

## Chapter 6

# Summary and Conclusion

**Abstract** Determining when to apply what model is a major challenge in statistical forecasting of a chaotic signal. The chaotic behavior of signals, like that of Caspian Sea level time series, renders the confidence band estimation and forecast updating components of forecasting quite significant for the forecast performance. In this chapter, a brief summary and conclusions are provided for the monograph “Long-Range Dependence and Sea Level Forecasting”.

**Keywords** Long-range dependence • Sea level forecasting • Chaotic signals • ARFIMA models

Determining when to apply what model is a major challenge in statistical forecasting of a chaotic signal. The chaotic behavior of signals, like that of Caspian Sea level time series, renders the confidence band estimation and forecast updating components of forecasting quite significant for the forecast performance. The long-range dependence concept, and the methodologies for the estimation of long-range dependence index (Hurst Number) were presented in [Chap. 2](#). The forecasting methodology for ARFIMA models, the uncertainty estimation of forecasts and the updating as new data become available were provided in [Chap. 3](#). It was shown that the forecast confidence interval size depends on the probability distribution of the residuals, forecast lead time, the difference parameter  $d$ , and the autoregressive and the moving average coefficients for ARFIMA models.

In [Chap. 4](#), the forecasting performance of the ARMA, ARIMA, ARFIMA and TL-ARFIMA models were investigated for the annually averaged Caspian Sea level data, which are available since 1837. The forecast confidence bands and the forecast updating performance of ARFIMA models were shown to be superior compared to those of ARMA or ARIMA models. The updating component of the long memory model makes the forecasting model more reliable as shown in the Caspian Sea level example. Considering the level of uncertainty in AOGCM forecasts, the pure statistical forecasts such as for the Caspian Sea level case reported

here may give valuable insights about the future sea levels without utilizing the computationally intense AOGCM approach.

While in hydrology various authors have considered long range dependence either by means of stationary long memory models [for example, the fractional Gaussian noise model of Mandelbrot and Van Ness (1968) and Mandelbrot and Wallis (1968)], or by nonstationary time trends (such as in Klemes (1974)), the signal of the Caspian Sea level time series seems to contain both a long term secular trend as well as long range dependent behavior. The example of the Caspian Sea level time series has shown that both the long range dependence and some secular long term trend may exist together in geophysical phenomena, and statistical modeling of a time series may be performed by the combination of a trend component and a long memory component.

Instead of the infinitely long differencing lengths, finite differencing lengths for the ARFIMA models were utilized due to the finite duration of the available sea level record. Sample ACFs of the residuals were compared for various differencing lengths, and the one that minimizes the correlation structure in the sample ACFs was selected. Confidence intervals and the forecast updating methodology, provided for ARIMA models in Box and Jenkins (1976), were modified for the ARFIMA models. The confidence intervals of the forecasts were estimated utilizing the probability densities of the residuals without assuming a known distribution. In the literature, normal distribution of the residuals is usually assumed for the estimation of the confidence intervals.

In order to check the statistical model reliability, portmanteau lack of fit and cumulative periodogram tests as model diagnostic tools (Box and Jenkins 1976) were introduced and utilized in the Caspian Sea level example case.

Sea level change has been also studied by AOGCMs (Gregory et al. 2001; Meehl et al. 2007a). There is substantial variability and uncertainty in the spatial distribution of sea level change among all GCMs (Meehl et al. 2007a). Climate models provide credible quantitative estimates of future climate change, particularly at continental scales and above (Randall et al. 2007). However, due to their coarse spatial grid resolution, their description of the spatial variation of the sea level change at regional and smaller spatial scales is too coarse.

In the case study of Peninsular Malaysia and Sabah-Sarawak coastlines on the assessment of sea level change along the coastlines of Peninsular Malaysia and Sabah-Sarawak, as reported in Chap. 5, the spatial variation of the sea level change was estimated by assimilating the global mean sea level projections from the AOGCM simulations to the satellite altimeter observations along the Malaysian coastlines (Ercan et al. 2013). The determination of the variation of the sea level change with respect to the spatial location along the Peninsular Malaysia and Sabah-Sarawak coastlines was based on the linear trend analyses of the observed satellite altimetry data. Using the observed monthly satellite altimeter data and using monthly twentieth century global mean sea level projections of various AOGCM models, a regression equation at each satellite altimeter location for each AOGCM was written, and the corresponding regression coefficients were estimated. The highest sea level rise occurs at the north-east and north-west regions of Peninsular Malaysia and at north and east sectors of Sabah.

In the future, sea inundation studies with fine resolution topographic maps can be performed based on the sea level projections of Caspian Sea and of the coastlines of Malaysia with priority given to urban, industrial, agricultural, touristic, and historical areas. Based on these projections, the impact of the sea level change in these regions can then be evaluated.

Statistical modeling and forecasting approaches may be investigated for other geophysical time series which may also exhibit a trend and a long memory component, as was found in the historical mean sea level data of the Caspian Sea levels.

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