# Drawing on Understanding of Workplace Practice to Inform Design of General Mathematics Curricula

**Geoff Wake**

In this article, I explore how we might develop general mathematics education curricula that reflect understanding of the nature of activity in relation to mathematics in both schools and workplaces. In doing so, I like many other researchers in the field of workplace mathematics, adopt a sociocultural theoretical perspective. Ideas of expansive learning and developmental transfer appear to offer potential for vocational and possibly well-focused prevocational education. For more general mathematics education, it is the nature of mathematical activity in horizontal and vertical senses that perhaps provide a way forward. Further, I suggest that learning communities need to not only consider the content of the curriculum but also to reconsider the didactical contract in their mathematical activity.

## 1 Introduction: The Transfer Problem

This chapter reflects on research that explored the boundaries between workplace practice and school/college mathematics (Wake and Williams 2001). In the development of about a dozen case studies, each of which focused on the practice of a particular worker, in a range of different settings we engineered 'breakdown' moments (Pozzi et al. 1998) by asking workers to explain their activity to researchers and students with their teachers. This generated understanding not only of the activity of the worker and its relation to mathematics but also provided insight into the use of academic mathematics and the affordances and constraints of current curricula. More recently, my involvement in research into proposed curriculum changes in mathematics in England, and into transitions into

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'mathematically demanding' courses in university has provided additional insight into the central issue of transfer: that is, the use, or application, of mathematics in a range of different settings.

In the last two decades or so many researchers have contemplated the overarching question of how 'transfer' might be conceptualised and better supported. Extreme positions in the debate might be characterised as those of, on the one hand, proponents of situated cognition (that is, that knowledge is developed in social settings by individuals in interaction with others and is dependent on the cultures, traditions and values of the community: see, for example, Lave (1988) and Lave and Wenger (1991)), whilst on the other hand, proponents of the classical psychology and information processing perspective (with knowledge being abstract generalisable and applicable in a range of different situations (e.g. Anderson et al. 1996). Our own research in many ways appeared to support the former contention as we found that students appeared ill-equipped to use their knowledge of school mathematics to understand workplace practices. We found their mathematical competence to be very much situated in a school culture that values technical competence with well-defined procedures for solving problems in familiar, and often mathematical, settings. This particular genre of mathematics appeared not well suited to support transferability, or transformation, of mathematics into unfamiliar settings. To illustrate this consider the following brief vignette which I have written about in more detail elsewhere.

As part of one case study we investigated the work of Alan, a railway signal engineer, who explained some of his day-to-day workplace activity to a group of students on a prevocational engineering course. As part of his work, Alan checked calculations of colleagues of where to place signal boards to give advance warning to train drivers that, at a signal ahead, they may be required to bring their train to a halt. The example training calculations that show how to find the average gradient over a number of sections of track proved particularly problematic for the students to understand. Alan explained that such averaging is required so that account can be taken of the gradient of the track when positioning signal boards because uphill gradients assist, and downhill gradients oppose, the braking of the train. Crucially the students were unable to bring together their understanding of the concepts of gradient and of averages to make sense of the process illustrated. This brief extract from the transcript of the discussions between researcher and one student from a group of three illustrates this.

| | |
|---|---|
| Researcher | Yes… So can you just explain what's going on in there [indicating a table which systemised using different gradients to find the total rise/fall over sections of track of different lengths] |
| Student | used different gradients for each slope and he's averaged it out… |
| R | yes can you sort of explain the detail … |
| S | you started adding them together—adding the gradients together and divide by two |
| R | Perhaps if we describe what each column is doing |

Here, the student appears to associate finding an average with the school mathematics procedure of "adding the values together and dividing by the number of values" rather than finding the total fall of track and dividing by its total length. The ensuing discussion was lengthy requiring the researcher to explain the basic concept of gradient, by drawing a number of diagrams.

Throughout our case studies we found students similarly ill-equipped to understand how workers were using mathematics in relation to their day-to-day work. In this article, therefore, I revisit the findings from the project that generated this vignette and reflect on these in light of current understanding of the nature of workplace activity involving use of mathematics with the aim of considering how this might inform future development of mathematics curricula in general education. At a time when many countries seek to encourage more of our young people to be motivated towards further study in science and technology-based subjects this provides an important challenge in strategic design (Burkhardt 2009).

## 2 The Nature of Mathematics in Workplaces

In summary we noted the following important features relating to the practices of workers as they went about those of their day-to-day activities that involved use of mathematics (in some way):

- Knowledge is often crystallised (e.g. Hutchins 1995) in artefacts, including tools and signs, often as a result of reification by workplace communities (Wenger 1998).
- Use of mathematics is often 'black-boxed' (Williams and Wake 2007) and engagement with mathematics often only occurs at 'breakdown' moments.
- The fusion (Meira 1998) of mathematical signs (in the sense of Pierce) with the reality they represent reduces cognitive effort.

Further, we identified the following important issues relating to mathematics content and competences that might usefully inform future development of both (pre-)vocational and general mathematics curricula:

- School/college mathematics is just one genre of mathematics and should be recognised as such with attention being drawn to the diversity of ways in which mathematics might appear elsewhere. This suggests that it is important to focus clearly on key mathematical concepts and principles and for students to experience how these can be applied in a variety of different situations using a range of different notations, inscriptions and so on.
- Mathematics is used in a rich variety of contexts both in workplaces and more generally in communicating information in all walks of life; these contexts are often complex and detailed, although often simplified to allow mathematical analysis. Mathematics curricula should allow time and space for students to

experience using their developing mathematical knowledge, skills and understanding in increasingly complex situations.

- Students appear armed with competencies in relation to mathematics that sees them particularly inadequately prepared to engage in using mathematics in workplaces. Particularly important in this regard is their lack of skill in making sense of the "mathematics of others". This is something that many workers have to do, given that they often take over parts of the work process that have previously been established. We note that our research pointed to a number of strategies useful in this regard (Wake 2007) and these should be highlighted in curriculum specification.
- Workers are often so immersed in their practice that the mathematics becomes 'fused' with the workplace reality it models. Underpinning assumptions are not made explicit but workers fully understand how a change in these will affect outcomes in terms of workplace processes. Curricula should provide students with experiences of working with mathematics in complex situations that mirror such scenarios with particular attention being paid to interpretation, variation and adaptation of models.
- Our research identified seven general mathematical competences (for example, interpreting large data sets, costing a project) (Wake and Williams 2001) each of which we saw in use across a number of different workplaces. Fundamental to these is the expectation that technology is integral as a tool when mathematics is being applied. This is mirrored in other recent research that identifies and organises mathematics around techno-mathematical literacies (Hoyles et al. 2007). Curricula should recognise and emphasise such competences.

At the time of this research, and in much of my research in schools/colleges since, it seems the case that mathematical content, often due to curriculum specification, is seen as compartmentalised around major content themes such as number, algebra, geometry, statistics and probability with statements of requirements and resulting curriculum implementation often being atomised. Resulting pedagogies are often transmissionist (Pampaka et al. 2011) and concerned with the development of instrumental, rather than relational, understanding (Skemp 1976). Consequently consideration should be given to how connections can be made across mathematical content areas cognisant of how concepts are often blended in workplaces and other areas of application.

## 3 Theoretical Perspectives

To understand the different practices we observed, and to explore the relationship between workers, mathematics and their socially constituted workplace practices, our analysis of case studies from a sociocultural perspective drew in particular on Cultural Historical Activity Theory (CHAT). Before attempting to synthesise the above to reach a strategic overview that might inform future curriculum development I, therefore, reflect further on considerations and understanding of 'transfer' from a CHAT perspective.

The well-known schema of Fig. 1 (Engeström 1987) draws our attention to the complexity of the interacting factors that mediate the actions of an individual within the activity of a system such as a workplace or school community. Building on Vygotsky's (1978) conceptualisation of artefact mediated actions of the individual in the upper triangle, in second generation Activity Theory, Leont'ev (1981) and followers expanded the unit of analysis to take into account the activity of the collective, as represented by the lower part of the triangular structure. This draws attention to how an individual's actions are socially constructed and mediated by rules (both implicit and explicit and which are historically evolved) and the division of labour between members of the community. Further, in third generation activity theory, Engestrom (2001) and others consider the interaction of two or more activity systems leading to notions of boundary objects (Star 1989) (artefacts that have 'currency' in each system) and boundary crossing (Engeström et al. 1995) by individuals who move between systems.

It is in the 'boundary space' between workplace and college that Hoyles et al. (2010) identify the potential for workplace training or perhaps, 'education for the workplace'. In such situations there is the potential for expansive learning and developmental transfer as participants in activity systems question and develop their practices in each and with each system consequently learning from the other. However, in general curriculum provision for mathematics that might better prepare students to be able to transform, mathematics into a *range* of different practices in which they might engage in the future, the development of such a boundary space does not at first sight appear to provide a solution. It is this unknown future participation that poses the problem: how can we second-guess the nature of the knowledge, skills and understanding that might be needed?

I suggest that the way forward is to consider the generality of the actions of individuals that emerged in the boundary space activity that our research, like that of others, developed. In particular, I draw attention to how, as an outsider introduced to a novel practice, one is required to strive to make sense of the mathematical activity as it has been historically and culturally constituted. This requires skills that allow one to de-construct these existing practices; in other words, to decouple the mathematics and the reality it models, prior to being able to reconstruct and if required build on these practices. It is also important to recognise the mathematics itself may comprise of a blend of mathematical concepts across domains as in the vignette reported here. This suggests that ideas of horizontal and

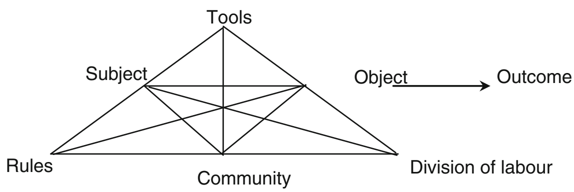


**Fig. 1** Activity system schema

vertical mathematisation, in the sense of Freudenthal and colleagues (e.g. Treffers 1987), are important, with 'models of' situations being generative of 'models for' further mathematical development. Crucially here mathematical models and mathematical modelling competencies here have an important role to play.

This is perhaps best exemplified by referring back to the illustrative vignette introduced above. Here we have a workplace activity relying on the blending of two important mathematical concepts (each potentially difficult in their own right): those of gradient and average. The students in the 'boundary space' afforded by the research activity struggled to unearth their understanding of these concepts beyond a most elementary procedural understanding. A more productive attempt to make sense of the mathematics would have understood that what had been developed was in effect a *model for* an average gradient over the whole length of track. It is in the development of this model that we detect vertical mathematisation as models *for* gradients of contributing sections of track are deconstructed to provide values for total rise/fall and total length of track. In terms of mediated actions we might view this as the learner having to switch from mathematics as being initially a tool for understanding the workplace artefact of gradient for a section of track and then becoming the object of study itself as the student/researcher uses it to reconstruct average gradient over a number of sections. In either of these interpretations of student action it is essential that the student is able to switch back and forth between the 'model for' and 'model of' the situation with ease and with understanding of one reinforcing understanding of the other.

This suggests that we require a curriculum formulation that

1. develops understanding of mathematical concepts *in addition to* procedural fluency with techniques
2. prioritises competencies in mathematical modelling and applications
3. requires students to engage with making sense of, and developing existing mathematical models (of others) of non-trivial situations.

In general, this suggests that mathematical modelling needs added emphasis in future curricula, and that in addition to developing a meta-cognitive understanding of modelling, it is also important that students engage with, and grapple with coming to understand, the models of others. It is important that students develop the enquiry skills that those in our research project lacked as it is such critical inquiry that is quintessential to the curriculum development proposed.

This draws attention to the nature of the activity of the learning community and how this needs to be reformulated: in essence a renegotiation of the didactic contract (Brousseau 1997). It was noticeable how in our workplace research and more recent research in classrooms, workshops and lectures in schools, colleges and universities in their implementation of the curriculum teachers adopt mainly transmissionist teaching practices (Pampaka et al. 2011). This suggests that teachers generally believe that competence in transfer is achieved through learners firstly acquiring technical and procedural competence prior to application (which in many cases can be an afterthought if it is suggested at all). The proposal here that the curriculum requires, at least in part, substantial connection with a non-mathematical reality, and that exploring this becomes the focus of collective activity building from individual actions, suggests a very different approach and consequently adoption of different roles by both teachers and learners affecting the division of labour in learning communities as it is generally currently constituted.

## 4 Conclusion: The Challenge for Curriculum Design

In summary my proposal is for future general mathematics curricula to introduce new practices for students that prioritise making sense of, and developing further, the mathematical models of others. This provides a major challenge in strategic curriculum design and I draw attention to three major factors that such design needs to take into account.

1. Specification. Mathematics curricula are often specified by stating mathematical content that should be learned. In developing a curriculum as suggested here it is important to emphasise expected outcomes in terms of new skills and competencies that learners require in understanding, and being able to develop, mathematical models. There is currently limited understanding of how learners might best de-couple mathematics and reality in ways that allow them insight into each. This an under-researched area, but one that might be informed by the substantial body of research into use of mathematics in workplaces.
2. Support. Because of the novel nature of the proposed curriculum there is a need to identify a range of rich resources to support the required activity: again there is perhaps the potential to draw on workplace research case studies to develop some of these.
3. Pedagogy. It is important that teachers do not consider how they might reduce the expected enquiry activity to a set of new heuristics. It is likely that they may need to re-conceptualise their role to become a supporter of joint enquiry rather than transmitter of mathematical expertise. This suggests a major shift in the typical didactic contract with mathematics learning being considered a workshop-situated and inquiry-based activity.

The approach proposed here tackles issues of transfer directly by placing the study of how mathematics models reality at the core of the curriculum. Uncoupling mathematical concepts from the situation they model and making sense of each and their interrelatedness becomes central to the mathematical activity of the student. Consequently students will be expected to explore how mathematics can be transformed to meet the needs of a range of diverse situations, with at times a focus on the coupling, and at other times a focus on the development of the mathematics itself. This suggests the need for continued research and development in this area of major importance to future worker expertise and adaptability.

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