Chapter 6
Synchronization Between Human Resources in Home Health Care Context

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Abstract This paper deals with the scheduling and routing problem in a Home Health Care structure, when synchronization is needed between two types of human resources, and time windows are considered for patients and caregivers. Our objective is to minimize the total waiting time of caregivers between patients. We give a mathematical formulation of this problem as a mixed integer linear program. We present some experiments in order to analyze the execution time and test the capability of the MILP to solve the problem in real cases, within reasonable execution times, to measure the impact of the proportion of synchronized visits, and to analyze the average workload of an operator.

6.1 Introduction

Home Health Care (HHC) is defined as medical and paramedical services delivered to patients at home. It helps patients to maintain and improve their life conditions. HHC have seen a significant evolution in France, as well as in several other countries. Among the reasons of this development, we can cite economic factors, ageing of populations, increase in the number of people with chronic diseases, congestion of the hospitals, improvements in medical technologies, and choices of the patients.

Due to its numerous specificities (resources mobility, human resources with specific skills and constraints, importance of the quality of service, uncertainties, …), HHC has become a particularly important application area for Industrial engineering. In this paper, we are interested in the scheduling and routing problem of HHC staff (i.e. deciding which human resource visits which patient at what time).

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We focus on the case when two human resources are required at the same time for some cares. This synchronization between resources occurs for example when a nurse and an auxiliary nurse are simultaneously required for a patient who needs help to get in or out of bed.

This paper is organized as follows: we define our problem in Sect. 6.2, and discuss of related work in Sect. 6.3. Section 6.4 proposes then a mathematical formulation for our problem as a mixed integer linear programming model, and some experiments are presented in Sect. 6.5.

6.2 Problem Description

As pointed out in [1–4], the synchronization, in HHC context, between the visits of the different stakeholder is a difficult and crucial problem.

Here a pair wise synchronization is studied between nurses and auxiliary nurses. The coordination of human resources corresponds to the pair wise synchronization constraints for multiple traveling salesman problem formulation. Synchronization implies temporal constraints.

Furthermore, hard constraints as working hours of human resources and time windows of patients must be taking into account. Time windows of patients represent either wishes of the patient or medical constraints.

In this study, short term planning is treated. Per time period considered, each patient must have exactly one visit. For this visit either a nurse is required or an auxiliary nurse is required or a nurse and an auxiliary nurse are required.

Our objective is to minimize the total waiting time of operators between patients. This objective is very important in practice. Indeed, the nurses and auxiliary nurses, besides the medical tasks also have preparatory tasks and administrative tasks to realize. It is thus important that the nurses and the auxiliary nurses have time at the beginning and at the end of a tour to realize these tasks and for that they should not waste too much time waiting between patients. Let us note that this waiting time is all the more important within the framework of this study as, on one hand, windows of patients are considered and, on the other hand, synchronization between the nurses and the auxiliary nurses are taken into account. As far as we know, in the literature, this objective is never used as an objective function.

We propose a Mixed Integer Linear Programming Model, and the data used for the tests is inspired by real data and is created in a random way.

6.3 Literature Review

In the literature, several issues are considered while dealing with resource planning of HHC, such as the resource dimensioning, partitioning of a territory into districts, allocation of resources to districts, assignments of care providers to patients, or the visits and the resource scheduling and routing. The most frequently treated issue is the last one, routing and scheduling. Readers can refer to Yalçındağ et al. [5] for a review of papers addressing the scheduling and routing problem as a Travelling Salesman Problem (TSP) or Vehicle Routing Problem (VRP) in the HHC context.

We focus here on the papers addressing the problem of coordination between human resources.

There are some papers considering "shared visits" [6–8]. They all consider human resources with the same qualifications, who should sometimes be more than one for some visits.

Here, we are more especially interested with papers dealing with synchronization constraints between human resources who have different qualifications.

Bredström and Rönnqvist [1] develop a general branch and price algorithm for routing and scheduling problem with time windows. The problem is formulated as a set partitioning problem, considering synchronization constraints. LP relaxations are used in order to solve the problem. In [2], they go further by considering both synchronization and precedence constraints as temporal constraints. They use a multi criteria objective function, minimizing preferences, travelling time, and maximal workload difference, and propose a heuristic to solve their model.

Kergosien et al. [3] propose an integer linear programming model and propose some technical improvements to solve the routing problem in HHC context. They consider the problem under the constraint of synchronization, disjunction (some operators cannot work together), time windows for operators and patients, and continuity of care. They formulate the problem as a multiple traveling salesman problem with time windows with some additional constraints with the objective of minimizing total travelling distance. They test the model on randomly generated instances with Cplex solver. Results show that the proposed integer linear program is not able to deal with instances of real size.

More recently, Rasmunssen et al. [4] consider four temporal constraints. They formulate the problem as a set partitioning problem, and develop visit clustering schemes for home care personnel, in order to explore how much they decrease run times, and how much they compromise optimality. They propose LP-based branch and price framework. The algorithm is tested with real life problem instances. Results show that visit clustering schemes decrease the execution time significantly but cause a loss of quality for a few instances. Furthermore they outline that visit clustering schemes allow finding solutions that could not be solved to optimality.

Note that all these papers dealing with synchronization or shared visits use total travelling cost/distance as objective function. Some of them [1, 2, 4] consider a multi objective function, considering also visit time preference for [1], referential operator for [2, 4], number of uncovered visits for [4], and workload difference for [2]. None of them considers the same objective as ours, namely minimization of the total waiting time of operators between patients, although it is very important in practice, as explained above. As far as the constraints are concerned, these papers take into account most of the constraints that a HHC center has to deal with in practice, as we also do in our problem.

6.4 Model Description

6.4.1 Assumptions and Notations

- **Operators**: We consider two kinds of operators: nurses and auxiliary nurses and assume that there are $\boldsymbol{N}$ nurses and $\boldsymbol{M}$ auxiliary nurses. We denote by $\boldsymbol{IN}$ the set of nurses, and by $\boldsymbol{IM}$ the set of auxiliary nurses.

  We assume that nurses and auxiliary nurses might have different working hours.

  Nurses and auxiliary nurses can take care of a patient related to their qualification. All the nurses have the same qualification, and all the auxiliary nurses have the same qualification.

  Each operator has to start/finish his/her work at Home Health Care Center (HHCC), which is denoted by $\boldsymbol{0}$ when it is the starting point, and by $\boldsymbol{d}$ when it is the ending point.
- **Patients**: There are $\boldsymbol{P}$ patients, needing each exactly one visit per time period considered. Among them, there are $\boldsymbol{PN}$ patients who need a care by a nurse, $\boldsymbol{PAN}$ patients who need a care by an auxiliary nurse, and $\boldsymbol{PS}$ patients who need a simultaneous care by a nurse and an auxiliary nurse (synchronization of two different operators). We denote by $\boldsymbol{IP_I}$ the set of patients that need a care by a nurse: $\boldsymbol{IP_I} = \{1, \ldots, PN\}$, by $\boldsymbol{IP_{AN}}$ the set of patients that need a care by an auxiliary nurse: $\boldsymbol{IP}_{\boldsymbol{AN}} = \{PN+1, \ldots, PN+PAN\}$, by $\boldsymbol{IP_{sync}}$ the set of patients that need a simultaneous care by a nurse and an auxiliary nurse: $\boldsymbol{IP}_{\boldsymbol{sync}} = \{PN+PAN+1, \ldots, PN+PAN+PS\}$ and by $\boldsymbol{IP}$ the set of all the patients ($\boldsymbol{IP} = \boldsymbol{IP_{NI}} \cup \boldsymbol{IP_{AN}} \cup \boldsymbol{IP_{sync}}$)

Each patient has a time window within which the operator(s) has to arrive at the patient's house. Duration of care for each patient and travelling time between each patient are fixed. Note that for these parameters, HHCC is considered as a patient.

6.4.2 Problem Formulation as Mixed Integer Linear Programming Model

6.4.2.1 Indexes

$\boldsymbol{n}$**:** $1, \ldots, N$, for nurses
$\boldsymbol{m}$**:** $1, \ldots, M$, for auxiliary nurses
$\boldsymbol{i}$**:** $1, \ldots, P$ for patients

6.4.2.2 Parameters

$[a_n, b_n]$: Working hours of nurse $\boldsymbol{n}$, $n \in IN$
$[c_m, d_m]$: Working hours of auxiliary nurse $\boldsymbol{m}$, $m \in IM$
$[e_i, l_i]$: Time window of patient $\boldsymbol{i}$ for the arrival time, $i \in IP_{sync} \cup IP_N \cup IP_{AN} \cup \{0, d\}$
D_i: Duration of care for patient $\boldsymbol{i}$, $i \in IP_{sync} \cup IP_N \cup IP_{AN} \cup \{0\}$
T_{ij}: Travelling time between patient $\boldsymbol{i}$ and patient $\boldsymbol{j}$, $i \in IP_{sync} \cup IP_N \cup IP_{AN} \cup \{0\}$; $j \in IP_{sync} \cup IP_{AN} \cup IP_N \cup \{d\}$
A: Big number with $\boldsymbol{A} \geq \max_{m,n} \{\boldsymbol{a_n}, \boldsymbol{c_m}\}$
B: Big number with $\boldsymbol{B} \geq \max_{m,n} \{\boldsymbol{b_n}, \boldsymbol{d_m}\}$

6.4.2.3 Decision Variables

Nurses

$$X_{nij} = \begin{cases} 1 \; if \; nurse \; n \; takes \; care \; of \; patient \; j \; immediately \; after \; patient \; i \\ 0 \; otherwise \end{cases}$$

$$\forall i \in IP_{sync} \cup IP_N \cup \{0\}; \forall j \in IP_{sync} \cup IP_N \cup \{d\}; i \neq j; \forall n \in IN$$

$\boldsymbol{t_{nj}}$: Arrival time of nurse $\boldsymbol{n}$ to the house of patient $\boldsymbol{j}$.

$$\forall j \in IP_{sync} \cup IP_N \cup \{0, d\}; \forall n \in IN$$

Auxiliary Nurses

$$Y_{mij} = \begin{cases} 1 \; if \; auxiliary \; nurse \; m \; takes \; care \; of \; patient \; j \; immediately \; after \; patient \; i \\ 0 \; otherwise \end{cases}$$

$$\forall i \in IP_{sync} \cup IP_{AN} \cup \{0\}; \forall j \in IP_{sync} \cup IP_{AN} \cup \{d\}; i \neq j; \forall m \in IM$$

$\boldsymbol{s_{mj}}$: Arrival time of auxiliary nurse $\boldsymbol{m}$ to the house of patient $\boldsymbol{j}$.

$$\forall j \in IP_{sync} \cup IP_{AN} \cup \{0, d\}; \forall m \in IM$$

Waiting Time

$\boldsymbol{Q_{nij}}$: Waiting time of nurse $\boldsymbol{n}$ between patient $\boldsymbol{i}$ and patient $\boldsymbol{j}$.

$$\forall i \in IP_N \cup IP_{sync} \cup \{0\}; \forall j \in IP_N \cup IP_{sync} \cup \{d\}; i \neq j; \forall n \in IN$$

$\boldsymbol{S_{mij}}$: Waiting time of auxiliary nurse $\boldsymbol{m}$ between patient $\boldsymbol{i}$ and patient $\boldsymbol{j}$.

$$\forall i \in IP_{AN} \cup IP_{sync} \cup \{0\}; \forall j \in IP_{AN} \cup IP_{sync} \cup \{d\}; i \neq j; \forall m \in IM$$

6.4.2.4 Mathematical Formulation

$$\min \sum_n \sum_i \sum_j Q_{nij} + \sum_m \sum_i \sum_j S_{mij}$$

Subject to:

$$\sum_n \sum_i X_{nij} = 1 \qquad \forall n \in IN; \forall i \in IP_{sync} \cup IP_N \cup \{0\};\ \ \forall j \in IP_{sync} \cup IP_N;\ \ i \neq j \tag{6.1}$$

$$\sum_m \sum_i Y_{mij} = 1 \qquad \forall m \in IM; \forall i \in IP_{sync} \cup IP_{AN} \cup \{0\};\ \ \forall j \in IP_{sync} \cup IP_{AN}; i \neq j \tag{6.2}$$

$$\sum_j X_{n0j} = \sum_j X_{njd} = 1 \qquad \forall j \in IP_N \cup IP_{sync};\quad \forall n \in IN \tag{6.3}$$

$$\sum_j Y_{m0j} = \sum_j Y_{mjd} = 1 \qquad \forall j \in IP_{AN} \cup IP_{sync};\quad \forall m \in IM \tag{6.4}$$

$$X_{n0d} = 0 \qquad \forall n \in IN \tag{6.5}$$

$$Y_{m0d} = 0 \qquad \forall m \in IM \tag{6.6}$$

$$\sum_j X_{nij} = \sum_k X_{nki}. \qquad \forall i \in IP_{sync} \cup IP_N;\quad j \in IP_{sync} \cup IP_N \cup \{d\}; \\ k \in IP_{sync} \cup IP_N \cup \{0\};\quad i \neq j;\quad i \neq k;\quad \forall n \in IN \tag{6.7}$$

$$\sum_j Y_{mij} = \sum_k Y_{mki} \qquad \forall i \in IP_{sync} \cup IP_{AS}; j \in IP_{sync} \cup IP_{AS} \cup \{d\}; \\ k \in IP_{sync} \cup IP_{AS} \cup \{0\}; i \neq j; i \neq k; \forall m \in IM \tag{6.8}$$

$$l_i * \sum_j X_{nij} \ \geq\ t_{ni} \geq e_i * \sum_j X_{nij} \qquad \forall i \in IP_N \cup IP_{sync}; \\ j \in IP_N \cup IP_{sync} \cup \{d\}; i \neq j;\quad \forall n \in IN \tag{6.9}$$

$$l_i * \sum_j Y_{mij} \ \geq\ s_{mi} \geq e_i * \sum_j Y_{nij} \qquad \forall i \in IP_{AN} \cup IP_{sync}; \\ j \in IP_{AN} \cup IP_{sync} \cup \{d\}; i \neq j;\quad \forall m \in IM \tag{6.10}$$

$$t_{nj} + (1 - X_{nij}) * l_i \geq\ t_{ni} + (D_i + T_{ij}) * X_{nij} \\ \forall i \in IP_{sync} \cup IP_N \cup \{0\};\quad \forall j \in IP_{sync} \cup IP_N \cup \{d\}; i \neq j;\quad \forall n \in IN. \tag{6.11}$$

$$s_{mj} + (1 - Y_{mij}) * l_i \geq s_{mi} + (D_i + T_{ij}) * Y_{mij}$$
$$\forall i \in IP_{sync} \cup IP_{AN} \cup \{0\}; \quad \forall j \in IP_{sync} \cup IP_{AN} \cup \{d\}; i \neq j; \quad \forall m \in IM \tag{6.12}$$

$$t_{n0} \leq t_{nj} + A * \left(1 - \sum_i X_{nij}\right)$$
$$i \in IP_N \cup IP_{sync}; \quad \forall j \in IP_N \cup IP_{sync} \cup \{d\}; i \neq j; \quad \forall n \in IN \tag{6.13}$$

$$s_{m0} \leq s_{mj} + A * \left(1 - \sum_i Y_{mij}\right)$$
$$i \in IP_{AN} \cup IP_{sync}; \quad \forall j \in IP_{AN} \cup IP_{sync} \cup \{d\}; i \neq j; \quad \forall m \in IM \tag{6.14}$$

$$t_{nd} \geq t_{nj} \qquad \forall n \in IN; \quad \forall j \in IP_N \cup IP_{sync} \cup \{0\} \tag{6.15}$$

$$s_{md} \geq s_{mj} \qquad \forall m \in IM; \quad \forall j \in IP_{AN} \cup IP_{sync} \cup \{0\} \tag{6.16}$$

$$t_{n0} \geq a_n \qquad \forall n \in IN \tag{6.17}$$

$$s_{m0} \geq c_m \qquad \forall m \in \mathrm{IM} \tag{6.18}$$

$$t_{nd} \leq b_n \qquad \forall m \in IM \tag{6.19}$$

$$s_{md} \leq d_m \qquad \forall m \in IM \tag{6.20}$$

$$\sum_n t_{nj} - \sum_m s_{mj} = 0 \qquad \forall j \in IP_{sync} \tag{6.21}$$

$$t_{nj} - (t_{ni} + D_i + T_{ij}) \leq Q_{nij} + B * (1 - X_{nij})$$
$$\forall i \in IP_N \cup IP_{sync} \cup \{0\}; \quad \forall j \in IP_N \cup IP_{sync} \cup \{d\}; \quad i \neq j; \quad \forall n \in IN \tag{6.22}$$

$$s_{mj} - (s_{mi} + D_i + T_{ij}) \leq S_{mij} + B * (1 - Y_{mij})$$
$$\forall i \in IP_{AN} \cup IP_{sync} \cup \{0\}; \quad \forall j \in IP_{AN} \cup IP_{sync} \cup \{d\}; \quad i \neq j; \quad \forall m \in IM \tag{6.23}$$

$$X_{nij} \in \{0,1\} \qquad \forall i \in IP_{sync} \cup IP_N \cup \{0\}; \quad \forall j \in IP_{sync} \cup IP_N \cup \{d\}; i \neq j; \quad \forall n \in IN \tag{6.24}$$

$$Y_{mij} \in \{0,1\} \qquad \forall i \in IP_{sync} \cup IP_{AN} \cup \{0\}; \quad \forall j \in IP_{sync} \cup IP_{AN} \cup \{d\};$$
$$i \neq j; \quad \forall m \in IM \tag{6.25}$$

$$t_{nj} \in R_{+} \cup \{0\} \qquad \forall j \in IP_{sync} \cup IP_{N} \cup \{0, d\}; \quad \forall n \in IN \tag{6.26}$$

$$s_{mj} \in R_{+} \cup \{0\} \qquad \forall j \in IP_{sync} \cup IP_{AN} \cup \{0, d\}; \quad \forall m \in IM \tag{6.27}$$

$$\begin{aligned} Q_{nij} \in R_{+} \cup \{0\} \quad & \forall i \in IP_{N} \cup IP_{sync} \cup \{0\}; \\ & \forall j \in IP_{N} \cup IP_{sync} \cup \{d\}; \; i \neq j; \; \forall n \in IN \end{aligned} \tag{6.28}$$

$$\begin{aligned} S_{mij} \in R_{+} \cup \{0\} \quad & \forall i \in IP_{AN} \cup IP_{sync} \cup \{0\}; \\ & \forall j \in IP_{AN} \cup IP_{sync} \cup \{d\}; \; i \neq j; \; \forall m \in IM \end{aligned} \tag{6.29}$$

Our objective is to minimize the sum of waiting times of operators between patients. We must remember that waiting time of each resource at HHCC is not considered. This occurs in two different ways. First, an operator starts his/her work at HHCC and waits before visiting a patient. Second, an operator finishes his/her work at HHCC before ending working time. We do not deal with these two cases, because a resource can spend the waiting time at HHC with paper works.

Constraint sets (6.1) and (6.2) ensure that each patient j that needs a care from a nurse (resp. an auxiliary nurse) is visited by only one nurse (resp. an auxiliary nurse).

Constraint sets (6.3) and (6.4) ensure that each nurse (resp. an auxiliary nurse) has to leave HHCC and return to HHCC. Constraint sets (6.5) and (6.6) avoid that an operator visits only $\{0,d\}$ that corresponds to HHCC, making sure that each operator works.

Constraint sets (6.7) and (6.8) ensure that if a nurse (resp. an auxiliary nurse) enters a patient's house, he/she has to leave it.

Constraint sets (6.9) and (6.10) ensure that each operator can arrive at a patient's house respecting his/her time window.

Constraint sets (6.11) and (6.12) formulate arrival time to the patients. Here we deal with the duration of care and travelling time of an operator between two patients that he/she takes care of, considering that these two patients are visited one after the other one.

Constraint sets (6.13) and (6.14) make sure that if an operator visits patient j, his/her arrival time will be greater or equal to the arrival time to HHCC. When an operator does not visit a patient, his/her arrival time to this patient will be zero. Constraint sets (6.15) and (6.16) force arrival time of operators to HHCC at the end of the day to be greater or equal than the arrival time to any patient j. Constraint sets (6.13), (6.14), (6.15) and (6.16) also force each operator to start/finish at HHCC. We need these constraints for our objective.

Constraint sets (6.17) and (6.18) ensure that each operator can start his/her work after his/her beginning of working time. Constraint sets (6.19) and (6.20) ensure that each operator has to finish his/her work before his/her ending of working time.

Constraint sets (6.21) are for the patients who need a simultaneous care by a nurse and an auxiliary nurse (synchronization between two different operators). Constraint sets (6.21) force two different operators to arrive to the patients at the same time.

Constraint sets (6.22) and (6.23) formulate the waiting time of operators between two patients that he/she visits consecutively.

Constraint sets (6.24) and (6.25) force decision variables to take binary values. Constraint sets (6.26), (6.27), (6.28) and (6.29) ensure that decision variables take positive real number values.

6.5 Experiments

6.5.1 Data Generations

The data, namely working hours of operators, time windows of patients, duration of care and travelling time, is generated from a real case of the region Rhône-Alpes in France: Grenoble HHCC. This specific case is one of the biggest of this region. Our objective is to test the limits of MILP and show that our MILP is capable to solve a real and big case. The data is generated according to the answers collected by a survey done during the regional project "OSAD" [9] and, more particularly, the interviews done with Grenoble HHCC.

Operators: We need to determine beginning and end of working hours for each operator (nurses and auxiliary nurses). The data generation for nurses and auxiliary nurses is the same in nature. We define two cases in order to deal with the working hours of part time operators. Each generated operator belongs either to the first case or to the second case. The choice between these cases depends on a random number between 0 and 1. If the random variable belongs to [0, 0.5], the operator belongs to the first case, else it belongs to the second case.

Case 1: For each operator belonging to case 1, we determine the beginning of his/her working hours, while the end of his/her working hours is fixed to 300 min. The beginning of working hours of each operator is determined according to a random number which is between 0 and 1. To avoid short working hours, we give 40% chances to start at 0 min, 30% chances to start at 60 min, 20% chances to start at 120 min, 10% chances to start at 180 min.

Case 2: For each operator belonging to case 2, we determine the end of his/her working hours, while the beginning of his/her working hours is fixed to 0 min. The end of working hours is determined by a random variable which is between 0 and 1. As in case 1, in order to avoid short working hours, we give different chances for the end of working hours: Each operator has 10% chances to finish at 120 min, 20% chances to finish at 180 min, 30% chances to finish at 240 min, 40% chances to finish at 300 min.

Patients: We can group the patients within three categories, according to the required care giver: first, patients needing a care by a nurse; second, patients needing a care by an auxiliary nurse, and third, patients needing a simultaneous care by a nurse and an auxiliary nurse. The data generation of each patient is the same in nature. As data, we need the time window of the operator arrival time, and the duration of care for each patient.

Time window of arrival is composed of the following data: the earliest arrival time and the latest arrival time. They are determined as follows: the value of the earliest arrival time depends on a random number between 0 and 1. It can be 0, 30, 60, 90 or 120, and each one has an equal probability of 0.2.

The latest arrival time is determined in a different way: we first determine the length of the time window. This length is added to the earliest arrival time, in order to calculate the latest arrival time. The length of time window depends on a random variable between 0 and 1. It can be 60, 90, 120, 150 or 180 min and each one has an equal probability of 0.2.

Duration of care ranges between 20 and 180 min. The majority of cares take 45 min. We decide to determine the care duration as follows: we suppose that we have three different intervals for care duration. The first interval of care duration is between 20 and 35 min. 8% of the considered patients belong to this interval. The second one is between 35 and 55 min. 84% of the patients belong to this second interval. The third interval is between 55 and 180 min 8% of patients belong to this interval.

The care duration belonging to each of these intervals is determined by a uniform random number between the limits of the interval in question.

Travelling Time: We define an area for the locations of patient's house and HHCC. We consider a Cartesian coordinate system which is limited between 0 and 40 km for x axis and between 0 and 40 km for y axis. This area is inspired from Grenoble HHCC. We suppose that the area takes place in the positive side of the coordinate plane. Each patient (including HHCC) is defined as a point in the Cartesian coordinate system in two dimensions (x, y). We suppose that HHCC is located in the middle of the coordinate system, which is the point (20, 20). The location of each patient is determined randomly by generating one random number between 0 and 40 for each axis x and y.

Note that the travelling distance between each location corresponds to the travelling time. The travelling time between each patient, and between each patient and HHCC is calculated as the Euclidian distance between two points of the plane with Cartesian coordinates (x_1, y_1) and (x_2, y_2).

Instances Generation: In order to create the test problems, we create pools of operators and patients. We have two operator pools; a nurse pool containing 20 nurses and an auxiliary nurse pool containing 20 auxiliary nurses.

We have three patient pools, according to the required care giver. First pool is formed by 16 patients needing a care by a nurse; second pool is composed of 16 patients needing a simultaneous care by a nurse and an auxiliary nurse, and the third pool is formed by 16 patients needing a care by an auxiliary nurse.

Each operator and patient in each pool has a reference number. For each instance, a random number is generated in order to choose the operators and patients according to the reference number. Selection of operators (i.e. nurses and auxiliary nurses) and patients (three different categories according to the required care giver) is done separately.

For the experiments, we define the size of the problem as (number of nurses, number of auxiliary nurses, number of patients, and percentage of synchronized care). For example, instance (5, 5, 20, 30%) means that we randomly chose 5 nurses (resp 5 auxiliary nurses) among the 20 within the nurse pool (resp. the auxiliary nurse pool), and 20 patients among the 3*16 patients within the three patient pools. These 20 patients are chosen by considering the wished percentage of synchronized care, which means here that 30% of these 20 patients belong to the second pool (patients needing a simultaneous care by a nurse and an auxiliary nurse)

6.5.2 Results

We first analyze the execution time and test the capability of the MILP to solve the problem of the HHCC in the Rhône Alpes region, within reasonable execution times, second we measure the impact of the proportion of synchronized visits for the 40 patients' problem size. Next we analyze the average workload of an operator. The percentage of his/her time spent taking care of patients, the percentage of his/her time spent travelling and percentage of his/her time spent in HHCC.

These experiments are done by taking into account the following information: number of operators, working hours of operators, number of patients, time windows of patients, duration of care and travelling time.

ILOG on CLPEX 12.2 OPL STUDIO is used to solve the test problems. The resolution time is limited to 1 h for ILOG. The experiments have been conducted with CPU 3 GHz, 4 Go of RAM and Windows 7 (64 bits).

6.5.2.1 Analysis of Execution Times

We note that according to the generated data, an operator works on average 240 min. The time horizon is 300 min. That means that an operator works on average 80% of the time horizon. Table 6.1 shows the test problem sizes, the number of instances solved within 1 h, and minimum, maximum and average execution times of solved instances.

For the instances (5, 5, 20, 30%), 80% of instances are solved within 1 h. 70% of instances are solved within 5 min while 10% of them are solved in more than 15 min. We increase the size of the problem and for the instances (10, 10, 30, 33%). 40% of instances are solved in 10 min and 10% of instances are solved in more than 50 min. The instances are solved on average within 18 min. Lastly we test the

Table 6.1 Number of instances solved within 1 h for different problem sizes and execution times of instances in minutes

Problem size	Percentage of instances solved within 1 h	Execution time in minutes		
		Minimum	Maximum	Average
(5, 5, 20, 30%)	80	0.6	16.4	3.5
(10, 10, 30, 33%)	70	2.9	57.9	17.3
(20, 20, 40, 30%)	90	2.5	32.3	13.9

Table 6.2 Number of instances solved within 1 h for different proportion of synchronized visits

Problem size	Percentage of instances solved within 1 h
(10, 10, 30, 33%)	70
(10, 10, 40, 30%)	0
(10, 10, 40, 20%)	10
(10, 10, 40, 10%)	80
(15, 15, 40, 30%)	20
(15, 15, 40, 20%)	70
(15, 15, 40, 10%)	100

instances (20, 20, 40, 30%), and 90% of instances are solved in less than 1 h, 20% of instances are solved within less than 5 min while the rest of them are solved between 10 min and 33 min. The average execution time is 14 min.

6.5.2.2 Impact of the Proportion of Synchronized Visits on the Execution Times

We tested several examples and, as we could expect, we observed that the impact of this proportion is really important. For example, as shown in Table 6.2 when we consider the example (10, 10, 30, 33%) and we try to increase the number of patients to 40, we observe that the proportion of synchronized visits has to be reduced until 10% in order to solve successfully those instances. Finally for the instances with 15 nurses, 15 auxiliary nurses and 40 patients, 20% of synchronized visits can be solved successfully within 1 h.

Note that the proportion 10% of synchronized visits is the proportion that is usually used in the literature. So we can conclude that the proposed MILP is able to solve the problems with until 40 patients in the conditions of the literature.

6.5.2.3 Analysis of the Average Workload of Operators

In order to analyze the workload of operators, three indicators are determined:

- % duration of care on working hours
- % travelling time on working hours
- % time spent in HHCC on working hours

Table 6.3 Values of three indicators for different size of the problems

	(5, 5, 20, 30%)			(10, 10, 30, 33%)			(20, 20, 40, 30%)		
Instances	Max	Min	Average	Max	Min	Average	Max	Min	Average
% duration of care on working hours	63	44	49	49	36	44	29	26	28
% travelling time on working hours	26	21	23	20	18	19	29	26	28
% time spent in HHCC on working hours	35	11	27	46	31	36	63	57	58

As it can be seen in Table 6.3, the time spent at patients' house on average decreases while increasing the size of the problem. The time spent on average for travelling first decreases and then increases while increasing the size of the problem. The range of this indicator is really small for each size. The time spent in HHCC on average is significantly increased while number of operators and patients increase. This can be explained because of the significant increase in the number of operators.

6.6 Conclusion and Future Research

In this study, we proposed a mathematical formulation for the problem coordination of human resources in home health care context. We tested the limits of the Mixed Integer Linear Programming. As a result, the MILP is able to solve different sizes of problems within 1 h. But a heuristic is required for bigger sizes of the problem. We measured the impact of the proportion of synchronized visits. As a result, the proportion of synchronized visits impacts the number of instances solved within 1 h because it affects the number of visits as well. For the problems with 40 patients, number of instances solved within 1 h is increased while reducing the proportion of synchronized visits. The average workload of an operator is analyzed.

For future works, material resource planning can be added to the problem. In this work we dealt with short term planning. Our problem can be extended to the midterm planning so that care continuity is considered. Stochasticity can be included into data generation as demand of patients, duration of care or working hours of operators.

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