Chapter 10
Self-Similarity of Network Traffic

Everybody wants to live longer but nobody wants to grow old.

—Jules Rostand

In 1993, it was found out that there are modeling problems with using Markovian statistics to describe data traffic. A series of experiments on Ethernet traffic revealed that the traffic behavior was fractal-like in nature and exhibit self-similarity, i.e. the statistical behavior was similar across many different time scales (seconds, hours, etc.) [1, 3]. Also, several research studies on traffic on wireless networks revealed that the existence of self-similar or fractal properties at a range of time scale from seconds to weeks. This scale-invariant property of data or video traffic means that the traditional Markovian traffic models used in most performance studies do not capture the fratal nature of computer network traffic. This has implications in buffer and network design. For example, the buffer requirements in multiplexers and switches will be incorrectly predicted. Thus, self-similar models, which can capture burstiness (see Fig. 10.1) over several time scales, may be more appropriate.

In fact, it has been suggested that many theoretical models based on Markovian statistics should be reevaluated under self-similar traffic before practical implementation potentially show their faults.

Self-similarity is the property of an object which "looks the same" when viewed at different scales [4].

Self-similarity describes the phenomenon where a certain property of an object is preserved with respect to scaling in space and/or time. That is, as one zooms in or

M.N.O. Sadiku and S.M. Musa, *Performance Analysis of Computer Networks*, DOI 10.1007/978-3-319-01646-7_10,

Fig. 10.1 An example of a burst traffic

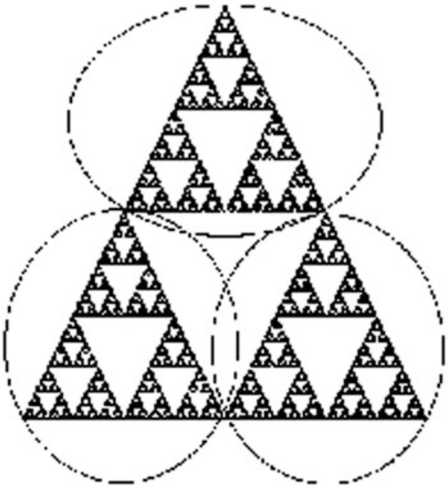

Fig. 10.2 The Sierpinski triangle

out the object has a similar (sometimes exact) appearance. For example, if an object is self-similar or fractal, its parts, when magnified resemble the shape of the whole. This idea is easily illustrated using the Sierpinski triangle (also known as Sierpinski gasket named after the Polish mathematician) shown in Fig. 10.2. The triangle S consists of three self-similar copies of itself, each with magnification of 2. We can look further and find more copies of S. The triangle S also consists of nine self-similar copies of itself, each with magnification of 4. Or we may cut S into 27 self-similar pieces, each with magnification factor 8. This kind of self-similarity at all scales is a hallmark of the images known as fractals.

Another example is the well known Koch snowflake curve shown in Fig. 10.3. As one successively zooms in the resulting shape is exactly the same no matter how far in the zoom is applied. A far more common type of self similarity is an approximate one, i.e. as one looks at the object at different scales one sees structures that are recognizably similar but not exactly so.

This chapter attempts to account for the self-similar traffic. We begin by first introducing the mathematics of self-similar process. We then present Pareto distribution as a typical example of a heavy-tailed distribution. We investigate the behavior of single queueing system with interarrival times having a large variance. We finally consider wireless networks with self-similar input traffic.

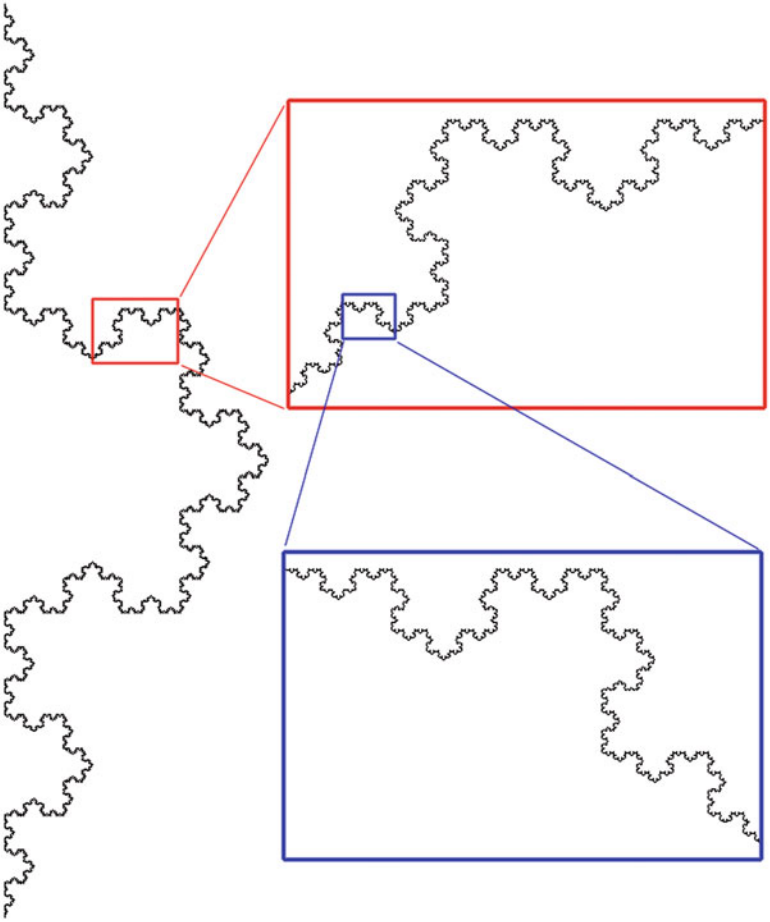

Fig. 10.3 Koch snowflake curve

10.1 Self-Similar Processes

Self-similar processes are stochastic processes, which can be described mathematically. They can be described by their characteristic of being scale-invariant. They are also characterized by fractal (i.e. fractional) dimensions, of which a number have been defined. One of these dimensions is the correlation dimension. A perfectly self-similar process on the average looks exactly the same regardless of the time scale observed.

Self-similarity manifests itself in a variety of ways: traffic appearing fractal-like, a spectral density obeying a power-law behavior, long-range dependence, slowly decaying variance, etc. [5]. The degree of self-similarity of a process is typically specified by the Hurst parameter H, where $0.5 < H < 1.0$. 0.5 represents non-self-similar behavior and the closer H is to 1, the more long-range dependence the process is.

A continuous stochastic process X(t) is self-similar if $a^{-H}X(at)$ has exactly the same second-order statistics (i.e. distribution) as X(t) for any real a > 0 and Hurst parameter H. The key idea is that a direct scaling of time yields a related scaling of the series regardless of what scale is chosen. A stochastic process X(t) is statistically self-similar if for a > 0, the process $a^{H}X(at)$ has the same statistical properties as X(t). The relationship may be expressed by the following three conditions [6, 7]:

$$1.\quad E[X(t)] = \frac{E[X(at)]}{a^H} \quad \text{(mean)} \tag{10.1}$$

$$2.\quad \mathrm{Var}[X(t)] = \frac{\mathrm{Var}[X(at)]}{a^{2H}} \quad \text{(variance)} \tag{10.2}$$

$$3.\quad R_X(t,s) = \frac{R_X(at,as)}{a^{2H}} \quad \text{(autocorrelation)} \tag{10.3}$$

The Brownian motion process and fractional Brownian motion process satisfy our definition of self-similarity. The fractional Brownian motion (FBM) is a continuous, zero mean, Gaussian process with parameter H, 0 < H < 1. FBM reduces to Brownian motion when H = 0.5.

A discrete-time definition of self-similarity may be given as follows. Let $X = (X_t : t = 0, 1, 2, \cdots)$ be a covariance stationary (also called wide-sense stationary) stochastic process—a process with a constant mean $\mu = E[X_t]$, finite variance $\sigma^2 = E[(X_t - \mu)^2]$, and autocorrelation function

$$R(k) = \frac{Cov(X_t, X_{t+k})}{Var(X_t)} = \frac{E[(X_t - \mu)(X_{t+k} - \mu)]}{\sigma^2}, \quad k = 0, 1, 2, \cdots \tag{10.4}$$

that depends only on k. A new aggregated time series $X^{(m)} = (X_k{}^{(m)} : k = 1, 2, 3, \ldots)$ for each m = 1, 2, 3, ... is obtained by averaging non-overlapping blocks of size m from the original series X. In other words,

$$X_k^{(m)} = \frac{(X_{km-m+1} + \ldots + X_{km})}{m}$$

For example,

$$X_k^{(3)} = \frac{X_{3k-2} + X_{3k-1} + X_{3k}}{3}$$

A process X is self-similar with parameter β (0 < β < 1) if

$$\mathrm{Var}\left[\mathrm{X}^{(m)}\right] = \frac{\mathrm{Var[X]}}{m^\beta} \quad \text{(variance)} \tag{10.5a}$$

$$R_{X(m)}(k) = R_X(k) \quad \text{(autocorrelation)} \tag{10.5b}$$

We also assume that X has autocorrelation function of the form

$$R(k) \sim L(t)k^{-\beta} \quad \text{as} \quad k \to \infty \tag{10.6}$$

where $0 < \beta < 1$, the symbol $\sim$ means "behaves asymptotically as," and L(t) is "slowly varying" at infinity, i.e.

$$\lim_{t \to \infty} \frac{L(tx)}{L(t)} = 1 \tag{10.7}$$

This self-similar process has self-similarity Hurst parameter

$$H = 1 - \beta/2 \tag{10.8}$$

There are two important characteristics of self-similar processes [6–10]. The first feature has to do with the their *long-range dependence* (LRD), i.e. their autocorrelation function decays hyperbolically (less than exponentially fast). Equation (10.5a) implies this. In spite of the serious effects of this characteristic on queueing behavior, it cannot be accounted for in Markovian traffic models. For short range dependent (SRD) processes, such as the traditional traffic models, their functions show a fast exponential decay. The two concepts of self-similarity and long-range dependence are often used interchangeably to mean the same thing.

The second feature of self-similar process is the *slowly decaying variance* (SDV). The variance of the sample mean decays more slowing than the reciprocal of the sample size:

$$\text{Var}\left[X^{(m)}\right] \approx a_1 m^{-\beta}, \quad m \to \infty \tag{10.9}$$

a_1 is a positive constant and $H = 1 - \beta/2$. This result indicates that the process has infinite variance. However, this result differs from traditional Markovian models where the variance is given by

$$\text{Var}\left[X^{(m)}\right] \approx a_1 m^{-1} \tag{10.10}$$

10.2 Pareto Distribution

Another issue related to self-similarity is that of heavy-tailed distribution. In fact, to produce self-similar behavior, the traffic model should employ heavy-tailed distribution with infinite variance. A distribution is heavy-tailed if [11]

$$\text{Prob}[X > x] = 1 - F(x) \approx \frac{1}{x^\alpha} \tag{10.11}$$

where $1 < \alpha < 2$. One of the distributions that are heavy-tailed is the Pareto distribution, which is defined as

$$\text{Prob}[X > x] = \left(\frac{\delta}{x}\right)^{\alpha} \tag{10.12}$$

where δ is a parameter which indicates the minimum value that the distribution can take, i.e. $x \geq \delta$ and α is the shape parameter ($1 \leq \alpha \leq 2$), which describes the intensity of self-similarity. α also determines the mean and variance of X. Thus, the cumulative distribution function is

$$F(x) = 1 - \left(\frac{\delta}{x}\right)^{\alpha} \tag{10.13a}$$

while the probability density function is

$$f(x) = \frac{\alpha}{\delta}\left(\frac{\delta}{x}\right)^{\alpha+1} \tag{10.13b}$$

The mean value of the Pareto distribution is

$$E(X) = \delta\frac{\alpha}{1-\alpha} \tag{10.14}$$

For our purposes, it is convenient to set $\delta = 1$.

It is common in simulating self-similar traffic to assume that the packet interarrival times are independent, identically distributed according to a Pareto distribution [12, 13]. The Pareto distribution is a distribution with memory, heavy tail, and strong burstiness. It can have finite mean and infinite variance depending on the value of one of its parameters. It has been shown that the ON/OFF source model with heavy-tailed distribution reproduces the self-similar traffic [14]. The lengths of the ON-periods are identically distributed and so are the lengths of the OFF-periods. Traffic obtained through infinite radix multiplexing of ON/OFF source traffic so that the ON interval or the OFF period follows a Pareto distribution is not as Fractional Gaussian Noise (FGN).

Example 10.1 Let there be a queue with time-slotted arrival process of packets. The load is 0.5 and there is a batch arriving according to Bernoulli process such that

$$\text{Prob}[\text{there is a batch in a time slot}] = 0.25$$

so that the mean number of arrivals in any batch is 2. Calculate the probability of having more than x arrivals in any time slot if the batch size is: (a) exponentially distributed, (b) Pareto-distributed.

Solution

(a) $\text{Prob}[\text{batch size} > x] = e^{-x/2}$

so that

$$\begin{aligned}\text{Prob}[> 10 \text{ arrivals in a time slot}] &= \text{Prob}[\text{batch size} > 10] \\ &\quad \times \text{Prob}[\text{there is a batch in a time slot}] \\ &= e^{-10/2} \times 0.25 = 0.001684\end{aligned}$$

(b) In this case, assuming δ = 1,

$$E[X] = 1\frac{\alpha}{\alpha - 1} = 2$$

or

$$\alpha = \frac{E[X]}{E[X] - 1} = 2$$

Thus,

$$\text{Prob}[\text{batch size} > \text{x}] = \left(\frac{1}{\text{x}}\right)^2$$

$$\begin{aligned}\text{Prob}[> 10 \text{ arrivals in a time slot}] &= \text{Prob}[\text{batch size} > 10] \\ &\quad \times \text{Prob}[\text{there is a batch in a time slot}] \\ &= \left(\frac{1}{10}\right)^2 \times 0.25 = 0.0025\end{aligned}$$

For the two distributions, the probability is of the same order of magnitude. This indicates that for a batch size of greater than 10 arrivals, there is not much difference between the two distributions. However, there would be significant difference is we try more than 100 arrivals. For exponential case,

$$\text{Prob}[> 100 \text{ arrivals in a time slot}] = e^{-100/2} \times 0.25 = 4.822 \times 10^{-23}$$

and for Pareto case

$$\text{Prob}[> 100 \text{ arrivals in a time slot}] = \left(\frac{1}{100}\right)^2 \times 0.25 = 2.5 \times 10^{-5}$$

10.3 Generating and Testing Self-Similar Traffic

A proper way of modeling network traffic is a prerequisite for an adequate design of networks. Several approaches have been developed for modeling self-similar traffic. These include the random midpoint displacement algorithm, on-off model, and wavelet transformation [15].

10.3.1 Random Midpoint Displacement Algorithm

This algorithm is used for generating Fractional Brownian Motion (FBM) with Hurst parameter $H \in (0.5,1)$ in a given time interval. If the trajectory of FBM Z(t) is to be computed in the interval [0,T], we start by setting Z(0) = 0 and Z(T) from a Gaussian distribution with mean 0 and variance T^{2H}. Next Z(T/2) is calculated as the average of Z(0) and Z(T) plus an offset δ_1, i.e.

$$Z(T/2) = \frac{1}{2}[Z(0) + Z(T)] + \delta_1 \tag{10.15}$$

where δ_1 is a Gaussian random variable with zero mean and a standard deviation given by T^{2H} times the initial scaling factor s_1, i.e.

$$\Delta_1 = T^{2H}.s_1 = \frac{T^{2H}}{2^H}\sqrt{1 - 2^{2H-2}} \tag{10.16}$$

The two intervals from 0 to T/2 and from T/2 to T are further subdivided and we reduce the scaling factor by $\frac{1}{2^H}$ and so on. At the nth stage, a random Gaussian variable δ_n is added to the midpoint of the stage n − 1 with a variance.

$$\Delta_n = \frac{T^{2H}}{(2^n)^H}\sqrt{1 - 2^{2H-2}} \tag{10.17}$$

Once a given point has been determined, its value remains unchanged in all later stages. As H goes to 1, Δ_n goes to 0 and Z(t) remains a collection of smooth line segment connecting the starting points.

10.3.2 On-Off Model

This traffic model is aggregated by multiple single ON/OFF traffic source. In other words, traffic is generated by a large number of independent ON/OFF sources such as workstations in a large computer network. An ON/OFF source is a burst traffic source which alternates active (ON) with silent (OFF) periods. During an active period (that is, a burst), data is generated at a fixed peak rate, while during silent periods no data is generated. Every individual ON/OFF source generates an ON/OFF process consisting of alternating ON- and OFF-periods. The lengths of the ON-periods are identically distributed and so are the lengths of OFF-periods. The ON/OFF source model with the “heavy-tailed” (Pareto-like) distribution reproduces the self-similar traffic. In other words, the superposition of many independent and identically distributed (i.i.d.) ON/OFF sources results in self-similar aggregate traffic.

Suppose there are N traffic sources, let the ON time of the ith traffic by $\tau^{(i)}$ and OFF time be $\theta^{(i)}$. The random variables $\tau^{(i)}$ and $\theta^{(i)}$ are i.i.d.; they satisfy

$$P(X > t) \sim at^{-\alpha}, \quad \text{with } t \to \infty, 1 < \alpha < 2 \tag{10.18}$$

where X is the length of the ON or OFF period. Since Pareto distribution is the simplest example of a heavy-tailed distribution, we may say that X follows Pareto distribution with finite mean and infinite variance.

There are several statistical methods that can be used for testing the time scale of self-similarity in traffic generation. These methods are used in the estimation of the Hurst parameter. They include R-S (Rescaled adjusted Range statistic) analysis and Variance-Time analysis.

Variance-Time Analysis

The method applies the following fact. The process X is said to be exactly *second-order self-similar* with Hurst parameter

$$H = 1 - \frac{\beta}{2} \quad (0 < \beta < 2) \tag{10.19}$$

if, for any m = 1, 2, 3, . . .,

$$Var\left(X^{(m)}\right) \propto m^{-\beta} \tag{10.20}$$

We take advantage of this equation. Taking the logarithm of both sides results in

$$\log\left[Var\left(X^{(m)}\right)\right] = c_1 - \beta\log(m) \tag{10.21}$$

for some constant c_1. Plotting log[$Var(X^{(m)})$] versus log(m) (i.e. a log-log graph) for many values of m of a self-similar process will result in a linear series of points with slope $-\beta$ or 2H − 2. This plot is known as a *variance-time plot*.

R-S Analysis

This is rescaled-adjusted range method. It obtains H based on overlapped data windows. Define a sequence $X_i (i = 1, 2, 3, \ldots, M)$. Let $\overline{X}_M$ and S(M) be the sample mean and the sample variance of the sequence respectively. We evaluate

$$W_0, W_m = \sum_{i=1}^{m} X_i - m\overline{X}(m), \quad m = 1, 2, 3, \ldots, M \tag{10.22}$$

The adjusted range is defined as

$$R(M) = Max(W_m) - Min(W_m), \quad 0 \leq m \leq M \tag{10.23}$$

The ratio R(M)/S(M) is called the rescaled adjusted range or R/S statistic. The log of R/S statistics (for several values of M) plotted against log(M) will have an asymptotic slope, which is the approximation of H.

10.4 Single Queue

Classical modeling techniques of queues assume Poisson arrival rates. However, several different types of input processes have been found to exhibit self-similar or fractal-like behavior. In this section, we consider the performance of a single server queue with interarrival times having a large variance [16, 17].

Let X be the random variable denoting the interarrival time of packets. X is assumed to have a Gamma distribution, i.e. the packet interarrival times are assumed to have a Gamma distribution.

$$f_X(t) = \frac{r\lambda(r\lambda t)^{r-1}}{\Gamma(r)} e^{-r\lambda t}, \quad \lambda, t > 0, 0 < r < 1 \tag{10.24}$$

Packet interarrival times which have a Gamma distribution with a specific range of parameter values give large values of variances. The service time is assumed to be exponentially distributed with parameter μ. The results of the G/M/1 queue can be readily used. Let p_n be the probability that k packets are in the queue at the arrival moment. Then

$$p_n = (1 - \sigma)\sigma^k \tag{10.25}$$

where σ is the unique root of

$$\sigma = F_X(\mu - \mu\sigma), \quad 0 < \sigma < 1 \tag{10.26}$$

$F_X(s)$ is the Laplace transform of $f_X(t)$.

$$F_X(s) = \int_0^\infty f_X(t) e^{-st} dt = \left(\frac{r\lambda}{s + r\lambda}\right)^r \tag{10.27}$$

If W_q is the random variable which denotes the waiting time of a packet in the queue, the mean and variance of W_q are respectively

$$E(W_q) = \frac{\sigma}{\mu(1-\sigma)} \tag{10.28}$$

$$\text{Var}(W_q) = \sigma_{W_q}^2 = \frac{1-(1-\sigma)^2}{\mu^2(1-\sigma)^2} \tag{10.29}$$

The complimentary queue waiting time distribution is

$$\text{Prob}(W_q > t) = \sigma e^{-\mu(1-\sigma)t}, \quad t \geq 0 \tag{10.30}$$

It remains to solve for σ. The value of σ is evaluated as follows. Using Eqs. (10.26) and (10.27),

$$\sigma = \left(\frac{r\rho}{1-\sigma+r\rho}\right)^r \tag{10.31}$$

where $\rho = \lambda/\mu$. If we define

$$z = \frac{r\rho}{1-\sigma+r\rho} \tag{10.32}$$

then

$$\sigma = 1 + r\rho - \frac{r\rho}{z} \tag{10.33}$$

From Eqs. (10.31) to (10.33), we obtain

$$z = \frac{r\rho}{(1+r\rho)} + \frac{z^{r+1}}{(1+r\rho)} \tag{10.34}$$

which can be evaluated using Lagrange series. Now we let

$$z = a + \xi\phi(z), \quad a = r\rho/(1+r\rho), \quad \xi = 1/(1+r\rho), \text{ and } \phi(z) = z^{r+1} \tag{10.35}$$

in the Lagrange series expansion, we get

$$z = \sum_{n=0}^{\infty} \frac{\xi^n \Gamma(nr+n+1)}{n!\Gamma(nr+2)} a^{nr+1} \tag{10.36}$$

This series can be summed by letting

$$z = \sum_{n=0}^{\infty} d_n \tag{10.37}$$

where

$$d_n = \frac{\xi^n}{n!} \frac{\Gamma(nr + n + 1)}{\Gamma(nr + 2)} a^{nr+1} \tag{10.38}$$

The values of d_n can be evaluated recursively as follows.

$$\begin{aligned} d_0 &= a \\ d_1 &= \xi a^r d_0 \\ d_2 &= \xi a^r (r + 1) d_1 \\ d_n &= b_n d_{n-1}, \quad n \geq 3 \end{aligned} \tag{10.39}$$

where

$$b_n = \xi a^r (r + 1) \prod_{k=1}^{n-2} \frac{(nr + k + 1)}{(nr - r + k + 1)}, \quad n \geq 3 \tag{10.40}$$

Only a finite number of terms in Eq. (10.37) is needed in practice. Once we calculate z using Eq. (10.38), we use Eq. (10.33) to obtain σ.

One should keep in mind that the application of self-similar traffic model does not mean that traditional queueing analysis is now irrelevant. It only means that under certain conditions, performance analysis critically depend on taking self-similarity into account.

10.5 Wireless Networks

Although self-similarity was originally found for Ethernet traffic [1, 2, 18], research has shown that the same holds for wireless networks [19]. This implies that simulating a wireless network with Poisson distributed input traffic will give wrong results.

A logistic function or logistic curve can be described by the following differential equation.

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{10.41}$$

where P is population size, K is capacity, and t is time. Setting x = P/K in Eq. (10.41) gives

$$\frac{dx}{dt} = rx(1 - x) \tag{10.42}$$

Logistic map is a discrete representation of Eq. (10.42) and is written as recurrence relation as follows:

$$x_{n+1} = rx_n(1 - x_n) \tag{10.43}$$

This equation has been used to obtain self-similar time sequence which could be used for traffic generation for wireless network systems [19]. Values of r in the range $3.50 < r < 3.88$ and $0 < x_0 < 0.5$ have been used.

10.6 Summary

1. Studies of both Ethernet traffic and variable bit rate (VBR) video have demonstrated that these traffics exhibit self-similarity. A self-similar phenomenon displays the same or similar statistical properties when viewed at different times scales.
2. Pareto distribution is a heavy-tailed distribution with infinite variance and is used in modeling self-similar traffic.
3. The most common method of generating self-similar traffic is to simulate several sources that generate constant traffic and then multiplex then with ON/OFF method using heavy-tailed distribution such as Pareto.
4. We analytically modeled the performance of a single server queue with almost self-similar input traffic and exponentially distributed service times.
5. Logistic map for self-similar traffic generation is used for wireless network.
6. OPNET can be used to simulate the network traffic's self-similarity [20].

Problems

10.1 (a) Explain the concept of self-similarity.
(b) What is a self-similar process?

10.2 Show that the Brownian motion process B(t) with parameter H = 1/2 is self-similar. Hint: Prove that B(t) satisfy conditions in Eqs. (10.1) to (10.3).

10.3 Show that the Eq. (10.14) is valid and that the variance of Pareto distribution is infinite.

10.4 If X is a random variable with a Pareto distribution with parameters α and δ, then show that the random variable Y = ln (X/δ) has an exponential distribution with parameter α.

10.5 Evaluate and plot σ in Eq. (10.24) for $0 < \rho < 0.2$ with r = 0.01.

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