Bi-Objective Network Equilibrium, Traffic Assignment and Road Pricing

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1 Traffic Assignment and User Equilibrium

Traffic assignment models the route choice of users of a road network. Given a set of origin-destination (OD) pairs and demand for travel between these OD pairs, it determines how many users choose each of the available routes, and thereby the amount of traffic on each section of the road network. Conventional traffic assignment is based on the assumption that all users want to minimise their travel time, or more generally, a generalised cost function

$$c(x_p) = m(x_p) + \alpha t(x_p), \tag{1}$$

where x_p represents traffic flow on route p, t is travel time, which is dependent on flow, and m is a monetary cost comprising of tolls, vehicle operating cost etc. that may also depend on flow and α is value of time. A user will choose the route between their origin and destination that has the least value of $c(x_p)$.

The traffic assignment problem is based on Wardrop's principle of user equilibrium [9], which can be stated as follows: *Under user equilibrium conditions traffic arranges itself in such a way that no individual trip maker can improve their generalised cost by unilaterally switching routes.* In other words, at equilibrium, the generalised cost of any used route between an OD pair must be equal and less than that of any unused route.

It is important to note that (1) is the linear combination of two components, time and monetary cost. In fact these are two different objective functions. Several authors

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have recognised this and suggested bi-objective traffic assignment models, see the references in [8]. However, these models are restrictive, by keeping the assumption of the existence of an additive generalised cost (or sometimes generalised time) function (1). Moreover, there is evidence, that users in reality do not behave according to this assumption, see references in [8]. In [8] we have suggested a more general bi-objective user equilibrium condition, that assumes that all users have the two objectives of minimising travel time and minimising toll cost.

Under bi-objective user equilibrium (*BUE*) *conditions traffic arranges itself in such a way that no individual trip maker can improve either his/her toll or travel time or both without worsening the other objective by unilaterally switching routes.*

We have shown that, even if considering all possible values of time, i.e. $\alpha \in [0, \infty)$, in (1), bi-objective models based on generalised cost provide only a subset of all possible solutions to traffic assignment that satisfy the BUE condition. Hence the definition of BUE provides an appropriate general framework for the study of traffic assignment in tolled road networks.

Furthermore, in [7] we have suggested the time surplus maximisation concept as a new route choice model that addresses the stochastic nature of route choice behaviour and the variability among users on their willingness to pay. It is based on the idea of time surplus. We assume that a user has in his mind a maximum time he is willing to spend in traffic, given any level of toll. If τ_p^k is the toll on route p for OD pair k and the travel time is $t(x_p^k)$ then the time surplus on route p for individual i is

$$t_{ip}^s = t_i^{max}\left(\tau_p^k\right) - t\left(x_p^k\right). \tag{2}$$

We assume that the higher the toll, the shorter the maximum time willing to spend, i.e. we assume that t_i^{max} is a strictly decreasing function of τ_p^k. This function t_i^{max} is an indifference curve between time and toll for user i. The time surplus maximisation concept stipulates that all users maximise their time surplus. This gives rise to a user equilibrium condition: *Under the time surplus maximisation user equilibrium* (*TSmaxBUE*) *condition traffic arranges itself in such a way that no individual trip maker can increase their time surplus by unilaterally switching routes.*

In order to find a solution of the TSmaxBUE problem, we employ a route-based formulation of the equilibrium condition and follow [4] to formulate this as a non-linear complementarity problem, which is solved by minimising an associated gap function. Notice that because time surplus is maximised, but the NCP formulation requires a cost function to be minimised, we need to write this cost function as $\eta_p^{ki} := M - t_i^{max}(\tau_p^k) + t(x_p^k)$ with a sufficiently large M in the NCP model.

2 Road Pricing

Road pricing is a common instrument to reduce congestion and has successfully been implemented in many cities around the world, e.g. in Singapore, Stockholm

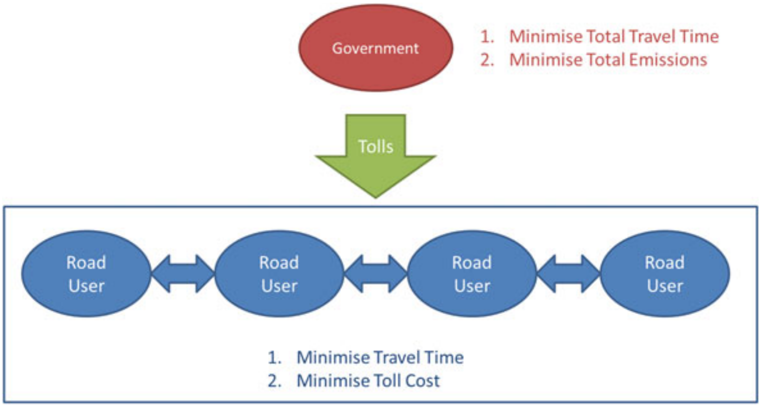

Fig. 1 The bilevel concept for road pricing

and London. The idea of congestion pricing is to charge tolls such that users are paying the marginal social cost rather than the average private cost for their trips. This induces changes in travel behaviour such that the total system travel time is minimised.

In todays world the idea of sustainable transport systems is gaining importance internationally. Sustainability encompasses the dimensions of economic, social and environmental sustainability [2]. The European Conference of Transport Ministers has defined a comprehensive catalogue of objectives of sustainable transport policy [3]. Among those, the objectives of creating wealth, reducing congestion, and reducing greenhouse gas emissions are relevant for this paper, the first in terms of economic sustainability, the second for both economic and environmental, and the last for environmental sustainability.

We suggest that, apart from considering tolls as a means to reduce congestion, road pricing can be an important instrument to reduce vehicle emissions. Hence the roading authority would pursue two objectives by charging road users: To minimise total travel time and to minimise total emissions. Road users on the other hand, will react to the imposed tolls and attempt to minimise their own travel time and toll cost. This framework is illustrated in Fig. 1.

At the government level there is, however, a dilemma. It is well known that tolls that minimise total travel time do not necessarily minimise emission levels [5, 10]. Hence the problem becomes that of the determination of efficient tolls such that neither the total travel time nor the total emissions can be reduced without worsening the other, which is a bi-objective optimisation problem.

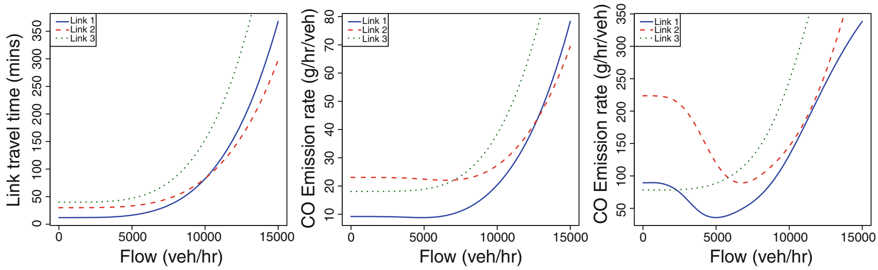

Fig. 2 Travel time (*left*) and CO emissions (*middle* and *right*) as functions of traffic flow

3 A Bi-level Bi-objective Optimisation-Equilibrium Model

Here, we first formalise the two objective functions at government level. The first objective function is to minimise total travel time:

$$\min z_t(x(\tau)) = \sum_{a \in A} x_a(\tau) t_a(x_a(\tau)), \tag{3}$$

where $t_a(x_a) = t_0 \left[1 + \alpha \left(\frac{x_a}{C_a}\right)^\beta\right]$, is a typical link travel time function [1]. Here t_a is the travel time on link a, which depends on link flow x_a. Also, t_0 is the free-flow travel time and C_a the practical capacity of link a. The values of $\alpha = 0.1$, $\beta = 4.0$ are typical, and we adopt them in the example of Sect. 4. The left plot of Fig. 2 shows three examples of travel time functions used in the example of Sect. 4.

The second objective function is the minimisation of total CO emissions.

$$\min z_e(x(\tau)) = \sum_{a \in A} x_a(\tau) e_a(v_a(x_a(\tau))). \tag{4}$$

Here, v_a is the traffic speed, which depends on link flow x_a and e_a is the CO emissions, which in turn depends on speed, on link a. Unfortunately, there is no consensus on the exact form of the emission function e_a. In Fig. 2 we show the functions proposed by Yin and Lawphongpanich [10] in the middle and by Niemeier and Sugawara [6] on the right.

4 A Three Link Example

We demonstrate our bilevel bi-objective-equilibrium model on a simple three link network. The three links (or routes) connect a single origin-destination pair and have the following characteristics. For route (link) 1, an expressway of 20 km length, we

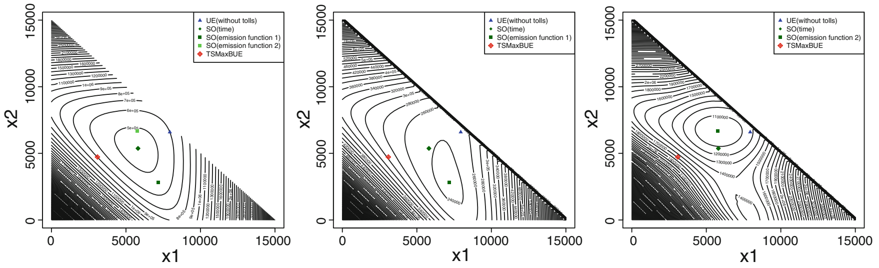

Fig. 3 Contour plots of travel time and emissions over feasible flows

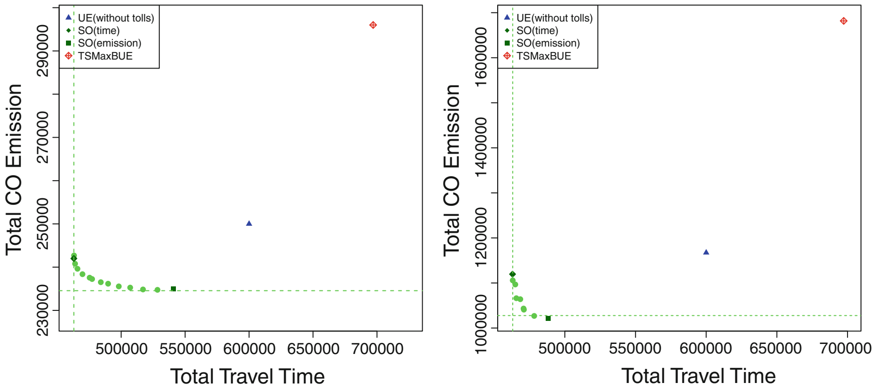

Fig. 4 The trade-off between total travel time and total CO emissions for efficient tolls

set $v_0 = 100$ km/h, $t_0 = 12$ min, and $C_a = 4,000$ vehicles per hour in the function (3). For route (link) 2, a highway of 50 km distance these values are $v_0 = 100$, $t_0 = 30$ and $C_a = 5,400$. Finally, route (link) 3, an arterial route of 40 km length, has $v_0 = 60$, $t_0 = 40$ and $C_a = 4,800$.

The travel time and emission functions for the three links of this network are illustrated in Fig. 2. In Fig. 3 we show contour plots of the travel time and emission functions over feasible flows together with the social optimum solutions for travel time and emissions, illustrating that these are indeed different. We also show the traffic flows at the untolled user equilibrium solution, and the TSmaxBUE solution at the toll values of $\tau_1 = 40$ and $\tau_2 = 20$.

To find the efficient tolls, we observe that we must have that the tolls are such that $\tau_1 > \tau_2 > \tau_3$ and assume that $\tau_3 = 0$. We assume that t^{max} is uniformly distributed between 10 and 25, 30 and 45, and 60 and 90 min on the three links, respectively. Moreover, we allow τ_1, τ_2 to be in the range between 1 and 40 in discrete steps of 1.

The resulting total CO emissions versus total travel time, for both emission functions, are plotted in Fig. 4, clearly illustrating the tradeoff between the two objectives as well as the difference to the untolled user equilibrium solution and the TSmaxBUE solution for $\tau_1 = 40$, $\tau_2 = 20$.

5 Conclusion

In this work we have proposed a bilevel framework for road pricing to support sustainable transport systems. On the upper level we consider a bi-objective optimisation problem of minimising total emissions and total travel time, whereas on the lower level we consider a bi-objective user equilibrium model with users who minimise their own travel time and toll cost. We have proposed the concept of time surplus maximisation as a way of dealing with the bi-objective user equilibrium. In future work, we will develop algorithms to solve the problem, based on the NCP formulation of the TSmaxBUE problem and using a multi-objective evolutionary algorithm to integrate this with the bi-objective optimisation problem on the upper level.

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