Improving the Industrial/Mathematics Interface

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Mathematics is ubiquitous in the sciences and engineering. Arguably a science is considered to have come of age when it has become sufficiently mathematical as illustrated by the burgeoning areas of mathematical biology and mathematical finance. Despite all the potential applications, some mathematicians have moved away from industry and from real applications. From the late 1960s under the influence of the Oxford group [Alan Tayler and Leslie Fox, see Ockendon and Ockendon (1998), Tayler (1990)], interest in modeling real industrial problems has steadily grown. Industrial mathematics and its near synonyms is problem-driven mathematics for the sake of the sciences while pure mathematics may be regarded as being mathematics for its own sake. In this context 'industry' is interpreted in a very broad sense: the remit of these groups includes more than collaboration with industry (problems may come from anywhere in the sciences, e.g. mathematical biology, mathematical finance, the environment). Collaborations between 'industry' and mathematicians can prove extremely successful; study groups with industry are an excellent example. These are typically weeklong intensive sessions involving mathematicians and industrialists/scientists who propose the problems. But in order to fully exploit industrial activities such as these, with the intention of improving the interface between industry and mathematicians, we must introduce modifications in the training of mathematicians. We suggest that there should be an element of industrial mathematics in every third level mathematics group. We will discuss the development of such activities in the context of experience with the 'Mathematics Consortium for Science and Industry (MACSI)', a network of applied mathematical modellers across Ireland.

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The terms 'industrial mathematics', 'applied mathematics' and 'mathematical modelling' are often used as near synonyms to make the distinction with pure mathematics whose central tenet is formal proof and which is not generally concerned with real problems arising outside of mathematics. An applied mathematician is a kind of lapsed pure mathematician in the sense that (s)he would like to prove every result formally but is sometimes unable to do so and must make intuitive leaps in the search for understanding. Mathematical modellers excel at paring problems down to their essence and using mathematical tools to discover the essential mechanisms, which govern processes. Otherwise, for example, the flow of air over airplane wings would not be understood. Classical applied mathematics is associated with names such as Archimedes, Newton, G. I Taylor, Stokes, Reynolds, Kelvin all of whom regularly used mathematics to understand phenomena in the physical world, essentially operating as mathematical modellers.

The Mathematics Subject Area Group (2002) was unanimous in identifying three skills which it believes every mathematics graduate should acquire: the ability to conceive a proof, to solve problems using mathematical tools and to *model a situation.* In point of fact, many groups do not recognize the latter as a key skill. The concept of modelling is central to the practical applications of industrial and applied mathematics. The UK Smith Institute (2004) identifies mathematical modelling and simulation as the core outlet for applied mathematics in Europe. Furthermore this document states that: 'Mathematics now has the opportunity more than ever before to underpin quantitative understanding of industrial strategy and processes across all sectors of business. Companies that take best advantage of this opportunity will gain a significant competitive advantage: mathematics truly gives industry the edge'.

1 Evolution of the Training

Many mathematics courses do not include a real modelling component and this neglect continues through to postgraduate level and beyond. Of course, many courses claim to have a significant modelling component but this is often achieved by relabelling an old course (typically a differential equations course). Unfortunately, some mathematicians pretend to be modellers and to solve real problems in order to obtain research funding.

In fact, they pick out problems which they can deal with but are typically of no interest to the industrial collaborator. The reality is that employers seeking mathematical graduates would like them to have genuine modelling skills, to really understand the discipline which they are modelling, to care about the significance of the results which they obtain and hence to have a broad scientific background. It is important that students gain exposure to suitable texts, e.g. Fowler (1997), Ockendon et al. (1999), Mattheij et al. (2005), which espouse the real philosophy of modelling. Modelling starts at the application level and a key part of the process is the formulation of the mathematical problem. Very few undergraduate third-level courses even touch on this extremely difficult topic. Most mathematics students are used to being presented with nicely formulated mathematical problems, which have equally elegant solutions. In reality, the hardest part of the process often involves asking the right question! Once the problem has been formulated and solved (hopefully), just as important is the interpretation of the solutions for the benefit of the industrial/scientific partner. What do the solutions really mean? How can the knowledge gained be put to use?

Genuine applied mathematical modelling deals almost exclusively with problems which arise outside of mathematics: in physics, chemistry, biology, finance, economics and industry in general. In many traditional pure mathematics undergraduate courses the emphasis is placed almost completely on mathematical rigour and technique with no genuine attempt to develop physical intuition and a feeling for real problems. In effect, the current generation of pure mathematicians is training the next generation to be like itself, to be logically rigorous and to prove theorems. In this philosophy, mathematics is closer to logic than science or engineering. There is nothing wrong with this: such mathematics involves significant intellectual activity. However, what industry needs is mathematicians who are genuine scientists and who are interested in solving real problems. In the controversial words of Dr. Bernard Beauzamy (2002), trained to doctoral level in pure mathematics, and later to set up his own mathematical consultancy company, ‘it (pure mathematics) brings solutions which nobody understands to questions nobody asked’. As already mentioned, applied mathematics is organically linked with the sciences, engineering and industry. Strengthening links with industry requires a well-designed training: while providing students with a fundamental mathematical education (logic, rigour etc.), courses can also provide them with a set of practical skills which are useful in the real world. This means, naturally, that courses must provide them with more genuine mathematical modelling skills and at least introductory courses in the connected disciplines. The degrees should involve industrial internships and even exposure to experimental work. Students must learn that the genuine application of mathematics involves starting with a real problem (physical, chemical…) and will usually mean communicating with engineers, physicists or any other scientist with practical knowledge of the problem, translating that problem into mathematics, solving the mathematical problem and interpreting the solutions in such a way that is meaningful for the practitioners at the source of the problem. Educating present and future mathematicians about the close links between applied mathematics and industry is not limited to the classroom. This important aspect can take many forms. A natural route is to set up an industrial mathematics group in every mathematics department (Friedman and Lavery 1993). This has been the approach at the university of Limerick, where MACSI (www. macsi.ul.i.e.) has developed a range of activities to stimulate links between present or future mathematicians and industry. These include: *Outreach Lectures* for the younger students and the general public, to stimulate interest in industrial applications of mathematics, to show the importance and ubiquity of mathematics in everyday life and that mathematics is the underpinning technology for modern society; a *Summer School*, organised every year to introduce secondary school students and first year university students to modelling concepts, including during the week always a site visit to one of our industrial partners; a summer *Internship Programme* in MACSI, where undergraduate students are confronted with the reality of applied mathematics and its industrial applications and work in conjunction with an experienced researcher on a topic suitable for their background; *Modelling Workshops*, which are organised on a regular basis with a typical session lasting up to a day and involving an industry representative, post-graduate students and research staff; and finally, *Modelling Classes* given by experienced practitioners occur several times a year.

The course topics can range from standard modelling techniques like asymptotic and non-dimensionalisation to the presentation of study group reports and examination of real scientific problems. Coupled with an adequate and varied under-graduate curriculum, these activities familiarise mathematicians of all levels with real-life problems and promote long-term links with industry.

2 Study Groups with Industry

Study groups with industry are organised on a regular basis on all continents. In the European context, study groups with industry (as initiated in Oxford University in the 1960s and continued under the umbrella of the European Consortium for Mathematics in Industry (ECMI)) are week long meetings where groups of industrialists, mathematicians and other scientists work intensively on problems proposed by the industrialists.

The study group format is standard. Mathematicians and other scientists gather for approximately a week with industrial collaborators to find solutions to a set of problems proposed by the latter. The first morning, industry representatives present the problems. One must appreciate that the problems presented are usually not mathematical problems to begin with. Typically, they are descriptions of a complicated industrial process, which is not well understood from a scientific point of view. Usually, there is a specific question of the type 'How might we prevent this happening?' Sometimes, the request is vaguer to the effect that if we can help to model the situation, something useful may come from the mathematical solutions. When all problems have been presented, the academic/scientific participants select the problem(s) they would like to work on. The first afternoon, subgroups of the scientific participants meet with each industry representative and ask far more detailed questions. Ideally, at the end of the day, the team should have defined in broad terms the approximate goals for the week. It is important to realise that in some cases, a successful outcome at the end of the week may be a properly formulated mathematical problem (i.e. the correct mathematical question).

During the rest of the week, the group works on the problems and progresses towards a solution. Participants may choose two strategies. Either they focus on a single problem all week or they may decide to hover between different problems. It is really a matter of taste. Some people like to work intensively on one problem:

other prefer to make smaller contributions to a number of problems. The industrial partner may or may not attend all sessions. (S)he also should be easy to reach if more information is required. A mid-week progress report may be required when the groups present their results. On the last day, all groups present their results to the industry representatives and the other academics. A report describing the work of the group is written in the weeks following the study group and given to the industrial partner.

In a study group, one is confronted with real applications of mathematics and must focus on getting significant results quickly. However, the first study group one attends can be an overwhelming experience, particularly for students. For this reason, specially designed students sessions are often organised. The goal is to familiarise students with the concept of a study group and with some of the standard techniques and ideas, which commonly occur. To achieve this, before the study group, an experienced mathematician runs a session where (s)he uses one or several past (and generally simplified) study group problems with the students. (S)he plays the part of the industrial representative and presents the problem to the students. Using the solution previously obtained, (s)he can guide the students and help them rediscover the results. Working in groups, the students all contribute to the final result and this constitutes an excellent first contact with industrial problems.

MACSI organised two study groups, ESGI62 and ESGI70 in January 2008 and June 2009 respectively. Topics included *Fluid mechanics*: spin coating and self-assembly of di-block copolymers, initiating Guinness; *Chemistry*: improvement of energy efficiency for wastewater treatment; *Biology:* lubrication of an artificial knee; *Electronic*: blowing up of polysilicon fuses, the effect of mechanical loading on the frequency of an oscillator circuit; *Engineering:* polymer laser welding, polishing lead crystal glass, solar reflector design; *Environment:* designing a green roof for Ireland; *Finance:* on the estimation of the distribution of power generated at a wind farm using forecast data; uplift quadratic programming in Irish electricity price setting.

These examples reflect traditional subjects arising in study groups. Optimisation, population modelling and medical problems are other traditional topics. One of the most unusual problems was submitted to mathematicians in Bristol in 2003. Participants were asked to study the artificial incubation of penguin eggs. (In the end, this involved modelling the fluid mechanics inside the eggs.) The problems submitted to study groups are extremely varied but they reflect the skills that are expected from a mathematician doing modelling: although daunting at first, they may be split in a series of subtasks and interim models which are much easier to tackle.

A successful study group is extremely beneficial to all the parties involved:

- Mathematicians and scientists are introduced to new and original problems. A single experience at a study group is enough to convince most mathematicians that industry is an extremely fertile source of interesting problems. They get the opportunity to apply their skills to new exciting problems and get the opportunity to see if the mathematics really works;

- Industrial collaborators obtain direct access to academic expertise. Universities, among other things, are repositories of knowledge and excellence but too often these resources are not easily available to the people who need them most. In the short term, a problem is usually at least partially solved for the industry (or at least some insight has been gained) and this often encourages them to look at their problems from a new perspective and to look for newer and innovative solutions. In the long run, the research carried out during a study group can deliver real solutions for the industrial partners and may lead to patents and genuine financial gains. Potential clients of the industrial partner are often impressed by the improved scientific approach;
- Study groups are unique scientific occasions and are more productive than traditional conferences. Excellent work relationships often develop leading to long-term collaborations.

Study groups are a very important way of improving the industrial/mathematics interface. If study group participants are completely alien to most of the concepts necessary and do not have the appropriate scientific training to solve industrial problems, this sort of initiative is bound to fail. The interface between industry and mathematics can only improve if appropriate training is offered to young (and older) mathematicians.

3 Conclusion

Developing a successful interface between mathematicians and industry requires much effort but the rewards are tangible. Industry provides new and interesting scientific problems; mathematicians provide insights, which allow the industrialists to improve their products. We note in passing that many scientific councils are now including economic relevance among the key requirements in proposals for research funding. Mathematical modellers hold a unique position in the scientific world with their ability to interact with practitioners in so many different areas in industry, the sciences and engineering. In our opinion, to date this advantage has not been exploited to its fullest extent, partly because of the divide in the mathematical world between pure and applied mathematics. It is our thesis that the way forward involves changing mathematics curriculums and placing more emphasis on real mathematical modelling. Study groups with industry are clearly one of the most significant ways of strengthening the industry/mathematics interface but such interactions can only further develop if the mathematicians develop a set of skills more suited for real industrial problems.

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