## **Chapter 1 Introduction**

**Abstract** The stochastic approaches have been valuable in hydrological, geophysical and climatological research for representing a wide range of time series variability, uncertainty estimation, and generating future alternatives. Long-range dependence characteristics of the geophysical time series have drawn attention of scientists since Hurst phenomenon was introduced. In this study, in an effort to forecast sea levels, various statistical forecasting strategies will be discussed: ARMA (Mixed Autoregressive-Moving Average process), ARIMA (Autoregressive Integrated Moving Average process), ARFIMA (Autoregressive Fractionally Integrated Moving Average process), and trend line combined with ARFIMA (TL-ARFIMA) combination models that shall be applied to the Caspian sea level record, while applying regression to assimilate the GCM projections of sea level change to the region of Peninsular Malaysia and Malaysia's Sabah-Sarawak northern region of Borneo Island.

**Keywords** Long-range dependence • Long memory • ARFIMA models • Time series analysis • Sea level change • Regression techniques

Statistical and stochastic approaches are utilized extensively in applications of geophysical and climatological research to characterize and quantify spatial and temporal variability of the parameters of interest. These approaches include regression techniques (Davis [1976;](#page-3-0) Wright [1984](#page-4-0)), analysis of variance (Box et al. [1978;](#page-3-1) Cochran and Cox [1957\)](#page-3-2), dimensionality reduction (Tenenbaum et al. [2000;](#page-4-1) Gamez et al. [2004\)](#page-3-3), principal component analysis or Empirical Orthogonal Function analysis (Preisendorfer [1988;](#page-4-2) Von Storch and Zwiers, [1999;](#page-4-3) Jollife, [2002\)](#page-3-4), Principal Oscillation Pattern analysis (Hasselman [1988](#page-3-5); Von Storch et al. [1995;](#page-4-4) Von Storch and Zwiers, [1999](#page-4-3)), Canonical Correlation Analysis (Hotelling [1936\)](#page-3-6), fractional Gaussian noise (Mandelbrot and Van Ness [1968](#page-3-7); Mandelbrot and Wallis [1969;](#page-3-8) Mandelbrot [1971;](#page-3-9) Koutsoyiannis [2002\)](#page-3-10) and autoregressive fractionally integrated moving average (ARFIMA) models (Granger and Joyeux [1980;](#page-3-11) Hosking [1981;](#page-3-12) and Geweke and Porter-Hudak [1983](#page-3-13)). The ARFIMA models are generalization of autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. Comprehensive information on ARMA and ARIMA models is provided in Box and Jenkins ([1976\)](#page-3-14). The stochastic approaches have been valuable in practice for representing a wide range of hyroclimatic time series variability, uncertainty estimation, and generating future alternatives (Salas et al. [1980](#page-4-5); Beran [1994](#page-3-15); Srikanthan and McMahon [2001;](#page-4-6) Sveinsson et al. [2003;](#page-4-7) and Koutsoyiannis [2011\)](#page-3-16).

Long-range dependence or long memory characteristics of the geophysical time series have drawn attention of scientists since 1960s (Mandelbrot and Van Ness [1968;](#page-3-7) Mandelbrot and Wallis [1968,](#page-3-17) [1969\)](#page-3-8) when the so called Hurst phenomenon (Hurst [1951](#page-3-18)) was discussed and explained. In addition to hydrology, long memory models have been used in several fields including astronomy, economics, and mathematics (Beran [1994\)](#page-3-15). The explanation of the presence of long memory in hydrologic processes was attempted by physical mechanisms such as climate nonstationarities (Potter [1976](#page-4-8)), storage mechanisms (Klemes [1974](#page-3-19), [1978](#page-3-20)), groundwater upwelling (Shun and Duffy [1999](#page-4-9)), and spatial aggregation (Mudelsee [2007\)](#page-4-10). Long memory, that may be present in sea level records, may be due to the combination of all of the above mechanisms as the oceans are part of the earth's water cycle which is influenced by each of these mechanisms.

Sea level change has been studied by Atmosphere–Ocean coupled Global Climate Models (also called General Circulation Models) (AOGCMs) (Gregory et al. [2001](#page-3-21); Meehl et al. [2007a\)](#page-4-11) or by analyses of the historical observations of the sea level by tidal gauges (Church et al. [2004;](#page-3-22) Church and White [2006;](#page-3-23) Bindoff et al. [2007\)](#page-3-24) or by satellite altimetry (Cazenave and Nerem [2004;](#page-3-25) Bindoff et al. [2007\)](#page-3-24). Based on the analyses of the tidal gauge records, Church et al. [\(2004](#page-3-22)) determined a global mean sea level rise of  $1.8 \pm 0.3$  mm/yr during the 1950–2000 period, and Church and White ([2006\)](#page-3-23) determined a mean sea level rise of  $1.7 \pm 0.3$  mm/yr for the twentieth century. Considering these results and allowing for the upward trend in recent years by satellite altimetry observations, Bindoff et al. [\(2007\)](#page-3-24) assessed the global mean sea level rise rate to be  $1.8 \pm 0.5$  mm/yr for the 1961–2003 period, and  $1.7 \pm 0.5$  mm/yr for the twentieth century.

While various authors have considered long range dependence either by means of stationary long memory models (for example, the fractional Gaussian noise model of Mandelbrot and Van Ness [1968](#page-3-7) and Mandelbrot and Wallis [1968](#page-3-17)), or by nonstationary time trends (such as in Klemes [1974](#page-3-19)), the signal of Caspian Sea level time series seems to contain both a long term secular trend as well as long range dependent behavior. As shall be shown in the following chapters, even after removing the long term trend from Caspian Sea level time series, the residual time series still demonstrate long range dependent behavior. The example of Caspian Sea level time series demonstrates that both the long range dependence and some secular long term trend may exist together in geophysical phenomena.

In this study, in an effort to forecast sea levels, various statistical forecasting strategies will be discussed: ARMA (Mixed Autoregressive-Moving Average process), ARIMA (Autoregressive Integrated Moving Average process), ARFIMA (Autoregressive Fractionally Integrated Moving Average process), and trend line

combined with ARFIMA (TL-ARFIMA) combination models that shall be applied to the Caspian sea level record, while applying regression to assimilate the GCM projections of sea level change to a particular region. The standard ARFIMA models will be applied to the annually averaged sea level observations. Finite differencing lengths for the ARFIMA models will be utilized due to the finite duration of the available observed sea level record. Sample ACFs of the residuals will be compared for various differencing lengths, and the one that minimizes the sample ACFs will be selected. Confidence intervals and the forecast updating methodology, provided for ARIMA models in Box and Jenkins [\(1976\)](#page-3-14), will be modified for the ARFIMA models. The confidence intervals of the forecasts will be estimated utilizing the probability densities of the residuals without assuming a known distribution. ARFIMA models will also be utilized to the residuals of the linear trends; and the trend line and ARFIMA combination models will be referred to as TL-ARFIMA models. The forecasting performance of ARMA, ARIMA, ARFIMA and TL-ARFIMA models will be investigated by comparing against the observed Caspian Sea level.

Meanwhile, for the region of Peninsular Malaysia and Malaysia's Sabah-Sarawak northern region of Borneo Island, long sea level records do not exist. In such case the Global Climate Model (GCM) projections for the twenty-first century can be downscaled to the Malaysia region by means of regression techniques, utilizing the short records of satellite altimeters in this region against the GCM projections during a mutual observation period. There is substantial variability and uncertainty in the spatial distribution of sea level change among all GCMs (Meehl et al. [2007a](#page-4-11)). Climate models provide credible quantitative estimates of future climate change, particularly at continental scales and above (Randall et al. [2007\)](#page-4-12). However, due to their coarse spatial grid resolution, their description of the spatial variation of the sea level change at regional and smaller spatial scales is too coarse. Therefore, a prudent projection could use the AOGCM (Coupled atmospheric-oceanic GCMs) projections for the global average sea level change, but then distribute these projections in space over regional scales according to the observed patterns based on observed sea level data by means of regression. This approach will be demonstrated for a case study along the Peninsular Malaysia and Sabah-Sarawak coastlines (Ercan et al. [2013](#page-3-26)).

The rest of this monograph is organized as follows: Long-range dependence concept is explained, methodologies developed in the literature for the estimation of long-range dependence index (Hurst Number) are provided and ARFIMA models are introduced in [Chap.](http://dx.doi.org/10.1007/978-3-319-01505-7_2) 2. Then, the forecasting methodology, the uncertainty estimation in the forecasts and the updating, as new data become available, are provided in [Chap.](http://dx.doi.org/10.1007/978-3-319-01505-7_3) 3. Afterwards, the results of the ARMA, ARIMA, ARFIMA, and TL-ARFIMA forecasting applications to the Caspian Sea level are discussed in [Chap.](http://dx.doi.org/10.1007/978-3-319-01505-7_4) 4. In the following chapter, the global mean sea level projections from the AOGCM simulations are assimilated to the satellite altimeter observations along Peninsular Malaysia and Sabah-Sarawak coastlines (Ercan et al. [2013\)](#page-3-26). In this chapter, statistical approaches are combined with AOGCM simulation results. Conclusions drawn from each case study are provided at the end of each case study.

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