An Application of Myriad M-Estimator for Robust Weighted Averaging

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Abstract. The method of signal averaging is such technique that allows the repeated or periodic waveforms which are contaminated by noise to be enhanced. The most often used operation for averaging is the arithmetic averaging and its different variations. Unfortunately the mean operator is sensitive for outliers. In this work the well known myriad M-estimator is applied for averaging. The myriad weighted averaging allows to suppress the impulsive type of noise. In order to evaluate the proposed method, artificial impulsive noise is generated with using the symmetric α-stable distributions. The impulsive noise component is added to the deterministic signal with known value of geometric signal-to-noise ratio (GSNR) which is equivalent of ordinary SNR. The experiments show usefulness of the proposed method for weighted averaging of periodic signals like ECG signal.

Keywords: weighted averaging, outliers, myriad.

1 Introduction

Averaging is one of the basic methods in statistical analysis of experimental science especially in the case when the system response is periodic [2]. This procedure is frequently applied for estimating the location of data in the presence of random variations among the observations which can be removed by application of this procedure [8]. There exists special reason for application of averaging. The traditional linear filtering schemes fail when the signal and noise frequency spectra significantly overlap [11]. Such situation takes place in analysis of biomedical signals like electrocardiograms (ECG), electroencephalograms (EEG) or other. Signal

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averaging allows to separate a repetitive cycles from noise without introducing signal distortion [11]. The signal averaging is based on the following assumptions: the signal waveform (cycle) is repetitive, the noise has to be random and uncorrelated with the signal and the temporal position of each cycle must be precisely known [2, 11]. Under the assumption that the noise is stationary with zero mean and not being correlated with the signal [7] the noise-reduction factor is equal to $\sqrt{N}$, where N is the number of averaged signals [3].

However presented arithmetic averaging is affected by a quite serious drawback which is sensitive to outliers caused by spikes artifacts and bursts of noise. Then the noise components have impulsive nature which is quite different from Gaussian distribution of noise. The impulse kind of noise is that noise which causes that the linear filtering technique lets down. Non-gaussianity results in significant performance degradation for systems optimized under the gaussian assumption [10]. This disadvantage is unacceptable in many situations because effective process of noise reduction is a first step in every signal processing system. Precision of all later actions (i.e. detection, classification, measurement, etc.) performed on the signal depends on quality of noise-reduction algorithms [7].

Additionally, traditional averaging method assumes that the noise power is constant, however most types of noise are not stationary. In reality, it can be noticed some variability of noise power which can vary from period to period. For these reasons the robust, weighted averaging method should be applied.

The objective of this work is to establish the robust method of weighted averaging with the connection of the myriad cost function. This paper presents a new robust myriad weighted averaging method. The paper is divided into four sections. Section 2 presents the idea of the weighted averaging method based on the minimization of the scalar criterion function and introduces the proposed method. Section 3 describes the numerical experiment and contains some results. Finally, the conclusions are given in Section 4.

2 The Weighted Averaging Method

2.1 Idea of Criterion Function Minimization

In [7] it is presented the weighted averaging method based on criterion function minimization (WACFM). The idea of this method is following. We start with the description of used denotation. Let us consider N cycles of periodic signal where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{iM}]^T$ is the ith signal cycle which consists of M samples and $1 \leq i \leq N$, $\mathbf{v} = [v_1, v_2, \ldots, v_M]^T$ is the averaged signal, $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ is the weight vector which satisfies the following condition

$$\underset{1 \leq i \leq N}{\forall} w_i \in [0,1], \sum_{i=1}^{N} w_i = 1 \; . \tag{1}$$

The scalar criterion function is defined as:

$$I_m(\mathbf{w},\mathbf{v}) = \sum_{i=1}^{N} (w_i)^m \rho(\mathbf{x}_i - \mathbf{v}) \;, \tag{2}$$

where $\rho(\cdot)$ is a measure of dissimilarity for vector argument, $m \in (1,\infty)$ is a weighting exponent parameter. Scalar criterion function (2) can be regarded as a measure of total dissimilarity between $\mathbf{v}$ and signal cycle x_i weighted by $(w_i)^m$, where $m > 1$. The task of searching for an optimal averaged signal $\mathbf{v}^*$ and an optimal weight vector $\mathbf{w}^*$, can be formulated as follows:

$$I_m(\mathbf{w}^*,\mathbf{v}^*) = \min_{\mathbf{w},\mathbf{v}} I_m(\mathbf{w},\mathbf{v}) \;. \tag{3}$$

Minimization of (3) with respect to $\mathbf{w}$ yields:

$$\underset{1\le 1\le N}{\forall} w_i = \frac{\rho\left(\mathbf{x}_i - \mathbf{v}\right)^{1/(1-m)}}{\sum_{i=1}^{N} \left[\rho(\mathbf{x}_j - \mathbf{v})\right]^{1/(1-m)}} \;. \tag{4}$$

The robust property of the weighted averaging strictly depends on the measure of dissimilarity $\rho(\cdot)$. The square function $\rho(\cdot) = ||\cdot||_2^2$ is frequently used [7]. If the weight vector $\mathbf{w}$ is given then the criterion function's (2) gradient with respect to an averaged signal $\mathbf{v}$ is set to zero and we obtain:

$$\frac{\partial I_m(\mathbf{w},\mathbf{v})}{\partial \mathbf{v}} = -2\sum_{i=1}^{N} (w_i)^m (\mathbf{x}_i - \mathbf{v}) = 0 \tag{5}$$

In this case the averaged signal $\mathbf{v}$ is given as:

$$\mathbf{v} = \frac{\sum_{i=1}^{N} (w_i)^m \mathbf{x}_i}{\sum_{i=1}^{N} (w_i)^m}, \tag{6}$$

and vector of the weights $\mathbf{w}$ is estimated as:

$$\underset{1\le 1\le N}{\forall} w_i = \frac{\left[||\mathbf{x}_i - \mathbf{v}||_2\right]^{2/(1-m)}}{\sum_{i=1}^{N} \left[||\mathbf{x}_j - \mathbf{v}||\right]^{2/(1-m)}}. \tag{7}$$

The optimal solution for minimization (2) in the case of the square function $\rho(\cdot)$ is obtained from the application of the iterative Picard algorithm with the formula (4) for $\mathbf{w}$ and (6) for the averaged $\mathbf{v}$ signal. This method is called weighted averaging based on criterion function minimization (WACFM). In this paper $m = 2$ which results in greater decrease of medium weights [7].

2.2 Myriad Weighted Averaging

The scalar criterion function (2) can use the alternative form of the dissimilarity function $\rho(\cdot)$ provided that satisfied the following properties [7]: 1) $\rho(\mathbf{0}) = 0$, 2) $\rho(\mathbf{y}) = \rho(-\mathbf{y})$, 3) $\forall_{1\leq j\leq p} \quad y_j \leq z_j \Longrightarrow \rho(\mathbf{y}) \leq \rho(\mathbf{z})$ – monotonicity, where all vectors $\mathbf{y},\mathbf{z} \in \Re_p$. One of the function which satisfies above conditions is the following function:

$$\rho(x) = \log\left(1 + x^2/K^2\right) , \tag{8}$$

where K is the linear parameter. The function presented in (8), known as the cost function is often used to define a maximum likelihood estimator of location in robust signal processing. This function is connected with Cauchy distribution and the myriad filter [1, 5, 6]. The linear parameter K controls the robustness of the myriad estimator. For small value of K, the myriad value tends to favour values near the most populated clusters of input samples. The case $K \to 0$ leads to highly robust selection location estimator called mode-myriad. The other special case takes place when $K \to \infty$ and signal samples satisfy Gaussian distribution. Then myriad estimator of location behaves like the arithmetic mean estimator [4].

Using (2) and (8) the scalar criterion function can be rewritten as:

$$I_m(\mathbf{w},\mathbf{v}) = \sum_{i=1}^{N} (w_i)^m \log\left(1 + \left(\frac{\mathbf{x}_i - \mathbf{v}}{K}\right)^2\right) . \tag{9}$$

If $\mathbf{v} \in \Re$ then Lagrangian of (9) with constraints from (1) is:

$$L(\mathbf{w},\lambda) = \sum_{i=1}^{N} (w_i)^m \log\left(1 + \left(\frac{\mathbf{x}_i - \mathbf{v}}{K}\right)^2\right) - \lambda\left[\sum_{i=1}^{N} w_i - 1\right] , \tag{10}$$

where λ is the Lagrange multiplier. Assume that the Lagrangian gradient is set to zero:

$$\frac{\partial L(\mathbf{w},\lambda)}{\partial \lambda} = \sum_{i=1}^{N} w_i - 1 = 0 \tag{11}$$

and

$$\underset{1\leq j\leq N}{\forall} \frac{\partial L(\mathbf{w},\lambda)}{\partial w_j} = m(w_j)^{m-1} \log\left(1 + \left(\frac{\mathbf{x}_i - \mathbf{v}}{K}\right)^2\right) - \lambda = 0 . \tag{12}$$

From (12) we can write:

$$w_j = \left(\frac{\lambda}{m}\right)^{1/(m-1)} \left[\log\left(1 + \left(\frac{\mathbf{x}_i - \mathbf{v}}{K}\right)^2\right)\right]^{1/(1-m)} \tag{13}$$

From (11) and (13), we get:

$$\left(\frac{\lambda}{m}\right)^{1/(m-1)} \sum_{i=1}^{N} \left[\log\left(1+\left(\frac{\mathbf{x}_i-\mathbf{v}}{K}\right)^2\right)\right]^{1/(1-m)} = 1 \,. \tag{14}$$

And finally, from (14) and (13), we obtain:

$$\underset{1\le i\le N}{\forall} w_i = \frac{\left[\log\left(1+\left(\frac{\mathbf{x}_i-\mathbf{v}}{K}\right)^2\right)\right]^{1/(1-m)}}{\sum_{j=1}^{N}\left[\log\left(1+\left(\frac{\mathbf{x}_j-\mathbf{v}}{K}\right)^2\right)\right]^{1/(1-m)}} \,. \tag{15}$$

If we assume that $\mathbf{v}$ is fixed, the next step of algorithm is estimation of averaged signal $\mathbf{v}$. Let the criterion function's gradient (9) with respect to averaged signal $\mathbf{v}$ is set to zero, then we get:

$$\frac{\partial I_m(\mathbf{w},\mathbf{v})}{\partial \mathbf{v}} = \left(\sum_{i=1}^{N}(w_i)^m \log\left(1+\left(\frac{\mathbf{x}_i-\mathbf{v}}{K}\right)^2\right)\right)' = 0 \,. \tag{16}$$

For a given data $\mathbf{x}$, the solution of (16) can be solved by using fixed-point search algorithm which can be written as:

$$\mathbf{v}_{(k+1)} = \frac{\sum_{i=1}^{N}\phi(\mathbf{v}_{(k)})\mathbf{x}_i}{\sum_{i=1}^{N}\phi(\mathbf{v}_{(k)})} \,, \tag{17}$$

where

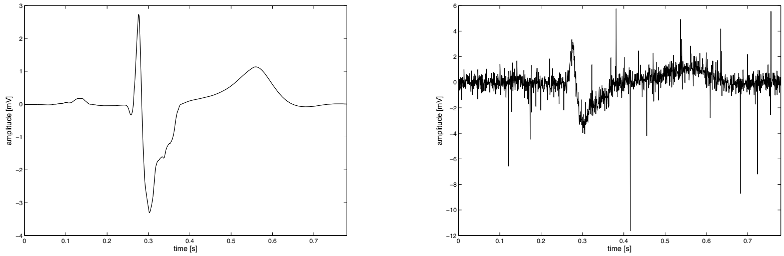
$$\phi(\mathbf{v}_{(k)}) = \frac{(w_i)^m}{\left(1+\left(\frac{\mathbf{x}_i-\mathbf{v}_{(k)}}{K}\right)^2\right)}, \tag{18}$$

and where the subscript denotes the iteration number. The algorithm is taken as convergent when $||\mathbf{w}_{(k+1)} - \mathbf{w}_{(k)}|| < \delta$ and δ is a small positive value ($\delta = 10^{-6}$). On the basis of (15) and (17) the new method of robust averaging is obtained that can be called WACFMMy.

3 Numerical Experiments and Results

Performance of the proposed method is evaluated in comparison with the trimmed-mean averaging (TMA) and WACFM method from [7]. The method based on minimization of scalar criterion function are initialized with the vector of all ones and $m = 2$. For a computed averaged signal the quality of tested methods is evaluated by the maximal absolute difference between deterministic component and the averaged signal (MAX). The averaging process should not deform the signal. For that reason, the presented methods are evaluated using the root mean-square error (RMSE) between the deterministic component and the averaged signal. All experiments are done in MATLAB environment.

For testing requirements the ECG signal from [7] is chosen. This signal is obtained by averaging 500 real ECG cycles (sampled at 2 kHz with 16-bit resolution) with a high signal-to-noise ratio (Fig. 1(a)). Before averaging these cycles are time-aligned.



(a) (b)

Fig. 1 Original ECG signal – deterministic component (a) and ECG cycle corrupted with simulated impulsive noise (b)

The purpose of this experiment is to investigate the proposed in this paper method in the presence of impulsive noise. This kind of noise is modelled on the basis the symmetrical α-stable (SαS) distribution [4, 9]. In order to simulate the real conditions of acquisition a series of 100 ECG cycles is generated with the same deterministic component and an impulsive noise with known four values of the generalized signal-to-noise ratio GSNR [9] which is equivalent of ordinary SNR, but in the case of the Gaussian distribution of noise. For the first, second, third and fourth 25 cycles, the GSNR values are 5, 10, 20 and 40 dB. The level of impulsiveness in SαS process is controlled with the characteristic exponent α and in this paper α changes from 1 to 2 with step 0.1. An example of ECG cycle corrupted with an impulsive noise modelled with SαS process with GSNR $= 5$ [dB] and $\alpha = 1.6$ is presented in the Fig. 1(b).

The RMSE and the maximal value (MAX) of residual noise for all tested methods are presented in Table 1 (the best results are bolded). An example of averaging of ECG cycles is presented in the Fig. 2.

The best noise reduction for the evaluated methods is obtained for trimmed-mean averaging but only for very impulsive case $\alpha = 1.0$. When $1.1 \leq \alpha \leq 1.8$ the best results of RMSE factor are obtained for the proposed method WACFMMy. But for $\alpha = 1.9$ and $\alpha = 2.0$ the best results are obtained for the WACFM method. These results show effectiveness of the WACFM method in the presence of Gaussian noise but this method fails when the level of impulsiveness is higher. The reason of such fact is the application of square function as the dissimilarity function. The proposed method WACFMMy uses the dissimilarity function which is more robust. Unlike the

Table 1 RMSE $[\mu V]$ and MAX $[\mu V]$ values for averaged signals in environment of an impulsive noise

	RMSE $[\mu V]$			MAX $[\mu V]$		
α	TAM $p = 25$	WACFM $m = 2$	WACFMMy $K = 0.1$	TAM $p = 25$	WACFM $m = 2$	WACFMMy $K = 0.1$
1.0	**17.9**	39.21	27.28	**59.7**	546.34	171.29
1.1	18.8	34.08	**17.66**	**66.0**	419.40	175.18
1.2	18.6	24.84	**13.22**	**69.2**	315.41	123.91
1.3	18.0	7.58	**5.78**	**60.7**	117.36	68.94
1.4	18.4	6.56	**5.74**	**55.7**	57.59	70.54
1.5	17.6	7.09	**5.29**	56.8	**55.86**	73.87
1.6	18.1	3.86	**3.46**	58.7	**22.42**	31.89
1.7	17.8	2.79	**2.76**	62.5	**14.72**	17.24
1.8	18.2	2.09	**2.05**	71.9	8.87	**8.74**
1.9	18.1	**1.81**	1.88	58.4	7.28	**7.10**
2.0	18.2	**1.62**	**1.62**	53.3	**5.05**	**5.05**

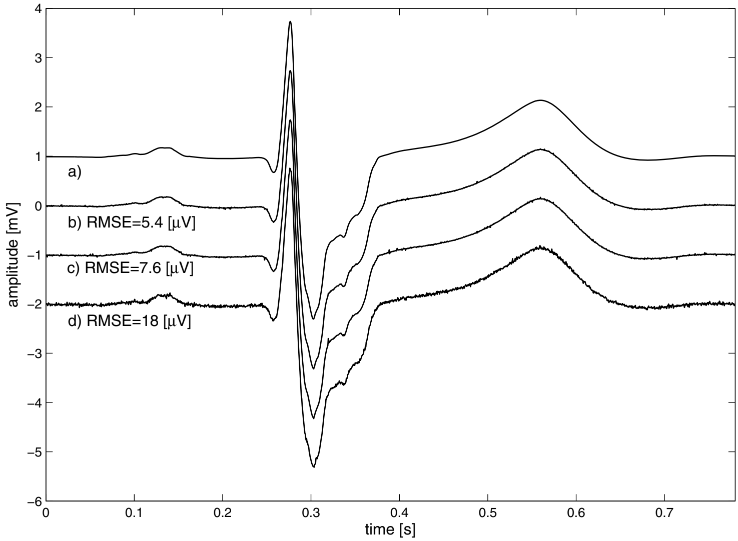


Fig. 2 Results of averaging: a) original signal, b) WACFMMy, c) WACFM, d) trimmed-mean. Signals are shifted vertically for better presentation.

results obtained for RMSE, the best values of MAX factor are obtained for trimmed-mean method for $(\alpha \in \langle 1.0, 1.4 \rangle)$, but for $\alpha \geq 1.5$ the best values are for WACFM method. The proposed method WACFMMy reaches the best results of MAX factor when $\alpha \geq 1.8$.

4 Conclusion

In this work the new method of robust weighted averaging of periodic signals is presented with example of ECG cycles averaging. The proposed method uses the minimization of scalar criterion function with the robust dissimilarity function of the form known from the myriad maximum likelihood estimator. The robustness of the proposed method can be controlled with one parameter. The obtained results show the usefulness of the presented robust myriad weighted averaging method for ECG signal processing. Therefore this method leads to the best results in a wide range of impulsiveness changes. It should be pointed out, that this method allows to suppress the impulse type noise in periodic or quasi-periodic signals.

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