Chapter 2 Seeing Reality in Perspective: The "Art of Optics" and the "Science of Painting"

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This chapter examines the adaptive assimilation and innovative conceptual prolongations with practical applications of the classical Greek–Arabic science of optics in Renaissance perspectival pictorial arts, as mediated by European mediaeval optical theories and experimentations. This line of inquiry gives a historical account of the epistemic bearings of the connections and distinctions between the exact sciences and the visual arts, with an emphasis on the role of classical optics in the art of painting, and the function of pictorial art in pre-modern natural sciences. A special focus will be set on examining the optical and geometrical legacy of the eleventh century Arab polymath, al-Hasan ibn al-Haytham (known in Latinate renditions of his name as "Alhazen" or "Alhacen"; d. after 1041 CE). This investigation considers the fundamental elements of his theories of vision, light, and space in the context of his studies in optics and geometry, while taking into account his use of experimentation and controlled testing as a method of demonstration and proof. This course of analysis will be furthermore linked to the adaptation of Ibn al-Haytham's research within the thirteenth century Franciscan optical workshops, while scrutinizing the impress that his transmitted texts had on Renaissance perspectival representation of spatial depth and its entailed organization of architectural locales and spaces.

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R. Lupacchini and A. Angelini (eds.), *The Art of Science*, DOI 10.1007/978-3-319-02111-9__2, © Springer International Publishing Switzerland 2014

2.1 Art of Science and Science of Art

The *dictum*: "*ars sine scientia nihil est*" ("art without knowledge [science] is nothing"), which was attributed to the fourteenth century French architect Jean Mignot [\(Ackerman 1949\)](#page-20-0),^{[1](#page-1-0)} and echoed in Martin Kemp's *The Science of Art* [\(Kemp](#page-21-0)) [1990\)](#page-21-0), is inverted by Annarita Angelini and Rossella Lupacchini in the thematic orientation of *The Art of Science*, which tacitly asserts that "knowledge [science] without art is nothing" ("*scientia sine arte nihil est*"). This state of affairs situates us within a *liminal* place in-between two propositions that are separated while at the same time being gathered in a dialectical unity: "*l'art n'est rien sans la science' 'la science sans art n'est rien*". The entangled relationships between the exact sciences and the visual and plastic arts date back to ancient times. In the antique epoch, the multivolume *De architectura* (ca. 15 BCE) of the Roman architect and polymath Marcus Vitruvius Pollio constituted one of the early treatises that demonstrated how the various disciplines that formed classical knowledge impacted architectural thinking and the architectonics of place-making.

Arithmetic, geometry, surveying-mensuration, mechanics, optics, astronomy, and natural philosophy were amongst the principal domains of inquiry that influenced pre-modern architecture as a synthesizing field of intellective reflection, and as an applied sphere of practice within material culture, which in itself offered an idealized embodiment of the visual and plastic arts.

In historical and epistemic terms, the entanglement of art with science found some of its most explicit manifestations in the theoretical treatises of Renaissance scholarship, and in the diverse modes of their architectonic and practical applications in the expansion and articulation of material culture. The boundaries that may have separated art from science became creatively blurred in the Renaissance; especially against the background of the deconstruction of the classical Aristotelian physics. What may be pictured as an epistemic or disciplinary "crisis" in classical natural philosophy offered opportunities for the flourishing of artistic and architectural imagination and thinking, wherein art and architecture opened up the horizons of inquiry and the landscapes of curiosity through freer forms of exploration and inventiveness. The successive epochs of the Italian Renaissance were marked by an affirmation of "the *art* of science" and "the *science* of art" at the same time. The scientific grounds of the visual-plastic arts and the artistic underpinnings of the exact-natural sciences were co-entangled. Such dynamics were evident in the context of reflections on the connection and distinction between the *perspectiva naturalis* of visual perception, and the *perspectiva artificialis* of the pictorial representation of the perceptual field of vision. The leitmotifs of *perspectiva* offered an optimal context for investigating the relationships between science and art, in terms of probing the optical and geometric foundations of

¹This *dictum* has been reported in connection with an anecdote about a dispute that took place over the assessment of the structural integrity of the elevation of the *Duomo di Milano* of the Santa Maria Nascente.

the pictorial representation of natural phenomena, while experimenting with the manner painting and drawing in perspective would contribute to the construction of legitimate and reliable knowledge about the visible reality.

When scientific images are radically removed from the familiarities of natural visual perception they necessitate the establishment of complex representational spaces that render the conditions of their observational perceptibility possible. Such modes of picturing reality find their roots in the entanglement of art with science through the course of the unfolding of Renaissance thought and its spheres of praxis. However, if the processes of knowing, proving, and representing are connected with imagining and imaging, does this signal symmetrical relations between science and art instead of asymmetries?

The refinement of representational space, which is pivotal in the enactment of the production of science and art, depended on variegated explorations that were set forth in pursuit of the "*costruzione legittima*" ("legitimate construction") of linear and central single-point perspective within the pictorial art of the Renaissance. Such artistic endeavours were mediated by investigations that also rested on the classical traditions of optics and geometry in the exact sciences. The entanglement of the elements of the pictorial art with the scientific taxonomies in the Renaissance may have been animated at its core by ontological–theological intentions in establishing metaphorical and symbolic connections between scriptural-textual exegesis and the presupposition of visual atonement in measuring reality via the "*visio intellectualis*". Despite the fact that the visual illusory depiction of spatial depth, in the geometric construction and projection of perspective, alluded also to higher orders of "reality", which transcended the way the "real" manifested itself empirically and experientially in visual perception, what concerns us in this line of inquiry is an investigation of the connection and distinction between art and science in relation to the *perspectiva* traditions [\(El-Bizri 2007a,](#page-20-1) [2010a](#page-21-1)[,b\)](#page-21-2), and not the explicit reflection on their theological bearings.

The pictorial order is intrinsically implied within the visual elements of the science of optics and of geometry. It also rests on these sciences in the projections and constructions that underpin its representational depiction of spatial depth in artificial perspective.

Examining the visualization of reality and the picturing of the world through the agency of perspective, in terms of natural vision and pictorial representation, constitutes an inquiry into the "art" of optics and the "science" of painting. The epistemic concerns that animated the Renaissance disputations around linear central single-point perspectives in the pictorial arts of the *Trecento*, *Quattrocento*, and *Cinquecento*, all offer concretized historical settings for such line of inquiry as set against the principal theories of the classical sciences of optics and geometry.

To situate this study in a deeper historical *milieu* that underpinned many facets of the *episteme* of Renaissance pictorial arts, I will principally focus on elucidating the key elements of the optical and geometrical legacies of the eleventh century Arab polymath al-Hasan ibn al-Haytham (known in Latinate renderings of his name as "Alhazen" or "Alhacen"; born in Basra ca. 965 CE, and died in Cairo ca. 1041 CE), with a particular focus on the seven books that constituted his monumental optical opus: *Kitab al-Manazir, The Book of Optics* [\(Ibn al-Haytham 1983,](#page-21-3) [1989\)](#page-21-4).[2](#page-3-0)

Even though Renaissance scholars were more often inclined theoretically to follow Euclid, Ptolemy, and Vitruvius, they nonetheless relied in optics and in selected aspects of geometry on the transmitted traditions that were associated with Ibn al-Haytham as these were adaptively assimilated and mediated by mediaeval European opticians and mathematicians, leading ultimately to the transformation of the *natural visual theory* into a *pictorial theory*. However, before we become directly engaged in an exegetical and hermeneutic interpretation of the *perspectiva* traditions in connection with Ibn al-Haytham's research in optics and geometry, we need still to probe more closely in the following section some of the entailments of the representational space of pictorial art and scientific imaging.

2.2 Representational Space

The epistemic, veridical, and apodictic criteria of *scientia*, as a source of reliable and sound rational knowledge when conducted within the parameters of precision in logical reasoning and experimenting, are not dependent on personal choices, as it is for instance the case with the spheres of theory and praxis in art, which do not necessitate strict rules of proof and demonstration. This liberal aspect in the explorative horizons of the visual and plastic arts opened up new spheres of inquiry that were imaginatively inventive and relatively freed from the need to follow with stricture the principles of scientific logic and its methodological directives. This state of affairs assisted in the constitution of imaginary models of empirical reality through pictorial representational spaces, which themselves offered contexts for informing the spatial and architectonic qualities of actualized physical architectural locales, specifically through the agency of design and its approximation of the realization of its own formal-material hypotheses.

The rigorous rationality that underpins the coherence of representational space in modelling an imaginative reality within the spectacle of linear central perspective is based on an inner geometric system of points, angles, axes, converging lines, and triangles. The representational space of pictorial perspective is imagined, and then depicted afterwards, or in a succession through the structuring order of geometric construction and projection. Such pictorial space is furthermore refined by way of colour and the anatomy of figurative forms of human and living beings, with their gestures and choreographies, which all manifest a virtual new reality that is saturated with communicative visual metaphors and symbolic meanings. These become vital in their turn in terms highlighting the role of imagination in pictorial and figurative representation, and in the un-concealment of hidden physical and mathematical

²I used simplified transliterations for all the Arabic terms throughout the text without the noting of diacritical vocalizing marks.

principles of reality. The science that grounded the pictorial arts became itself served by the unfolding of their applications, in founding the role of imagery in the scientific modelling of realities that remained otherwise imperceptible in the course of lived experiential and empirical ambient settings of our human sensibility and its sensorial conditions.

The designer or painter–architect contemplates and imagines certain spatial and architectonic possibilities, which belong to reflections on a given pictorial or architectural context and are mediated via concepts that set down the theoretical hypotheses of design. Such processes unfold through conjectures and the exploration of the most probable possibilities by testing them through drawing, drafting, tracing, and in terms of scaled-models, as physical "*maquettes*". These procedures enact calculative, intuitive, and imaginative strategies that attempt to approximate in actualization what can possibly be done in tangible terms within physical reality. The logic of geometry, physics (statics), architectonics, material mechanics, formal, and spatial qualities, atmosphere in imagined sensorial experiences, all bring science and art together in design, while also being oriented by the agency of language in articulating thinking and the manner it depicts the gradual emergence of a composite of form and matter in making. Artistic visions are therefore all along co-entangled with scientific abstractions.

The pictorial representational space that is depicted through artificial linear central perspective makes the seeming sense of infinity manifest in virtual visual terms. The material paintings on the surfaces of canvas appear as windows that are carefully opened up into given regions of imagined worlds, which are chosen through the agencies of the painters and their inherence in history, culture, and language, and are also offered as a complex web of narratives to the observers, be it those who are contemporaneous patrons, or eventually as anonymous spectators that are yet to come in posterity. A human viewpoint on the world is established by *seeing reality in perspective*. A relationship is set between the finite distance of the painter–observer from the surface of the painted canvas, and the implied sense of infinity within the representational virtual space of the depicted portion of imagined reality in the painting.

Two pyramids-cones of visibility intersect in seeing by way of perspective: the finite pyramid-cone of vision of the *perspectiva naturalis*, as studied in optics in connection with direct visual perception, and the pyramid-cone of the *perspectiva artificialis* in the pictorial order, which seemingly tends towards infinity. The pyramid-cone of vision in the *perspectiva naturalis*, as entailed by direct visual perception, is finite and determined by the nearness of its vertex (which is at the centre of the eye of the painter–observer) to its base. As for the pyramid-cone in the *perspectiva artificialis* pictorial order, it gives the semblance of tending towards infinity through the converging geometric lines that meet in the centring-vanishing point on the horizon line. This can be illustrated by the geometric projections in perspective as shown in Fig. [2.1.](#page-5-0) Let the position of the eye of an observer be seen in a top-view plan as point **O**. Let the lines extended out from this point **O** delimit a cone of vision *CV* that encompasses a given box-shaped object of vision with a vertical surface **a** as it is also seen in a top-view plan. Let *PP* be the picture plane

Fig. 2.1 Geometric projections in perspective

(namely the surface of the canvas) as seen in the top-view plan. Project point **O** into a centring-vanishing point **O**¹ that appears in a front-view elevation on a horizon line *HL* at the height of the eye of the observer above the ground level *GR*. Let h be the height of the surface **a** of the object of vision, which when projected in perspective will be encompassed by the lines extending out from O_1 in such a way that the surface **a** appears in perspective in the shape **A**.

The geometry of the configuration shown in Fig. [2.1](#page-5-0) is embedded in the singlepoint linear and central construct of pictorial perspective, which is established from the viewpoint of a fixed angle of vision, which is determined in the form of a triangle when looking at the cone of vision *CV* in a top-view plan.

The *perspectiva artificialis* is static and marked by fixity, in contrast with the manner the eyes continually move and vibrate in scanning the visual field in the *perspectiva naturalis*. The representational space that is depicted via the *perspectiva artificialis* is itself static and fixed, while opening up to a sense of seeming infinitude. The single-point linear and central construct of pictorial perspective, with the fixity and static quality of its representational order, offer an idealized context for abstractness in geometric space, which is unlike what is brought into appearance within the horizons of natural visual perception. Artificial perspective reveals a symbolic order that is modulated by the exact rules of geometry, and it grants an abstractive viewpoint on what remains hidden from natural sight in the concrete fields of empirical and sensible experience. Artificial perspective lets something "omnipresent" appear, through its geometric order; and yet, there is also the virtual sense by which the painter–observer is also looked at from within the painting when gazing at it.

The contemplation of the painting reveals a virtual viewpoint from a seeming infinity, which looks back at the painter–observer, and is situated at the vertex of the pyramid-cone of the *perspectiva artificialis* within the pictorial space; namely, at the centring-vanishing point where parallels in pictorial-depth tend towards as the seeming "infinite", while meeting in it as geometric lines traced on a twodimensional surface. As if the painter–observer is also supposedly seen from infinity in a gaze coming from within the painting that remains "*omnivoyant*", given the fixity of the angle of vision in the geometric representational structure of the single-point linear and central pictorial perspective. This outlook is densely expressed in Nicolaus Cusanus's "*Figura paradigmatica*", in his *De coniecturis* (*On conjecture*; ca. 1440 CE), which offers an analysis of two intersecting pyramids, one of light (*lux*), as the *pyramidis lucis*, and the other of shadow (*tenebrae*), as the *pyramidis tenebrarum*, which respectively evoke the ideas of unity and manifoldness. Perspective is posited in this context as a channel of communication between divinities and mortals, "God and man" [\(Cusanus 1514,](#page-20-2) [1972;](#page-20-3) [Carman](#page-20-4) [2007\)](#page-20-4). As if the idealized representational space of pictorial perspective carries also a deeper sense of reality in unveiling the geometric order that grounds and structures the visible universe. In opening up to the infinite, the virtual reality of the painting, as an object of sensible experience, in its materiality as paint-pigments brushed on a canvas surface, becomes itself a portion of a much wider world that is enacted in the pictorial art with its communicative meaningful and symbolic internal complexities.

2.3 Optics

In the earlier sections I advanced various observations concerning the conceptual aspects that emerge from reflecting on the connection and distinction between art and science in relation to the roles played by the debates and the explorations of perspective in the Renaissance artistic and architectural *milieu*. I also signalled the significance that is attributed to the adaptive assimilation and interpretive use of optics, as a science of the *perspectiva naturalis*, in informing the epistemic disputations and technical reflections on the "*costruzione legittima*" of the *perspectiva artificialis*. To situate this inquiry in a setting that entangles the history of science with the history of art and architecture, I will mainly focus in this present section on the fundamental aspects of the optical tradition of Ibn al-Haytham (Alhazen) as primarily embodied in his *Kitab al-Manazir* (*Book of Optics; De aspectibus or Perspectiva*), in view of exploring some of its propositions that are relevant to Renaissance "perspectivism".

The most poignant revolution in the classical science of optics, from the times of Ptolemy to those of Kepler, is embodied in the research of Ibn al-Haytham, who devised a scientific solution to ancient controversies over the nature of vision, light,

and colour, which were disputed between the classical mathematicians (exponents of Euclid and Ptolemy) and the Aristotelian physicists. Ibn al-Haytham's research in optics (including his studies in catoptrics and dioptrics, respectively on the principles and instruments of the reflection and refraction of light) also benefited from the investigations of his predecessors in the Archimedean-Apollonian tradition of ninth century Arab polymaths, like the Banu Musa and Thabit ibn Qurra, and of tenth century mathematicians, like al-Quhi, al-Sijzi and Ibn Sahl.³

Ibn al-Haytham's *Kitab al-Manazir* was translated from Arabic into Latin towards the end of the twelfth century under the title: *Persepctiva*, [4](#page-7-1) or *De Aspectibus*. A fourteenth century Italian version of Ibn al-Haytham's *Optics*, titled: *Prospettiva*, acted as the main reference in optics for the Renaissance sculptor and theorist Lorenzo Ghiberti.

The Latin version of Ibn al-Haytham's *Optics* impacted the research of Franciscan scholars of optics in the thirteenth century, mainly in the 1260s and 1270s, of figures such as Roger Bacon, John Peckham, and Witelo.^{[5](#page-7-2)} Ibn al-Haytham's tradition also influenced the investigations of fourteenth century opticians, like Theodoric (Dietrich) of Freiburg (d. ca. 1310) in Europe, and Kamal al-Din al-Farisi (d. ca. 1319 CE) in Persia; both scholars offered correct experimentally oriented explications of the phenomenon of the rainbow and its colouration, while basing

³While Ibn al-Haytham's optical research proved to be a revolutionizing tradition in the course of development of the scientific discipline of optics up to the seventeenth century, other legacies in this science existed in the history of ideas in the classical Islamic civilization. One of these principal traditions is attributed to the research of the Arab philosopher al-Kindi (d. ca. 873), who partly influenced the optical investigations of Robert Grosseteste (d. ca. 1253) through the Latin version of his treatise in optics, entitled: *De Aspectibus*. However, this optical tradition was primarily Euclidean and Ptolemaic, like it was also later the case with the research of the Persian mathematician and philosopher, Nasir al-Din Tusi (d. ca. 1274). It is also worth noting in this regard that the philosopher and physician Ibn Sina (Avicenna, d. 1037 CE) developed a physical "intromission" theory of vision that is akin to that of Aristotle. Ibn Sina's contributions in optics were not as influential as those of Ibn al-Haytham. Nonetheless, his research on the anatomy of the eye in his *al-Qanun fi al-tibb* (*The Canon of Medicine*) impacted the evolution of ophthalmology up to the sixteenth century, and his research in meteorology inspired Kamal al-Din al-Farisi's revision of Ibn al-Haytham's *Optics* in terms of offering a reformed explication of the reality of colours and the rainbow. Furthermore, Ibn Sina's theory of perception was ecumenically influential in Islamic civilization and European mediaeval scholarship, particularly in terms of elucidating philosophical meditations on the nature of the soul (*al-nafs; De anima*) and the bearings of its cognitive faculties in terms of visual perception [\(Al-Kindi 1950–53,](#page-20-5) [1997;](#page-20-6) [Hasse 2000\)](#page-21-5).

⁴The manuscript of the fourteenth century Italian version of Ibn al-Haytham's *Optics*, entitled: *Prospettiva*, is dated on 1341 CE, and it is preserved in the Vatican under the following cataloguing details: Ms. Vat. At. 4595. Folios 1–177.

⁵In a critical analysis of Alistair C. Crombie's thesis that "modern" scientific methodology is attributable to the tradition of Robert Grosseteste, and to thirteenth century opticians like Roger Bacon, John Peckham, and Witelo, Alexandre Koyré argued that the scientific method found its earlier roots in the legacy of Ibn al-Haytham (Alhazen) in optics, which resulted in the flourishing of the perspectivism of Franciscan scholars in the European Middle Ages, in addition to the application of their experimental methods [\(Koyré 1948;](#page-21-6) [Crombie 1953;](#page-20-7) [Simon 1997;](#page-22-0) Federici Vescovini [1990,](#page-21-7) [2008\)](#page-21-8) .

their studies on reformed revisions of Ibn al-Haytham's theory of colours as noted in his *Optics* [\(Federici Vescovini 1990,](#page-21-7) [2008\)](#page-21-8).[6](#page-8-0) For instance, Kamal al-Din al-Farisi conducted an experiment on a large spherical glass vessel modelling a rain-droplet, which was subjected to light in a controlled environment within a *camera obscura* (*al-bayt al-muzlim*), to demonstrate the decomposition of white light into a spectrum of colours, in view of explicating the phenomenon of the rainbow in meteorological optics [\(El-Bizri 2009\)](#page-20-8). Ibn al-Haytham's tradition in history of science in Islam continued to be subsequently influential through the investigations of the Syrian astronomer at the Ottoman court, Taqi al-Din Muhammad Ibn Ma'ruf (d. ca. 1585 CE; [El-Bizri 2005a\)](#page-20-9).

The Latin translations of Ibn al-Haytham's *Kitab al-Manazir*, in addition to works associated with geometry and conics, in relation to Arabic sources in mathematics, also impacted Renaissance scholars of the calibre of Biagio Pelacani da Parma [\(Pelacani da Parma 2002\)](#page-21-9),^{[7](#page-8-1)} Francesco Maurolico, Ettore Ausonio, Egnatio Danti, and Francesco Barozzi.⁸ Ibn al-Haytham's *Optics* was also assimilated in Renaissance scholarly circles, partly through the mediation of thirteenth century Franciscan opticians, and it influenced the perspective theories of Leon Battista Alberti in the *De pictura*, and impacted more directly the propositions of Lorenzo Ghiberti in the *Commentario terzo* [\(Federici Vescovini 1998\)](#page-21-10). A printed edition of Ibn al-Haytham's Latin version of the Optics was established by Friedrich Risner in 1572 in Basle, under the title: *Opticae Thesaurus*, which was eventually consulted by seventeenth century scientists and philosophers such as Kepler, Descartes, Huygens, and possibly even Newton. The recognition of Ibn al-Haytham's *œuvre* is also evident in the high station he was accorded by the seventeenth century German scientist Johannis Hevelius, whereby the frontispiece of the latter's *Selenographia sive Lunae Descriptio* (dated 1647) depicts Ibn al-Haytham standing on the pedestal of *ratione* (reason), with a compass in his hand and a folio of geometry, while Galileo stands on the pedestal of *sensu* (observation), holding a telescope.

An investigation of the historical and epistemic entailments of Ibn al-Haytham's tradition in optics elucidates some of the dynamics that are at work in the emergence and development of novel scientific rationalities. His legacy established the principal scientific foundations of mediaeval *perspectiva* in the European traditions, and, through them, it grounded in part selected Renaissance theories of vision and

⁶Phenomena that were originally treated as topics of meteorology were studied based on new models of "reformed" optics. For instance, Kamal al-Din al-Farisi's (d. ca. 1319 CE) explication of the phenomenon of the rainbow (*qaws quzah*) constituted a part of his commentary on Ibn al-Haytham's *Optics* in *Tanqih al-manazir*; namely, a treatise entitled: *The Revision of [Ibn al-Haytham's] Optics* [\(Al-Farisi 1928–29\)](#page-20-10).

⁷This is particularly the case with the *Quaestiones perspectivae* of Biagio Pelacani da Parma.

⁸He is also known as "Franciscus Barocius", and this particular discussion figures mainly in his *Admirandum illud Geometricum Problema tredecim modis demonstratum*—Raynaud and Rose discussed some related elements of the adaptive assimilation by Renaissance theorists of Arabic mathematical sources on conics and their applications in optics [\(Raynaud 2007;](#page-22-1) [Rose 1970\)](#page-22-2).

perspective, while continuing furthermore to influence the unfolding of the science of optics up to the seventeenth century.

Ibn al-Haytham's scientific method consisted of combining mathematics with physics in the context of experimental demonstration, verification, proof, and controlled testing (*i'tibar muharrar*), including the design and use of scientific instruments and installations [\(El-Bizri 2005a\)](#page-20-9). Ibn al-Haytham investigated the veridical conditions of visual perception to ground the observational data of his experimental research, along with setting rigorous parameters for the application of optics in astronomy and meteorology.

One of the principal aspects of Ibn al-Haytham's reforming of the science of optics is encountered in his ingenious resolution of the longstanding ancient dispute between the mathematicians (*ashab al-ta'alim*; Euclidean and Ptolemaic) and the physicists (*ashab al-'ilm al-tabi'i*; Aristotelian) over the nature of vision and light. Ibn al-Haytham showed that vision occurs by way of the introduction of physical light rays into the eye in a configuration that is geometrically determined in the form of a pyramid-cone (*makhrut*) of vision, with its vertex at the centre of the eye and its base on the visible lit surfaces of the object of vision; while taking into account the rectilinear propagation of light in the homogeneous transparent medium between the observer and the seen object. He thus rejected the "extramission" theory of the ancient mathematicians, which holds that vision occurs by way of the emission of a subtle and non-consuming ray of light (akin to fire) from the eye that meets the lit medium, which, as a physical phenomenon, is structured in the form of an actual pyramid-cone of light. In view of explicating the process of vision, Ibn al-Haytham retains the structural form of a pyramid-cone of vision, in terms of geometric modelling, while emphasizing that it is abstracted from matter, and that the lines determining its outline and configuration were purely mathematical (virtual and postulated) rather than being physical. Moreover, he refuted the physicists' theory of vision (as inspired by Aristotle's *Physics* and *De anima*), which ambivalently conjectured that the sight results from the "intromission" into the eye of the form of the visible object without its matter when the transparent medium (*al-shafif; diaphanes*) is actualized by physical illumination. Ibn al-Haytham demonstrated that vision occurs by way of the introduction of light into the eye, while showing that this physical phenomenon was geometrically structured in the shape of a virtualmathematical cone of vision [\(Nazif 1942–43;](#page-21-11) [Federici Vescovini 1965;](#page-21-12) [Sabra](#page-22-3) [1978,](#page-22-3) [1989;](#page-22-4) [Rashed 1992\)](#page-21-13). Consequently, he distinguished vision from light, and devised novel methodological procedures that brought the certitude and invariance of geometrical demonstration to bear with isomorphism instead of mere synthesis on his research in physical optics [\(El-Bizri 2005b\)](#page-20-11). He moreover subjected the resultant mathematical-physical models and hypotheses to experimentation by way of controlled empirical procedures of testing, including the devising and use of experimental instruments and installations, like the *camera obscura* [\(Nazif](#page-21-11) 1942– 43; [Schramm 1963;](#page-22-5) [Omar 1977\)](#page-21-14). Moreover, his experimentation did not consist of a simple element of empirical methodology, rather it was theoretically integral to his proofs, and granted an apodictic value to his inquiries in optics [\(Rashed 2005;](#page-21-15) [El-Bizri 2005b\)](#page-20-11).

Ibn al-Haytham's geometrical, physical, physiological and meteorological studies in optics were also related to his psychology of visual perception, and to his analysis of the faculties of judgement and discernment (*al-tamyiz*), of cognitive comparative measure (*al-qiyas*), of (*eidetic*) recognition (*al-ma'rifa*), imagination and memory (*al-takhayyul, al-dhakira*). He thus distinguished the *immediate* mode of perception by way of glancing from *contemplative* perception (*Optics*, II.4: 5, 20, 33 \degree while reflecting on the manner of perceiving particular visible properties (*al-ma'ani al-mubsara; intentiones visibiles*—*Optics*, II.3: 43–48).[10](#page-10-1) Pure sensation only perceives light *qua* light and colour *qua* colour (*Optics*, II.3: 50–52; II.4: 22), while vision depends primarily on exercising the *virtus distinctiva (al-quwwa al-mumayyiza*; faculty of discernment), which perceives all visible 22 properties (*Optics*, II.3: 1–25), while being aided by imagination and memory, and usually operating without deliberate and excessive effort (*Optics*, II.4: 12–15, 22). Ultimately, the light introduced into the eyes results neurologically and physiologically in sensations in the last sentient (*al-hass al-akhir; sentiens ultimum*) in the frontal part of the brain (*muqaddam al-dimagh*; *Optics*, I.6: 74).

Ibn al-Haytham's observations rested on anatomical examinations of the structure of the eye (*Optics*, I.5: 1–39) and the investigation of binocular vision (*Optics*, I.6: 69–82). "*Why do we see a single object of vision instead of two, even though we look at it with two eyes?*" The image formed on the crystalline of the eye (*aljalidiyya*) passes through the vitreous ocular humour (*al-zujajiyya*) and reaches the hollow optic nerve (*al-'asaba al-jawfa'*), which connects to the common nerve (*al- 'asaba al-mushtaraka*; optic *chiasma*) and reaches the last sentient as a sensation in the anterior part of the brain. Under normal conditions of binocular vision, the observer perceives a single visible object with two sound eyes instead of having two images of one and the same object. Binocular vision does not readily result in double vision, unless this is due to errors in vision (which Ibn al-Haytham examined in detail in Book III of his *Optics*). The form of a single visible object occurs on the surface of the crystalline of each of the eyes. Looking at that object, its form is received in each one of the eyes. This can be illustrated as shown in Fig. [2.2.](#page-11-0) Let a given point **O** on the object of vision, respectively, reach the right and left eyes in points O_1 and O_2 that become united into a fused " O_{image} " via the common optical nerve. Consequently, two forms (such as those entailed by points like O_1 and O_2), occur on the crystalline of each of the eyes, passing via the vitreous to the hollow nerves, and then, as sensations, become unified in the common nerve, and reach the last sentient as an ordered single form of a sensible visible object (namely as an "**O**image").

⁹References that are hereinafter made to Ibn al-Haytham's *Optics* in the body of the text indicate the numbering of the Book with its chapters, as these correspond with the Arabic critical edition of the text [\(Ibn al-Haytham 1983\)](#page-21-3) and its annotated English translation [\(Ibn al-Haytham 1989\)](#page-21-4).

¹⁰Ibn al-Haytham enumerated twenty-two particular visible properties (*Optics*, II.3: 44), while Ptolemy restricted their number to seven [\(Lejeune 1948;](#page-21-16) [Sabra 1966\)](#page-22-6).

O*image*

 \mathfrak{Q}_1

 $O₂$

 \overline{z}

Apart from binocular vision, "*why a single object of vision appears as one and not many? And how do multiple light rays, which result in manifold visual data, get received into the eye in an ordered structure?*" The object of vision is seen by way of the introduction into the eye of light rays that are emitted from its visible lit surfaces, which propagate rectilinearly across the transparent medium that is between the eyes of the observer and this object, while the reception of these light rays in the eye is structured geometrically in the shape of a *virtual* cone of vision (*makhrut al-shu'a*), with its vertex at the centre of the eye and its base on the seen and lit surfaces of the visible object (*Optics*, I.2, I.3). The light rays that are structured within this mathematical model travel rectilinearly from every point on the lit and appearing surfaces of the visible object in a punctiform-corpuscular configuration, with a spherical irradiation that is emitted in rectilinear trajectories from each of these points through the transparent medium, and in all directions. This phenomenon reflects a point-by-point correspondence between each point on the lit and visible surface of the object of vision and each correlative point on its image that occurs on the crystalline, which ultimately secures the ordering of the visible aspects of this seen object. Only the light rays that meet the outer surface of the crystalline humour (*al-rutuba al-jalidiyya*) perpendicularly (centrally at a normal) are admitted into the eye in terms of this point-by-point correspondence between the lit and visible surfaces of the object of vision and the image they have on the crystalline. This is shown in the Fig. [2.3,](#page-11-1) whereby a hypothetically given "point" **O** on the object vision will have its corresponding image admitted within the eye as a single hypothetically given "point" **X**.

This phenomenon was also analysed by Ibn al-Haytham in terms of studying the geometrical properties of the outer surface of the crystalline as an optical lens (spherical section) in dioptrics, as set in Book VII of his *Optics*, which focused on the mathematical properties of refractive surfaces (with differing indices of refraction), and derived from varied spherical, cylindrical, and conical sections

(circle, ellipsis, parabola, hyperbola). This line of study was partly based on the tenth century research of Abu al-'Ala' Ibn Sahl on geometrically modelled lenses, mainly the latter's *Kitab al-Harragat* (*Burning Instruments*).^{[11](#page-12-0)}

The impressions of the luminous physical rays on each of the eyes in binocular vision are ultimately interpreted in the brain as a single integral image of one object of vision, which is not the same as a mental image *per se*. On Ibn al-Haytham's view, vision is a physiological, neurological, and psychological/cognitive phenomenon that is not simply reducible to the order of geometric and physical analytics in optics. Vision is investigated in this regard from the standpoint of the epistemic dimensions of cognition, and not solely from the standpoint of mathematicalphysical models. This nuance indicates the possibility of translating his theory of vision into two forms of "visual cultures": one that imitates the visible realm in "pictorial representations", and the other conducts thinking through the agency of "mental pictures". Respectively, these correlate with the representational spaces of art and science.

2.4 Renaissance Perspectives

The Renaissance perspective geometric constructs aimed at reproducing with twodimensional approximation and pictorial reinterpretation the three-dimensional spatial spectacle that is naturally perceived by eyesight. This procedure rested on the science of optics, though finding novel spheres of its application and significance in the context of the visual fine arts. Objects of vision are to be depicted as they appear. This effort started with *Trecento* artists in terms of representing what we see in pictorial terms, without yet having developed a rigorous geometric method to construct linear perspective. Even though such aspects of depicting things as they appear may have had some much earlier manifestations in examples from antiquity (including murals in some of the Roman villas in Pompeii), the preoccupations of the *Trecento* painters were addressed via conscious studies rooted in the classical science of optics.

Geometry was used in the science of optics, within the tradition of Ibn al-Haytham, in isomorphism with physics and controlled experimentation. This endeavour aimed at scientifically explaining the nature of visual perception, and the laws of the rectilinear propagation of light in a homogeneous transparent medium, the reflection of light on polished surfaces (catoptrics), and the refraction of light when passing from a transparent medium into another that differs from it in subtlety and in its refractive indexing (dioptrics). In the case of art and architecture, geometrical constructions and projections eventually acted as tools for the depiction

¹¹This aspect had implications on studying spherical aberration; namely, when beams of light, which are parallel to the axis of the lens (as a spherical section), yet that also vary in terms of their distance from it, become all focused in different places, which results in the blurring of the resultant image.

of spatial depth in pictorial representations, and served also as design directives in the organization of architectural space and the articulation of its architectonic features in concrete physical settings.

In *Le due Regole della prospettiva*, first published in 1583, Jacopo Barozzi (known as Vignola) dedicated a chapter to refute the idea of constructing linear perspective through two vanishing points that correspond with binocular vision [\(Barozzi 1611\)](#page-20-12). In this, he deployed arguments that accorded with what Ibn al-Haytham demonstrated in terms of the psychological–neurological–physiological aspects of vision, by way of accounting for binary visual perception, and the fusionunification of the visible form of the object of vision, when the light rays emitted from the visible lit surfaces of that object make their final impress, via the eyes and the optical nerves, on the last sentient located in the anterior part of the brain (principally as noted in Chap. 6 of Book I of the *Optics*). This aspect of binocular vision, and its implications in terms of thinking about the method of constructing linear perspective in pictorial representational art, attracted also the comments of the Renaissance mathematician Egnatio Danti who sustained similar views as those of Jacopo Barozzi, as he commented on the latter's opus [\(Raynaud 2003,](#page-21-17) [2004\)](#page-21-18). Danti displayed also signs of awareness with regard to these observations in the science of optics, and in lines that accorded with Ibn al-Haytham's theories. Ultimately, linear perspective is said to have a single centring-vanishing point instead of two, hence, being mono-focal and central, without contradicting the nature of binocular vision. However, the traditions practiced in the *Trecento* pictorial renderings, based on asserting binocular vision, tended to posit two vanishing points that are correlative with the two eyes of the observer, without being in this "bifocal" in the sense of having a two-point perspective that is associated with relatively more modern constructs (like the ones that are also "trifocal", or "curvilinear", etc.). Notwithstanding, the science of optics, as exemplified by Ibn al-Haytham's theory of visual perception, and his analysis of binocular vision, allowed for two pictorial interpretations: the *first* consists of positing a single centring-vanishing point in mono-focal central linear perspective, which correlates with the presupposition of a single cone of vision receiving the seen spectacle by the observer, and taking into account the fusion of impressions on the eyes through the common optical nerve (as discussed earlier and shown in Fig. [2.2\)](#page-11-0), while the *second* allows the positing of two vanishing points in asserting binocular vision. The latter was exemplified in what we may call: "the heterodox [*Trecento*] perspectives", which posits two vanishing points; like, for instance it was the case with Lorenzo Ghiberti's *Christ Amongst the Doctors* (fourth panel of the North door at the Baptistery of San Giovanni).

The *problématique* of the "*costruzione legittima*" (legitimate construction of perspective) centred on the consequences of doubling the unique centring-vanishing point of central perspective, and on debating the risks of distortions, or of compromising the spatial unity of the representational pictorial field. The manipulation of heterodox two-point perspectives, in terms of depicting central foreground figures against architectural background settings, to neutralize the effects of diplopia, did not always succeed in avoiding visual distortions, or in securing the unity of the painted representational space [\(Raynaud 2004\)](#page-21-18).

The pictorial interpretation based on the heterodox positing of two-vanishing points (like it was the case with Gentile de Fabriano's *The Tomb of Saint Nicholas*), reflects an optical awareness of the need to accommodate binocular vision instead of monocular sight. However, this consciousness does not account for the fusional convergence of the two images formed on the crystalline of the eyes, and their unification in terms of the physiological-neurological-psychological determinants of vision, as analysed by Ibn al-Haytham. Rather, this practice rests on an analysis of binocular vision that attempts to overcome the effects of double vision, diplopia and parallax phenomena, under normal physiological conditions of eyesight. Such dimensions were also carefully studied in Ibn al-Haytham's optics, in terms of investigating the implications of distance in vision (nearness to the eyes in particular), and of optical convergence or its insufficiency, of visual alignments and misalignments, of parallax phenomena and stereopsis, with their various effects on the positioning of the eyes and the physiological-ocular effort in focusing sight on certain objects within a given spectacle, with the potential also of generating errors in visual perception (principally as studied in Chap. [2](#page-0-0) of Book III of Ibn al-Haytham's *Optics*).

Binocular diplopia, commonly known as "double vision", entails the simultaneous perception of two quasi-displaced-images of a single object, which results from the misalignment of the two eyes relative to one another, and due to "convergence insufficiency". This is not an ocular disorder when the object of vision is brought at a near distance to the eyes and results from normal physiological conditions of optical convergence, which require additional effort in focusing the two eyes on an object that is very close to them, and seeing it against the background of other more distant objects. Binocular vision is normally accompanied by *singleness in vision or binocular fusion*, in which one and a single image is seen despite each eye having its own image of the object of vision. Moreover, stereopsis exploits the parallax, in terms of the displacement of a single object viewed via two different lines of sight of the eyes, along with the binocular fusion of these two resultant images, leading ultimately to seeing spatial depth.

The theoretical presuppositions guiding the construction of mono-focal linear perspective are grounded on a sound optical analysis of binocular vision and the singleness of vision in terms of binocular fusion, as analysed by Ibn al-Haytham. This relies on the psychological, physiological, and neurological determinants of visual perception, which result, under normal conditions of vision, in the fusion of two disparate images-forms of a single visible object, as received by each of the eyes of the observer, and being unified in the brain through the agency of the optical nerves, the common optic *chiasma*, and the exercising of cognition in effecting sight.

Alberti and Ghiberti animated the discussions concerning these optical directives in terms of how they underpinned the legitimate methods of constructing perspective, as these embodied varying levels of adapting Ibn al-Haytham's optical legacy and its reception by mediaeval and Renaissance perspectivists. These elements of debate also continued to preoccupy figures such as Piero Della Francesca in his *De Prospectiva Pingendi*, with applications in his *Flagellation* painting (see Fig. 1.5 in chapter 1) that rendered it exemplary amongst the perfected perspectival constructs. Celebrated linear perspectives were also associated with Masaccio's

Santa Trinita (in Santa Maria Novella, Firenze), Donatello's *Banquet of Herod*, or Raphael's remarkable *Scuola di Atene* (in the *Stanza della Segnatura*, Vatican apostolic palace). Investigations that focused on the perfection of the depicted pictorial representational spaces, in the projections and constructions of linear central perspectives, combined with in-depth studies in geometric optics, resulted eventually in perspectival approaches to Euclidean geometry, which culminated in the seventeenth century in advanced legacies of geometric perspectivism, as for instance embodied in Girard Desargues' *Œuvres mathématiques*, and in his projective geometry [\(Desargues 1647\)](#page-20-13).

In order to grasp the invention of linear perspective based on optics, it is vital to also consider novel ways of accounting for "the visibility of space" since the times of Ibn al-Haytham, by way of considering the mediated reception of prolongations of his legacy in Renaissance circles, and the definition of representational space with an ordered geometry of its own that unifies its imagined visual field. The novel reorientation of the development of perspective in relation to the spatial order of the seen spectacle that is depicted rested on a newly defined "looking space" [\(Vesely 2004\)](#page-22-7), which required the controlled counterbalancing of the problematic aspects of visual illusions or errors via geometrical structuring measures that facilitated the location of objects and their interrelations within the visual field. This ushered a new phase in the debate over orthogonals, viewing points, vanishing points, and the visual cone-pyramid, which became foundational concepts for the invention of geometrized perspective and its presupposition of a "mathematized space". The idea of perspective as a pictorial representational construct rested on the fundamental notions of a "geometrized space" and "the visibility of spatial depth" (both rooted in Ibn al-Haytham's mathematical and optical research, as we shall highlight in the following section below).

2.5 Geometrical Place as Spatial Extension

Ibn al-Haytham presented his geometrical conception of place as a solution to a long-standing problem that remained philosophically unresolved, which, to our knowledge, also constituted the first viable attempt to geometrize "place" in history of science. This corresponded with Ibn al-Haytham's foundational endeavour to "mathematize physics" in the context of experimental research in optics. Ibn al-Haytham aimed at promoting a geometrical conception of place that is akin to *spatial* extension in view of addressing selected mathematical problems that resulted from the unprecedented developments in geometrical transformations (similitude, translation, homothety, affinity, etc.), the introduction of motion in geometry, the anaclastic research in conics and dioptrics in the Apollonian-Archimedean Arabic legacy since the ninth century [\(El-Bizri 2004,](#page-20-14) [2007b\)](#page-20-15).

Besides the *penchant* to offer mathematical solutions to problems in theoretical philosophy that were challenged by longstanding historical obstacles and epistemic impasses, Ibn al-Haytham's endeavour in geometrizing place was undertaken in view of sustaining and grounding his research in mathematical analysis and synthesis (*Fi al-tahlil wa-al-tarkib*),^{[12](#page-16-0)} and in response to the needs associated with the unfurling of his studies on knowable mathematical entities (*Fi al-ma'lumat*)[,13](#page-16-1) and in order to reorganize most of the notions of geometry and rethinking them anew in terms of motion *(al-haraka, al-naql)*. Consequently, he had to critically reassess the dominant philosophical conceptions of place in his age, which were encumbered by inconclusive theoretical disputes over Aristotle's *Physics* [\(Aristotle 1936\)](#page-20-16).

Even though Aristotle affirmed that *topos* has the three dimensions of length, width and depth (*Physics*, IV, 209a 5), he defined *topos* as: "the innermost primary surface-boundary of the containing body that is at rest, and is in contact with the outermost surface of the mobile contained body" (*Physics*, IV, 212a 20–21). Contesting this long-standing Aristotelian *physical* conception of *topos*, Ibn al-Haytham posited *al-makan* as "imagined void" (*khala' mutakhayyal*; postulated void) whose existence is secured in the imagination (like it is the case with invariable geometrical entities). He moreover held that the "imagined void" *qua* "geometrized place" consisted of imagined immaterial distances that are between the opposite points of the surfaces surrounding it [\(Rashed 1993,](#page-21-19) [2002\)](#page-21-20). He furthermore noted that the imagined distances of a given body, and those of its containing place, get superposed and united in such a way that they become the same distances (*qua* dimensions) as mathematical lines having lengths without widths-breadths.

From a philosophical viewpoint, we could say that Ibn al-Haytham's geometrical determination of place was "ontologically" neutral. This is the case given that his mathematical notion of *al-makan* was not simply obtained through a "theory of abstraction" as such, nor was it derived by way of a "doctrine of forms", nor was it grasped as being the (phenomenal) "object" of "immediate experience" or "common sense". It is rather the case that his geometrized place resulted from a mathematical isometric "bijection" function between two sets of relations or distances [\(El-Bizri](#page-20-15) [2007b\)](#page-20-15).[14](#page-16-2) Nothing is thus retained of the properties of a body other than *extension*, which consists of mathematical distances that underlie the geometrical and formal conception of place [\(Rashed 2002\)](#page-21-20).

To give an example of Ibn al-Haytham's mathematical refutation of Aristotle's physical definition of *topos*, we could consider the case of his geometric demonstration based on the properties of a parallelepiped (*mutawazi al-sutuh*; a geometric solid bound by six parallelograms; a cuboid). If this given parallelepiped were to be divided by a rectilinear plane that is parallel to one of its surfaces, and is then recomposed, the cumulative size of its parts would be equal to its original

 12 The Arabic critical edition (based on four manuscripts) and the annotated French translation of this treatise (*Fi al-tahlil wa-al-tarkib; L'Analyse et la synthèse*) are established in [Rashed](#page-21-20) [\(2002,](#page-21-20) pp. 230–391).

¹³The Arabic critical edition (based on two manuscripts) and annotated French translation of this treatise (*Fi al-ma'lumat; Les connus*) are established in [Rashed](#page-21-20) [\(2002,](#page-21-20) pp. 444–583).

^{14"}Bijection" refers to an equivalence relation or function of mathematical transformation that is both an "injection" ("one-to-one" correspondence) and "surjection" (designated in mathematical terms also as: "*onto*"') between two sets.

Fig. 2.4 The magnitude of a parallelepiped divided along the lines a and b would increase in surface area by a quantity equal to $2ab$; the magnitude of the same parallelepiped carved out of a cube with a side c would increase in surface-area by a quantity $4c^2$, whilst it would decrease in volume

magnitude prior to being divided, while the total sum of the surface areas of its parts would be greater than its surface-area prior to being partitioned. Following the Aristotelian definition of *topos*, and in reference to this divided parallelepiped, one would conclude that: an object divided into two parts occupies a place that is larger than the one it occupied prior to its division, since its total surface area increased with its division. Hence, the magnitude of the place of a given body increases while the size of that body does not; consequently: "objects of equal magnitudes are contained in unequal places", which is an untenable proposition [\(Rashed 2002;](#page-21-20) [El-Bizri 2007b\)](#page-20-15). Likewise, if we consider the case of a parallelepiped that is carved, then, its bodily magnitude is diminished while the total sum of its surface area would increase. Following the Aristotelian definition of *topos*, and in reference to this carved parallelepiped, one would conclude that: an object that diminishes in magnitude occupies a larger place, which is untenable.

For example, as shown in the Fig. [2.4,](#page-17-0) the magnitude of the middle parallelepiped that has been divided along the lines a and b would increase in surface area by a quantity equal to $2ab$. As for the carved parallelepiped to the right side in the figure (Fig. [2.4\)](#page-17-0), if a cube with a side c were to be cut out from it, then its magnitude would decrease, whilst its surface-area increases by a quantity $4c²$.

Moreover, using mathematical demonstrations, in terms of geometrical solids of equal surface-areas (isepiphanic), and figures that have equal perimeters (isoperimetric), Ibn al-Haytham showed that the sphere is the largest in (volumetric) size with respect to all other primary solids that have equal surface-areas (*al-kura a'zam al-ashkal al-lati ihatatuha mutasawiya*). So, if a given sphere has the same surfacearea as a given cylinder, then they occupy equal places according to Aristotle, and yet, the sphere would have a larger (volumetric) magnitude than the cylinder; hence unequal objects occupy equal places, which is not the case.

Ultimately, Ibn al-Haytham's critique of Aristotle's definition of *topos*, and his own geometrical positing of *al-makan* as an "imagined void" (*khala' mutakhayyal*), both substituted the grasping of the body as being a totality bound by physical surfaces to construing it as a set of mathematical points that are joined by geometrical line-segments. Hence, the qualities of a body are posited as an *extension* that consists of mathematical lines, which are invariable in magnitude and position, and that connect points within a region of the *three-dimensional space* independently of the physical body.

The geometrical place of a given object is posited as a "metric" of a region of the so-called "Euclidean" *qua* "geometrical *space*", which is occupied by a given body that is in its turn also conceived extensionally, and corresponds with its geometrical place by way of "isometric bijection". Consequently, Ibn al-Haytham's geometrical determination of place points to what later was embodied in the conception of the "anteriority of spatiality" over the demarcation of a metric of its regions by means of mathematical lines and points, as explicitly implied by the notion of a "Cartesian space" [\(Rashed 2002;](#page-21-20) [El-Bizri 2007b\)](#page-20-15). The scientific and mathematical significance of the geometrization of place was confirmed through the unfolding of mathematics and physics in seventeenth century conceptions of place as extension (namely as a volumetric, three-dimensional, uniform, isotropic and homogeneous space), particularly in reference to Descartes' *extensio* and Leibniz's *analysis situs*, and the emergence of what came to be known in periods following Ibn al-Haytham's age as being the "Euclidean space" (namely, an appellation that is coined in relatively modern times, and describes a notion that is historically posterior to the geometry of figures as embodied in Euclid's *Stoikheia* [*The Elements; Kitab Uqlidis fi al-Usul*]).^{[15](#page-18-0)}

Ibn al-Haytham's reflections on the notion of space in his *Kitab al-Manazir (Optics)* were commensurable with his mathematical conception of place in his *Qawl fi al-makan (Discourse on Place)*. Ibn al-Haytham asserted that spatial depth is a visible property (unlike the eighteenth century immaterialism of George Berkeley, who denied the visibility of space).^{[16](#page-18-1)} Ibn al-Haytham also argued that: in order that the distance, which separates the observer from the object of vision, gets estimated, the thing being perceived ought to be near objects that are ordered and contiguous (*Optics*, II.3: 76–80), as well as share a common unified terrain with the observer.

To demonstrate this situational and phenomenological condition, Ibn al-Haytham established an experimental installation that consisted of a wall dividing a given hall into two distinct spaces S_1 and S_2 (as shown in Fig. [2.5\)](#page-19-0), which are visually linked through a pinhole aperture **a** (*thuqb*), piercing the wall separating them, in such a way that the floor and ceiling in space S_1 could not be seen when looking through [a] from S_2 . The concealed space S_1 receives objects that could only be viewed by observers in this experiment from S_2 through aperture [a]. If two screen-walls $w1$

¹⁵After all, the expression deployed by Euclid that is closest to a notion of "*space*" as denoted by the Greek term: "*khôra*", is the appellation: "*khôrion*", which designates "an area enclosed within the perimeter of a specific geometric abstract figure", as for instance noted in Euclid's *Data (Dedomena; al-Mu'tayat)* Proposition 55 (as also related to: *Elements*, VI, Proposition 25): "if an *area* [*khôrion*] be given in form and in magnitude, its sides will also be given in magnitude" [\(Euclid 1956,](#page-21-21) [1883–1916\)](#page-21-22).

¹⁶This question preoccupied Maurice Merleau-Ponty in the twentieth century, in terms of reaffirming the visibility of spatial depth in his *Phénoménologie de la Perception* [\(Merleau-Ponty](#page-21-23) [1945;](#page-21-23) [El-Bizri 2004\)](#page-20-14).

Fig. 2.5 Experimental installation conceived by Ibn al-Haytham

and **w2** were to be introduced into the concealed space S_1 at different distances from the dividing wall, then, looking through the aperture [a], the observers in space S_2 could not detect the difference between the distances of the screen-walls **w1** and **w2**; and when these screen-walls were subjected to an intense light, the observers were not able to even distinguish them from each other (*Optics*, II.3: 80–84). The same applies also for judging the distance that separates a vertical rod \mathbf{r} in S_1 from the observer in S_2 , which cannot be determined accurately.

As Ibn al-Haytham argued, the relation with the common ground that is shared between the observer and the object of vision is measured through the spatiality of the body of the observer. The feet (*al-qadamayn*) in pacing, the stretched forearm (*dhira'*) and the hand (*yad*) in grasping, as well as the scale of the human embodiment *(al-qama)* all act as measure determinants in a pre-reflexive and nonintentional manner (*Optics*, II.3: 150–155). Ultimately, the estimation of distance in seeing spatial depth was not restricted to topics in optics, rather they had applications that were also significant in terms of Ibn al-Haytham's explication of his observational data in astronomy, like his treatment of the question concerning the moon-illusion; namely when the moon appears larger at the horizon than at its zenith.

Ibn al-Haytham's geometrization of place, and his affirmation of the visibility of spatial depth, resonated with Renaissance and Early-Modern conceptions of spatiality and extendedness. The definition of place as "space" corresponded also with the manner architecture and perspective shared a sense of coherent spatiality

as embodied in the "idealized representation" of the notions of the "*room*" and of "a looking space" [\(Vesely 2004\)](#page-22-7), which acquired the characteristics of the "isotropic space of geometry".^{[17](#page-20-17)}

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¹⁷This development was perhaps "anticipated" in the "perspectivity" of architecture with the "parallelism" of its structuring components (columns, pillars, walls) and the "axial regularity" of its spatial articulations [\(Vesely 2004;](#page-22-7) [El-Bizri 2010b\)](#page-21-2).

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