Chapter 5
Dynamic Buckling Criteria

Dynamic stability, or better, dynamic buckling (sometimes referred to as a dynamic response) means a loss of stability of the structure subjected to pulse load. Especially, it can act along the axis of the column or in the plane of the plate. It should be mentioned that for the ideal uniformy compressed structures (without any geometrical imperfection) the critical buckling amplitude of pulse loading leads to infinity. Therefore, the dynamic buckling can be analysed only for structures with initial geometrical imperfections. In such a case, the critical value of dynamic load does not have a bifurcation character, thus it has to be determined on the basis of the assumed criterion. Some the most popular and some new criterion allowing to determine dynamic buckling is presented in this chapter.

There are authors (e.g. [22]) who claim that for the thin-walled structures with flat walls (structures with stable postbuckling equilibrium path) dynamic buckling does not occur—the dynamic response can be analyse only.

Raftoyiannis and Kounadis [20] divided the criteria into the geometric and energetic ones. Geometric criteria are those in which a loss of dynamic stability refers usually to deflection or shortening, whereas the energy criterion is the one in which the critical value is determined by the potential and/or kinetic energy of the system.

Another division of criteria distinguishes the criteria for structures with a stable postbuckling equilibrium path (plate structures) and the criteria for structures with an unstable postbuckling equilibrium path or having a limit point (shells, rods). For structures with an unstable postbuckling equilibrium path, it is possible to derive mathematically the relation allowing for finding the critical value, describing the dynamic buckling load [11]. However, for plate structures the mathematical derivation is not possible and the criteria have been formulated from observations of behaviour of such structures [2] or on the basis of experiments [1, 6, 8]. Therefore, from the moment when the dynamic buckling problem appears in the literature, a number of dynamic stability criteria has been developed.

In the literature, in addition to displacement and energy criteria, failure criteria can be found. Such a criterion has been formulated by Petry and Fahlbusch [19]. In failure criteria for the dynamic buckling stability, proper hypotheses to determine

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the equivalent stress state are needed. The equivalent stress state can be compared with the following material properties: yield limit or ultimate stress for tension or compression. It should be noted that in the majority of works (also here), the assumed material properties were obtained from static tests.

5.1 Volmir Criterion

Volmir in his work [28] presented the behaviour of a simply supported rectangular plate subjected to different pulse loads. He analysed the pulse of infinite duration, linearly increasing load and pulses of finite duration. The pulses taken into consideration had rectangular and exponentially decreasing shapes. Volmir solved the dynamic buckling problem using the Bubnov-Galerkin method for determining the buckling and postbuckling state for statically loaded structures. Then, after setting the deflections and load as a function of time, the equations of motion were obtained. The equations of motion are solved using the Runge-Kutta method. During his analysis, Volmir took into consideration not only tensile load pulses but also the shear type. Probably due to computational difficulties, Volmir proposed a very simple but time-consuming method for determining the "critical" dynamic load. This method consists in assuming the buckling mode and dynamic response analysis. This assumed buckling mode (a number of sine halfwaves describing out-of-plane deflection) was taken as the critical one for uncoupled dynamic buckling if the dynamic response of the plate subjected to a given amplitude of pulse resulted in an increase of deflection in the shortest time. Volmir considered the buckling problem which can be described by a theoretical system with one degree of freedom. For the critical mode, "a factor of dynamism K_D" (further referred to as the Dynamic Load Factor—DLF) was determined. The K_D factor was defined as a ratio of the pulse amplitude of the critical load to the static critical load. On the basis of his study, Volmir suggested a very simple criterion for the dynamic stability loss, assuming that a loss of stability of the plate subjected to pulse load occurs when the maximum deflection of the plate is equal to the assumed constant value. Usually the critical deflection value was assumed to be equal to the thickness of the plate or half of its thickness.

5.2 Budiansky–Hutchinson Criterion

One of the first displacement criteria was formulated by Budiansky and Hutchinson [4, 11]. This criterion involves structures with geometrical imperfections and an unstable postbuckling equilibrium path or a limit point. The authors of that criterion analysed cylindrical shells and axially loaded rods. A similar criterion was formulated for cylindrical shells loaded transversely by Budiansky and Roth [5]. They considered pulse load of finite or infinite duration and derived the relationship which

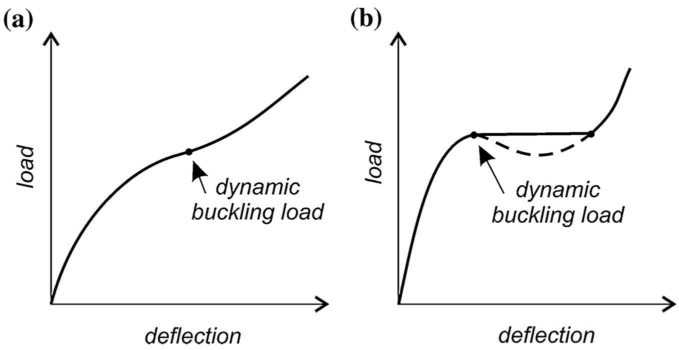


Fig. 5.1 Equilibrium paths for pulse loaded structures with large (**a**) and small (**b**) initial geometrical imperfections

allows for determining the critical load. It has been noted that the critical load corresponds to the inflection point on the curve presenting the load as a function of deflection (Fig. 5.1). Budiansky and Hutchinson defined their criterion in the following form:

A loss of stability of structures subjected to pulse loading occurs when there is an unlimited increase of deflection for small increments of load.

Many authors have adopted the above criterion for plated structures.

On the basis of the time history of the structure deflection, for a given pulse shape and its duration, a graph of the maximum deflection amplitude as a function of the load is built (Fig. 5.2). The Budiansky-Hutchinson criterion adopted for plated structures states:

A dynamic stability loss occurs when the maximum plate deflection grows rapidly with a small variation of the load amplitude.

5.3 Ari–Gur and Simonetta's Criterion

Ari-Gur and Simonetta [2] conducted a series of experiments and theoretical analyses of thin plates clamped on all edges and subjected to pulse load with a halfwave of sine shape (with finite duration). They noted that for a perfectly flat plate, the pulse load intensity L (the force F or the shortening U), which would result in a loss of stability, was infinitely large. The Ari-Gur and Simonetta's critical load value was set as depending on the following parameters:

- deflection—measured in the middle of the length and the width of the plate,
- load intensity L_m defined as the force pulse amplitude F_m or the shortening U_m.

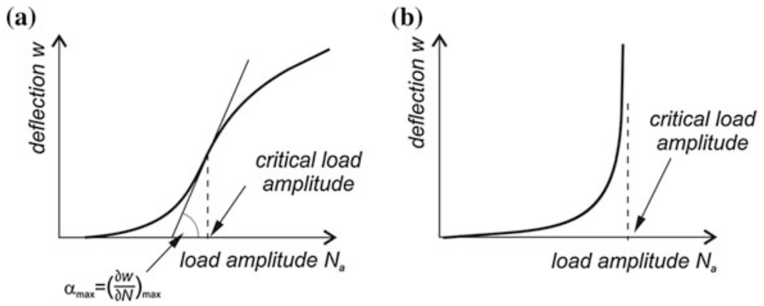


Fig. 5.2 Typical curves representing deflections as a function of load for the local (**a**) or global (**b**) buckling mode

On the basis of the analytical and numerical study, four dynamic buckling criteria are proposed in [2]. The first one (Fig. 5.4a) is based on the observation of the deflection w_m (Fig. 5.3) and the intensity of the pulse load L_m. It was formulated as follows:

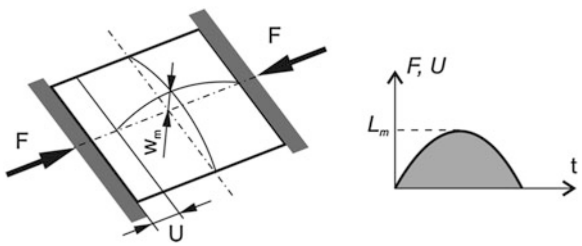
Dynamic buckling occurs when a small increase in the intensity of the pulse load L_m causes a significant increase in the value of the deflection w_m.

The above criterion is analogous to the criterion defined by Budiansky and Hutchinson (Sect. 5.2). Ari-Gur and Simonetta have noticed that the criterion can be used for loads in the form of a force or a displacement and for different pulse load durations. However, for a very short period of the pulse duration, a very high amplitude value in deflections is required (a large amplitude in comparison to the static critical load) to obtain a rapid increase, which—as is known—can cause a change in the mode of deflections during the pulse load. These observations have led to the formulation of the second criterion which is based on the analysis of the maximum value of the pulse load L_m and the deflection value w_m (Fig. 5.4b). This criterion is as follows:

Dynamic buckling occurs when a small increase in the amplitude of the pulse load L_m causes a decrease in the value of the deflection w_m.

The next two criteria are failure criteria which are based on a response of the loaded edge of the considered plate, namely shortening (for a force as a pulse) or

Fig. 5.3 Analysed pulse load and measured parameters



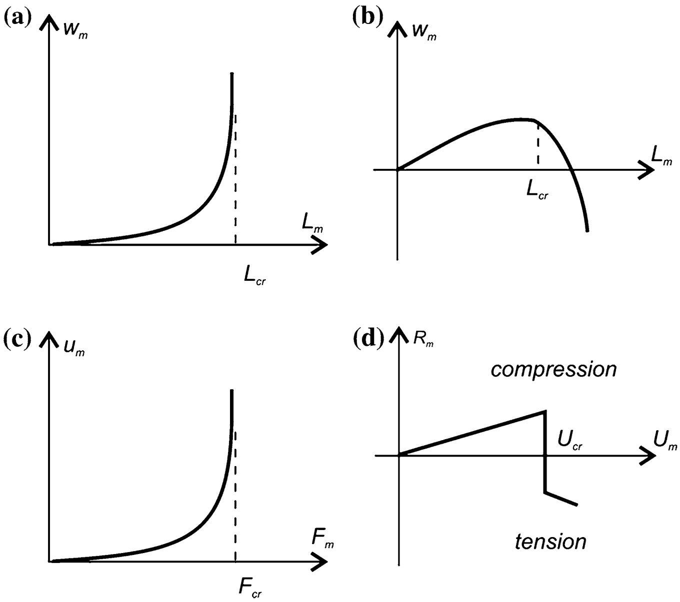


Fig. 5.4 Graphs presenting Ari-Gur and Simonetta's dynamic buckling criteria

reaction (for a displacement as a pulse load) on loaded edges. In the case of the shortening analysis, a similarity to the increasing deflection analysis (the first Ari-Gur and Simonetta's criterion) can be noticed. The third criterion (Fig. 5.4c) was formulated as follows:

Dynamic buckling occurs when a small increase in the amplitude of the force pulse F_m causes a sudden increase in the shortening u_m of loaded edges of the plate.

A significant increase in deflection of the plate will reduce its stiffness, so the load which causes a significant increase in the shortening is defined as the critical load (Fig. 5.4c).

The last criterion applies to the case when the load is defined not by a force but by an impulse of displacement (shortening of loaded edges) and defines the critical U_m displacement pulse intensity. The fourth criterion (Fig. 5.4d) given by Ari-Gur and Simonetta is:

Dynamic buckling occurs when a small increase in the pulse displacement intensity U_m of the loaded edge causes a change in the sign of the value of the reaction R_m at the edge of the plate.

As is well known for the deflected plate, the reaction distribution on the loaded edge has a sinusoidal shape (Fig. 5.5), thus in the case of large deflections, the tension appears in the middle of the loaded edge which maintains the straightness of the edge under load. The value of the resultant tensile force can be greater than the compressive forces that occur outside the central part of the edge under load.

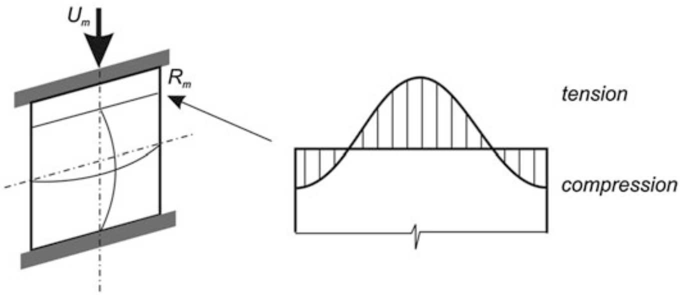


Fig. 5.5 Force distribution along loaded edges of the plate with initial imperfections

5.4 Kleiber–Kotula–Saran Criterion

Kleiber, Kotula and Saran in their work [13] presented the problem of dynamic stability for rod systems. The problem was solved with the finite element method. The paper quoted the criterion of stability in the Lyapunov sense, which requires disturbing the initial conditions and the analysis of the system behaviour. The aim of the mentioned work [13] was to formulate such a criterion that would allow the dynamic stability analysis based only on solving basic equations. It is well known that FEM solutions for static loaded structures are unstable when the tangent stiffness matrix is singular. Kleiber, Kotula and Saran began to analyse the tangent stiffness matrix for a system of rods subjected to pulse load with infinite duration (Heaviside pulse loading). First, they defined the critical pulse duration t_{cr}. For the time t less than the critical one, structural deflections are very small and oscillate around the position of equilibrium. When the time is longer than the critical t_{cr}, deflections of the structure begin to grow. This growth of deflections can be limited or unlimited. Then, they noticed that for every moment of time t less than the critical time t_{cr}, the tangent stiffness matrix [30] was positively defined. The above allows for formulating the following eigenvalue problem:

$$(\mathbf{K_T} - \chi\mathbf{M})\vartheta = 0, \tag{5.1}$$

where $\mathbf{K_T}$ is the tangent stiffness matrix, $\mathbf{M}$ is the mass matrix of the structure, χ is an eigenvalue and ϑ is the vector of eigenvalues.

Analysing (5.1) for the time t equal to the critical value t_{cr}, it is found that the smallest eigenvalue $\chi_{min}(t_{cr})$ changes its sign from positive to negative, which corresponds to the growth of the structure deflection. The above analysis shows that the zero eigenvalue corresponds to singularity of the tangent stiffness matrix. In order to formulate the stability criterion, it should be determined whether an increase in the deflection at the time $t > t_{\mathrm{cr}}$ is limited or unlimited. On the basis of the examples presented in [13], it has been noted that the smallest eigenvalue for the times $t > t_{\mathrm{cr}}$ reaches the minimum value of χ'_{min} and then begins to rise and again changes its sign to the positive one and reaches the maximum value χ''_{min}.

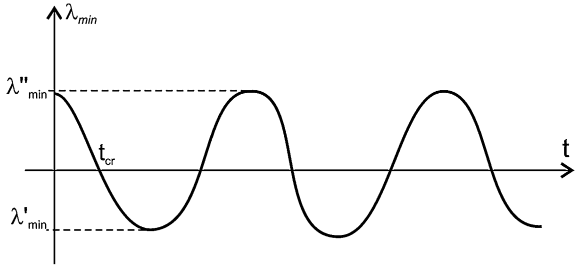


Fig. 5.6 Exemplary time history for the minimum eigenvalue

On this basis, Kleiber, Kotula and Saran defined a quasi-bifurcation criterion of dynamic stability for the structures subjected to pulse loading. It says that the structure loses its stability (deflection begins to grow indefinitely) when for $t = t_{cr}$ the determinant of the tangent stiffness matrix is equal to zero and the absolute value of the smallest eigenvalue χ'_{min} of (5.1) is greater than the maximum absolute value of the next smallest eigenvalue χ''_{min}(Fig. 5.6), i.e., $\left|\chi'_{min}\right| > \chi''_{min}$.

In summary, in order to consider the load as critical (causing an infinite deflection growth) for the given time $t = t_{cr}$, the two following conditions have to be simultaneously satisfied:

$$\mathbf{K_T} = 0 \wedge |\chi'_{\mathrm{min}}| > \chi''_{\mathrm{min}}. \tag{5.2}$$

The tangent stiffness matrix in the stability theory of dynamical systems [12] corresponds to the Jacobi matrix.

5.5 Author's Criterion

The above-discussed criteria can be applied to plates, shells or beam-columns and they have been formulated for the non-coupled buckling mode. In complex thin-walled structures, the stability loss has often a coupled form—one buckling mode enhances or accelerates the creation of another one and, as a result, a new buckling mode appears. Such multi-modal modes of the stability loss should also be taken into account in the analysis of thin-walled structures subjected to pulse load. As is well known from the dynamic buckling literature, the pulse duration equal to the period or half a period of natural vibrations with the mode corresponding to the buckling mode is considered.

Coupling of various buckling modes is associated with different vibration frequencies (and, thus, periods of vibration) corresponding to different buckling modes. This often leads to situations where for one buckling mode, the established pulse duration corresponds to the dynamic load and for another one, the period of vibration is so long that the pulse load should be treated as quasi-static. In the global buckling mode case, the deflection of the structure grows to infinity. Taking above into account, the following questions appear:

- how to determine the pulse duration in the case of interactive buckling?
- which value should be taken as the critical one in the global buckling mode—the deflection asymptote or the deflection for which the theory is no longer valid?

To answer the first question, numerous calculations and simulations have been performed. It has led to the conclusion that the worst case is when the pulse duration corresponds to the first flexural vibration mode—for short beam-columns it corresponds to the local mode. For longer structures, the first flexural vibration mode corresponds to the global mode, which finally appears when different buckling modes interact.

While answering the second question and taking the above-mentioned into account, one should note that the new dynamic buckling criterion could be formulated especially for all cases when the deflection grows rapidly to infinity. However, this new criterion could also work for the remaining buckling modes.

The equations of motion solution for the multi-mode (3.42) or single-mode (3.43) buckling allow for determining the deflection change of the analysed structure in time and, thus, the function that describes the behaviour of the system subjected to pulse load. Similar equations are solved in problems of stability of motion. Therefore, it has been decided to look for an analogy in determining the critical value describing dynamic buckling. The quasi-bifurcation criterion of dynamic stability for pulse load (Heaviside type) proposed by Kleiber, Kotula and Saran [13] seemed to be close to the dynamic buckling (impact) problem. The direct application of this criterion for estimation of the critical amplitude of pulse loading (of finite duration) for analysed thin-walled structures did not give expected results. The obtained dynamic critical load values were smaller than the results obtained from other well-known criteria [11, 28]. The discrepancies can be explained by different structures and different pulse types.

The further analysis concentrated on the Jacobi matrix calculation in a similar way as it is done in the dynamic stability problem for the periodical solution [12], where the values of the characteristic roots of the Jacobi matrix were checked. On the basis of numerous examples and many studies (the results are presented in Chap. 6.2), it was noticed that for the thin-walled structures subjected to pulse loading, which lose their stability according to the Budiansky-Hutchinson criterion or the Volmir criterion, the maximal radius $r_{\max}$ calculated from the characteristic root $\chi = a + jb$ (where $j = \sqrt{-1}$) of the Jacobi matrix was equal or greater than unity in the complex plane (Fig. 5.7). It was also noted that it is was sufficient to analyse the eigenvalues at any time between 0 and $1.5{\cdot}T_p$. Therefore, the dynamic buckling criterion for thin-walled structures can be formulated as follows:

Thin-walled structures subjected to pulse loading of finite duration lose their stability even if one characteristic root $\chi = a + jb$ of the Jacobi matrix found for every time moment from 0 to $1.5T_p$ lies in the complex plane outside the circle with the radius equal to unity.

The developed criterion has some limitation in application. The limitation was found during the dynamic buckling analysis of the plate subjected to rectangular pulse load [16]. For the mentioned case, it was observed that for pulse duration less

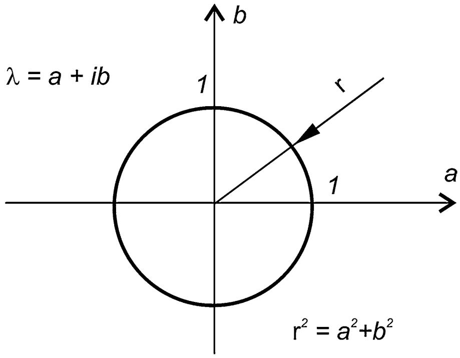


Fig. 5.7 Geometric representation of the proposed criterion

than $0.75T_p$ (T_p period of natural vibration), the results were independent of the pulse duration, thus for this case, it could be used only for pulse load with duration no longer than $0.75T_p$.

5.6 Teter Criteria

A similar way to find a dynamic buckling criterion as described in previous Section 5.5 was used by Teter [26]. Teter analysed the behaviour of thin-walled columns with stiffened open cross-sections subjected to pulse loading. The coupled buckling was analysed. As found out by Teter, the criterion proposed by the author of this monograph is not sufficient to estimate the critical amplitude of pulse load for long columns with stiffened open cross-sections [26], particularly if the duration of the pulse is equal to half a period of the vibration in the considered structure. Taking above into account, he attempted again to modify the Kotula-Kleiber-Saran criterion and formulated the following one:

A dynamic stability loss occurs when during the tracing time of solutions all eigenvalues of the Jacobi matrix are not positive simultaneously and at any moment one can find two negative eigenvalues.

The main disadvantage of this criterion is the same as in the previous one (Sect. 5.5)—a need to determine the Jacobi matrix (the tangent stiffness matrix) and its eigenvalues. This possibility exists for analytical-numerical methods or other open source software. Therefore, Teter decided to propose a new phase plane criterion. It is based on the results of numerical calculations obtained from any software to determine the displacement and the velocity. This criterion states:

The dynamic buckling load for the tracing time of solutions has been defined as the minimum value of pulse load such that the phase portrait is an open curve.

The criterion for dynamic buckling structures subjected to unbounded pulse load based on the phase plane was used by Schokker et al. [21], Hutchinson and Budiansky [11] and Hsu [10]. More details can be found in [3].

5.7 Petry–Fahlbusch Criterion

Petry and Fahlbusch [19] have noted that the Budiansky-Hutchinson criterion originally formulated for shells, widely used by many authors in the analysis of the dynamic behaviour of plates, does not allow one to use fully the capacity of the plate structure. In practice, this leads to a conservative determination of the critical dynamic load for the plate. These authors believe that the critical dynamic load should be based on the stress state analysis for the structures with a stable post-buckling equilibrium path. Analysing the stress state in any moment of time for the structures subjected to pulse load, it is possible to determine the dynamic load leading to a failure. On the basis of this approach, Petry and Fahlbusch have formulated the following criterion of dynamic buckling:

A dynamic response of the structure subjected to pulse load is dynamically stable if the condition that the equivalent stress is less than or equal to the assumed limit of stress is satisfied at any time and any point of the structure.

In the case when a deformable body is taken into consideration, they suggest assuming the yield limit as a limit of stress in the proposed criterion. Petry and Fahlbusch have redefined also the dynamic load factor to the following form:

$$DLF_f = \frac{N_F^{dyn}}{N_F^{stat}}, \tag{5.3}$$

where the dynamic failure load N_F^{dyn} and the static failure load N_F^{stat} in the sense of the limit stress are introduced. Petry and Fahlbusch in their work present results for the structures made of isotropic materials, in which, as is well known, the equivalent stress is calculated according to the Huber-Mises hypothesis. Nowadays, composite materials with orthotropic or even anisotropic material properties are very often used for different structures, so the Petry-Fahlbusch approach should be modified by an application of proper failure criteria for such materials.

5.8 A New Approach to Dynamic Buckling Load Estimation

Examining typical curves of the dynamic buckling problem (Fig. 5.8b) and the postbuckling behaviour of thin-walled structures subjected to static load (Fig. 5.8a), one can notice some similarity.

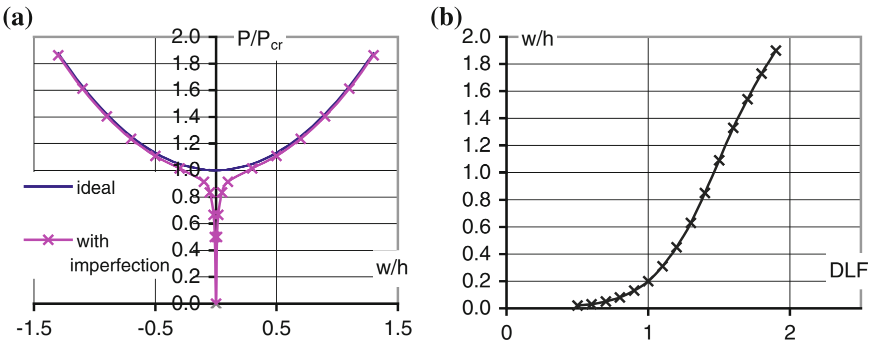
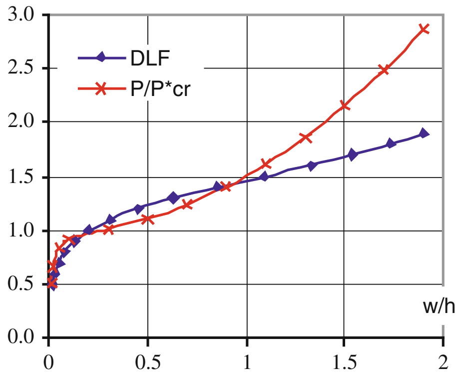


Fig. 5.8 Postbuckling equilibrium paths (**a**) and nondimensional displacement as a function of the dynamic load factor (**b**)

Fig. 5.9 Postbuckling behaviour in the case of static and dynamic load



The curves shown in Fig. 5.9 (i.e., the nondimensional static load P/P^*_{cr} and the nondimensional dynamic load DLF as a function of nondimensional displacement) are similar, especially when the states are changed from prebuckling to post-buckling. Taking into account this similarity, it has been decided to check the suitability of the well-known methods for determining critical static loads, based on the results of experimental tests. The most popular methods are:

- Soutwell method [3, 18, 23, 24, 29].
- mean-strain (P-ε_m) method [7, 23, 24, 27];
- method of straight-lines intersection in the plot of mean strains [7, 23, 24, 27];
- alternative method P-w^2 [7, 23, 24, 27];
- P-w curve inflection point method [7];
- "top of the knee" method [15];
- Tereszkowski method [25];
- Koiter method [9, 14].

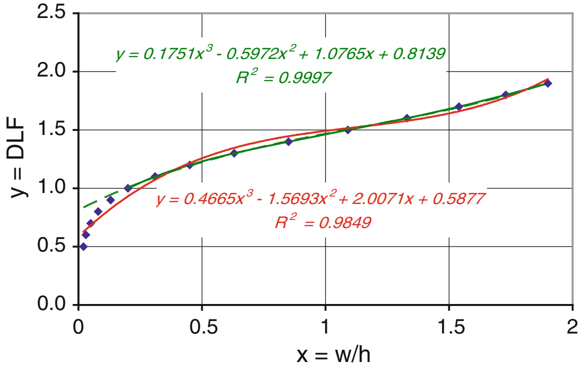


Fig. 5.10 Trend lines received from the results (points) of the numerical calculations

Kubiak and Kowal-Michalska [17] have decided to employ two well-known methods for identification of the critical load that are usually applied to the results of experimental investigations. The inflection point method (*P*-*w*), which is very similar to the "top of the knee" method, and the alternative (*P*-w^2) method have been used.

To find the inflection point, the approximation equations are found. The post-buckling equilibrium paths for structures with initial imperfections are the third order polynomial. It has been decided to adopt the same order function to fit the curve on the basis of points (Fig. 5.10) received from the numerical calculations. Thus, this function has the following form:

$$DLF = a_3\left(\frac{w}{h}\right)^3 + a_2\left(\frac{w}{h}\right)^2 + a_1\left(\frac{w}{h}\right) + a_0. \tag{5.4}$$

Approximating DLF(*w*/*h*) with function (5.4), it is very easy to find the inflection point and its coordinates:

$$\begin{gathered}\left(\frac{w}{h}\right)_{cr} = \frac{-2a_2}{6a_3},\\ DLF_{cr} = a_3\left(\frac{w}{h}\right)^3_{cr} + a_2\left(\frac{w}{h}\right)^2_{cr} + a_1\left(\frac{w}{h}\right)_{cr} + a_0.\end{gathered} \tag{5.5}$$

As presented in Fig. 5.10, there are two different approximation curves (trend lines) and their equations—the first curve (the red one, bottom equation in Fig. 5.10) is obtained including all points from the numerical calculations and the second one (green, upper equation—Fig. 5.10) is based on points for DLF equal or higher than 1. The obtained results based on the above-mentioned curves are presented in Table 5.1.

Comparing the results presented in Table 5.1, it can be said that the critical values of DLF do not depend on the number of points taken into consideration for determining the trend line equation—it is enough to take all points for $DLF \geq 1$. More results obtained employing this approach and comparisons with other criteria will be presented in Sect. 6.2.

Table 5.1 An influence of taken approximate equations on critical value of DLF

Case	Constant in (5.4)				DLF_{cr}
	a_3	a_2	a_1	a_0	
1) All points from the numerical calculations taken to obtain an approximate equation	0.4665	−1.5693	2.0071	0.5877	1.523
2) Points with DLF ≥ 0.7	0.3252	−1.1071	1.5800	0.6832	1.526
3) Points with DLF ≥ 0.8	0.2655	−0.9081	1.3899	0.7291	1.527
4) Points with DLF ≥ 0.9	0.2159	−0.7396	1.2236	0.7722	1.527
5) Points with DLF ≥ 1.0	0.1751	−0.5972	1.0765	0.8139	1.523

To use the alternative (P-w^2) method, the relation between the dynamic load factor and dimensionless deflections should be changed into a curve or an equation describing the relation of *DLF* versus the square of the dimensionless deflection, and next, the point of intersection of the straight line representing the postbuckling state with the vertical axis can be found (Fig. 5.11). For exemplary results obtained for a simply supported compressed plate with the initial geometrical imperfection equal to 1/100 of the plate thickness (Fig. 5.11), the critical value of DLF_{cr} is equal to 1.14, according to the proposed alternative method (P-w^2).

The next suggestion concluded in [17] is to calculate the dynamic load factor as a relation between the amplitude of the pulse load and the critical load for an imperfect structure. According to the obtained results [17] (also presented in Sect. 6.2), it can be said that for small imperfections (i.e., less than or equal to 1/100 of the plate or wall thickness), there are no differences in the results after assuming the traditional definition of the dynamic load factor DLF. However, for higher imperfections (i.e., greater than 1/10 of the plate thickness), the relations between the nondimensional deflection and the dynamic load factor are similar to static postbuckling equilibrium paths, which could mean that for highly imperfect structures the dynamic responses are similar to deflection for statically loaded structures. More examples are presented in Sect. 6.2.

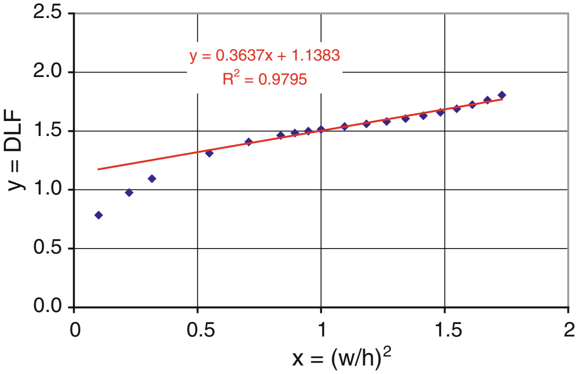


Fig. 5.11 Linear trend line for the alternative method

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