Chapter 11
Discussion of Issues in Chapters in Part II

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11.1 Introduction

This chapter is the second 'space for reflection' in this book; an opportunity for Jon and Luc to comment on John's three chapters and Luc's chapter. Richard Noss, a noted scholar and designer in the constructionist tradition in the area of mathematics and digital tools, has kindly agreed to join the discussion that led to this reflective chapter.

To structure the discussion John designed seven questions under four headings and Jon, Luc and Richard responded as they saw fit. The bulk of the text below presents the questions and the responses. 'I/my' refers to John and the questions follow the sequence of chapters in Part II.

11.2 Space for Alternative Conceptions on the Development of Tools

Chapter 7 provides a 'potted history' of the development: of tools; in understanding of the place of tools in activity; in mathematics education (of tool use in this field). Chapter 7 is my interpretation and, as such, is open to bias from my experiences, understandings and interests. In Sect. 7.2 I focus on the period from the 1960s to the present as a period which witnessed a flowering of ideas and technological developments relevant to tool use in mathematics education. My temporal focus here may simply reflect my own development as it was the period when I grew up.

Question 1
Is the period from the 1960s to the present a period which witnessed a flowering of ideas and technological developments relevant to tool use in mathematics and mathematics education?

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Responses

Richard: Well it's certainly true that the sixties represented the beginning of time: the first point at which anyone could reasonably claim that computer-use might mediate learning of anything, let alone maths. I think one can reasonably make the case that although tools were a topic in what there was in maths education (of course much less than now—and mercifully so perhaps), it was the computer—its expressive power and now its ubiquity—that has disrupted mathematical learning design and teaching practice to the point where 'tool' is hardly broad enough to characterise it.

Luc: To answer this question, we have to distinguish between mathematics and mathematics education. "For me, there are four elements leading to 'the flowering of ideas and technological developments relevant to tool use in mathematics education': the evolution of tools as supports of thinking, the evolution of schooling, the evolution of 'who is using tools'? and the evolution of perspectives in mathematics education". For example:

- Evolution of tools as supports of thinking: see the creation of writing and the developments of tools for mathematics learning in the scribal schools or the invention of printing
- Evolution of schooling: the necessity of addressing a large audience of heterogeneous students leads to the introduction of blackboards in school (and subsequent discussions on their legitimacy, as they replace oral interaction by written interactions)
- Evolution of 'who is using tools': the discussion is all the more important that the first users of tools are far from the math teachers (see the discussion on the abacus or on calculators in classroom)
- Evolution of perspectives in mathematics education: see the beginning of the twentieth century, where the mathematicians pleaded for a more active way of teaching mathematics

The feature of the period 'from the 1960' is that it meets these four evolutions: digital metamorphosis, generalisation of instruction, 'digital natives', inquiry-based mathematics teaching. Probably the first time in history where these four conditions meet with such an intensity.

Jon: I think the current cascade of new technological resources has much to offer and the ride has just begun. I hope I have illustrated this in my Chapter on homo habilitation mathematicus. That said, as I have responded in question 3, I suspect the long-term consequences remain to be identified. Moreover larger sociotechnological issues dominate which technologies flourish—if Facebook or Google sees the merit in a current tool then it will be developed but if not it is very hard for the community to find the level of resources needed to ensure successful robust and accessible implementation.

Remaining in Sect. 7.2, I am rather scathing about Piaget on tools, that he said nothing about them. Maybe Piaget had so many other important things to say that he simply didn't have time to focus on tools. Further to this, I ascribe a form of 'tool blindness' to researchers who continued work along Piagetian lines (starting from radical constructivists)—maybe I am simply unaware of post-Piagetian research on tool use in ontogenetic development.

Question 2
Am I being unfair on Piaget and post-Piagetians?

Responses
Richard: Well yes, you're being a bit unfair, although the failure to conceptualise tools (or contexts) limits the generalizability of Piaget's findings. This is the key contribution of Papert's work.
Luc: Yes, a bit unfair. Actually, for Piaget, learning comes from interactions with objects in various contexts. He probably underestimates the importance of mediations (of tools as well of institutions, mainly schools). This is the key contribution of Vygotsky's work.

11.3 On Theory and Theories

In mapping the content of this book, Luc and I took an early decision that there were three 'movements' in mathematics education that were particularly interesting with regard to tool use: constructionism; activity theory (AT); and work originating in twentieth century French didactics. I shall come on to questions specific to each of these movements in the next section but here I would like to consider the place and importance of 'theories' (constructionism and activity theory could be called 'theories' and the chapter on French didactics outlines several approaches that could be called 'theories'). Before framing my question I'd like to note that I think 'theoretical considerations' (including stating epistemological and ontological assumptions and principles regarding what it is to do and to learn mathematics) are important but (1) the theories used in mathematics education are quite different things to theories in the physical sciences and (2) theories do not exist without people to interpret them and different people may interpret a theory in different ways. I mention this simply to note that I do not see theories in mathematics education as being without problems.

Question 3
To what extent is a theory needed to understand tool use in mathematical activity?

Responses

Richard: I don't know the answer to this question but I'm sure that anyone attempting to answer it ought first to have read diSessa and Cobb (2004).

Luc: A complex question, that I could subdivide into different issues: is a theory needed to understand a specific aspect of human activity? Is a theory needed to understand this *specific* aspect of human activity that is 'tool use in mathematical activity'? Is a theory needed to understand this *more specific* activity that is 'tool use in mathematics education'? Is a *specific* theory needed to understand this specific aspect of human activity that is 'tool use in mathematical activity'? And, at least, what does 'to understand' mean?

Some elements of a personal point of view:

- Each human, aiming to accomplish a given activity needs to *understand* it. No need for a theory, but effective need for developing a reflective point of view on this activity (supported by social practices, in school, in a community of practice, etc.).
- The purpose of a science is not only to understand a given phenomenon, but to make it socially understandable.
- In this perspective, different theories could allow one to understand what is at stake in 'tool use in mathematical activity' (as in this book). . .
- I do not think that a single theory is able to grasp the whole complexity of tool use in mathematical activity: personally, when I think 'didactical situations', I have in mind the theory of didactical situations; when I think 'institutions', I have in mind the anthropological theory of didactics (ATD), when I think 'mediation', I have in mind Vygotskian theory. . .
- This kind of theoretical ubiquity is viable only if, for the particular topic I am working on—as it happens, the interaction between teachers and resources, I am able to build a kind of theoretical ecosystem, combining diverse theoretical approaches, being aware that this combination is relevant only within the perimeter of the topic I am working on.

Jon: To 'understand' I suppose one must have a theory. But as with the logical foundations of mathematics which are central to the interests of mathematical philosophers and logicians, explicit theorising has little direct impact on either mathematics teachers or researchers.

As long as education faculties function largely independently from mathematics departments and as long as evidence-based educational theory remains unusual, I do not expect things to change. There will be

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periodic edicts from ministries of education and changes to curricula in university, none of which will have the intended impact. Eventually, various tools will become embedded in educational practice whether or not their impact is understood and whether or not the teachers are sufficiently expert to properly exploit their use. Since the media and technologies are still in rapid evolution, it may well be impracticable to expect more. Have we as educators yet properly integrated the Gutenberg revolution into our teaching style?

I also suspect that for profound cultural reasons the answer to this question looks quite different in each of, say, Hong Kong, France and Australia.

11.4 Constructionism and Activity Theory

I had an interesting experience in writing the chapters on constructionism and on activity theory. I felt I knew them (and the mathematics education literature related to tool use they stimulated) quite well before I started each chapter and I expected them both to say a great deal about tool use in mathematics. But in my reading, and the subsequent synthesis of this reading for each chapter, I was a little disappointed with what they had to say about tools. I summarise these little disappointments as follows:

11.4.1 Constructionism

Mindstorms is a fascinating book but it says very little about tools per se. *Windows on mathematical meanings* (WMM) gives greater insight into tool use in mathematical activity than anything that went before and, oddly, after—that is, the constructionist community (of which Richard is a part) post WMM (1996) did not take 'the place of tools in learner meaning making in mathematical activity' beyond anything done in WMM.

11.4.2 Activity Theory

I did not find an AT view on tools in mathematical activity but, instead, found multiple AT views on tools in mathematical activity. In retrospect I should not have been surprised because the 'unit of analysis' matters a great deal in consideration of

tool use (basically that AT provides and insight on tool use when the unit of analysis has mediated action tools but when the unit of analysis is the activity system itself, AT does not provide great insight on tool use).

Question 4 (for Richard)
Is my 'little disappointment' with constructionism (from a tool use perspective) justified?

Richard's Response
Yes it is justified. Constructionism isn't really a 'theory' in the sense of, say, constructivism or 'evolution' or 'string theory' (yes I know science and social science have different criteria and meaning for the word). But it is true that the constructionist community has so far manifestly failed to situate the idea into the broader theoretical culture—a great failing of 'Windows' too: one day we will finally say more!

I also had a question on activity theory but there was only a short comment from Richard, 'I alternate between thinking it's mainly obvious and that it's used too formulaically to be useful (triangles!!)'.

11.5 On French Didactics

I found the chapter on French schools of thought fascinating in terms of the context provided. I have followed these schools of thought for several decades, so there was little new for me in terms of what theoretical frameworks say, but in terms of contextualising these frameworks within wider mathematical and educational movements I learnt a great deal. Of the many questions I could ask I have selected two. These questions are rather specialised and so I do not really expect anyone except Luc to answer them.

My first question relates to similarities and differences constructionism (as advanced in *Windows on mathematical meanings*—WMM), the theory of didactical situations (TDS) and the ATD with regard to the place of tools in learner meaning making in mathematical activity. My interpretation of the similarities and differences in these three frameworks is briefly summarised as follows. All three frameworks are centrally interested in learners' mathematical actions. In WMM the focus is on the joint design of tasks and tools, which allow learners to make connections/mathematical relationships. In TDS the teacher designs the milieu (which includes tools) to facilitate learners formulating and validating a pre-determined mathematical understanding. ATD is also interested in the milieu (which includes ostensives) but individual meaning making is viewed via institutional practices which overshadows individual meaning making.

Question 5
What is your reaction to my summary?

Luc's Response
I certainly agree with John's implied point that constructionism and didactiques are fully compatible as theories, although this might be because they belong to different forms of theory (see the diSessa and Cobb paper mentioned above).

- Two nuances: For the TSD, the *milieu* is not done once for ever. Students interact with the milieu, and, in this measure, contribute to its design. In the tread of this theory, Sensevy (2009) and his colleagues developed a theory of the *joint action* of a teacher and her students, all of them having a responsibility to the progression of the knowledge in a given classroom.
- For the ATD, instead of 'institutional practices', I would speak of 'institutional constraints', that influence the relationships of the individuals to knowledge and the way they accomplish tasks, using various ostensives.

My second question on French didactics concerns the instrumental approach and its relation to Leont'ev's approach to activity theory (activity–actions–operations). A strength of the instrumental approach is that it makes few assumptions but has a wide field of application. Leont'ev's approach to activity theory can be used, as was seen in Chap. 9, to shed light on the relationship between learners and their environments including the process by which an artefact becomes a tool for learners. The instrumental approach has the potential to enhance our understandings of the action and operation aspects of Leont'ev's approach without compromising any of its basic assumptions.

Question 6
What is your reaction Luc? Can the instrumental approach and Leont'ev's approach to activity theory be 'networked'?

Luc's Response
Before answering to your question, I would like to be sure to correctly understand what do you mean by 'the instrumental approach makes few assumptions'. For me, precisely, it can be used 'to shed light on the relationship between learners and their environments including the process by which an artefact becomes a tool for learners'. Perhaps we need to distinguish the Rabardel's approach, and the result of its appropriation by some French didacticians?

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For my own experience, the limitation of the instrumental approach is in its consideration of social processes (even if Rabardel himself, in his seminal book in 1995, evokes social schemes, or a social part of schemes). This 'missing resource' leads my doctoral students, who aimed to capture social processes, to use other theoretical frameworks, as communities of practice (Sabra, 2011) or activity theory (Hammoud, 2012). The communities of practice framework were useful for its concepts of participation and reification whilst the activity theory framework was useful for its notion of rules and division of labour. In a recent paper Gueudet et al. (2015), we have used both the documentational approach of didactics and CHAT to study the collective design of an e-textbook, analysing both the activity system of the community of designers, and the documentational genesis of the designed resources.

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