Chapter 13
The Calculator Debate

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13.1 Introduction

Questions about the use of calculators in the learning and/or assessment of mathematics have been raised for many decades and this, in the opinion of us three authors, warrants a chapter on the calculator in this part of the book devoted to issues with regard to tool use in mathematics. I chose the slightly emotive title 'The calculator debate' for this chapter, over a more neutral title such as 'hand-held computational technology', because the digital artefact known as 'the calculator' does, it appears, stir the emotions of many people (on both sides of the debate on the affordances and constraints of this artefact for school students learning mathematics). This affective dimension of tool use in mathematics is not an issue that should be ignored in this book. This artefact, the calculator, is, by my Sect. 1.3.1 definition of a tool, only a tool when it is used to do something and it can be used for non-arithmetic purposes including such as drawing a straight line or as a paperweight. The often extreme valuations (positive and negative) of this artefact as an arithmetic tool are interesting and one of the aims of this chapter is to explore these valuations. Extreme valuations of artefacts for the study of arithmetic are not, historically, restricted to the calculator debate. Buisson (1911), a dictionary of pedagogy and instruction, in a section on the use of the abacus in primary mathematics,[1] reports on a late nineteenth century abacus debate and cites a professor at the Polytechnic of Zurich:

> Le boulier corrompt l'enseignement de l'arithmétique. La principale utilité de cet enseignement est d'exercer de bonne heure, chez l'enfant, les facultés d'abstraction, de lui apprendre à voir de tête, par les yeux de l'esprit. Lui mettre les choses sous les yeux de la chair, c'est aller directement contre l'esprit de cet enseignement.

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[1] See http://www.inrp.fr/edition-electronique/lodel/dictionnaire-ferdinand-buisson/document.php?id=2204.

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> The abacus corrupts the teaching of arithmetic. The main purpose of this teaching is to exercise in early childhood the faculties of abstraction, to teach him to see in the head, through the eyes of the mind. It put things in front of the body, goes directly against the spirit of this teaching.[2]

I structure this chapter in three sections. In Sect. 13.2 I position ‘the calculator’ within ‘portable hand-held computational technology’ and I briefly review: calculator use; the research literature on the use of the calculator; and the ‘calculator debate’. In Sect. 13.3 I consider the calculator with regard to Wertsch’s (1998) ten *properties of mediated action*. The last section speculates on a possible future of the calculator debate.

13.2 Hand-Held Computational Technology

When I say ‘calculator’ in this chapter I use an everyday term to refer to an everyday object and I start by attempting to put some precision on a tacit understanding of this term. Two ways to position the calculator within portable hand-held computational technology are: diachronically, over time; synchronically, at a moment in time. I shall attempt to do both and I start with the synchronic description so as to have a working definition of what I mean by ‘the calculator’.

By ‘hand-held computational technology’ I refer to digital electronic artefacts with batteries that can be held in a human hand. There are an awful lot of these around: electrical probes; weighing scales; etc. These are calculators but they are not what one regards as ‘calculators’ so I limit the domain to such artefacts which can execute arithmetic operations. This still leaves a lot of artefacts: mobile phones; small tablet computers; etc. These again are calculators but they are not really what one regards as ‘calculators’ so I further limit the domain to artefacts in which the main purpose is for the user to execute/evaluate mathematical operations/functions. This last definition captures, I feel, what is meant by the term ‘calculator’ but the definition has lost its extensionality, by which I mean that the first two definitions referred only to the external properties of the artefact but the last one, which captures the meaning of the everyday word, includes human motives for using the artefact.

I thus take a calculator to be a hand-held digital electronic artefact with batteries whose main purpose is for the user to execute/evaluate mathematical operations/functions. This gives us a set of plastic and metal artefacts: arithmetic calculator; scientific calculator; graphic calculator; and symbolic calculator. The calculator debate concerns all types of calculator but it is usually centred on the first two. There are two main differences between arithmetic and scientific calculators. (1) Arithmetic calculators execute arithmetic operations as they are written whereas scientific calculators execute the expression entered prior to pressing the ‘enter’ or

[2] Translated by Luc Trouche.

'=' key. This makes no difference for expressions such as $2 \times 3 + 4$ but it does for the expression $2 + 3 \times 4$ where the arithmetic calculator will give the wrong answer (20 instead of 14) according to the conventions of mathematics. (2) Scientific calculators have a number of mathematical and statistical functions embedded within keys that arithmetic calculators do not have. Calculators can be viewed as composite tools instead of unitary tools, by which I mean: an arithmetic calculator can be viewed as a tool for four-function arithmetic, for storing intermediate results (via the memory key), for calculating percentages, etc.; the scientific calculator can be viewed as including all the tools of an arithmetic calculator as well as being a tool for doing trigonometry and a tool for statistical calculations, etc. The graphic calculator can be viewed as including all the tools of a scientific calculator as well as being a tool for displaying graphs of functions, etc. The symbolic calculator can be viewed as including all the tools of a graphic calculator as well as being a tool for algebraic manipulation and for calculating derivatives and integrals, etc. There are many interesting issues specific to the use of graphic and symbolic calculators such as their potential for multiple forms of representation but this chapter mainly focuses on arithmetic and scientific calculator.

Diachronically the calculator is simply a recent, in historical time, aid to calculation. Our species has, for millennia, used tools for calculations and in Chap. 4 I considered abaci, the method of prosthaphaeresis, logarithms and slides rules. The quote above from Buisson (1911) concerning abaci shows that calculators are not alone in generating debate on the use of tools for doing arithmetic. I now move on to consider calculator use.

A consideration of calculator use could be lengthy but my purpose here is a 'broad brush stroke' account of the dimensions of calculator use. I start by distinguishing in-school and out-of-school use. In an educational institution, a school, a learner may be directed by another (a teacher) to use or not use a calculator. This is control of the use of an artefact in a public place and there are many examples of this: no smoking; no parking; no ball games; etc. A student has greater personal agency on the use of artefacts outside of school (where they can ignore a demand 'do not use a calculator' in a homework assignment). In adult work activity, restrictions on calculator use are rare but the object of the activity in a workplace setting is rarely on learning mathematics. In the late twentieth century calculator use in the workplace was common in shops, offices and engineering sites. This continued into the twenty-first century but calculator use is increasingly replaced by the use of more advanced digital technology.

I now consider in-school use of calculators. I begin with my own impressions of use (seven dimensions) and then report on a recent survey.

1. Age of the learners: There are often restrictions on the use of calculators with young children. When calculator use is allowed young children are often presented with an arithmetic calculator. Scientific calculators are the norm in secondary schools; implicit and/or explicit restrictions on use may still be applied but these are generally less stringent than they are in primary schools.

2. Tasks: Many tasks from the pre-calculator period have not changed as a direct result of the introduction of calculators. Where calculators have replaced tables of logarithms, the numbers used in tasks are not so likely to be designed for ease of calculation. Some tasks are specifically designed for calculator use (see 6 and 7 below).
3. Attributes of the teacher: Apart from school and/or curricula exhortations to use or not use a calculator, the use of a calculator in a classroom is largely determined by the teacher. There is great variation over mathematics teachers in their attitude to and their understanding of calculator use.
4. Regions: Calculator use varies over nations and, in federations, over states (see Tables 13.1 and 13.2 below).
5. Date: There is some variation in calculator use over time in different regions (restrictions on use may be relaxed at a later date).
6. Curriculum: Although there have been a large number of school-based projects in which the use of calculators is a prominent feature, national curricula have incorporated few new areas of mathematical content to account for calculator use. Numerical analysis (e.g. trial and improvement methods) is an exception.
7. Assessment: Calculator use in high-stakes assessment varies over nations and, in federations, over states. There is variation in grade restrictions (e.g. allowed after a certain age/grade) and examination papers (allowed, not allowed and two levels of papers, one allowed and one not). In calculator allowed papers there is variation as to whether calculators are permitted (no designed change of tasks) or expected (some tasks are changed to facilitate use of calculator).

Table 13.1 Example: fourth-grade calculator policies

Australia	Statements/policies vary by state. In most cases calculator use is encouraged but not mandated during mathematic instruction . . .
Austria	Calculators are not used until grade 5
Belgium (Flemish)	Students learn to use the calculator effectively in mathematics instruction
Botswana	Calculator use is permitted in examinations but not encouraged by teachers
Denmark	Calculator use is permitted in examinations

Source: Mullis et al. (2012, pp. 78–79)

Table 13.2 Example: eighth-grade calculator policies

Syrian Arab Republic	It is forbidden to use calculators on exams because mental arithmetic keeps student intellect alive. . . . Most students and schools do not have calculators
Tunisia	The use of calculators is permitted to solve simple problems or mathematical operations in instruction
Turkey	Calculator use is allowed in instruction for some objectives in the curriculum
United Arab Emirates	Calculators are allowed during mathematics classes but not during examinations
United States	Statements/policies vary by state. Some states have standards for calculator use in instruction and most states have standards for use in assessments . . .

Source: Mullis et al. (2012, pp. 80–81)

TIMMS 2011 Encyclopedia (Mullis et al., 2012) provides the most recent survey available at the time of writing. It provides tables (called Exhibits) on 'National Policies Regarding Use of Calculators in Mathematics Instruction and assessment' at grades 4 and 8. Exhibit 21 (fourth grade, pp. 78–79) reports on 52 countries of which 17 have no policy. Exhibit 22 (eighth grade, pp. 80–81) reports on 45 countries of which 8 have no policy. Both show variation with regard to my dimensions 1, 3, 4 and 7. For example, the stated policies (other than 'no policy') of the first five countries in Exhibit 21 and the last five countries in Exhibit 22 are replicated in Tables 13.1 and 13.2.

Calculator use in instruction and assessment globally clearly shows variation. This variation could cause us to question the meaningfulness of the term—if there are many different calculator uses, then is the term/construct 'calculator use' a valid construct? From the perspective on artefact and tools developed in Chap. 1 of this book, the calculator is an artefact that becomes a tool when it is used to do something (in a particular way). From this perspective the calculator (be it arithmetic, scientific, graphic or symbolic) becomes a myriad of tools through different usages and the term 'calculator use' becomes a collective term for these different tools. I now move on to the research literature on calculator use.

A thorough consideration of the research literature on the use of the calculator would be lengthy and my aim, as with calculator usage above, is to provide a 'broad brush stroke' based on my knowledge of the literature (which is open to claims of bias). I first note two trends in research on calculator use and then consider different types of research. The first trend is that most research on calculator use has been on school-based use. The second trend concerns the date of school-based research. Most research: on arithmetic and scientific calculators was conducted in the 1980s and 1990s: research on graphic and symbolic calculators was largely conducted in the 1990s and the early years of the twenty-first century. I conjecture that reasons for this are (1) graphic and symbolic calculators appeared after arithmetic and scientific calculators and (2) the twenty-first century has witnessed a marked increase in the availability and use of computers in school and researchers have largely turned their attention away from calculator use.

'Research on calculator use', like 'calculator use', is a term/construct that has several meanings ranging from randomised control experiments to scholarly (or -experience-informed) inquiry. Taking research as including all these forms leads to a range of types of publications on calculator use. Within this range I report on: experiments; descriptions of classroom activities and their consequences; meta-studies and explorations of student–calculator actions/activity.

Hedren (1985) reports on a longitudinal study of eight form 4 to form 6 (students developed from age 10 to age 12 over the course of the project) classes in Sweden. The quasi-experiment aspect of the study was only a part of this research which also included questionnaires, interviews with teachers and classroom observations. These eight classes, who could use calculators at any point in their lessons, formed the experimental group; three similar classes formed the control group. The form 4 pre-test items covered mental arithmetic, algorithms and word problems; the difference between the results of experimental and control classes was very small.

The form 6 post-test items included similar content but the items were designed by an independent group. The experimental classes scored significantly better in 47 items but significantly worse in 9 items. Hedren (1985, p. 175) states that experimental 'pupils did not get worse results overall in mental arithmetic [and] . . . achieved a better ability to solve word problems'. With regard to the latter, 'We maintain that these positive results are due to our pupils' greater opportunities to concentrate on the process of problem solving when they used their hand-held calculators for calculations'. In the conclusion Hedren (1985, p. 178) states 'we have drawn conclusions on the basis of the observed differences in test results, we cannot eliminate the possibility that the results might have been caused by factors other than the use of calculators'.

Quesada and Maxwell (1994) report on a USA pre-calculus college study conducted over three semesters. Students in the experimental group used a graphic calculator and a textbook written for the use of this calculator. Students in the control group used a scientific calculator and a traditional textbook. Data consisted of student responses to four tests, a final examination and weekly quizzes. 90+% of final examination questions were identical over the groups. One hundred and ninety-nine students in the experimental group and 335 students in the control group were included in the qualitative analysis. 'Statistical results obtained in this study indicated that the test scores of the experimental groups were significantly higher than those of the control groups' (Quesada & Maxwell, 1994, p. 212). However, 'It is not clear what really causes the improvement in scores when the graphing calculator is used' (Quesada & Maxwell, 1994, p. 214). Both Hedren (1985) and Quesada and Maxwell (1994) end with caveats concerning the explanation for the results. This is a common feature of research which relies on statistical methods.

Shuard, Walsh, Goodwin, and Worcester (1991) report on the calculator-aware number (CAN) curriculum and teacher development project, a British project that worked in collaboration with primary teachers in the 1980s. It is an example of a report on classroom activities and their consequences; the research element of this work is a by-product of the development work. It was a radical project in that, at a time when it was estimated that 80 % of primary mathematics time in Britain was devoted to pencil-and-paper practice with standard written algorithms. CAN advocated that: standard written algorithms should not be taught; children should have a calculator at all times; and they should be the ones who decide when calculator use is appropriate. Project teachers developed new tasks for children. The following is a teacher-designed task focused on place value for 6-year-old children.

Put a number inside a square. Then put a number at each corner so that the four 'corner' numbers add up to the number in the square.

Examples of children's work include:

173 in the square and 100, 70, 3 and 0 at the corners; 44444444 in the square and 11111111 at each corner (CAN project children started working with large numbers early in their education).

The CAN project did not simply downgrade one set of tools (traditional algorithms) and upgrade another set of tools (calculator methods) but also placed a great deal of emphasis on investigational work and mental calculations. CAN, its leaders proclaimed, witnessed 'a great flowering of mental calculation' (Shuard et al., 1991, p. 12) and non-experiment test results purported to show CAN project children outperforming non-CAN similar age children on a host of items including many for which a calculator did not appear useful (see Shuard et al., 1991, pp. 59–63 for details). Ruthven (1998), considered below, suggests reasons for the 'great flowering of mental calculation'.

Hembree and Dessart (1986) is a meta-study of 79 research reports which focuses on the effects of calculators on student achievement and attitude. The paper attends to criteria for the selecting and coding of the studies it considers, and also to the methods of analysis. The conclusions are calculator enthusiasts' dream and include:

> In Grades K-12 (except Grade 4), students who use calculators in concert with traditional instruction maintain their pencil-and-paper skills without apparent hard. Indeed, a use of calculators can improve the average student's basic skills with paper and pencil, both in basic operations and in problem solving ... Students using calculators possess a better attitude toward mathematics and an especially better self-concept in mathematics than noncalculator students. This statement applies across all grade and ability levels. (Hembree & Dessart, 1986, p. 96)

These conclusions are, I believe, one reason why Hembree and Dessart (1986) remains a highly cited paper on calculator use. Whilst I do not doubt the scientific integrity of the researchers, I feel it should be pointed out that: my comments on difficulties in explaining results of research which relies on statistical methods applies to this meta-study; the conclusion report on statistically significant results and there are a number of 'results' which are not reported as they are not statistically significant (including results on conceptual understanding); this meta-study reports on research reports and such data can be subject to the *Hawthorne effect* (that the novelty of being involved in a research project may have encouraged teachers of classes using calculators to put extra effort into their lessons).

Ruthven (1998) is a micro-study of the mental, written and calculator strategies of a sample of students' upper primary (10–11 years of age) schoolchildren and is an example of research which explores student–calculator actions/activity. The study also throws some light on CAN's 'great flowering of mental calculation'. Students were drawn from six schools, three of which had participated in the CAN project but CAN terminated before the student sample started at the schools. The contextual information on the schools makes it clear that the legacy of CAN remained in the post-CAN schools and not in the other schools. Ruthven (1998) reports on student responses to four arithmetic word problems such as S*tamps*:

A second-class stamp costs 19p.
How much would 5 second-class stamps cost?
How much change would you get from £5?

Table 13.3 Students use of written and calculator media, from Ruthven (1998)

Medium and level of use	Non-project students	Post-CAN students
No written use	15	24
Multiple written use	5	3
No calculator use	13	14
Multiple calculator use	9	4
No written or calculator use	5 (19 %)	11 (38 %)
Multiple written or calculator use	14 (52 %)	7 (24 %)
All pupils	27	29

The paper catalogues students' strategies. For example, *Stamps* resulted in six mental strategies, two written strategies and one calculator strategy (noting, however, that the trichotomy mental-written-calculator is simplistic and written strategies serve two cognitive functions, recording and spatially schematising). Ruthven (1998) then proceeds to quantitative analysis and my account skips to the part of this which aims:

> To explore more systematically how pupil characteristics might be associated with use of written column or calculator methods, aggregate levels of use were collapsed into three categories—no use, single use (on one problem only) and multiple use (on two or more problems)—and modelled using logistic regression . . . (Ruthven, 1998, p. 35)

Table 13.3 replicates part of the raw data from Ruthven (1998, p. 36) on students strategies in all problems. I have not included indices from logistic regression but I have marked 'interesting' percentages.

Twice the proportion (38 %) of students from post-CAN schools, compared to students from school which were not in the CAN project, employed only mental strategies in all four problems. Over twice the proportion (52 %) of students from schools which were not in the CAN project, compared to students from post-CAN schools, made multiple use of written or calculator strategies in all four problems. Ruthven comments on these figures:

> The greater use of mental strategies by pupils in the post-project schools is of particular interest, as it is consistent with the more positive attitude to mental calculation found amongst such pupils in the macro-study. (Ruthven, 1998, p. 37)

From my position, as someone with a particular interest in tool use in mathematics, this result is interesting because it shows that familiarity with a mathematical tool (a calculator) does not necessarily lead to a reliance on this tool in problem solving. Indeed, it can lead to a proclivity for mental arithmetic methods and perhaps even a 'great flowering of mental calculation'. It also questions policy statement such as 'It is forbidden to use calculators on exams because mental arithmetic keeps student intellect alive' (see Table 13.2 above).

Whilst research on calculator use is 'mixed' along several dimensions and does not 'prove results', the vast body of research points to in-school calculator use does more 'good' in terms of learning mathematics than 'bad'. But if one has a strong opinion on a subject (global warming, the economy or a tool such as the calculator) research may have little or no influence on that opinion. On this note I turn to the calculator debate.

What I call 'the calculator debate' is a series of questions or, more often, statements about calculator use in education and, in particular, in mathematics lessons and examination. The statements are sometimes categorical, 'calculators should not be used in examinations', and sometimes qualified, 'calculators should not be used until students have mastered written methods'. The national policy statements (other, perhaps, than 'no policy') in Tables 13.1 and 13.2 above are premised on national debates on calculator use and Exhibits 21 and 22 in Mullis et al. (2012) show wide variation across countries in the 2011 resolution of the calculator debate. The statements made in this debate: sometimes appear to be made in ignorance but are often carefully constructed arguments; sometimes focus on the calculator alone but often focus on the calculator in concert with innovative (or traditional) means of teaching mathematics. A local version of the latter hit the world news in the late twentieth century as *California's math wars*. Jackson (1997) reports on these 'wars' that centred on *Reform Mathematics*, of which calculators were only a part. Jackson (1997) reports that the anti-reformers believe 'the reformers have swung too far in the direction of "discovery learning" in which students discover mathematical ideas on their own rather than the teacher telling them' (Jackson, 1997, pp. 695–696). The debates/wars continue to rage (though with less publicity than the California's one). Indeed, as I write (2014) a ban (initially announced in 2012) in my country (England) on calculator use in mathematics examinations for 11-year-olds, has just been enacted. The reason that the Education and Childcare Minister Elizabeth Truss gave was that 'children were not getting the rigorous grounding in mental and written arithmetic they needed to progress' (Department for Education, 2012) but this rationale is tied up with concern for England's performance and ranking in international mathematics tests

- Tests for 10- and 12-year-olds in Massachusetts do not allow calculators. In Hong Kong, calculators are not allowed in tests for 9- and 11-year-olds. Elementary students learn how to perform basic arithmetic operations without using a calculator.
- Pupils in Massachusetts, Singapore and Hong Kong outperform pupils in England in international league tables at age 10 and age 14 ... (Department for Education, 2012)

But constructed arguments can also lead to polar position as we shall now see. Gardiner (1995) and Ralston (1999) have a number of similarities: they were written at about the same time; they are written by mathematicians with a keen interest in school mathematics education; they are accounts based on premises which value the culture of mathematics. But these two papers represent polar positions in the calculator debate. I now outline the argument in each paper.

Gardiner (1995) starts by listing four unresolved issues: what should be taught; why is it important; how should it be taught; what level of fluency is expected? Mathematics rests on the *language of expressions* and the fact that the objects and methods of mathematics are *absolutely exact*. A curriculum which embraces calculator use abandons exact forms/objects such as π and $\sqrt{2}$ and turns them into '*algorithms* to be *evaluated*' (Gardiner, 1995, p. 528). This does not mean that mathematics must not use $\sqrt{2} \approx 1.414$ but that 'what should be taught' should differentiate between $\sqrt{2}$ and 1.414, 'the "=" symbol conveys a *moral* message . . . not only. . . *exactly equal*' (Gardiner, 1995) but the person who writes it should be able to explain the transformation. The '=' button on the calculator is a 'completely different animal . . . [it is] like the magician's utterance [*abracadabra*], to focus attention on the effect, and hence distract the observer from looking for the true cause' (Gardiner, 1995). The calculator ceases to be a calculating aid and becomes a tool which 'controls, obscures, and distorts the meaning of the symbols and the operations' (Gardiner, 1995, p. 529). The calculator, to Gardiner, is not the sole culprit for the perceived sins of the curriculum he attacks, hand-in-hand with the calculator are artefacts which reduce the need for mathematical thought (tasks which consistently require only one step to obtain a solution) and artefacts which undermine the need for fluency in the language of expressions (formula books).

Ralston (1999) is in part I reaction to the *math wars* and Ralston claims that paper-and-pencil arithmetic (PPA) should 'no longer be a goal of elementary school mathematics' (Ralston, 1999, p. 173). Ralston argues that:

- PPA is not a useful life skill 'in a world where arithmetic is almost universally done using calculators' (Ralston, 1999, p. 177)
- PPA is not useful for professional mathematical pursuits because multi-digit arithmetic in these pursuits is done using calculators or computers
- Expertise in PPA methods does not promote expertise in calculator methods (and vice versa), so 'halfway houses are almost certain to be ineffective' (Ralston, 1999, p. 176)

So 'the argument in favour of learning PPA stands or falls insofar as this skill is necessary to learn subsequent mathematics' (Ralston, 1999, p. 177).

Ralston notes the importance of mental arithmetic and of algorithms for elementary mathematics but he argues that skill in PPA does not help mental arithmetic and the latter 'require that (personal) algorithms be developed and learned' (Ralston, 1999, p. 183). Ralston goes on to outline an elementary curriculum, which he considers mathematically challenging where 'mental arithmetic and calculators should not be the only tools . . . Manipulatives and other arithmetic models . . . should continue to play an important role' (Ralston, 1999, p. 185).

I expect that there are many unstated opinions (based on the authors' past experience) under the surface of the arguments that Gardiner and Ralston construct that lead to their polar positions but reading these two papers at face-value I am struck by their valuations of algorithms (which are artefacts). Gardiner clearly values specific algorithms above others and his dislike of calculators is partly due to the fact that calculators appear to undermine the algorithms he values. Ralston, on the other hand, values algorithms per se and 'it may be doubted that any real flavour of algorithms is imparted by the teaching of most PPA' (Ralston, 1999, p. 183).

I end this opening section of this chapter here, with a dispute amongst mathematicians. Perhaps mathematicians are too close to our subject to be neutral on the calculator debate. With this I turn to the writings of James Wertsch, who is not a mathematician.

13.3 Properties of Mediated Action

Chapter 2 of Wertsch (1998) examines ten 'basic claims that characterise mediated action and cultural tools' (Wertsch, 1998, p. 25). As I said in Chap. 1, I see the prefix 'cultural' as unnecessary as I do not see how any tool can be a cultural but the term 'cultural tool' has widespread use and I am content to regard it as a synonym for 'mediational means' or just 'tool'. I also do not see Wertsch's ten claims as exhaustive but Wertsch does not claim they are. But I do consider them well-considered and apply to all forms of mediated action. It is thus interesting to view the calculator, as a mediational means, in terms of general properties of any mediational means; the claims provide a means to view the particular (the calculator) in terms of the general (mediational means). I position Wertsch's book and his Chap. 2 before applying it to the calculator debate.

Wertsch is an educator but not a mathematics educator. His theoretical framework is sociocultural and he was an important figure in the 1980s in introducing Soviet activity theory in the West; see Wertsch (1981) which was considered in Chap. 9 of the book you are reading. Wertsch (1998) has six chapters: Chaps. 1 and 2 are introductory; Chaps. 3–6 consider narrative as a cultural tool for representing the past and this leads to a sociocultural analysis of official (Soviet) and unofficial histories of Estonia. Chapter 1 considers the task of sociocultural analysis, which he states is 'to understand how mental functioning is related to cultural, institutional, and historical context' (Wertsch, 1998, p. 3).Wertsch argues that this task is holistic and should go beyond the confines of individual disciplines. Ironically, given the focus on tools in the book you are reading, he writes:

> Dissatisfaction has grown with analyses that limit their focus . . . various traditions in the human sciences have had different and incommensurable ideas about the essence of human nature. Some traditions have viewed humans as political animals, others have argued that our essence lies in tool-using activities, still others define us as symbol-using animals (Wertsch, 1998, p. 3)

He takes academic inspiration for his quest from the writings of Vygotsky, Bakhtin and Kenneth Burke. In Chap. 2 he reformulates the task of a sociocultural approach to be:

> to explicate the relationships between human *action*, on the one hand, and the cultural, institutional, and historical contexts in which this action occurs . . . this involves focusing on agents and their cultural tools—the mediators of action (Wertsch, 1998, p. 24)

In the remainder of this section I state each claim in bold italics; summarise the claim; consider the implications for calculator use in mathematics education.

In stating Wertsch's claims I repeat some claims about artefacts and tool use made in earlier chapters of the book you are reading to retain the integrity of Wertsch set of claims.

13.3.1 Mediated Action Is Characterised by an Irreducible Tension Between Agent and Mediational Means

Wertsch's analysis of mediated action focuses on humans (agents) and mediational means (cultural tools). This is not to say that there are not other aspects which can/should be considered but this dyad, agent(s)-and-mediational-means, is, in the language I used in Chap. 9, his *unit of analysis*. A focus solely on either part of this dyad loses does not permit an analysis of human actions.

Consider the learner action of keying in $123 \times 45 =$ on a calculator and getting 5535. Did the child get the answer 5535?—Wertsch would answer 'no'. Did the calculator get the answer 5535?—Wertsch would answer 'no'. Did the child–calculator dyad get the answer 5535?—Wertsch would answer 'yes'. Wertsch would give similar responses if 'calculator' was replaced by 'standard written algorithm' or 'tables of logarithms' or . . . From this child–calculator dyad position it is meaningless to say that a calculator is either beneficial or detrimental for the learning of mathematics because such statements consider just one part of the essential dyad. It is, however, meaningful to speak of 'banning calculator use' because, with regard to this claim, this statement effectively means 'banning learner-with-calculator actions'. It could be (and is!) argued that some learner-with-calculator actions are beneficial or detrimental for learners at specific stages in their mathematical development and a calculator ban amounts to 'throwing out the good with the bad'.

Gibsons' *affordances* (considered in Sect. 7.3.1) is a relevant construct to introduce under this claim because the affordances of the environment are what it *offers* the agent (for good or bad); Wertsch focuses down to the agent-mediational means aspect of this environment. In mathematics similar (but not identical) artefacts can offer the child different affordances (dependent on the task). Consider the following task:

Copy and complete the following equations:

$1 + 2 = \_$

$4 + 5 + 6 = 7 + \_$

Write the next three equations.

Write down any patterns you notice.

The visual affordances of most arithmetic and scientific calculators for this task are different. In keying in, for example, $4 + 5 + 6$ on an arithmetic calculator the terms are lost as one key in the numbers and the final display is just 15. This does not happen on a scientific calculator, 15 is displayed but the expression $4 + 5 + 6$ is also displayed. This may or may not be important for a child attempting the task but it does indicate that the child–calculator dyad can be extended to the child–calculator-task triad and the type of calculator used in the task is worthy of note. This is related to what Bartolini Bussi and Mariotti (2008) call the *semiotic potential of an artefact*, the potential of an artefact to focus learners on relationships between signs.

13.3.2 Mediational Means Are Material

Wertsch (1998) notes that many mediational means have 'a clear-cut materiality in that they are physical objects that can be touched and manipulated . . . and they continue to exist as physical objects even when not incorporated into the flow of action' (Wertsch, 1998, p. 30) but 'In some instances, mediational means do not have materiality in the way that prototypical primary artifacts do' (Wertsch, 1998, p. 31). Language is the prime example of the second kind but the materiality of language is evident in its acoustic properties (language can be recorded). Establishing the materiality of mediational means is important to Wertsch because:

> The external, material properties of cultural tools have important implications for understanding how internal processes come into existence and operate. Such internal processes can be thought of as skills in using particular mediational means. The development of such skills requires acting with, and reacting to, the material properties of cultural tools. Without such materiality, there would be nothing to act with or react to, and the emergence of socioculturally situated skills would not occur. (Wertsch, 1998, p. 31).

This claim is important as terms such as 'conceptual tool' and 'psychological tool' are not uncommon in the mathematics education literature (e.g. Douady, 1985); my point here is not to proscribe such terms but to make their link the material world clear. The claim is not important to the calculator debate as the materiality of the calculator is not questioned. I would, however, add a rider to Wertsch's claim: behind the use of any material form of a tool there is also an *ideal* form of the tool.[3] Before an agent uses a tool, the agent must have an idea, which may be quite rudimentary, of what the tool is to be used for and how to act with the tool. The ideal form of a tool is not a Platonic ideal form but simply what an agent conceives prior to action with an artefact at a particular time. The ideal form of a calculator will vary across children and, in an individual child, will likely vary over the course of their mathematical development with a calculator. See Cole (1996, pp. 117–118) for further considerations about ideal and material forms of tools.

[3] This rider has similarities to (but is not identical to) Luc Trouche's distinction between an artefact and an instrument—see Luc's definition of a tool in Sect. 1.3.

13.3.3 Mediated Action Typically Has Multiple Simultaneous Goals

The main point behind this claim is that 'mediated action typically serves *multiple* purposes . . . [which] are often in conflict . . . [and] mediated action cannot be adequately interpreted if we assume it is organised around a single, neatly identifiable goal' (Cole, 1996, p. 32). This point is related to Leont'ev's distinction between 'goals' and 'motives' considered in Chap. 9. Wertsch considers the case of pole-vaulting where the obvious goal is to cross the bar but other goals may include impressing an audience or beating a particular opponent or many other things. Beyond the individual, a pole-vaulting event is set in a social setting which brings in collective goals.

In this section Wertsch considers calculations. The goal of performing a multiplication may be to get the right answer but this goal varies and/or splits into distinct artefact specific goals in different contexts. In a workplace context the right answer is usually an end-in-itself but in an educational context the goal is usually related to obtaining the goal with or without a specific artefact, such as using a standard algorithm or not using a calculator. Further to this, 'the goal of obtaining the right answer needs to be coordinated with other aspects of the sociocultural setting' (Cole, 1996, p. 34) such as, in an educational context, a test situation or practicing an algorithm in a classroom or conceptual understanding. In such settings 'the goal of the agent and the affordances of the mediational means [may] come into conflict' (Cole, 1996, p. 33). For example, the calculator does not afford developing the skills required to perform standard written algorithms.

With this claim Wertsch also illustrates that the calculator debate is but an instance of many disputes in society about the use of specific mediational means. For example, in sporting events, 'excellence' is often really 'excellence with regard to a specific and standard artefact'. For example, in the shot put, the weight of the shot is 7.26 kg for men and 4 kg for women and there are many other rules (e.g. the athlete must not wear gloves). Calls for bans on specific artefacts are fairly common over sporting history: fibreglass poles in pole-vaulting; specific types of tennis rackets; and specific types of golf balls. These calls for bans on the use of specific mediational means are associated with the affordances and constraints of the mediational means and the goals of the mediated action. Quite often the calls for bans celebrate the constraints of the old and castigate the affordances of the new.

13.3.4 Mediated Action Is Situated on One or More Developmental Paths

This claim is an elaboration of Vygotsky's (1978, pp. 64–65) assertion that, 'the historical study of behaviour is not an auxiliary aspect of theoretical study, but rather forms its very base'; the irreducible tension between agents and mediational means 'always have a particular past and are always in the process of undergoing further change' (Wertsch, 1998, p. 34). Wertsch contrasts aircraft design in the 1960s (with slide rules and drafting equipment) and in the 1990s (with computers) and asks 'What developed?' (Wertsch, 1998, p. 35). His answer is threefold: any developed intelligence goes hand-in-hand with development of mediational means; we cannot interpret development without some idea of a *telos* (end point); but development is not determined by a preordained end point, development is contingent on all sorts of things.

This claim provides means to understand the views of some players in the calculator debate. As we saw in the first part of this book, written methods, including 'standard algorithms', are simply a part of the history of human methods of calculating. Oral means and semiotic tools (abaci, tables of logarithms, . . .) have always been a part of our means of calculating and these have, over the centuries, been in a constant state of development. Some players (those who suggest banning calculators in some form) in the calculator debate do not appear to appreciate this historical development and/or implicitly consider that the end point 'arrived' with the standard written algorithms for calculation. Some other players in the debate appear to be taken with a sort of positive sense of the *telos*, that calculators are a positive force for development. I have been careful in the above to say 'some', I do not loosely attribute naivety to players in the debate. My own view is that this *telos*/development aspect of the calculator debate is important but complicates matters. It is important to know where we came from to understand where we are now (with regard to means to perform calculations) and to consider where we might be going. But we are simply at one point along a developmental path and the complication is that (1) there is no pre-determined *telos* but (2) we need to consider a *telos* to consider where the path might go.

13.3.5 Mediational Means Constrain as well as Enable Action

This claim is an elaboration of the Gibsons' construct of affordances and constraints to mediated action, 'even if a new cultural tool frees us from some earlier limitation of perspective, it introduces new ones of its own' (Wertsch, 1998, p. 39). Academics, Wertsch argues, who consider mediated action, 'can often be seen as falling into one of two basic camps, depending on whether one takes a "half-full" or "half-empty" perspective' (Wertsch, 1998). Regardless of one's perspective, 'the constraints imposed by cultural tools are typically recognised only in retrospect through a process of comparison' (Wertsch, 1998, p. 40).

The half-full and half-empty perspectives are often clearly marked in the calculator debate: Ralston is a half-full author and Gardiner is a half-empty author though I do not believe either fails to see the other perspective (they simply do not value the other perspective). Regarding constraints being recognised only in retrospect, I do not feel this really applies to the calculator debate; the accuracy constraints of log tables and, especially, slide rules were recognised long before the digital calculator arrived. But drawing attention to the Gibsons' construct of affordances and constraints is, however, almost always relevant in debates on mediated action. The Gibsons', in their many writings on affordances and constraints of agent–environment dyads, often note that these can be regarded as 'bad' or 'good'. 'Bad' and 'good' are, clearly, value-laden terms and not common terms in academic writing but they are common terms in public debates.

13.3.6 New Mediational Means Transform Mediated Action

Wertsch cites Vygotsky (1981, p. 137), 'by being included in the process of behaviour, the psychological tool [sign] alters the entire flow and structure of mental operations' and argues that this can be understood via consideration of the different *genetic domains* of phylogenesis, sociocultural history, ontogenesis and microgenesis. But, regardless of the genetic domain, 'the introduction of a new mediational means creates a kind of imbalance in the systematic organisation of mediated action, an imbalance that sets off changes in other elements . . . Indeed, in some cases an entirely new form of mediated action appears' (Wertsch, 1998, p. 43). I consider mental operations, genetic domains and imbalance in turn with regard to the calculator.

With regard to the flow of mental operations, consider teaching a class of children aged 7–8 how to write a fraction as a decimal. Shuard et al. (1991, p. 21) report on one CAN project teacher doing this with ¼. The teacher said 'The way we write it, it contains the numbers 1 and 4. What can you do with 1 and 4 on a calculator?' One child wrote:

$4 + 1 = 5$ $1 + 4 = 5$
$4 \times 1 = 4$ $1 \times 4 = 4$
$4 - 1 = 3$ $1 - 4 = -3$
$4 \div 1 = 4$ $1 \div 4 = 0.25$

He then said 'I think a quarter is 0.25' and checked it in two ways:

$0.25 + 0.25 + 0.25 + 0.25 = 1$
$0.25 \times 4 = 1$

This example is unspectacular inasmuch as the mathematics education literature teems with examples of students' idiosyncratic ways of tackling tasks with a wide variety of tools. But the example nicely illustrates the Vygotsky-Wertsch point that a new tool alters the flow of mental operations. Shuard et al. (1991) do not provide details of the child's actions beyond those I replicate above but it is reasonable to think that the child combined a systematic 'operation search' on the calculator, using the signs 1, 4, =, +, −, × and ÷, and then focused on $1 \div 4 = 0.25$. We have no idea what he was thinking at this point but whatever it was the focus of the thought was produced by child–calculator dyad. His subsequent 'double check' on his hypothesis also used the calculator. I say 'used the calculator' but it appears to me that this may be a weak term, the calculator appears to not only influence the flow of mental operations but to be an integral part of the mental operations. This phenomena often goes by the name 'distributed cognition' in the literature (Hutchins & Klausen, 1992), cognition in action is distributed over human and non-human agents. Notice that I have now edged into ascribing agency (in some form) to a tool. I shall put a stop on this speculation at this point as the example does not support further exploration but we shall come back to this issue at various points in the remainder of this book.

I now consider genetic domains and the imbalance which sets off changes in other elements. The first appearance of the digital calculator was in the genetic domain of sociocultural history—a new artefact/tool[4] appeared in society. Mathematics (used by workers in the field of electronics) contributed to this appearance but the appearance was not a part of mathematics education of the time. Before long this influenced the microgenetic actions of students engaged in arithmetic tasks, such as the actions of the student considered immediately above. Such moves from sociocultural history to microgenetic actions are not unique to the calculator, the introduction of tables of logarithms (considered in Chap. 4) led to a similar movement over genetic domains. This move from sociocultural history to microgenetic actions will occur as long as the new artefact/tool is used (in the case considered here, is used in mathematics) but the influence of this use on ontogenetic development is only possible if the use is sustained over a considerable period of time. The CAN project is an example of sustained calculator use over time and Ruthven's (1998) study appears to provide evidence of students' ontogenetic development, the use of the calculator appears to have influenced the development of students' mental calculation strategies. One could speculate on the influence on phylogenetic development, as Prenksy (2001) does with the ascription of twenty-first century children as *digital natives*, but this is a speculation too far for me at this moment in time.

The imbalance that Wertsch talks about, with regard to calculators, can be viewed as a *fallout* from the move from sociocultural history to microgenetic actions but it can also be viewed in terms of Leont'ev's (1978) triple (operations, actions, activity; considered in Chap. 9), calculator use by students transforms not only students' actions but their operations (keystrokes in place of written signs) and the activity of learning arithmetic itself. This imbalance can lead to fundamental 'what are we doing?' questions amongst those concerned with mathematics instruction—and we see differing responses to this question in Gardiner (1995) and Ralston (1999). The introduction of the calculator into mathematics classes also created curriculum and assessment imbalances. At the time the CAN project started the primary mathematics curriculum at the school level (almost all focused on

[4] I use the term' artefact/tool' in compliance with my distinction between artefacts and tools, an artefact becomes a tool in use.

arithmetic facts and algorithms) in England was organised with regard to age-related constraints on the size and type of numbers involved (e.g. learn whole number addition facts up to 20). The CAN project simply did away with this organisation, ‘Most children enjoy using the largest numbers they can handle confidently. Many CAN activities encourage children to use numbers of their own choice’ (Shuard et al., 1991, p. 13). In assessment the imbalances border at time on being absurdities such as ‘Calculators are allowed during mathematics classes but not during examinations’ (see Table 13.2 above). This says, in other words, the tool used for learning is not allowed in the examination of this learning. If a similar rule was applied to learning to drive, using the indicator for signalling turning in learning to drive a car but using hand signals when sitting a driving test, it would strike many people as strange.

13.3.7 The Relationship of Agents Towards Mediational Means Can Be Characterised in Terms of Mastery

This claim (and the next) address Wertsch’s interpretation of mediated action with regard to ‘internalisation’:

> . . . the process of internalisation consists of a series of transformations:
>
> (a) *An operation that initially represents an external activity is reconstructed and begins to occur internally . . .*
> (b) *An interpersonal process is transformed into an intrapersonal one . . .*
> (c) *The transformation of an interpersonal process into an intrapersonal one is the result of a long series of developmental events . . .*Vygotsky (1978, pp. 56–57).

With obvious respect towards Vygotsky, as a founder of sociocultural analysis, Wertsch (1998) is nevertheless critical of much discourse which invokes the term ‘internalisation’:

> It encourages us to engage in the search for internal concepts . . . [and] entails a kind of opposition, between external and internal processes, that all too easily leads to the kind of mind-body dualism that has plagued philosophy and psychology for centuries . . . it seems that many different interpretations [of internalisation] clutter the conceptual landscape and that these are tied to different exemplars. (Wertsch, 1998, pp. 48–49)

Wertsch uses the word ‘exemplars’ in the sense of Kuhn (1970) in a passage where Kuhn is reacting to the view that, in students learning, ‘Scientific knowledge is embedded in theory and rules; problems are supplied to gain facility in their application’ (Kuhn, 1970, p. 187). Kuhn argues that this view is wrong, ‘at the start and for some time after, doing problems is learning consequential things about nature. In the absence of such exemplars, the laws and theories [the student] has previously learned would have little empirical content’ (Kuhn, 1970, p. 188). The exemplars that Wertsch considers concern mediated action and he finds the word ‘mastery’ (know how) appropriate to this domain as it comes with less *conceptual* *baggage* than 'internalisation'. Further to this, it is not clear to Wertsch how many (most) forms of mediated action could be fully internalised (though some internal transformation may, of course, accompany mediated action).

I include this claim of Wertsch for completeness; it does not appear particularly relevant to the calculator debate as I am not aware of participants in the debate making positive or negative claims with regard to the internalisation of calculator use. For example, it is difficult to imagine how multiplying 343×822 with a calculator (or, indeed, with a standard algorithm) could be internalised. Mastery does appear to be a better word as we can be clear whether or not a student has mastered an arithmetic algorithm with a specific tool. With regard to clarity of exposition the 'anti-calculator' subset in the calculator debate appear to have the upper hand; the minister's statement cited above, that 'children were not getting the rigorous grounding in mental and written arithmetic they needed to progress' (Department for Education, 2012) may be open to criticism but it is a clear statement with regard to mastery.

13.3.8 The Relationship of Agents Towards Mediational Means Can Be Characterised in Terms of Appropriation

> In addition to being characterized by level of mastery, the relationship of agents to mediational means may be characterized in terms of "appropriation." In most cases, the process of mastering and appropriating cultural tools are thoroughly intertwined, but . . . this need not be the case. The two are analytically and, in some cases, empirically distinct. (Wertsch, 1998, p. 53).

Wertsch uses the term 'appropriation' in the sense of Bakhtin, 'taking something that belongs to others and making it one's own' (Wertsch, 1998, p. 53) Bakhtin's interest was language:

> The word in language is half someone else's. It becomes "one's own" only when the speaker populates it with his own interpretation, his own accent, when he appropriates the word, adapting it to his own semantic and expressive intention. Prior to this moment of appropriation, the word does not exist in a neutral and impersonal language . . . but rather it exists in other people's mouths, in other people's contexts, serving other people's intentions . . . (Bakhtin, 1981, p. 293)

But appropriation is also a relevant construct with regard to non-linguistic artefacts. There are many such artefacts that people appropriate: bicycles; clothes; guns; musical instruments; . . .; and calculators. Appropriation of an artefact can be viewed as a continuum, from non-appropriation to complete appropriation and all points between these poles. A cyclist may be someone who simply uses a bicycle but a positive assertion 'I am a cyclist' is likely to reflect a person who enjoys cycling, is proud that s/he is not polluting the atmosphere and may make modifications to their bicycle—being a cyclist to such a person is a part of their identity. I have never heard anyone say 'I am a calculatorist' but there are people who are proud of their calculator, set up special modes and have preferred sequences of keystrokes for certain types of problem. But it is difficult to imagine using a calculator being a part of someone's identity, this artefact, unlike bicycles or computers (with 'geeks'), does not seem to lend itself to intense positive personal ownership. I have, however, in the course of classroom-based research over the decades encountered students (generally high attaining students) who pride themselves on avoiding calculator use. Appropriation of (or aversion to) artefacts is not just an individual 'choice' as trends (peer pressure) in clothes, music and gadgets amongst different age group shows. In the case of the calculator a student may be influenced by the calculator debate itself, 'My teacher/parents say that calculators make you bad at maths'.

I posit that appropriation and mastery of calculators (and other artefacts) are, in general, interrelated in as much as low/high appropriation is often paired with a low/high level of mastery. An artefact that has been widely appropriated and mastered in the twenty-first century is the mobile phone. This artefact is not just a phone but a source of games, *apps* and enables the owner to access the internet—it is widely used and used for different purposes. The calculator is used for mathematics and this use is often restricted to a small set of its possible functionalities. I have seen students use an arithmetic calculator for arithmetic operations but they do not know what the memory button does, and students use a scientific calculator for trigonometric questions but they do not know what the statistical functions on the calculator are for. Mastery of their calculators in these students is localised to specific uses/functionalities and this is not likely to generate appropriation of the calculator as a useful tool for doing mathematics. Sheryn (2005), which reports on research which monitored the use of graphic calculators (GC) by six senior school students studying academic stream mathematics over 9 months, found similar behaviours, 'None of the six students were extremely proficient with their GC at the end of the year although some were confident using a very limited selection of features of the GC'. (Sheryn, 2005, p. 106) and goes on to say 'I have seen that only a few students appropriate their GC' (Sheryn, 2005, p. 107).

Low mastery and appropriation of calculators by students appears to set a serious problem for the 'pro-calculator' subgroup in the calculator debate. It may be that without large-scale integration of calculators coupled with the virtual abolishment of standard written algorithms (such as those enacted in CAN or proposed by Ralston), the majority of students will fiddle with keys they know to be useful for specific types of problems.

13.3.9 Mediational Means Are Often Produced for Reasons Other Than to Facilitate Mediated Action

Wertsch's eight claims above concern how mediational means are taken up and used. In this claim he considers how and why they are produced. Sometimes they are produced for the purpose for which they are used but this is not always the case, sometimes they are a *spin-off*. Wertsch cites fibreglass pole-vaulting poles. Fibreglass was developed by the military for reasons that had nothing to do with pole-vaulting. But once the material was produced it was available to be made into poles for pole-vaulting. Werstch also cites the QWERTY keyboard which was designed to slow down typists on manual typewriters so that the keys did not stick (a common phenomenon in nineteenth century pre-QWERTY keyboard typing). In mathematics education two of the most commonly used digital technologies have similar design histories: spreadsheets, which were designed for finance; the calculator which, as we have seen in Chap. 7, was produced because the technology to produce it was available.

It is tempting to throw out Wertsch's claim with 'So what, the technology is available, let's use it' but a consideration of the production of mediational means can provide insights into old and new mediational means. I now consider the standard written method of adding positive integers and contrast it with a calculator method and mental methods. A statement which many people on both poles of the calculator debate would agree with, though the valuations behind the agreement would differ, is 'calculator use does not reinforce skill with the traditional written method'. Let us take 363 and 448. The standard written method starts with the least significant digits, adds these (11), records the least significant digit of this addition (1) in the unit column and 'carries' the most significant digit of this addition (1) into the tens column, etc. The calculator method is key in 363, +, 448, =. This clearly does not reinforce the written method because it was not designed to do this (the technology to reproduce the visual design was not available when the first calculators were produced). Now let us consider performing 363 + 448 without writing anything down. Where do you start? Most likely with the most significant digits, which is the opposite to the standard written method. The standard written method appears to work against the mental method whereas the calculator method could be said to be neutral. The methods children use will, in general, be tied to the context of use. Selter (2001) investigates primary children's use of mental, informal written and standard algorithms in addition and subtraction tasks up to 1000 and found 'The written algorithms became the main method after they had been introduced not least because a high amount of time was devoted to them during the lessons' (Selter, 2001, p. 166). But Threlfall (2002), which focuses on mental calculations of similar aged children, reports on an orally presented task to 53 children, 45 + 48. Threlfall records eight solution types, none of which starts with the least significant digit. The point of relaying the information above is to point out differing perceptions of mediational means with regard to the time of their production. The written method itself was produced for a specific technology, pen and paper. It was produced so long ago that it is tempting to think that it is, somehow 'natural' but it is not, it is 'artefactual' just like the calculator method. The gulf between initial production of this written method and current consumption may also lead to a belief that it encourages skill in 'mental methods' but it appears that this is not so. The production of the calculator is more recent than the standard written method and there

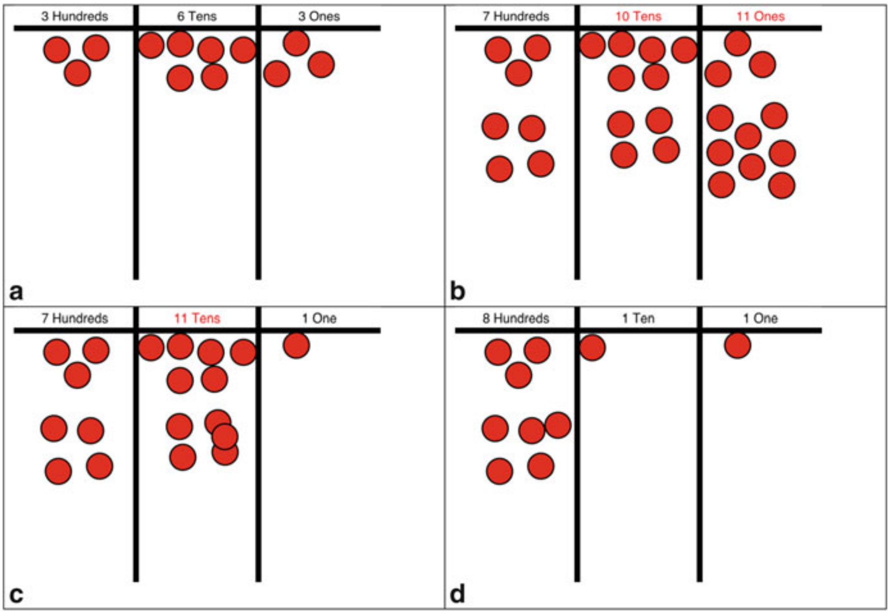

Fig. 13.1 Four images from the *app* Place Value Chart. *Instructions* Tap the screen to create tokens. Move tokens from field to field by dragging. Moving from a lower place to a higher place is only possible if you have enough tokens. Moving tokens from a higher place to a lower place is always possible. Moving one token, one field to the right will result in ten tokens there. Remove tokens by shaking the device or moving single tokens to the top

appears to be a belief that it is detrimental for skill in 'mental methods' but it appears that this is not so.

The production of mediational means develops over time. The production of twentieth century calculators resulted in linear output that did not highlight place value as both the written and mental methods above do. I write this in 2014 and I do not have a calculator that will highlight place value in the calculation of 363 + 448 but I do have an *ipod* (which is smaller than most calculators) with an *app* that will do this (see Fig. 13.1). The explanation of Fig. 13.1 is as follows: in frame (a) I have tapped the screen to obtain a visual representation of 363; in frame (b) I have tapped the screen to obtain a visual representation of 448 below the representation of 363; frame (c) shows the visual representation after I have 'swiped' the units column to the tens column; frame (d) shows the visual representation after I have 'swiped' the tens column to the hundreds column; By the time you are reading this such a calculator may exist. The production of future calculators may change the tenor of the calculator debate.

13.3.10 Mediational Means Are Associated with Power and Authority

A gun is associated with power and authority though, returning to Wertsch's first claim, it is the dyad man-with-a-gun that may wield power and authority. But Wertsch is interested in more subtle examples and cites scholars who 'have argued that the rise of print media and literacy have had a transformatory effect on how power is organised and exercised in society' (Threlfall, 2002, p. 65). In the field of education Freire (1993), amongst others, has attacked 'transmission teaching' as a means to control a population's thoughts and actions. Wertsch again looks to Bakhtin and language and how words can encourage or silence communication. The phrase 'knowledge is power' is not just appropriate to government level spies, instrumental sheet music often comes with suggested fingering but instrumental virtuosi often keep their fingerings to key pieces a secret. Mathematics is not immune from power relations: the linguistic artefact initiation-response-evaluation (what is $x + x$?-2-good) can be used to wield power in the classroom; and mathematicians, not school children, determine what counts as an elegant solution to a problem.

A classical text on authority is Weber (1947) who posited three types of authority (and responses to that authority): legal (rational obedience to the law); traditional (loyalty); and charismatic (devotion). Perhaps if Weber was alive today he would add a fourth type, 'artefactual' (obedience to digital technology). Amit and Fried (2005) examined two 8th grade mathematics teachers and their classes through observations and interviews. This study found evidence of the authority of friends and shared authority but 'The teacher's tremendous authority, in every sense of the word, was evident in all of the student interviews in both Danit's and Sasha's class' (Amit & Fried, 2005, p. 155).

Calculators in the mathematics classrooms have the potential to level out some power relations with regard to knowledge. The teacher is traditionally the possessor of knowledge in the mathematics classroom but, in a subset of mathematical problems, a calculator can give a student equal authority to the teacher as both can press a few keys and obtain the solution to $363 + 448$. In the case of symbolic calculators, the student-with-a-calculator may sometimes have more authority than the teacher with traditional tools, e.g. solving $\int \frac{1}{1+x^2}dx$ requires about the same number of keystrokes as solving $363 + 448$. This issue is not, to my knowledge, raised in the literature around calculator use in the classroom; perhaps this aspect of power associated with this mediational means is invisible to many.

One aspect of the calculator debate is about the authority associated with different mediational means for doing mathematics though 'authority' is usually implicit in the normative language of the debate, 'Once the moral imperative of the "=" symbol (*exactness in principle*) is lost, mathematics becomes no more than an experimentally based bag of tricks' (Gardiner, 1995, p. 529). A more overt aspect of the calculator debate associated with authority and mediational means are the various proposals to ban calculator use (legal authority). In the language of mediational means this is using one mediational means, a law, to proscribe the use of another mediational means, a calculator. This is a very common relationship between mediational means and accounts for a great many laws in every nation (laws on vehicles, on firearms, . . .). It is interesting that a digital artefact used in the mathematics classroom is singled out for this treatment. A compass is potentially more dangerous to the physical well-being of students but the perceived ‘mental dangers’ for students appear to be seen by some as more dangerous than these physical dangers. In stressing this point I am not suggesting that it is wrong for people to suggest a ban on calculators but not compasses, I am merely noting that it appears to class calculator use a mental hazard like pornography.

13.4 A Future for the Calculator Debate?

Wertsch’s claims do not, of course, resolve the calculator debate but they help us to see the calculator debate in the wider perspective of the influence of tools on and in practices. ‘Tool-X’ in the following could be ‘calculators’ or ‘fibreglass poles’ or many other tools.

> Tool-X came along at a certain period in time and people started doing things with it. Tool-X was, over time, incorporated into a specific practice but it came into practice that pre-dated the arrival of tool-X. When used, tool-X transformed that practice. Some people did not value the new practice as they valued the old practice though some preferred it.

When an artefact appears and someone does something with it, they do it for a reason but that is not to say that this reason for using the artefact was built into the artefact; it does not even mean there is a reason for the existence of the artefact other than it could be produced. When the artefact is used this use may be ascribed as ‘good’ (or ‘bad’) for, say, developing mathematical understanding but in reality: all artefacts have affordances and constraints; expecting an artefact used in doing mathematics to have any ‘natural’ link to the way the mind works is probably expecting too much. The use of a new artefact transforms mediated action. From the point of view of mathematics education, this transformation is at the heart of the calculator debate, whether the transformation is appreciated or not (and the forms of this appreciation will differ, even on one side of the debate, say the ‘anti-calculator’ side, over the participants, e.g. mathematicians and politicians).

Wertsch’s ten claims were not intended to be exhaustive. I would add three claims about mediational means relevant to the calculator debate: (i) a given mediational means enables action only in concert with other mediational means; (ii) mediational means exert agency; (iii) many mediational means have a finite useful lifespan. With regard to (i), calculator use in a classroom is coordinated with the use of other artefacts: tasks; black/whiteboards; computers; textbooks; pencil and paper; the structure of the lesson (the time sequence and the spatial arrangement of desks). All of these things have the potential to interrelate. With regard to (ii) I first note that all artefacts exert agency of some form. In some cases this is minimal and takes the form of 'resistance' (Latour, 2005), an artefact offers resistance when the artefact won't let me do what I want to do. At the other extreme, computer-management learning (CML), which, incidentally 'have a minimal effect on student mathematics achievement' (Cheung & Slavin, 2011, p. 17) are often programmed to give students a sequence of tasks which is determined by the students' test result; a CML (in concert with the design team) can exert a strong agency over the students. The calculator falls between these two extremes. My third claim (considered in the paragraph below) is, perhaps, the most relevant for the future of the calculator debate as it implies that the debate may simply go away.

There are mathematical tools that have withstood the test of time since they were first produced. The compass and the straight edge are examples. But tools that aid calculation have been replaced, at least at the global level (the abacus is still used in some, generally Far Eastern, educational practices and tables of logarithms and slide rules are occasionally used by history of mathematics enthusiasts). Given this history it may be reasonable to assume that the calculator will be replaced—but by what? In my discussion of Wertsch's fourth claim, mediated action is situated on one or more developmental paths, I stated that there is no pre-determined *telos* but we (mathematicians and mathematics educators) need to consider a *telos* in order to take the issue of calculator use in mathematics education forward. This is complicated for reasons related to what Engels (1894/1968, p. 694) states, 'Men make their history themselves, only they do so in a given environment, which conditions it, and on the basis of actual relations already existing'. But existing artefacts can give us a vision for future artefacts. I refer to Wartofsky's (1979; considered in Sect. 7.3.2) 'tertiary artefacts', 'artefacts of the imaginative construction of "off-line" worlds' (Wartofsky, 1979, p. 208). Even though our thoughts are bounded by our experience, these experiences allow us to imagine a world beyond our experiences. Figure 13.1 shows the output of an artefact that allowed me to imagine a touch-screen future artefact with visual output that could replace current calculator technology. The artefact which produced Fig. 13.1 was produced by Ulrich Kortenkamp, and is a feature of Ladel and Kortenkamp (2013, discussed in Chapter 9) who was inspired to produce the *app* on the basis of his work (his 'online praxis' in the language of Wartofsky) with young children engaging with mathematics on touch-screen technology. Such an artefact could (only could) bring the pole divisions in the calculator debate closer together or even make the calculator debate in its current form a thing of the past.

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