Which Color Space Should Be Chosen for Robust Color Image Retrieval Based on Mixture Modeling

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Summary. As the amount of multimedia data captured and published in Internet constantly grows, it is essential to develop efficient tools for modeling the visual data similarity for browsing and searching in voluminous image databases. Among these methods are those based on compact image representation, such as mixture modeling of the color information conveyed by the images. These methods could be efficient and robust to possible distortions of color information caused by lossy coding. Moreover, they produce a compact image representation in form of a vector of model parameters. Thus, they are well suited for task of a color image retrieval in large, heterogenous databases. This paper focuses on the proper choice of the color space in which the modeling of lossy coded color image information, based on the mixture approximation of chromaticity histogram, is evaluated. Retrieval results obtained when RGB, $I1I2I3$, YUV , $CIE\ XYZ$, $CIE\ L^*a^*b^*$, HSx, $LSLM$ and TSL color spaces were employed are presented and discussed.

1 Introduction

As the amount of multimedia data captured and published in Internet constantly grows in latest years, it is essential to develop tools for managing this enormous amount of information, to be suited for everyday use to growing number of users [1]. This huge amount of visual information is so far the largest and the most heterogenous image database, thus there is a question which features meaningfully describe its content. Color is a useful characteristic of our surrounding world, giving clue for the recognition, indexing and retrieval of the images presenting the visual similarity. However, the image representation in various color spaces can possibly yield different retrieval results due to the fact that employed color spaces can present different characteristics and thus they are suitable for different image processing tasks [2].

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The presented paper is organized as follows. Firstly the color space set, which was used for retrieval efficiency evaluation, is discussed. Then the retrieval methodology is presented and along with the discussion on the experimental results. Finally, conclusions are presented.

2 Analyzed Color Spaces

The color is uniquely defined in specified color space. The RGB color space is considered as fundamental and commonly used space, which is a base for many others obtained by it linear or nonlinear transformations. The color spaces evaluated by linear transformation of RGB (e.g. YUV, YIQ) are commonly associated with hardware color displays. On contrary, the color spaces obtained via nonlinear transformation of the RGB (e.g. HSV or $L^*a^*b^*$) are considered as reflection the characteristic of the human visual systems. Let us note that different color spaces derived from RGB by either group of linear or nonlinear transformation can reveal various performance. Thus it is important to determine which of the color spaces is the most desirable for image retrieval task. The most common RGB color space enables to describe each color in terms of red, green and blue components. Other color spaces can be derived from this space using linear or nonlinear transformations. This first group of spaces are obtained by using the specified transformation matrix.

First of the analyzed color spaces is the XYZ color space. It was derived by the International Commission on Illumination (CIE) in 1931 as a result of set of experiments on the human perception, [4]. The second analyzed color space $I_1I_2I_3$ is the result of the decorrelation of the RGB components using the $K-L$ transform performed by Ohta 1980, [5]. The next color space is the YUV color space used by color video standards. It consists of luminance (Y) component and chrominance components (U and V).

The $LSLM$ is a color space based on the opposite responses of the cones, i.e. black-white, red-green and yellow-blue. The following transformation matrix defines the $LMLS$ color space, [6]:

$$\begin{bmatrix} L \\ S \\ LM \end{bmatrix} = \begin{bmatrix} 0.209 & 0.715 & 0.76 \\ 0.209 & 0.715 & -0.924 \\ 3.148 & -2.799 & -0.349 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{1}$$

The nonlinear transformation of RGB color spaces are, among the others, HSV, TSL and CIE $L^*a^*b^*$. The TSL (Tint, Saturation, and Luminance) color space is widely used in face detection research field,[7, 8]. The transformation is based on the formulae:

$$T = \begin{cases} \frac{1}{2\pi} \arctan \frac{r'}{g'} + \frac{1}{4}, & \text{if } g' > 0 \\ \frac{1}{2\pi} \arctan \frac{r'}{g'} + \frac{3}{4}, & \text{if } g' < 0 \\ 0, & \text{if } g' = 0 \end{cases} \tag{2}$$

$$S = \sqrt{\frac{9}{5}\left(r'^2 + g'^2\right)}, \qquad L = 0.299R + 0.587G + 0.114B \tag{3}$$

CIE $L^*a^*b^*$ color space is a color-opponent space with dimension L for lightness and a and b for the color-opponent dimensions, [9]. The HSV (Hue, Saturation, Value) color space is related with the phenomenon of the human eye vision. This model assumes that all colors can be extracted from the white color, when some parts of the spectrum are absorbed by the illuminated object and other parts can be reflected.

3 Image Retrieval Scheme

The parametric color image description is used in many image analysis solutions. Although several proposed image retrieval techniques utilize the *Gaussian Mixture Model* (*GMM*) [10, 11] as color distribution descriptor [12, 13, 14], the aspect of the distortions caused by the lossy compression was not taken into account. These methods simply index all images in the database by fitting GMM to the data, according to some predefined rules. The *mixture* based retrieval scheme applied in this work is robust to distortions introduced by lossy compression and noise, [15, 17, 18, 19, 16], thus it is important to test whether the use of various color spaces in information modelling process will alternate the retrieval results,influencing the retrieval accuracy.

The first step in applying the applied methodology is to construct the histogram $H(x, y)$ in the chosen chromaticity space defined as $H(x, y) = N^{-1}\sharp\{r_{i,j} = x, g_{i,j} = y\}$, where $H(x, y)$ denotes a specified bin of a two-dimensional histogram with first component equal to x and second component equal to y, the symbol $\sharp$ denotes the number of samples in a bin and N is the number of color image pixels. As the pairs of the components representing analyzed color spaces the following pairs were chosen: $r - g$ (RGB), $I2I3$ (I1I2I3), $U - V$ (YUV), $T - S$ (TSL), $a - b$ ($L^*a^*b^*$), $H - S$ (HSV), and $S - LM$ (LSLM.)

The next stage of the presented technique is the modeling of the color histogram using the Gaussian Mixture Models (GMM) and utilizing the Expectation-Maximization (EM) algorithm for the model parameters estimation as described in details in [15, 16]. Let us assume the following probabilistic model: $p(x|\Theta) = \sum_{m=1}^{M} \alpha_m p_m(x|\theta_m)$, which is composed of M components and its parameters are defined as: $\Theta = (\alpha_1, \ldots \alpha_M, \theta_1, \ldots, \theta_M)$, with $\sum_{m=1}^{M} \alpha_m = 1$. Moreover, each p_m is a function of the probability density function which is parameterized by θ_m. Thus, the analyzed model consists of M components with M weighting coefficients α_m.

Finally after derivations shown in [11] the model parameters are defined as:

$$\alpha_m^{k+1} = N^{-1} \sum_{i=1}^{N} p(m|x_i, \Theta^k), \quad \mu_m^{k+1} = \frac{\sum_{i=1}^{N} x_i \cdot p(m|x_i, \Theta^k)}{\sum_{i=1}^{N} p(m|x_i, \Theta^k)}, \tag{4}$$

$$v_m^{k+1} = \frac{\sum_{i=1}^{N} p(m|x_i, \Theta^k)(x_i - \mu_m^{k+1})(x_i - \mu_m^{k+1})^T}{\sum_{i=1}^{N} p(m|x_i, \Theta^k)} \quad , \tag{5}$$

where μ and v denote the mean and variance, m is the index of the model component and k is the iteration number. The E (Expectation) and M (Maximization) steps are performed simultaneously, according to (4) and (5) and in each iteration, as the input data we use parameters obtained in the previous one.

The main idea of the application of the *GMM* technique lies in the highly desirable properties of this approach. The inherent feature of the GMM enables to approximate the distorted color histogram of the color image subjected to lossy compression, which is obtained through limited model complexity (7 components) and number of iterations (75) of E-M algorithm, as shown in [15, 17, 18, 19, 16]. Thus, this approach does not reflect exactly the corrupted (by e.g. lossy coding) data, but rather approximates it toward the chromaticity histogram of the original image. The lossy compression significantly corrupts the color distribution of an image and a lack of the application of any refinement techniques may lead to the high rate of false negative results, as images stored in lossy formats are considered as dissimilar on the basis of their corrupted color palette. In details, in the used method the *GMM* is used as a descriptor of the image color distribution. However, the very important aspect is the fact that the used method, based on weighted two-dimensional Gaussians is robust to distortions introduced by lossy compression techniques and therefore it can be used for the retrieval of images contained in the Web based databases, which very often store images in lossy compression formats, like JPEG and noise can be present. Due to this highly desirable properties of the GMM retrieval method it is important to support it efficiency by the proper choice of the color space, which describes accurately the color composition of the analyzed image without influencing the similarity of the GMM models for the visually related images. Having image indexed using the GMM signatures, it is important to choose the accurate similarity measure to compare the indexes associated with each of the images with that of the given query. In general, the Minkowski metrics can be used to compare point to point the color histograms. However, these measures are very susceptible to bin shifts, thus even highly similar images can be considered as dissimilar when corresponding bins of their chromaticity histograms are shifted and therefore, this group of similarity measures is not taken into account in further analysis. However, it is more suitable to generalize this concept toward distribution to distribution similarity. For that purpose the Kullback-Leibler based similarity measures as (D_G) [20] and that taking into account the correlation of components within each of the both mixture models, representing the analyzed images, as (D_V) [21], are employed.

Let us note that the first approach are designed to measure image similarity for image retrieval tasks whereas the second approach was initially designed for acoustic models used for speech recognition. The following formulae describe the distances as follows: $D_{KL}(G_i, G_j) = (\mu_i - \mu_j)^T \Sigma^{-1}(\mu_i - \mu_j) + TR(\Sigma_i \Sigma_j^{-1} + \Sigma_i^{-1}\Sigma_j)$ where G_i and G_j denotes normal distributions with mean values μ_i and μ_j, and covariance matrices Σ_i and Σ_j respectively. $D_G(G^a, G^b) = \sum_{i=1}^{k} \alpha_i^a (D_{KL}(G_i^a, G_{\pi(i)}^b) + \log \frac{\alpha_i^a}{\alpha_{\pi(i)}^b})$ where $\pi(i) = \arg\min_j (D_{KL}(G_i^a, G_j^b) - \log \alpha_j^b)$ is the matching function between components i and j of the Gaussian Mixture Models G^a and G^b and α_i^b denotes the weighting parameter α for i^{th} component of model G^a. The second similarity measure is defined as: $D_V(G^a, G^b) = \sum_{i=1}^{k} \alpha_i^a \frac{\sum_{i'=1}^{k} \alpha_{i'}^a e^{-D_{KL}(G_i^a, G_{i'}^b)}}{\sum_{j=1}^{k} \alpha_j^b e^{-D_{KL}(G_i^a, G_j^b)}}$.

The second group of analyzed similarity measures are the adaptive approaches such as *Earth Movers Distance* [22] (*EMD*), which is based on the assumption that one of the histograms reflects hills and the second represents holes in the ground of a histogram. The measured distance is defined as a minimum amount of work needed to transform one histogram into the other using a soil of the first histogram. As a measure of the distance between two distributions in EMD framework the Kullback-Leibler divergence was used, which is a measure of the dissimilarity between two probability distributions.

4 Experimental Results

In order to choose the most accurate color space for purpose of color image retrieval, the set of eight color spaces (*RGB*, *CIE XYZ*, *YUV*, *I1I2I3*, *LSLM*, *CIE* $L^*a^*b^*$, *HSx* and *TSL*) were analyzed. Firstly, the color images of database of Wang[3] (1000 color images categorized into 10 thematically consistent categories), were compressed to 25% of their file size and rescaled to 10% of their size, using IrfanView software (`www.irfanview.com/`). These images produced distorted histograms in comparison to the histograms of the original, uncompressed images. Next step was to model the color chromaticity histograms using the *GMM* methodology obtaining image signatures, according to formulae presented in previous Section. These image signatures were compared by the D_V ,D_G and *EMD* similarity measures due to the reasoning given in previous Section. However, to generalize retrieval observations, the *Precision* and *Recall* measures are employed. In more details, *Precision* is the fraction of retrieved images that are relevant, while Recall is the fraction of relevant instances that are retrieved. Fig. 1 illustrates the *Precision – Recall* plots evaluated for the entire analyzed Wang database, i.e. each of the database image was used as a query. The evaluated *Precision* values for each of 1000 queries were averaged (for each corresponding *Recall* value) producing the plots shown on Fig. 1 The criterion of the image similarity was the membership of the image to one of the 10 categories of this database. This Figure illustrates the comparison of the retrieval efficiency for the set of color spaces obtained by the linear (left) transformations of the RGB color space for *EMD* similarity meaures. The best performance

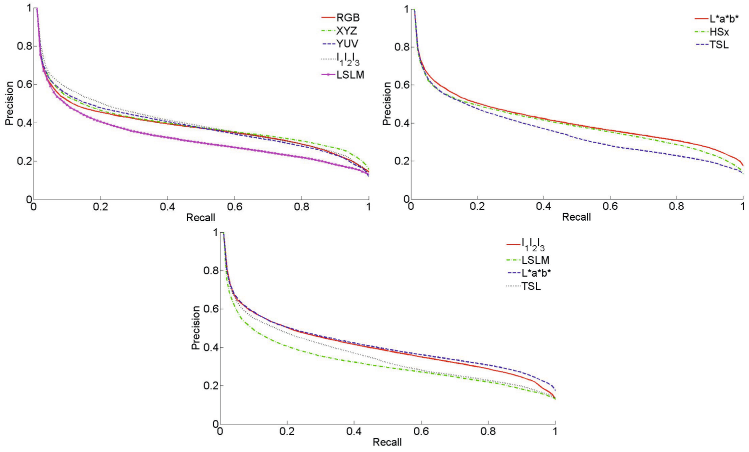

Fig. 1. The $Precision - Recall$ plots of the retrieval results evaluated using GMM methodology for set of color spaces obtained by linear (left upper row) and nonlinear (right upper row) transformations of the RGB color space. The comparison between best performing color space retrieval results is shown in bottom row. The comparisons of retrieval efficiency was evaluated using the EMD similarity measure. Plots were evaluated over all the images of the database of Wang [3].

presents the $I1I2I3$ color space, which is the result of the decorrelation of the RGB components, while the worst efficiency is associated with $LSLM$ color space. Then the nonlinear transformations of RGB are analyzed in (center), showing that the $CIE\ L^*a^*b^*$ color space provides the highest accuracy of retrieval results. The comparison of the retrieval performance of the color spaces related to the best and the worst results in terms of effectiveness shows that $CIE\ L^*a^*b^*$ (nonlinear) and $I1I2I3$ (linear) presents the comparable efficiency, however, the latter needs much simpler computation during transformation from RGB. As the D_G and D_V similarity measures seems to be not well suited for GMM based image retrieval scheme (as shown in Table 1), their efficiency was not shown on $Precision - Recall$ plots. The comparison between best performing linear and nonlinear color spaces is depicted on 1(bottom row) Let us note, that $Precision$ and $Recall$ tend to ignore the ranking of retrieved images, e.g. $Precision$ of 0.5 indicates that half of the retrieved images is relevant to the given query but without any further information if relevant images are the first half of the retrieved set or are the second half. In order to counteract this, it is advised to measure $Precision$ and $Recall$ at the specified points e.g. at the answer set of 1, 5, 10, and 25 images. These measurements are assumed to give the better idea of the behavior of the analyzed retrieval scheme for applied color spaces. This results show that D_G and D_V are not well suited color spaces when GMM based retrieval is considered, presenting much lower retrieval efficiency than retrieval based on EMD similarity measure. Thus, D_G and D_V similarity measures should not be used for mixture based retrieval techniques.

The used technique was also tested in order to evaluate it robustness to noise for various color spaces. Fig. 2 illustrates the $Precision - Recall$ plots (EMD similarity measure was used) for the set of 150 color images divided into 14 categories. Each category consist of original image, from the $LIVE$ database - without added noise (http://live.ece.utexas.edu/research/quality/) and its 9 versions. Original images were subjected to 9 noise distortions: "salt & pepper" where noise density is 2%, 10% and 20% of total image pixels affected; poisson noise; gaussian white noise of variance: 0.01, 0.03, 0.05, multiplicative noise of image values governed by I = I + I n where n is uniformly distributed random noise with (mean,variance): (0,0.04) and (0,0.2). The retrieval experiment was evaluated using the same scheme as for database of Wang. It can be noticed that the $CIE\ L^*a^*b^b$ (nonlinear), RGB and $I1I2I3$ (linear) color spaces offer the best retrieval efficiency. One must be aware that the retrieval efficiency is not only related with the retrieval method, but also with the content of the analyzed database and the used ground truth. In the case of the Wang database some of the categories share not only the semantic relation between images within the category but also the arrangement of the colors present on the images (e.g. "Beaches", "Horses"), but some do not as "Buses" and "Flowers", what influences the retrieval efficiency of applied technique. It is important to underline that this work does not test the effectiveness of the GMM based retrieval scheme, as it effieciency was elaborated and tested in other Author's work [15, 17, 18, 19, 16] as well as the comparison with other widely used retrieval schemes as $MPEG-7$ descriptors, correlograms and others. The previous work clearly show the usefullness of the GMM based retrieval technique, especially when lossy compressed images are analyzed. However there is an open question which color space is best suited for this kind of retrieval scheme. Presented paper addresses this problem.

The overall behavior of the proposed retrieval method can be specified by *average recall* ($\hat{R}$) and *average precision* ($\hat{P}$). The *average precision* is defined as a sum of $\frac{1}{rank(O_i)}$ divided by number of queries q:$\hat{P} = \frac{1}{q}\sum_{i=1}^{q}\frac{1}{rank(O_i)}$. In this approach only the *average precision* $\hat{P}$ is analyzed.

Table 1 summarizes the average precision for the entire database of Wang at the points of 1, 5, 10 and 25 retrieved images for D_G, D_V and EMD similarity measures. It can be observed that $L^*a^*b^b$ and $I1I2I3$ color spaces provide comparable results for small set of retrieved candidate images. The average precision values at the points related to 1, 5, 10 and 25 retrieved images should be chosen to examine the retrieval efficiency as user is usually more interested in relevance of the highly ranked candidate images than the overall success rate of the retrieval system.

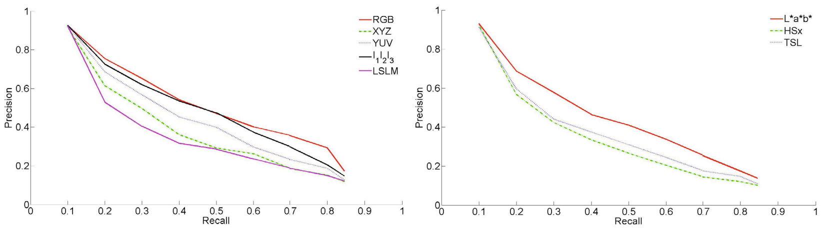

Fig. 2. The $Precision - Recall$ plots of the retrieval results evaluated using GMM methodology for set of color spaces obtained by linear (left) and nonlinear (right) transformations of the RGB color space, evaluated over all the images of the $LIVE$ database and their noisy versions.

Table 1. The Average precision ($\hat{P}$) values of the retrieval results evaluated using GMM methodology for set of color spaces. Average precision values at specified points related to 1, 5, 10 and 25 retrieved images were obtained over all the images of the database of Wang.

Color space	$\hat{P}_1$	$\hat{P}_5$	$\hat{P}_{10}$	$\hat{P}_{25}$	$\hat{P}_1$	$\hat{P}_5$	$\hat{P}_{10}$	$\hat{P}_{25}$	$\hat{P}_1$	$\hat{P}_5$	$\hat{P}_{10}$	$\hat{P}_{25}$
	EMD				D_G				D_V			
RGB	1	0.6052	0.5176	0.4368	0.3384	0.1659	0.1522	0.1174	0.1724	0.1077	0.1039	0.1250
XYZ	1	0.6225	0.5365	0.4429	0.3630	0.1963	0.1765	0.1757	0.1918	0.1207	0.1232	0.1371
YUV	1	0.6240	0.5505	0.4587	0.3929	0.2021	0.1826	0.1786	0.1923	0.1130	0.1079	0.1101
I1I2I3	1	0.6491	0.5833	0.4750	0.3440	0.1944	0.1798	0.1784	0.1526	0.0989	0.1012	0.1122
LSLM	1	0.5908	0.4925	0.3781	0.3984	0.1923	0.1674	0.1745	0.1584	0.1039	0.1063	0.1179
$L^*a^*b^*$	1	0.6491	0.5833	0.4750	0.3958	0.1848	0.1757	0.1528	0.1485	0.0956	0.0995	0.1086
HSx	1	0.6283	0.5559	0.4722	0.3766	0.1360	0.1204	0.1198	0.1845	0.1123	0.1143	0.1239
TSL	1	0.6374	0.5524	0.4462	0.3696	0.13675	0.1285	0.1193	0.1066	0.1254	0.1252	0.1323

5 Conclusions

In this work the problem of the choice of the most accurate color space for the GMM based retrieval scheme was analyzed. The conducted experiments shown that this decision plays a crucial role in the efficiency of the retrieval system. Thus, the best performance of the GMM based scheme is associated with CIE $L^*a^*b^*$ and $I1I2I3$ color spaces using the EMD similarity measure. As CIE $L^*a^*b^*$ slightly outperforms the $I1I2I3$ it should be taken into account that when image color information is given using RGB values, the linear transformation to $I1I2I3$ is less complicated than for CIE $L^*a^*b^*$ Due to the fact that user is, in general, interested in relatively small set of top ranked images, it is important to examine the system efficiency (in terms of $Precision$ and $Recall$) at points of e.g. 1, 5 and 10 retrieved images. This comparison (see Table 1) also indicates that CIE $L^*a^*b^*$ and $I1I2I3$ color spaces are the most accurate choice.

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