A Comparative Study of $PI^{\lambda}D^{\mu}$ Controller Approximations Exemplified by Active Magnetic Levitation System

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Abstract. The $PI^{\lambda}D^{\mu}$ DFOC was examined when applied to the Active Magnetic Levitation System. This research is based on the Prof. Ivo Petras Toolbox for fractional controller synhesis. The point of interest is the PID configuration applied at the simulation and experimental stages. The search for the optimal controller form is dependent on the quality measure in the transition phase when the external excitation load is activated. The digital control experiment was carried out in the MATLAB/Simulink using a USB I/O board. The controller realisations are compared and discussed.

Keywords: fractional order controller, real-time control, active magnetic levitation.

1 Introduction

The point of interest of this research is the design and experimental investigation of the $PI^{\lambda}D^{\mu}$ Discrete Fractional Order Controller (DFOC) and its implementation for Active Magnetic Levitation control. Some historical aspects along with an introduction to fractional order calculus can be found in [17]. The fractional order theory and fractional order PID controller is comprehensively studied and discussed in [11], [1], [14], [2].

2 Active Magnetic Levitation

2.1 Test-Rig

To carry out the experimental part of the DFOC research the MLS2EMi laboratory test-rig was used. The conventional MLS system was expanded with a bottom electromagnet. Moreover, a current driver was used instead of a PWM driver. Additionally, to enable the digital control in the soft-real time regime the RT-DAC USB2 I/O board was connected to the MATLAB/Simulink/RT-CON. The real-time mode is based on the multimedia timer and uses USB communication, and therefore the control task is executed with a software specific jitter. The AMLS was characterised by structural instability and sensitivity to real-time performance.

W. Mitkowski et al. (Eds.): *Theory & Appl. of Non-integer Order Syst.*, LNEE 257, pp. 231–241.
DOI: 10.1007/978-3-319-00933-9_21

2.2 Mathematical Model

A custom investigation stage was set up to identify a set of ML parameters. To determine the ball position, sensor characteristics was identified with reference to a fixed location vis-a-vis the electromagnet surface. The coil current characteristics was identified to determine scaling and saturation factors. Finally, the electromagnet constant was identified by means of a stabilisation experiment. Assuming that the coil current change is less than 0.5A, and the sampling period is relatively low with respect to this time constant, the electro-magnetic part of the MLS model can be disregarded. The AML operates as a current driven system (1). Otherwise, the model must be expanded with coil inductance modelling [8], [3].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -K_{em} i^2 m^{-1} x_1^{-2} + g + F_{Ext} m^{-1} \end{aligned} \tag{1}$$

where: x_1 [m] and x_2 [m/s] denote the sphere distance from the electromagnet and the sphere velocity, m is the sphere mass equal to 0.03776 [kg], g is the gravity equal to 9.81 m/s^2, K_{em} is the construction constant that characterises the electromagnet equal to $5.9490 \cdot 10^{-5}$ [$\mathrm{Nm}^2/\mathrm{A}^2$], i (measured in [A]) denotes the current of the electromagnet coil and F_{Ext} denotes the external force generated by the lower electromagnet in the same direction as the gravity force.

2.3 PID Control

To keep the ferromagnetic object in a stable position, a PD controller with appropriate stiffness and damping properties can be applied [9]. Moreover, the PID controller can be adjusted manually or optimally with the help of the genetic algorithms method [15]. The designed controller performance depends on the sampling period and and applied discretisation method [10]. The fractional PID controller under consideration is given in the form (2).

$$G(s) = K_p + \frac{T_i}{s^\lambda} + T_d \cdot s^\mu \tag{2}$$

The continuous Laplace operator s^a can be approximated by different methods [2]. Typically, the fractional order PID controller tuning methods are based on: frequency gain, pole distribution, genetic algorithms, particle swarm optimisation, and Taylor series.

2.4 Discussion of the Executing Hardware

In this research the sampling frequency is relatively low for demonstration, educational and research purposes of control in the soft real-time mode. The RT-DAC and MATLAB/Simulink interface enables the use the rapid prototyping method to close the control loop. The choice of the sampling period [16] must be adequate for the hardware properties and can be optimised [12]. For the fixed-point implementation, the constraints ought to be considered [13]. In the case of industrial applications, a hard real-time solution will be needed to guarantee the requisite performance [4], [6], [5]. In this case the point of interest is the controller form to be embedded into analog processors [7]. The $PI^{\lambda}D^{\mu}$ controller was embedded into analog processors and tested with oscilloscope measurements [1]. When such implementation is being considered, the DFOC controller form should be given in the lowest possible order due to limited hardware resources.

3 Study of DFOC Controller

3.1 Synthesis

The DFOC controller synthesis was realised using DFOD/I Toolbox for MATLAB [2]. With this Toolbox the following four approximation methods are available: CFE of Euler rule, CFE of Tustin rule, PSE of Euler rule, PSE of Tustin rule. Therefore the considered controllers were respectively designated: DFOC_CE, DFOC_CT, DFOC_PE, DFOC_PT. There are a few constraints depending on the chosen approximation method: for CFE Euler and CFE Tustin the order must be set at less than 5. For PSE Euler the minimal order is 100 and for PSE Tustin 20. As a result the discrete form of the fractional order controller in the form (3) was obtained and embedded into a Simulink diagram for the simulation and experiment tasks.

$$G(z) = \frac{b_0 + b_1 z^{-1} + ... + b_m z^{-m}}{a_0 + a_1 z^{-1} + ... + a_n z^{-n}} \tag{3}$$

The most important question was how to choose optimal parameters for the FDOC controller. The following method was proposed. For a set of PID controller parameters assuring the stable operation of the AML system, the external load was activated at time t_b for a period Δt using the bottom electromagnet. In the period $[t_b, t_e]$ covering the excitation event the quality of control $J2$ was investigated (4). The calculations were based on the simulation model solved with the discrete step size equal to the sampling period. For the chosen values of $\lambda \in (0,1)$ and $\mu \in (0,1)$ the search for minimal quality factor J_2 was performed. For the unstable operation the quality factor J_2 was set to 1 (see Fig. 1).

$$J_2 = \int_{t_b}^{t_e} e^2(t)dt \tag{4}$$

Note, that for λ and μ set to 1, the DFOC_CE form is not achieved (see Fig. 2a). The synthesis results in empty numerator and denominator set to 1. The DFOC_CT, and DFOC_PT results in the unstable behavior of the system (see Fig. 2b, d). Only, the PE approximation satisfies the stability criterion (see Fig. 2c). Finally, the optimal parameters were selected for controller implementation.

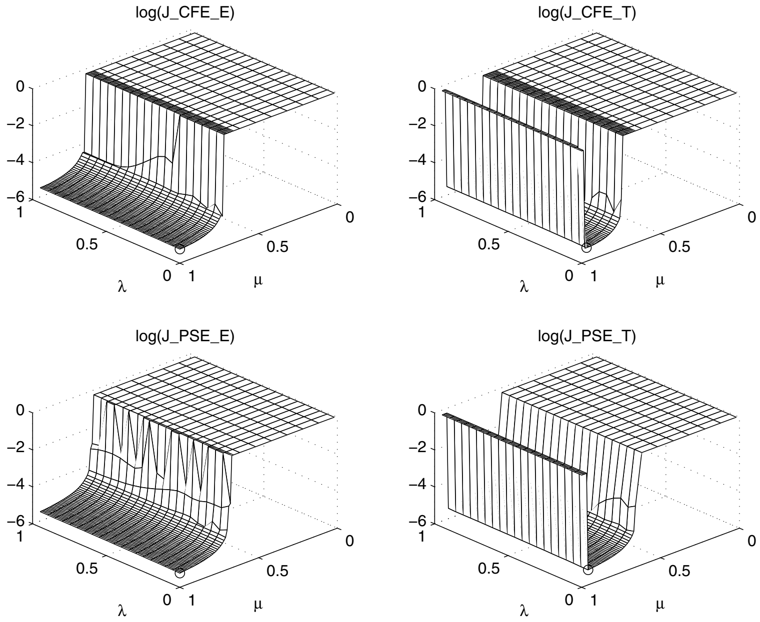

Fig. 1. Comparison of quality criterion vs. integration and derivative factors

Table 1. DOFC Synthesis parameters

Parameter	DFOC_CE	DFOC_CT	DFOC_PE	DFOC_PT
P	120	120	120	120
I	5	5	5	5
D	3	3	3	3
λ	0.1	0.1	0.1	0.1
μ	0.99	0.96	0.99	0.95
apr. ord.	1	1	100	20
J_2	$4.887{\cdot}10^{-6}$	$4.677{\cdot}10^{-6}$	$4.974{\cdot}10^{-6}$	$5.565{\cdot}10^{-6}$

The summary of optimal settings is presented in Table 1. It should be noted, that the minimisation of the quality criterion for the fractional order controllers tends towards the classical PD controller form.

Finally, the approximation order was considered for the obtained FOC parameters, and diagnosed in the available range (see Fig. 3). The minimum value for the quality criterion J_2 was obtained for the lower bound of the approximation order. In the case of DFOC_PE study, J_2 is constant (the standard deviation is $2.01{\cdot}10^{-21}$). It schould be noted, that for DFOC_CT the instability of the controller for $n > 1$ was observed.

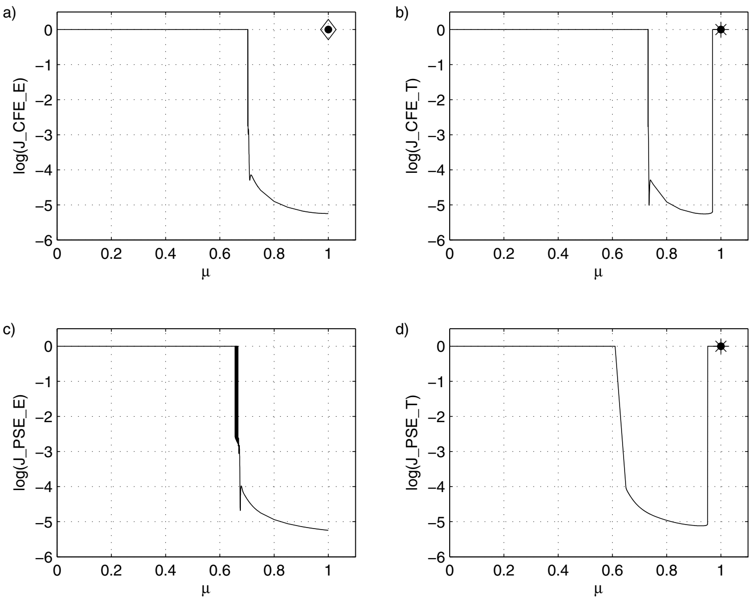

Fig. 2. Comparison of quality criterion vs. derivative factor at $\lambda = 1$

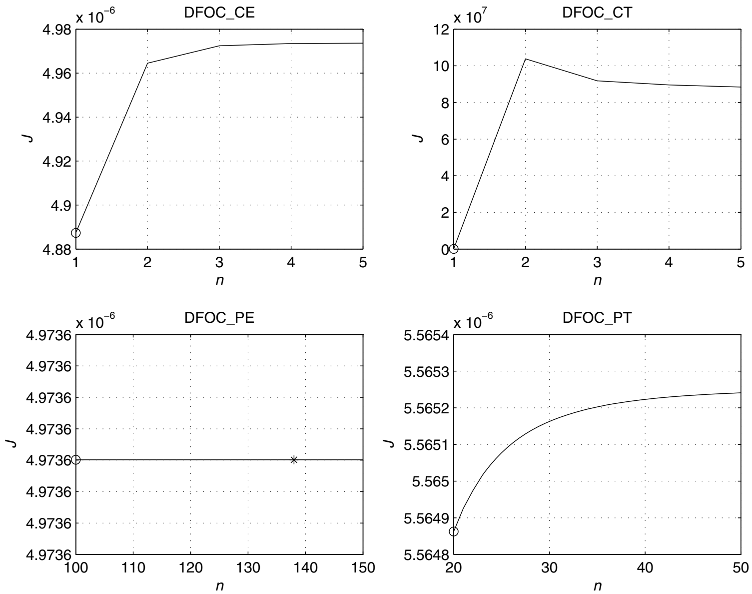

Fig. 3. Comparison of quality criterion vs. approximation method and order

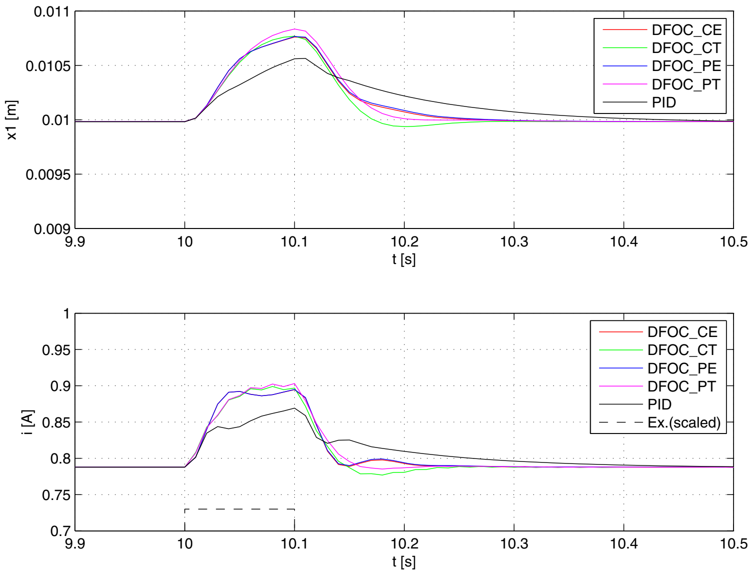

Fig. 4. Simulation: ball displacement and control signal during stabilisation and external excitation

The system response varies with the controller design method. The simulation results are very promising for experimental application. The control signal does not reach the constraints.

3.2 Real-Time Experiments

To test the DFOC controllers the following scenario was implemented. The standard PID controller with Euler backward derivative approximation was applied, so that the ball could reach a stable position, before switching to the DFOC controller, which was activated. Each experimental phase was enabled for 10 seconds. It should be noted, that all controller calculations were realised, because controller input was active and the displacement error was delivered for each sample time. During the stabilisation phase the external load generated by the bottom electromagnet was activated to pull down the ball and diagnose the generated control action by all controllers. This scenario was repeated four times to test all the DFOC controllers under consideration. The scenario concept is summarised in Table 2 and the complete record of the experiment is given in Fig. 5.

It was found that the DFOC controllers do not operate as expected. In order to have a closer look at their behaviour the experiment phases were extracted and are presented in Figs. 6 -6. Additionally, to demonstrate the properties of the real-time system, the difference in the sampling period intervals were acquired (using an RT-DAC USB hardware timer) and presented in the form of a time

Table 2. Experiment scenario

Exp. phase	t_b [s]	t_e [s]	Controller	t_{EM2on} [s]
A	0	10	PID	5.0-5.1
B	10	20	DFOC_CE	15.0-15.1
C	20	30	PID	25.0-25.1
D	30	40	DFOC_CT	35.0-35.1
E	40	50	PID	45.0-45.1
F	50	60	DFOC_PE	55.0-55.1
G	60	70	PID	65.0-65.1
H	70	80	DFOC_PT	75.0-75.1
I	80	85	none	no

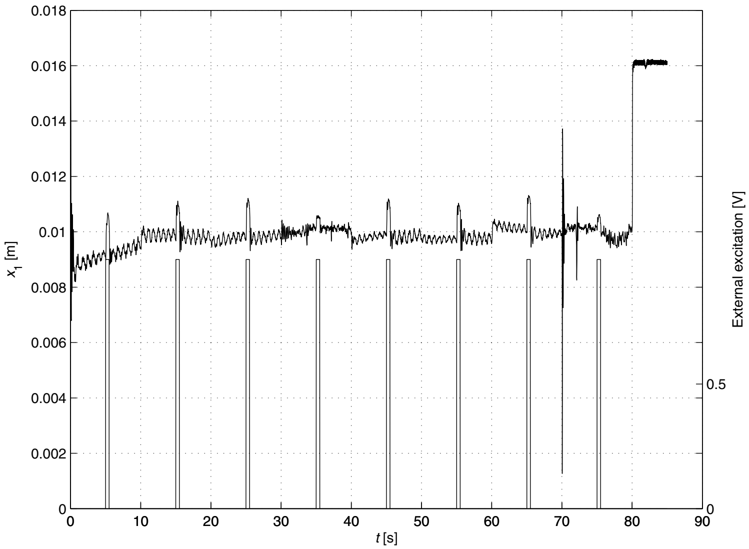

Fig. 5. Time diagram of the realised experiment

diagram and histogram. The existing control system jitter affects the calculation of derivative and integral approximations. Observing the jitter histograms, one can find two peaks at about 9.75ms and 10.75ms. Moreover, histograms provide an information about the computational load. For DFOC_CE and DFOC_CT realizations the sharp peaks are well separated due to the low controller order. For higiest order controllers: DFOC_PE and DFOC_PT the histogram is more fuzzy. Especially for the DFOC_PE the computation effort is high.

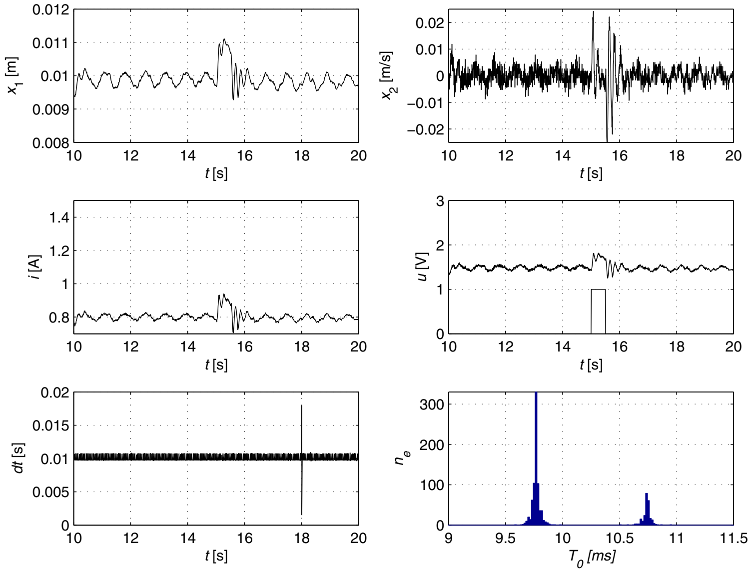

Fig. 6. Experimental stage of the DFOC_CE Controller

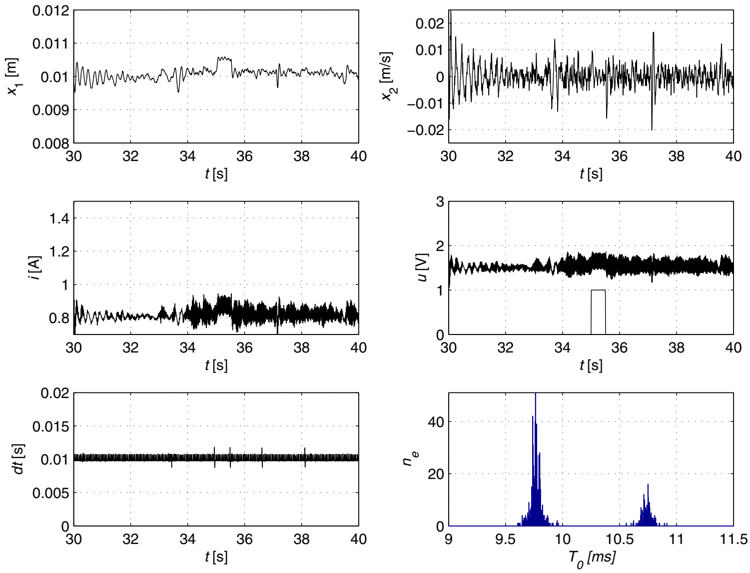

Fig. 7. Experimental stage of the DFOC_CT Controller

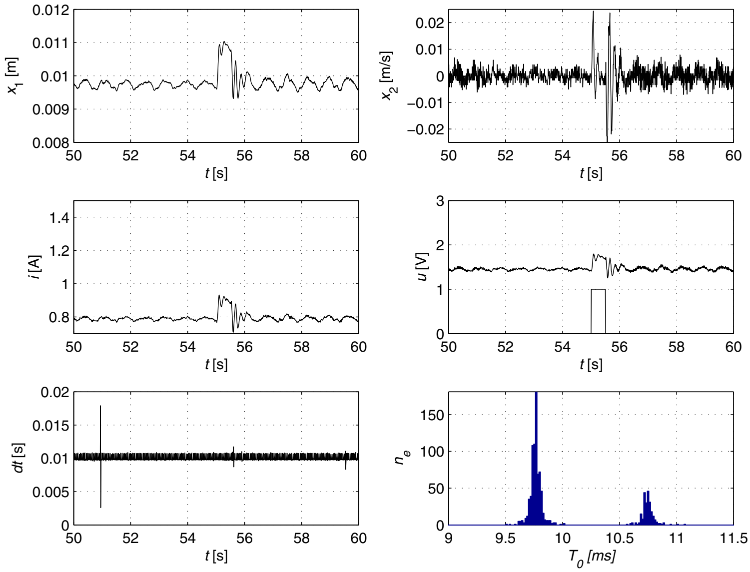

Fig. 8. Experimental stage of the DFOC_PE Controller

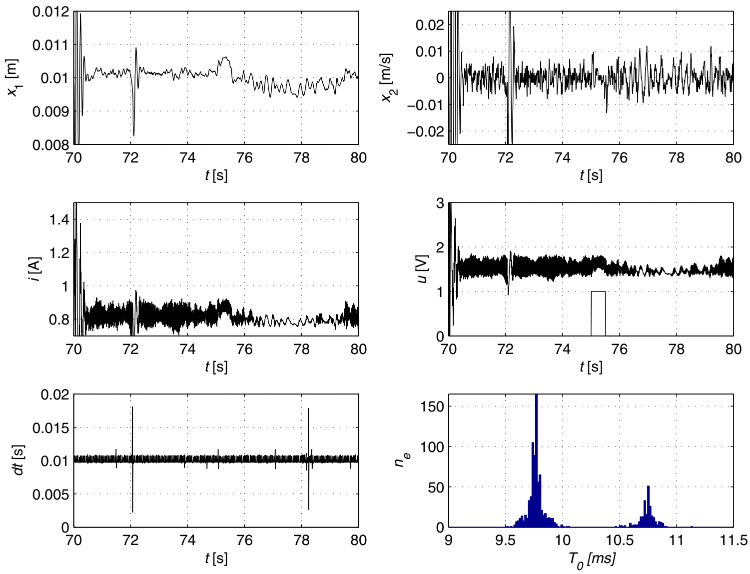

Fig. 9. Experimental stage of the DFOC_PT Controller

4 Discussion and Further Research

The DFOC was designed using a dedicated toolbox [2]. The controllers were practically implemented in the simulation and experimental stage. The numerical solution and ideal modeling do not enable a full diagnosis. Therefore the theoretical and numerical study should be expanded with sampling and quantisation effect. Another interesting area for research is FOC robustness. For the DFOC_PT realisation, the impact of the ball at the electromagnet was observed. This happened when the control action was toggled. The initial stage of the controller and its robustness should be analysed in detail. The soft-real time regime shows the DFOC's higher sensitivity (lower robustness) to the sampling jitter compared to that observed in classic PID controllers. Finally, the adjustment of the λ integration factor, μ derivative factor and approximation method affects the closed-loop dynamics and controller form. Therefore further research is planned to design an FOC with a specified closed-loop performance and to embed the designed controller into target platform [7].

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