to play on them, according to the dynamics of the classroom activity. Hoyles, Noss, and Kent (2004) give a good description of such a teacher's expertise, based on the collaborative work of a teachers' team. I will go back, regarding connectivity, to teachers work in Sect. 19.3.

19.2.3 Internet as a Connectivity Multiplier

The third occasion for meeting 'connectivity for students' happens 10 years after (around 2010) when I was in charge of a *e-culture teaching unit* for students (third university year) aiming to become mathematic teachers. The name itself of this unit

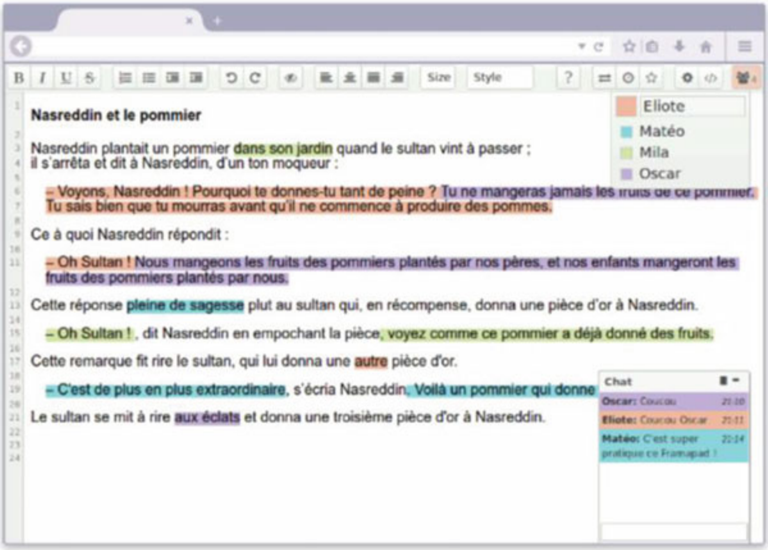

Fig. 19.4 An interface allowing students to share their ideas (screen copy of a designer's advertising)

(e-culture) indicates the major change happening at this time: the integration of Internet for teaching. During the whole sessions, students were connected to Internet and were free to use online software (mainly Geogebra). I integrated also an application[4] providing online collaborative sheets named Pad (Fig. 19.4), allowing students to discuss together (each contributor being identified by a colour, and students using also a chat for commenting their current work). It is up to the teacher: to organise such a sheet for the whole class, or for pairs of students; to be part, or not, of the discussion. Obviously, in such a context, all the tools are not under the teacher's control, as the students can use their own tools for communicating between them... or with somebody else outside!

In this period, I welcomed a Mexican PhD student, aiming to analyse students' work and associated orchestrations in the Internet era (Betancourt, 2014). It was for me a good opportunity to look at connectivity through the eyes of an advanced student.

Betancourt's thesis is related to 'learning of linear algebra supported by digital resources'. In his work (Betancourt, 2014), he related a practical work he proposed, in the context of this e-culture teaching unit, to the students working by pairs: students working together, intentionally, did not seat next to each other, then they had to use the Internet facilities to communicate. The mathematical problem at

[4] Framapad: https://framapad.org

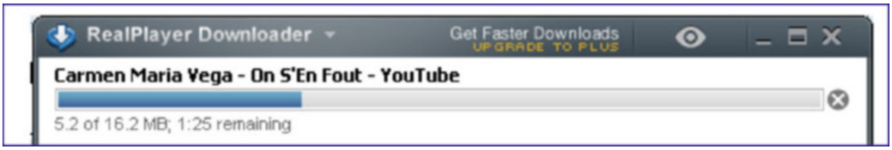

Fig. 19.5 Interface for following the video downloading (Betancourt, 2014, p. 82)

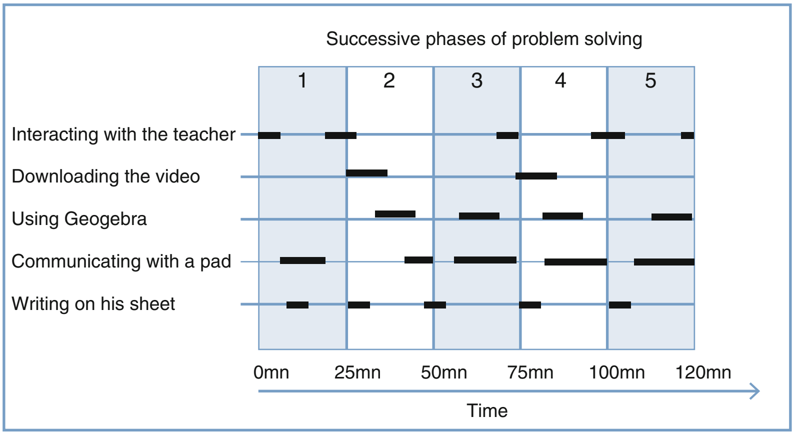

Fig. 19.6 Structure of a student's activity and tools used over 2 h (adapted from Betancourt, 2014, p. 105)

stake consists in modelling the process of a video downloading through Internet. The two questions asked to the students, using an interface for analysing the downloading process (Fig. 19.5), were: is the downloading velocity constant? How, according to you, does the application compute, at any moment, the remaining time for achieving the downloading (for example, Fig. 19.5, the remaining time to download 16.2 Mo is 1.25 s).

The activity proposed to students (Betancourt 2014, p. 143) was divided in five phases (Fig. 19.6): the teacher introducing the problem and the tools to be used; each student downloading the video, getting data and integrating them in Geogebra; discussing within the pair and trying to model the process; downloading again the video for checking the model; final discussion and conclusion.

I cannot, in the frame of this chapter, analyse the content of the students' activity, but I would like to underline some elements of structure over these five phases, following the activity of a given student (Fig. 19.6):

– Phase 1: he lessens to the teacher, then interacted, using the pad, with his pair colleague, expressing some doubts about the constant velocity of the downloading process. At the end of this phase, he interacted with the teacher (Betancourt, 2014, p. 121):

The student S: In my opinion, the velocity of the downloading process is not constant.
The teacher T: Perhaps, but how could you justify this opinion for your colleagues?
S: The downloading velocity, at a given moment, depends on the number of persons downloading the video at the same moment.
T: Perhaps, but how can you evidence this using the data you will pick up from the analysis of the real downloading of the video?

– Phase 2 is dedicated to the analysis of the downloading process and the use of Geogebra for displaying the data.
– During the following phases, the pad and Geogebra seem to be the essential supports of the student's reflection, combining individual mathematics manipulations using the dynamic geometry software, and collective mathematical discussion using the online writing tool.

The use of the Internet, compared to the calculators network configuration, clearly changes the connectivity regulation. The students' interactions are not monitored via the teacher's computer and displayed on the common workspace, but the students' pairs freely organised their work: on their own screen, it is up to them to manage the part of their working *space* dedicated to Geogebra, and the part dedicated to the Pad. With respect to the phases of the mathematics activity orchestrated by the teacher, the two students can negotiate the organisation of their working *time*, and eventually split their work in two parts, one for each of them.

The question at stake—studying the behaviour of Internet through the velocity of a downloading process—illustrates in some way the metamorphosis of the mathematics learning landscape due to the emergence of Internet: Internet appears as a *multiplier of the teacher's orchestration choices* (he can organise students in groups of two, three, or more; with students face-to-face, or at a distance; he can use a common class working space, for showing to the whole class the work of a group, or several groups of students... Internet appears as *rebalancing teacher and students' responsibilities* towards the progress of knowledge in the classroom. Last but not least, Internet appears as a *connectivity multiplier*, opening opportunities for connecting students to students (in this example via a Pad) and for connecting students to Internet resources (in this example Geogebra).

Actually, as underlined Betancourt (2010, p. 127), Geogebra was not the sole resource to be exploited. Students tried also, using their browser, to get direct answers to the problem at stake, with, as I noticed myself, keywords extracted from the teacher's question, as 'modelling downloading process velocity', aiming to find a direct answer... But, doing that, in this case, they did not succeed to find relevant resources.

The Betancourt's experience, with the associated artefacts (essentially a Pad and a dynamic geometry software), could induce the idea of a double connectivity level: for manipulating mathematics objects, students work individually with a given software; for discussing their methods and results, students work collectively.

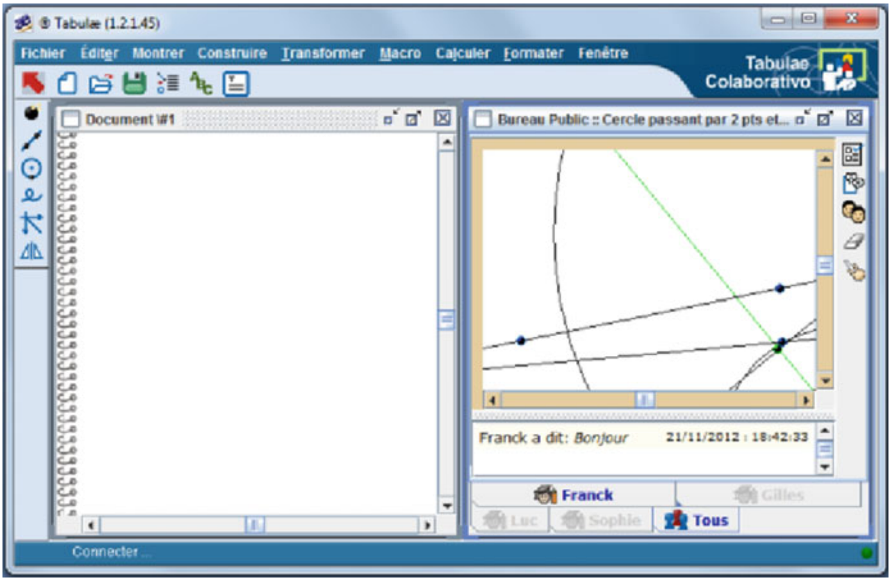

Fig. 19.7 An interface combining individual and collective geometrical work (Bellemain, 2014, p. 31)

What happens effectively in this particular experience is due to the artefacts available for the students. It could work differently with other artefacts. It was the case, in the frame of the same e-culture teaching unit, sometimes after this first experiment, when an invited researcher, Franck Bellemain (2014), proposed a new activity based on a new collaborative dynamic geometry software, Tabulae.[5] This environment (Fig. 19.7) proposes a window combining a space (left) for the individual student's work, and a space (right) for designing collaboratively a mathematical figure and exchanging comments. In such an environment, both mathematical work and associated discussion can be done collaboratively.

In this section, from the sherpa-student configuration to the calculator network configuration, and then to the Internet universe, I evidenced, over 20 years, the emergence of *connectivity* at technological, social and cognitive levels, as a major *potential* factor for renewing students' mathematical activity. This connectivity implies an added complexity for the teacher, that has to conceive and manage orchestrations making profit of these new opportunities. In which way connectivity could also benefit to teachers work? This is the purpose of the following section.

[5] Developed by Luiz Carlos Guimarães at the LIMC laboratory (Laboratório de Pesquisa e Desenvolvimento em Ensino de Matemática e Ciências, http://www.limc.ufrj.br/site/limc_olaboratorio.html) in the Federal University of Rio de Janeiro.

19.3 Connecting Teachers and Mathematics Teaching Knowledge Through Internet Resources

In this section, I follow again the thread of my own experience for analysing the potential and real effect of connectivity for 'teachers working with teachers'. For this purpose, I choose three entries: the experience, from 2000 to 2006, of a teacher training organisation, the SFoDEM; the experience, from 2001, of a teacher online organisation; and the recent experience (2014) of a MOOC aiming to develop the usage of tools in mathematics teaching.

19.3.1 The SFoDEM, Monitoring Teachers for Collaboratively Design Teaching Resources

The SFoDEM[6] was developed in the region of Montpellier, France, from 2000 to 2006, by the local Institute of Research on Mathematics Teaching (IREM, http://www.irem.univ-montp2.fr). The considerations for designing such on organisation were that, in spite of many institutional actions and the enthusiasm of pioneering teachers, in spite of the rapid evolution of technological tools and equipment, integration of ICT into mathematics teaching was slowly increasing in France. Guin and Trouche (2005), pp. 1023–1024 explained the SFoDEM objective, and described its organisation:

> [...] the main objective of SFoDEM was to provide a continuous support for teachers in the *conception*, *appropriation and experimentation* of pedagogical resources to get over the crucial transition to the pedagogical act. This requires a collaboration to be built between teachers with different teaching experiences aiming to support their day-to-day practice. Various themes were chosen (transition from numerical to algebraic setting and ICT; graphic and symbolic calculators; experiments of teaching sequences towards dynamic geometric diagrams; simulation of random experiences; and cooperative problem solving via Internet) to find invariants in distance training viable beyond the organization and these studied themes [...]
>
> SFoDEM is piloted by a leadership team of three researchers and its platform is managed by an administrator. About 15 trainers are involved in the training network and every year since September 2000, about 100 teachers volunteer to participate in this project. The *training committee* (composed of the leadership team, the administrator and the network of trainers) manages the *coordination* of the five themes: first experiments on distance teaching have pointed out the necessity of compensating distance with an established structured and controlled organization and showed the crucial role of planning and *regulation* [...]. The organization alternates face-to-face meetings and distance periods (the trainers of each theme have a face-to-face meeting each week, the training committee each month, and each theme—trainers and trainees—meets four times a year).

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[6] SFoDEM stands for *Suivi de Formation à Distance des Enseignants de Mathématiques*, what could be translated by « Distant follow-up of Mathematics Teachers Training »

The five themes reveal the feature of this period, a transition one between the calculator era and the Internet era. In some sense, I could say that the SFoDEM rested on *social connectivity*, *teaching and training connectivity*, and *mathematics connectivity* to develop the integration of ICT in mathematics teaching. By social connectivity I mean the efforts made for connecting: teachers with trainers on a continuous way (both face-to-face and at distance); the leadership team; and the training committee. By teaching and training connectivity, I mean the efforts made for connecting the day-to-day teaching practice and the training one, the training consisting in designing pedagogical resources to be experimented in each trainees' classroom. By mathematics connectivity, I mean the efforts made for connecting different mathematical fields (calculus, algebra, geometry and statistics) and different artefacts (calculators, dynamic geometry software, Internet) to find invariants of a training organisation aiming to foster teachers' use of ICT.

The SFoDEM objective was quite ambitious, justifying its long time duration. Its pilots draw some main lessons in a CDRom (Guin, Joab, & Trouche, 2006), organised in two parts: a *design path*, and a *library of pedagogical resources*:

- The design path organised in five steps untitled '*Exploring*', '*Defining*', '*Thinking*', '*Exchanging*', '*Revising*', evidenced the central place of Internet for supporting the collaborative design of resources. For example, the first step, 'Exploring', consists, before beginning a new design, in (Fig. 19.2): visiting the main existing repositories, particularly the IREM one and the *Mathenpoche* one (see Sect. 19.3.2); reading already published reports of designing/using resources; searching with a browser and relevant keywords existing resources able to inspire a new design. The four following steps ('*Defining*', '*Thinking*', '*Exchanging*', '*Revising*') needed the use of an online platform dedicated to the interactions between the members of the project. Finally the achievement of the design path leaded to the development of a *technological connectivity* (Fig. 19.8).
- The library of pedagogical resources evidences the importance of a *common model of pedagogical resources* for facilitating both the *design*, the *exchange* and the *appropriation* of a given resource. This common model was composed,

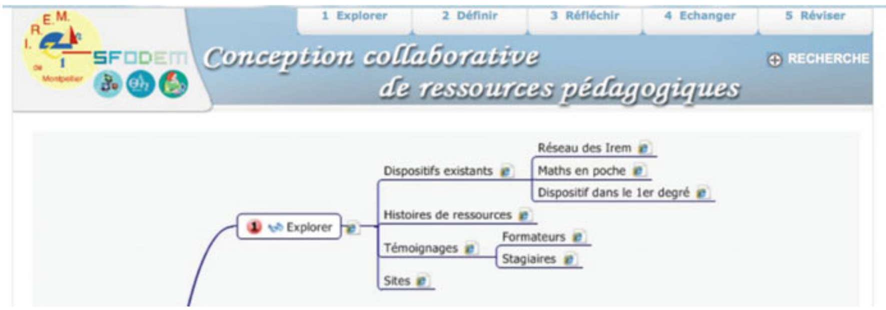

Fig. 19.8 The first step, 'Exploring' of the design path (Guin, Joab, & Trouche, 2006)

at the end of the SFoDEM experiment, of: an identification sheet (including metadata aiming to situate the resource in a larger repository), a student sheet (explaining the mathematical task at stake), a teacher sheet (underlining didactical challenges), scenarios of use and usage reports (enriched over the successive implementation of the resource), traces of students work (evidencing some critical points), a technical sheet (supporting the implementation of the resource in different technological environments) and a CV ('curriculum vitae' of the resource, tracing the main step of its evolution). Finally, the achievement of the library of pedagogical resources leaded to what I could name a *documentation connectivity* (documentation seized in the sense introduced Sect. 15.3.2): the documentation connectivity of a given resource should be defined as its potential for connecting it to different possible usages and associated traces, to different technological possible environments, to different didactical difficulties, and for relying it to its own genesis (where does the resource come from?) and to its different designers.

Such a technological and a documentation connectivity do not develop on a continuous way over the whole life on SFoDEM. Guin and Trouche (2005), p. 2024 underline some major difficulties:

– From a technological point of view: 'this organisation has rapidly revealed that schools equipment [in terms of Internet access] is frequently *inadequate* or *inaccessible*'.
– From the trainers point of view: 'usual trainers' strategies were essentially based on *imitation* strategies where trainees were asked to take the position of a student'.
– From the process of design itself:

> Moreover, initial resources provided by trainers, often expert resources, were too complex for an experimentation by trainees in their own class. Then, there was an evolution towards simpler resources, easier to implement and towards *virtual workshops* of trainees creating resources from initial ideas, named 'germs of resources'. This evolution may be considered as an evolution from a top-down approach towards a bottom-up approach.

Finally, I retain, from the SFoDEM experience, three major lessons: obtaining significant results in terms of integrating ICT in classroom practices needs a strong organisation mobilising over the time researchers and trainers; in this process, social connectivity, technological connectivity, and documentation connectivity seem to jointly develop (other examples can be found in Gueudet & Trouche, 2011); the development of both technological and social connectivity seems to rebalance the responsibilities of trainers and trainees with respect to the design of resources (see the virtual workshops of trainees), recalling the phenomena arising in connected classrooms (Sect. 19.2.3).

Some difficulties encountered seem to be linked to *a period of transition* characterised both by the *emergence of Internet* (just beginning to be a tool available in schools) and the *emergence of online communities*, not so easy among teachers The following section proposes another case study of an online community developing in the same period, but without any institutional support.

19.3.2 Sésamath, Teachers Connecting Teachers

Sésamath is a French association created in 2001. It gathers in-service mathematics teachers, aiming to 'freely distribute resources for mathematics teaching'. Its website front page (http://www.sesamath.net/) claims '*mathematics for everybody*', '*working together*, *supporting one another*, *communicating*!'. Its growth has been quite rapid: today, it gathers 100 subscribers, 5000 teachers participating in various projects, and its website proposes 45,000 digital resources for mathematics teaching and welcomes about one million visits, each month. One reason for this growth could be the existence of the French network of IREM, which has, in some sense, paved the way since 1970 (see Chap. 10). But the essential reason seems to be the way this association benefit from the development of Internet and adapt its way of functioning to this development.

The development of Sesamath follows a model (Fig. 19.9) evidenced, in the same period, by other online teachers associations (Gueudet & Trouche, 2012):

- A first group of teachers gathers, for sharing, essentially via Internet, resources.
- Then this founding group, I will call it the kernel, engages in a cooperative work (it is generally the moment of the formal creation of the association), sharing not only resources, but the work for designing them; doing that, it attracts a crown of teachers interested in making profit of these resources, some of them proposing their own resources for the benefit of the whole group.
- At last, the founding group deepens its cooperation for thinking together the whole process of designing the resources and developing the association, moving towards a real community of practice (Wenger, 1998); doing that, it attracts successive crowns of teachers, more or less close to the kernel, according to their engagement in the community project.

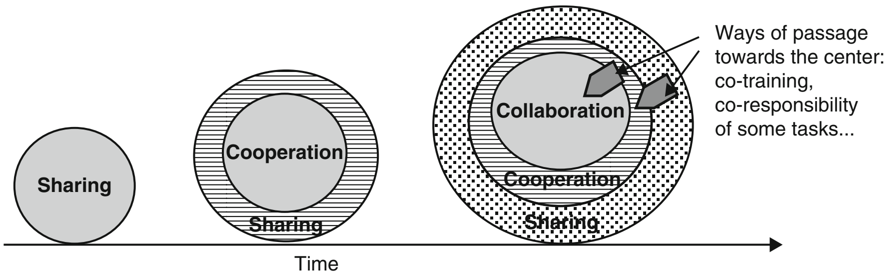

Fig. 19.9 Development of an online teachers community designing and sharing resources (Gueudet & Trouche, 2012, p. 311)

These crowns are not tight: the growing of the 'rolling stone' supposes, for the collaborating kernel, to carefully think ways of passage from the distant crowns towards the centre (Fig. 19.9).

In 2015, the current president of the Sesamath association, Hélène Gringoz, summarised, during a meeting of a research project,[7] the genesis of her association

> The association was established on October 31st 2001, [. . .]. At the very beginning, it's ten Mathematics teachers who were very, very fan of technology in general, for example calculator, or overhead projector. That was 15 years ago, so the computer was absolutely not as developed as today. And these teachers created websites, and they created resources they put on these websites for their own teaching. [. . .] They were actually teachers who met because they create resources they missed for their own teaching, they create them together and then put them available to all teachers. [. . .] For 15 years, it is this spirit that will prevail: the creation of collaborative resources made available to all, it is really the foundation of our association.
>
> The creation of resources took different forms. The best known is Mathenpoche[8]: i.e. the creation, in two years, of a set of interactive exercises that covered the range of teaching level from 6th to 9th grade (the French middle school). And the first printed edition of these exercises occurred in 2002 [. . .] And since, it works well, it was decided in 2005 to publish the first textbook, for the 7th grade [. . .] We were seen as precursors, as people a bit wacky, quite innovative but not really serious [. . .] This situation changed in 2005, since our textbook covered 15 % of the market.[9] And so, it became credible, since we were followed by a number of teachers [. . .] Today, 15 years after its creation, Sésamath hosts 45,000 resources, addressing all the teaching levels from 1st grade to the University.
>
> At the beginning, we had to face the distribution of resources, it was very complicated, Internet was not working very well. We were just teachers, and therefore, we trained each other so that the distribution of resources goes as well as possible. [. . .] This led us to create tools as mail servers, list servers, and a collaborative interface. When we began to write in 2003–2004, downloading a file was very heavy and the speed was very low. So, we created in 2003 an interface that can store files and send links automatically via emails, in order to avoid downloading them each time [. . .].
>
> As the basis of our association is the distribution of resources, gradually came the idea that all our online resources should be free [. . .].
>
> Our development allows us to propose, to all the mathematics teachers, a new interface, Labomep,[10] a mathematics laboratory where teachers can appropriate Sésamath resources,

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[7] It was the ReVEA project ('Living resources for learning and teaching', www.anr-revea.fr). The whole interview (audio) is available on the page presenting the ReVEA meeting http://ife.ens-lyon.fr/ife/recherche/groupes-de-travail/revea-collectif. The translation has been made by the author of this chapter.

[8] http://mathenpoche.sesamath.net. The English translation of Mathenpoche should be « Maths in the pocket »

[9] To be noticed: the online version of the Sésamath textbook are, from the beginning, free. Their printed versions are quite cheap (half the price of an 'ordinary' textbook), as the Sésamath authors do not get royalties for their work. The royalties, as low as possible, go to the association, for allowing it to hire the technicians necessary to develop its digital environment.

[10] Labomep (http://www.labomep.net/fiches/fiche26.php), meaning 'Laboratory for math in the pocket', is an interface opened for schools. Once a school is identified, each teacher, individually or collectively with her colleagues, can design her own resources in combining Sésamath resources. Then, she can, through the Sésamath interface, make these resources available for her students.

> combine them with their own resources as they wish. That is the way we develop Sésamath step by step, and that's it: Sésamath offers now a portal, fifteen websites, resources for classroom, for teacher and for student….

The Sésamath president interview is very illuminating, bringing out the way the association develops using the connecting Internet potential, and sometimes anticipating it. A complementary analysis of Sésamath is given by Pepin, Gueudet, Yerushalmy, Trouche, and Chazan (2015), evidencing how Sésamath develops a collaborative design involving a number of teachers—the *social connectivity* point of view—and improve its resources *documentation connectivity* over the design of successive textbooks (Sect. 19.3.1):

> The *mode of design* of these textbooks involves a large number of actors. Many teachers (approximately one hundred, for each textbook) have contributed to its design, in a *collaborative* and *iterative* way, as 'authors of content', or 'designers of didactical scenarios', or 'testers', or 'experimenters' in classes (a single teacher could have several roles, or change roles at different moments). The textbook resulting from this process is expected to fit the wishes and needs of a large number of teachers.
>
> Far from being a simple textbook, the Sesamath textbooks constitute a *hybrid* system of resources for teaching (i.e., including a classical structure in chapters, online supplements, animated corrections). Following their development helps to understand this systemic aspect:
>
> – The first model of Sesamath textbooks was a *single static book*, available both online (under a pdf, but also an odt format, allowing teachers to make modifications) and in hard copy, accompanied by separated animations on line, a set of Mathenpoche exercises, etc. (i.e. a real *resource system*, see Figure 19.1).
> – The second model was a *flexible and dynamic digital textbook*, which a teacher could organize according to his/her needs, with animation and extra exercises integrated in each chapter.
> – The third model was both a flexible and dynamic digital textbook *and* a laboratory for collaboratively adjusting the textbook to the needs and projects of the community (school, team of teachers). This laboratory, named *LaboMEP* allows teachers to develop and share their own lessons, but also to differentiate their teaching according to the results of their students.

As for the SFoDEM case, social connectivity, technological connectivity, and documentation connectivity jointly develop. Besides, some differences between the SFoDEM and the Sésamath cases are clear: nor researcher, or trainers, or institutional support in the second case. SFoDEM designed a limited numbers of resources for a limited number of teachers; on the contrary, Sésamath aimed to cover, with its resources, the whole range of the curriculum needs, and to be in touch with the biggest number of teachers. For guarantying the quality of its resources, SFoDEM relies on a careful didactical analysis by experts of the domain; Sésamath counts upon the contribution of multiple users, allowing the resources to be enriched (and sometimes corrected).

Roughly speaking, I could say that SFoDEM illustrates the *web.1 connectivity* and Sésamath the *web.2 connectivity*, characterised by more interactivity, simplicity and flexibility (O'Reilly, 2005). Is it possible to combine, in developing new forms of connectivity, both the monitoring of experts and the implication of a huge number of resources and users? It is one of the challenges of the MOOCs, I focus on it in the following section.

19.3.3 The MOOC Initiative, as a Connectivity Multiplier

I report in this section on my recent experience in MOOC, opening, for me, new horizon for thinking the connectivity potential in mathematics education.

Opening Wikipedia this morning (10 June 2015), I got this definition for MOOC:

> A massive open online course (MOOC/muk/) is an online course aimed at unlimited participation and open access via the web. In addition to traditional course materials such as filmed lectures, readings, and problem sets, many MOOCs provide interactive user forums to support community interactions between students, professors, and teaching assistants (TAs). MOOCs are a recent and widely researched development in distance education which was first introduced in 2008 and emerged as a popular mode of learning in 2012 (http://en.wikipedia.org/wiki/Massive_open_online_course).

Recently introduced in distance education (Cisel & Bruillard, 2012, Bozkurt et al., 2015), the MOOC have been, at the beginning, mainly developed by the most prestigious universities, benefiting of the well recognised expertise of some of their researchers (see for example the Stanford MOOC on mathematical thinking, taught by Keith Devlin https://www.coursera.org/course/maththink). The rapid development of this very new way of teaching/learning comes with the emergence of a lot of questions (see Fig. 19.10, on the wikipedia), none of them being really solved, at the time where these lines are written.

In 2014, based on the experience in this domain of the IREM network (see Chap. 10) and the IFÉ (French Institute of Education), was launched the MOOC eFAN Math (meaning: Teaching and Training Teachers for mathematics education

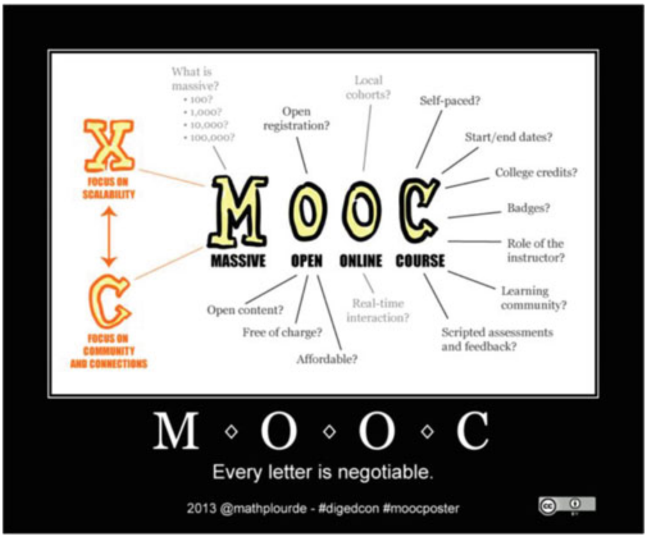


Fig. 19.10 Questioning the true nature of MOOC (2013 @mathplourde)

in digital environments[11]). Its targeted audience was teachers and teacher educators for primary or secondary schools and it aimed to support them for conceiving lessons where instruments and software effectively support students' mathematical activity. For this purpose, it develops a directory of teaching projects, based on the inputs of the participants, and enriched all over the 5 weeks MOOC duration. The orchestration of these 5 weeks clearly expresses the intentions of eFAN maths:

– Week 0: presentation of the MOOC, and constitution of teaching projects teams (the participants were supposed to come into the MOOC with a professional question, as 'how introduce symmetry with a dynamic geometry software?' giving matter to such a team; or to join a team already constituted on a question having sense for them).
– Week 1: presentation of a gallery of possible instruments for doing mathematics (the participants may comment, or/and add new instruments); each teaching project team has to decide which instruments could be used for developing its projects.
– Week 2: presentation of task design processes for reaching a given didactical objective in using a given set of instruments; each teaching project team has to design a relevant task according to its goal and to reflect on the effects of the selected technological environment on students mathematical thinking.
– Week 3: presentation of implementation processes of a given lesson in a given technological environment; each project team has to discuss the teacher's role in term of orchestration.
– Week 4: presentation of processes and tools for sharing a given resource with colleagues, for evaluating and revising it; each project team has to apply/discuss them to the light of its members experiments.

Each week begins with two short videos: a first one summarising the activities and issues of the previous week, the second one presenting the theoretical elements grounding the activities of the week to come, the tools to be used by the teams, the references to go further, and the work to be done. The description of projects in progress were available for all, and opened to comments. All the teachers following the MOOC had to answer, each week, a quiz questioning their understanding of the main notions at stake. Two main tools supported the eFAN Maths activities: the first one, the FUN platform,[12] is dedicated by the French Ministry of Higher Education to the French MOOCs; it hosted the general structure of eFAN Maths,[13] its videos and its quiz. As the FUN platform could not provide tools for collaborative design, a Moodle platform was opened for welcoming the work of the teaching projects teams.

[11] The MOOC eFAN Maths was hosted by two French institutions: Ecole Normale Supérieure de Cachan et Ecole Normale Supérieure de Lyon.

[12] The platform FUN (France Université Numérique http://www.france-universite-numerique.fr/moocs.html) is based on the open source technology EdX.

[13] https://www.france-universite-numerique-mooc.fr/courses/ENSCachan/20007/Trimestre_3_2014/about

It is possible to draw some lessons from the point of view of the eFAN Maths team, and from a questionnaire fulfilled by the participants (Gueudet, G. (coord.), 2015, Aldon, 2015). The eFAN pedagogical team was composed of 10 researchers and an engineer coming from the IREM network or from the IFÉ. They all consider this experience as very productive, but very time consuming (the estimate time for the whole process of conception and implementation of this MOOC was, for the whole team, 600 h), and needing to deeply renew the usual teacher training organisation; they estimate also that the available tools (mainly the FUN platform) were not at all adapted to the objective (interactivity and connectivity) of the MOOC. eFAN Math gathered at its beginning 3250 subscribers; the numbers of video downloading decreased from 2800 (first week) to 860 (fourth week); 169 teaching projects were developed and 500 participants were inscribed on the Moodle platform dedicated to the work on these projects. In this sense, eFAN Maths, compared to classical teacher training organisation, appears really as a connectivity multiplier. Finally 161 participants answered the final questionnaire; among them, 75 % estimated that eFAN Maths reached its objectives.

The decreasing number of participants is not surprising: for most of the MOOC, one estimates that the number of participants following the whole training is about 10 %. It was the case for eFAN Maths, if one considers that 'achieving the training' corresponds to 'achieving a teaching project'. The 161 answers to the questionnaire, corresponding more or less to 50 % of the active participants, are then to be considered carefully: 68 % of them wish a more focused training (closer to their teaching, in primary vs. secondary schools, closer to their teaching needs); globally, they wish to have more time for being able to fully conceive, share, experiment, discuss, and revise a teaching project; they wish to have a more effective support from the eFAN team when needed; they wish to dispose of more efficient collaborative tools for designing their projects and a more interactive platform for exchanging with participants and with the pedagogical team.

Some more analyses are certainly needed, for knowing more about the quality of the teaching projects developed during eFAN Maths (their documentation connectivity, particularly from the point of view of ICT integration), the results for teachers knowledge (in terms of cognitive connectivity), and practice. But some results appear critical: the need for time and the complexity of the new equilibrium to be found both in each teacher classroom and in the MOOC itself; the interest to base the training on the design of resources meeting the real teachers needs.

The will for fitting as close as possible the local learners needs and to better monitor their work needs could lead to move towards the notion of SPOC (Small Private Online Courses), as proposes Fox (2013). Effectively, the eFAN experience seems to evidence that, when teachers were working in the same school, they benefit better for the training.

There are also some contradictory tendencies to balance (and decision to be taken, see Fig. 19.10):

– Dillenbourg, Fox, Kirchner, Mitchell, and Wirsing (2014) propose diverse solutions for reinforcing social connectivity within a MOOC, balancing then the interests of MOOC and SPOC:

> How can we motivate MOOC learners and teachers when the teachers may be both geographically distant and socially disconnected from the learners? MOOCs could, for example, support reciprocal teaching, direct instruction, mastery learning, peer assessment and instruction, small-group/community interactions such as dynamic regrouping of learners to match learning styles and paces, and so on (p. 9).

– Social connectivity and technological connectivity are also to be carefully combined, as notice Dillenbourg et al. 2014 (p. 5):

> MOOCs take multiple forms. At one end of the spectrum is the xMOOC, which is characterised by a rather tight structure, little social interaction and mainly computer-marked assessments. At the other end is the cMOOC or Connectionist MOOC, which is almost entirely free of pre-provided content and relies instead on very high social interactivity to produce the course content and outcomes. Most current MOOCs lie between these extremes, with some structure (weekly content in the form of video and quizzes) and some important social interactions (discussions, peer-review of work, and so on).

Finally, looking back at the previous sections, the reader may realise how the experiences of SFoDEM and Sésamath were announcing, in some ways, the emergence of MOOCs, under their extreme tendencies, as new forms of fostering teachers professional development on the basis of their collaborative work on teaching resources. For these organisations to work effectively, technological connectivity and social connectivity appear as necessary ingredients.

Internet, both for students (Sect. 19.2) and for teachers (Sect. 19.3) clearly appears as a connectivity multiplier, from a technological as well as a social point of view. To what extent this connectivity improves also teacher documentation, knowledge and practice, as well as students learning and mathematical activity, is not a trivial question. I discuss, in the next section, the way the ICMI study connectivity panel (Hoyles et al., 2010) addressed this issue, mainly from the point of view of students.

19.4 Some Lessons from the ICMI Study Connectivity Panel

The ICMI study connectivity panel, chaired by Celia Hoyles, was based on four presentations (one of them has already been introduced in this chapter, Sect. 19.2.2). I will focus in this section on two of them, the first one concerning the effects of connectivity in a given classroom, the second one across classrooms. I draw then some general lessons from the panel.

19.4.1 Enacting Classroom Participatory Simulations

As I did Sect. 19.2.3, Uri Wilenski (Hoyles et al., 2010, pp. 452–455) exploits 'a neglected affordance of connectivity: the ability to give people a shared interactive experience in classroom contexts'. For this purpose, he presents an outline of his work with NetLogo,[14] using the notion of connectivity in two senses.

The first sense is a *macro-micro level connectivity*:

> In our many years of working with NetLogo in middle and secondary classrooms, we have endeavoured to bring to students descriptions of complex systems at a micro-level and connect those micro-level descriptions to macro-level and observable phenomena. Typically, when we have taught students about systems that can be constructed as complex, we have concentrated on aggregate equations that summarize system behaviour. For example, to describe the behaviour of ideal gases, we rely on equations such as PV = nRT. But agent-based modelling enables students to more directly control and examine the behaviour of elements of the system and connect this behaviour to the system emergent behaviour. Thus in NetLogo's GasLab model suite, students come to understand the ideal gas as composed of Myriad interacting gas molecules and see PV = nRT as an emergent result of these interactions. There are hundreds of NetLogo models we have used in classrooms. Students examine a range of phenomena such as the spread of a disease through a population, or the interaction of predator and prey in an ecosystem [. . .]

For example, for the interaction predator/prey (Fig. 19.11), students can vary essential parameters as the number of sheep, the number of wolves, the quantity of

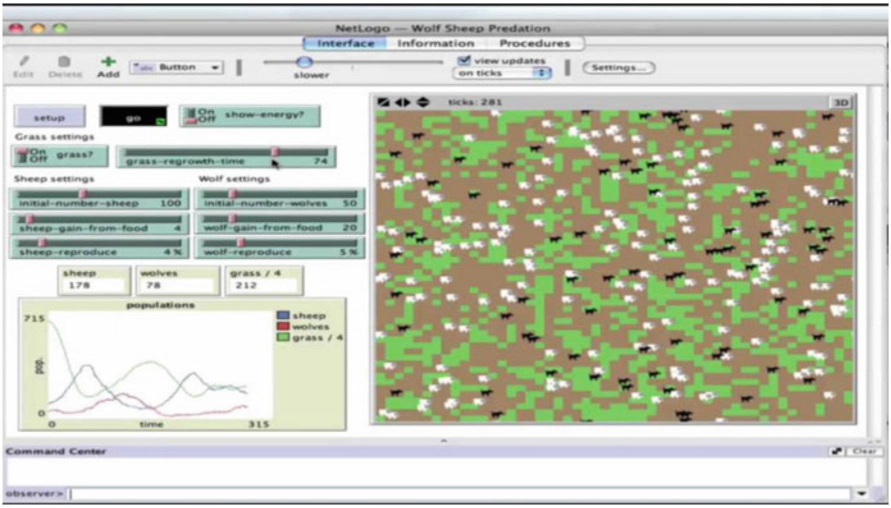


Fig. 19.11 The NetLogo interface for studying a model of predator and prey

[14] NetLogo (https://ccl.northwestern.edu/netlogo/) is a multi-agent programmable modelling environment, developed at the Center for Connected Learning of the Northwestern University. It is an extension of the Logo environment developed by Seymour Papert (http://en.wikipedia.org/wiki/Seymour_Papert)

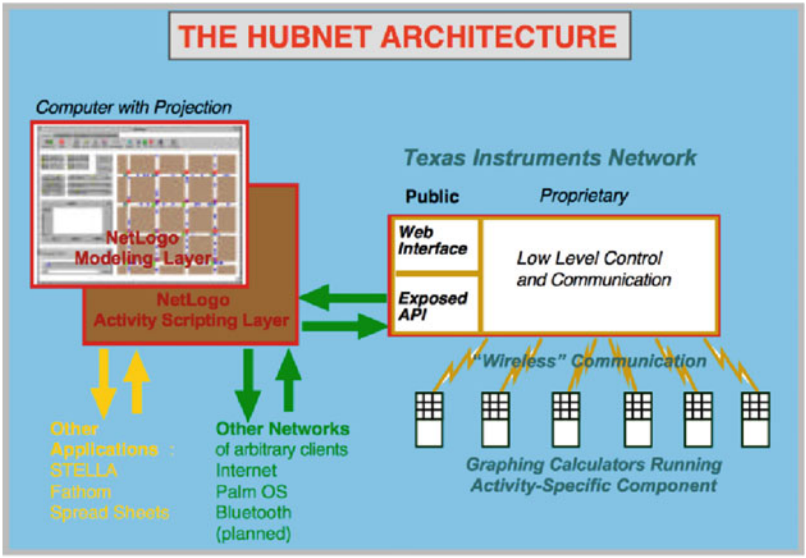


Fig. 19.12 The HubNet architecture (Hoyles et al., 2010, p. 454)

grass, and observe the evolution of the process. The hypothesis is that the micro-level connectivity managed by the application will facilitate students understanding of the system behaviour. Wilenski observes that it was not so simple: 'despite considerable efforts to "lower the threshold" of entry into agent-based modelling, it remains difficult for elementary students to master both the programing and modelling skills needed'. Even with the monitoring of a teacher, this approach 'leaves the student somewhat passive, as only a few can be engaged at any one time and they are limited to discussion of model behaviour'.

These difficulties lead him to develop connectivity in a second sense, a technological one, through the added module HubNet, enabling 'a host of devices to connect to a logo simulation and control agents within that simulation' (see Fig. 19.12 a set of calculators connected to the teacher's computer). The sole modelling activity is then transformed into a participatory simulation, in which each student may take part.[15]

Wilenski (Hoyles et al., 2010, p. 453) underlines the important benefits of such an application for learning:

[…] the modelling activity:

- Becomes more engaging—especially for younger learners. It becomes a social activity and captures much of the same draw as online games.
- Promotes greater student participation. Every student can be actively involved at the same time. Because they often require continuous action on the part of the students, they

[15] This application, through a cooperation with Texas Instruments, gave birth to the TI-Navigator network, that we describe Sect. 19.2.2.

are "in-the moment" motivated to participate. Such universal participation is very hard to achieve in a traditional classroom.

- Enables a shared experience of a complex system. There are very few opportunities, in the classroom or in life, for students to collectively witness the same complex system unfolding. Focal attention to such a system is hard to achieve outside of the virtual and, even when achieved, if the viewing does not connect the micro-level behaviour to the macro-level outcomes, then only the appearance is shared, not the mechanisms of action.
- Facilitates classroom discussion of the system and examination of "what-ifs". Student can suggest experiments with varying critical system parameters and/or agent-rules, hypothesize the observed behavioural change, run the simulation and refine the experiment.
- Scaffolds individual modelling and analysis. Once students have had several opportunities to collectively model and analyse complex systems, they are much better prepared (and motivated) to conduct such inquiry on their own. Often students have suggestions for model experiments that do not get explored in class. These questions are potent seeds of further student inquiry, experimentation and model revision.

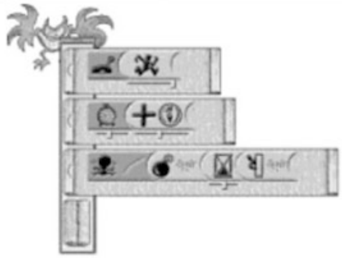
What I retained from this rich experiment is the interest of combining different level of connectivity: the *technological connectivity* (HubNet architecture) enables all the students to participate *in the same time* to the construction of a given phenomenon. This collective engagement (*social connectivity*) is stimulated by the student's awareness to be an actor of the *mechanisms of action*, and co-responsible of the final result. The system insures the connection between the micro-level and the macro-level, and the students, being involved in the whole process, incorporates the interrelations between these two levels (*cognitive connectivity*). All over the process, the teacher's orchestration is needed for regulating students' activity. This is made possible by the presence of all the actors in the same time in the same place. I analyse in the following section what could happen when such on activity occurs in different places.

19.4.2 Exploiting Connectivity Across Classrooms

The Noss and Hoyles's presentation (Hoyles et al., 2010, pp. 455–460) in the connectivity panel addresses actually the question of connectivity *within* and *across* classroom, through two projects co-directed by themselves: the Playground Project and the Weblabs Project. I will focus in this section on the first one.

The Playground project,[16] as its name indicates, aims to use the potential of games for stimulating children (from 4 to 8 years old) engagement and learning (about games and mathematics, see Chap. 18). Going beyond the simple 'playing

[16] Its website (http://playground.ioe.ac.uk) points out, on its front page: « The playground project is building computer environments for 4–8 year-olds to play, design and create games. A playground is a place to play with rules not just play by them. We aim to harness children's playfulness, creative potential and exploratory spirit, allowing them to enter into abstract and formal ways of thinking » (see also Chap. 18).



When the game starts, I change my speed to 22.5

When a second and half is up, I change my direction plus 25 degrees

When I am shot by a ray, I explode, I wait two second, I appear

(every object has an empty rule for making new rules)

Fig. 19.13 Stones combined for constituting rules defining a monster' behaviour (Hoyles et al., 2010, p. 455)

game', it aims to add a new dimension where children build their own games. Due to the age of the children, the project favours other modalities of interactions than words: mainly speech and direct manipulation. The authors describe the way the project allows children to design their own games:

> Children populated their games with objects which had 'behaviours'—sets of rules that determine their action. Behaviours were defined using collections of iconic rules, which could be viewed by opening a scroll of paper attached to an object (see Fig. 19.13 for rules defining a monster's behaviour). Each rule was expressed as a visible 'sentence' or string of graphic icons, which combined a condition and a series of actions to be executed whenever the condition was true. The icons representing the conditions and actions were represented as 'stones', small concrete manifestations of the concept that could be strung together to constitute a rule. Actions stones had a convex left side so that conditions with their concave right side could naturally fit to their left. Any object could accept any number of these iconic rules, all of which would be executed in parallel whenever the conditions for their execution were satisfied. (Hoyles et al., 2010, p. 456).

When the game starts, I change my speed to 22.5
When a second and half is up, I change my direction plus 25°
When I am shot by a ray, I explode, I wait two second, I appear (every object has an empty rule for making new rules)

The project gives then means for children for *constructing*, *expressing*, and *communicating* their own games. It offered a language allowing them to define rules in a synthetic and no ambiguous way. Once defined a game, the project leads the children to discuss it on two successive phases: sharing the game *through face-to-face interactions* in their own classrooms; sharing the game *using Internet*, either synchronously or asynchronously with a remote classroom. The project findings evidence that, over the two phases, 'children collaboratively came to explain phenomena arising from rules we characterised as either *player* (an agreed regulation), or *system* (a formal condition and action for the behaviour of the game)'

(Hoyles et al., 2010, p. 457). These findings underline also major differences between face-to-face an remote interactions:

> We found that in face-to-face collaboration, the children centre their attention on narrative, and addressed the problem of translating the narrative into system rules, which can be 'programmed' into the computer. This allows the children to debug any conflicts between system rules in order to maintain the flow of the game narrative.
>
> When *we added remote communication* to the system by enabling the sending and receiving of games from within the Playground system, we found that children were encouraged to add complexity and innovative elements to their games, not by the addition of socially-constructed or 'player' rules but rather through additional system rules which elaborate the formalism (games were created using two different kinds of programming systems, neither of which employed textual modalities). This shift of attention to system rules occurs at the same time, and perhaps as a result of, a loosening of the game narrative that is a consequence of the remoteness of the interaction.
>
> This phenomenon was particularly evident *in the case of asynchronous interaction* where, stripped of even the semantics of gestures, our extremely young students found it increasingly natural to try to communicate meaning via the various formalisms we provided. Thus a key historical claim for programming, that is offers a key motivation and model for immersion in a formal system, came to life as children struggled to modify and add rules of their programs that achieved the effects they desired. And it is worth stressing that asynchronous communication, while somewhat less attractive to the students at the time [...] allows students to reflect on, and therefore use more effectively, the formal rules of their games.

The main result I retain here is that 'The shift from narrative to system/formal rules does, in fact, seem to be a direct result of the necessity to formalise, in the absence of all the normal richness of interaction that characterises face-to-face collaboration' (Hoyles et al., 2010, p. 457). In this case, *technical connectivity*, understood as providing children means to communicate via Internet, leads to *cognitive connectivity*, leading the children to relate the implicit rules of the game to formal ones, parts of a system of rules. The discussion among children connects then a level of informal speech and a level of formal system of rules.

I would like to add extra personal comments.

The Playground project concerns a particular part of mathematics, linked to programing. This part will probably strongly develop in the future, supported both by the improvements of the software dedicated to 'children and programing' (see for example Scratch[17]), and by the evolution of curricula, favouring interaction between mathematics and programing (see Sect. 12.3.3).

Noss and Hoyles associate, in this experiment, 'connection to Internet', and 'remote interaction'. Of course, this association is note a necessary one, as connection to Internet and face-to-face interaction may jointly develop (see Sect. 19.2.3 or 19.4.1).

As the authors underline themselves, their project began in the previous century, where peer-to-per connectivity was quite limited. Today, the remote or face-to-face

[17] Scratch: "Create stories, games, and animations, Share with others around the world" (https://scratch.mit.edu), developed by the Massachusetts Institute of Technology, hosting 9,767,423 projects (on 12 June 2005)

interactions could combine texts (under different formats, more or less formal, from SMS to emails), pictures, as well as audio or video interactions.

However, I found useful to present, in the frame of this chapter, some lessons of this experiment, evidencing that even a limited use of connectivity, reduced to peer-to-peer interactions through the exchanges of texts via Internet could have important effects. From this connectivity panel held in the ICMI study, some more general lessons emerge, that I underline in the following section.

19.4.3 The ICMI Study Connectivity Panel, Some Lessons and Perspectives

In this section, I would like to focus on the main lessons drawn by the connectivity panel, then by the ICMI study itself.

Regarding the panel, Hoyles et al. (2010) retain both the potential of *technological connectivity* and the conditions for exploiting it:

– The potential is seen for developing *social and cognitive connectivity* (essentially regarding students):

> Digital technologies are already changing the ways we think about interacting with mathematical objects, especially in terms of dynamic visualisations and the multiple connections that can be made between different kinds of symbolic representations. At the same time, we are seeing rapid developments in the ways that it is possible for students to share resources and ideas and to collaborate through technological devices both in the same physical space and at a distance (p. 439).

The conditions for exploiting technological connectivity are quite largely described: 'Alongside overcoming not inconsiderable technical challenges, establishing an appropriate set of *socio-technical/mathematical norms* that prioritised collaboration [is] crucial regarding connectivity' (p. 460). Some years after, the point of view on technical challenges could seem quite optimistic… But the necessity of changing the socio-technical/mathematical norms clearly appears: the experiences presented during the panel stand at the fringes of the schooling system, and one measures the necessary distance for implementing them in the schooling system.

Among these conditions, even if this question was not addressed by all the panellists, rethinking the teacher's role in terms of new orchestrations appears actually crucial:

> […] here we are delineating new, even more demanding roles for the teacher, to be aware—across not only her own classroom but those in remote location—of the evolution of discussion, the mathematical substance of what is and what is not discussed, and the need all the while to find ways to keep students on task without removing the exploratory and fun elements of the work. This is, surely, a demanding set of roles for the teacher (p. 460).

Finally, the panellists shared the awareness that connectivity was a promising field of research, specially regarding the cognitive aspect, i.e. implications for learning, quoting Moreno-Armella, Hegedus, and Kaput (2008), suggesting how 'networks can link private cognitive efforts to public social displays thus—potentially at least—enhancing student's metacognitive ability to reflect upon their own work to reference to others'.

The connectivity panel was part of the 17th ICMI study aiming to 'rethink the terrain' of technology and mathematics education. The theme of connectivity appears certainly, within this study, as the one where further research was the most needed, as stated by Artigue in her concluding chapter (Hoyles & Lagrange, 2010, p. 473):

> The way digital technologies can support and foster today collaborative work, at the distance or not, between students or between teachers, and also between teachers and researchers, and the consequences that this can have on student's learning processes, on the evolution of teachers' practices is certainly one essential technological evolution that educational research has to systematically explore in the future [. . .] most of this space is still for us nearly terra incognita. We observe an intense creativity, which very often develops independently of research and this is a very stimulating situation. But we also have to be careful. As stressed by Richard Noss in the panel on connectivity, connectivity does not necessarily imply collaborative work and collaborative work does not necessarily imply better mathematics learning, or, I would add, better mathematics teaching. We are submerged by an avalanche of information, data and possibilities of connection and the way this avalanche can be organized, treated and transformed into knowledge or means for productive action is an open problem.

Since the time of this ICMI study (2006 for the conference, 2010 for its proceedings), connectivity has developed, at least at a technological level, for the students (Sect. 19.2.3) as well as for the teachers (Sect. 19.3.3). Which new lessons and perspectives could be drawn in this new situation? I propose some answers, and some new questions in the next discussion section.

19.5 Discussion

I call this section 'Discussion' rather than 'Conclusion' because the forms of connectivity are evolving so rapidly that I can offer no conclusion. I would like to underline the strong current evolutions, in terms of technologies and usages, then to question the links between connectivity and mathematics, and, last but not least, address the theoretical needs for analysing, in such contexts, mathematics learning and teaching processes.

19.5.1 Internet Uses as a Connectivity Multiplier and a Seamless Learning Tool

I had structured the two first parts of this chapter looking at the evolutions from the students' side, then from the teacher's side. For understanding the processes at stake, I have to embrace in our analyses the two sides in their interrelations. We showed (Gueudet & Trouche, 2012, pp. 313–316), in the case of Pierre, a teacher member of the Sésamath association, the synergy resulting of the interrelations between Pierre's work in/for his association, Pierre's work in/for his class and students' usages:

> He evinces a strong collective involvement both in his school and in Sesamath: he is 'teacher in charge of technology', treasurer of the school cooperative, responsible of the school's chess club. These activities are not all dedicated to mathematics. In Sésamath, as of 2008, he was a member of the board for 5 years. This meant that he spent approximately 1 h a day reading emails and participating in forums 'that engage the association life'. He was also a member of a Sésamath project developing a grade 6 textbook, which is still in progress at this time. He was, finally, the pilot of a new Sésamath project entitled 'mathematics files for primary schools'.
>
> Documentation work takes place within each of these collective involvements and each of them is part of Pierre's work, as he said: 'Consuming time in collective activities is a component of my teaching activity'. He particularly emphasizes the importance of the primary school project ('it gives a better understanding of what my pupils know when arriving at secondary school'), the Sésamath board ('it makes me aware of the questions asked to the profession as a whole') and the 'grade 6 textbook'. It is actually this last project, which appeared as fostering Pierre's documentation. For all the duration of the project (2 years), Pierre decided to have only grade 6 classes (three classes, for 6 h teaching in it), to 'align' his documentation work with the community documentation. Thus, the documentation work that Pierre accomplished in 2008–2009 for the grade-6 level *concentrated* his main efforts, and *connected* individual and community documentation [. . .]
>
> To this collaborative documentation corresponds a collaborative form of teaching [. . .]. Using online resources is an important feature of Pierre's documentation work, within or without his students (for preparing his teaching or collaborating in Sésamath projects). Within his classroom, a connected computer, a projector and an interactive whiteboard (IWB) are used to work with online resources. For example at the beginning of each lesson, the teacher opens Pronote, an application allowing displaying the students list, to note the absentees, to memorize what has been done, and what is still to do… Another example of this continuous Internet use: the teacher exploits Google to do any arithmetic operation exceeding students' capacities of mental computation (it was amazing to observe that handheld calculators remain in students' schoolbags!). For continuing to interact with his students outside of the classroom, he developed a collaborative website on which he regularly uploads mathematics problems (that he calls 'enigma'). Students try to solve them and write their solutions on a forum (Gueudet & Trouche, 2012, pp. 313–316).

The use of Google is particularly significant of these interrelations: Pierre uses Google for doing computations, because actually it corresponds to the students' usages. As they are more and more connected to Internet, at home, as well as in school, often through their mobile phones, they tend to use Google as an universal machine: they use the same procedure to answer to a geographical question ('what is the number of inhabitants of such a city?') and to a mathematical one ('what is the result of 45 times 59'?): in the two cases, Google is required to provide the answer. In such a procedure, the constructive aspect of mathematics practices ('yes, *I can* compute 45×59') is lost. It looks like if each result of any question was lying somewhere on the shelves and I had just to go to the relevant shelve and take it: that is the efficiency of Google to do that for us. It is also well known also that, Sesamath providing a wide number of resources covering the whole curriculum, some teachers restrict sometimes their documentation work to 'search on the shelves of Sésamath' what fits their needs of the day. This is of course not the sole aspect of the Internet usages, and there are a lot of productive aspects as I evidenced in Sects. 19.2 and 19.3, but this way of searching direct answers to direct questions is a real economical effect of being continuously connected to a wide repository of resources. The institutions try to control this continuous connectivity (see for example, Sect. 12.3.3, the development of 'machines to the test', excluding during the examination at least any connection between a calculator and 'outside'), but removing connectivity cannot be the sole answer: the development of new environments for communicating based on Internet appeals new kind of orchestrations: the case of MOOC (Sect. 19.3.3) evidences the interest, and the complexity, of such reflections on *Internet as a connectivity multiplier*.

Another aspect appearing in the description of Pierre's work is the dilution of the frontiers between working in school and working out of school, the teacher and his students interacting through a website and Internet resources (as LaboMep, see Sect. 19.3.2). It happens *during* the time of schooling and curriculum knowledge, but I happens also, from a general point of view, *after* the time of schooling, considering lifelong education. It leads to the notion of *seamless learning* (Wong, Milrad, & Specht, 2015),

> [Researchers] propose *seamless learning* as a learning approach characterized by the continuity of the learning experience across a combination of locations, times, technologies, or social settings, (perhaps) with the personal mobile device as a mediator. The basic rationale is that it is not feasible to equip students and knowledge workers with all the skills and knowledge they need for lifelong learning solely through formal learning (or any one specific learning context). Henceforth, student learning should move beyond the acquisition of curriculum knowledge and be complemented with other approaches in order to develop the capacity *to learn seamlessly* (p. xvii).

We are at the beginning of the analysis of this kind of learning: Chaps. 15 on teachers work with digital tools, 17 on the design tasks, 18 on using games open windows on leaning/teaching with Internet.

19.5.2 Connectivity and Mathematics

I have presented in various sections of this chapter (Sects. 19.2.2, 19.4.1 and 19.4.2) the potential of technological connectivity for linking different aspects of the teachers and students' mathematical activities, what I have named documentation connectivity, cognitive connectivity or micro and macro level connectivity. I would like to examine now to which extent mathematics practicing, learning and teaching requires connectivity and in which sense.

We have already met the necessity of 'connecting things' for *learning and teaching mathematics* in two senses: *connecting different representations of mathematical objects* through a specific activity of treatments and conversions, as a central activity for conceptualising (see the work of Duval, Sect. 12.3.3);

connecting ostensives and non ostensives, the last ones guiding the usage of first ones (see the work of Chevallard, Sect. 12.3.2).

More generally, if we consider that *doing* mathematics is solving problem, it appears clearly that doing this needs to connecting different point of views on objects and processes (see examples of proofs in Chap. 6; see Jon's examples on visual theorems, Sects. 3.2.1 and 3.4).

In a discussion between the three authors of the first draft of this chapter John asked Jon:

> I imagine that now, compared to the beginning of your professional life, that you might send out a Maple (or whatever) file to colleagues and say something to the effect "Look at this, there's something strange going on 'under the hood'". Are conjectures more of a shared 'thing' than they were 40 years ago?

Jon replied:

> Life is vastly different than forty years ago. I think to a significant degree this is covered in Chap. 3. It is certainly covered in an article "The Future of Mathematics 1965 to 2065." https://www.carma.newcastle.edu.au/jon/future.pdf. This just appeared as part of the MAA100th anniversary book. When David Bailey and I wrote our book *Mathematics by Experiment* between 2001 and 2004, it was already possible to be scholarly without ever visiting the library and when we revised the book in 2007 this was even more true. The level of connectivity is limited largely by one's imagination and willingness to contact people/ remember what resources may be available. The sociology of this—as with social media—has not yet stabilised. Perhaps it never will. So I am routinely sent stacks of papers by isolated researchers asking me to help them publish them along with even more intrusive requests. Yet on balance this is a wonderful time to be working in a subject which—despite the public image of a solitary researcher staring at a blackboard—has always thrived on and needed human interaction.

Indeed, in responding to one of the questions I posed in Chap. 6, Jon performed an Internet search.

Moving on (but keeping in the domain of mathematics) doing mathematics, since the first written practices (see Chap. 5 and Proust, 2014), has always dealt with highly structured texts. Reading such texts leads one to combine different registers of activity: learning, solving, classifying, archiving, exploring or inventing. Digital tools give us new means for combining these different registers. This links to my construct 'cognitive connectivity' (the internal—in the mind—rather than the external—in action—side of connectivity). Noss and Hoyles (1996), I posit, had similar ideas when, 20 years ago, they compared mathematical connectivity to the functioning of the Web, introducing the notion of webbing:

> Like the web of mathematical ideas, the Web (we will use a capital to denote the electronic network), is too complex to understand globally—but local connections are relatively accessible. At the same time, one way—perhaps the only way—to gain an overview of the Web is to develop for oneself a local collection of familiar connections, and build from there outwards along lines of one's own interests and obsessions. The idea of webbing is meant to convey the presence of a structure that learners can draw upon and reconstruct for support—in ways that they choose as appropriate for their struggle to construct meaning for some mathematics (Noss & Hoyles, 1996, p. 108).

The question is then how the use of the web could support mathematical connectivity. Trouche and Drijvers (2014, p. 6) proposes an approach combining the concepts of webbing and orchestration:

> In the webbing approach, conceptualization appears as a coordination process, 'the process by which the student infers meaning by coordinating the structure of the learning system (including the knowledge to be learned, the learning resources available, prior student knowledge and experience and constructing their own scaffolds by interaction and feedback)' (Hoyles et al., 2004, p. 319). In the instrumental orchestration approach, conceptualization appears as a command process, characterized by the conscious attitude to consider, with sufficient objectivity, all the information immediately available not only from the calculator, but also from other sources and to seek mathematical consistency between them (Guin & Trouche, 1999). 'Very sophisticated artefacts such as the artefacts 25 available in a computerized learning environment give birth to a set of instruments. The articulation of this set demands from the subject a strong command process. One of the key elements for a successful integration of these artefacts into a learning environment is the institutional and social assistance to this individual command process. Instrumental orchestrations constitute an answer to this necessity.' (Trouche, 2004, p. 304). It seems that there is a kind of intended internalization from an instrumental 30 orchestration, seen as an external process of monitoring students' instruments by the teacher, to an internal orchestration, seen as a process of self-monitoring the individual and personal instruments by a student. Coordination and control are certainly two facets of mathematical activity, particularly in a technological rich environment, and the two approaches seem to privilege, each, one of these facets.

With the notion of internal coordination and control comes a new reflection on curriculum resources. Recent analyses of e-textbooks, i.e. textbooks making profit of the digital potentialities, mainly from the point of view of connectivity, underline the necessity, for insuring their quality, to take into account connectivity and coherence (Pepin et al., 2015).

19.5.3 New Theoretical Needs

Describing recent experiences and, regarding for example interactive collaborative mathematical interface (Sect. 19.2.3) or MOOCs (Sect. 19.3.3), I was aware, as I said previously, that we are just at the beginning of the analysis of the connectivity aspects and effects. For developing analyses on new phenomena, sometimes new theoretical frames are needed in order to define new concepts and system of concepts. Taking into account connectivity as a major intellectual challenge has led to the creation of the *connectivism* frame, thus defined by Wikipedia (https://en.wikipedia.org/wiki/Connectivism):

> Connectivism is a hypothesis of learning which emphasizes the role of social and cultural context. Connectivism is often associated with and proposes a perspective similar to Vygotsky's' zone of proximal development' (ZPD), an idea later transposed into Engeström's activity theory (see Chap. 9). The relationship between work experience, learning, and knowledge, as expressed in the concept of 'connectivity, is central to connectivism, motivating the theory's name.

The definition of this theory as 'a learning theory for the digital age' (Siemens, 2005) indicates the emphasis that connectivism gives to technology's effect on how people live, communicate and learn. This is not the choice I had made until now, but I am sure that the development of this new domain will benefit to the other domains of research interested in connectivity.

Actually, for lighting the questions at stake, I am trying a theoretical networking approach as presented by Prediger, Arzarello, Bosch, and Lenfant (2008) (see also Chap. 9), connecting theoretical frameworks for understanding connectivity. In the case of teacher's work, it gave matter to the *documentational approach* (Sect. 15.3.2), crossing the domain of architecture information (Salaün, 2012) and instrumental approach. This approach is used in the frame of a French national project (www.anr-revea.fr) for analysing the evolution of teachers work with resources in a time of digital transition.

In the community of mathematics education, other theoretical approaches should be exploited in order to understand connectivity. My own view is that Sfard's construct 'commognition' is important in this regard. Sfard (2010), p. 432 defines thinking as:

> the *individualized version of interpersonal communication*—as a communicative interaction in which one person plays the roles of all interlocutors. The term *commognition*, a combination of *communication* and *cognition* comes to stress that inter-personal communication and individual thinking are two varieties of the same phenomenon.

According to this perspective, developing social and reflective connectivity is developing opportunities for improving mathematical thinking.

Finally, looking at connectivity in the mathematics education community leads to develop an interdisciplinary program of research, that is before us.

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