Chapter 1
Introduction

The subject of this monograph is a study of buckling and postbuckling behaviour of thin plates and thin-walled structures with flat walls, subjected to static and dynamic load. The investigations have been carried out in the elastic range. The presented method of solutions as well as the exemplary results of calculations with some conclusions based on the conducted analysis have been the results of author's investigations carried out for the last 10 years.

Buckling and postbuckling behaviour of different structures subjected to static load are very well described in the worldwide literature. In the case of dynamic load – when the dynamic buckling problem is considered – there are numerous papers dealing with shells, some dealing with single plates made of different materials and with different boundary conditions, but still there is a lack of papers describing the behaviour of thin-walled complex structures consisting of flat plates. Some papers dealing with dynamic buckling of thin-walled plate structures have been written by scientists from the Lodz University of Technology [42, 43, 85, 92, 97–101, 108–119, 133–136] and the Lublin University of Technology [89, 106, 181]. This gap in the literature devoted to complex thin-walled structures was the main reason why the author of this monograph has decided to survey the problem of dynamic buckling of thin-walled plate structures.

The basic assumptions, a review of the thin plate theory, the methods used to determine the buckling load and a postbuckling analysis of thin-walled structures subjected to static and dynamic load are presented in this study. Two methods employed for static and dynamic buckling investigations are introduced. The ANSYS commercial software based on the finite element method and own analytical–numerical method developed for about twenty years in the Department of Strength of Materials of the Lodz University of Technology have been used.

The application of two different methods allows for wider understanding of the phenomenon. Two different methods can also enable one to uncouple the phenomena occurring at the same time and to attempt to estimate their impact on the final result.

A general mathematical model, adopted in the proposed analytical–numerical method, enables the consideration of all types of stability loss, i.e. local, global

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(flexural, torsional, flexural–torsional, lateral and distortional) and interactive forms of buckling. The applied analytical–numerical method includes adjacent walls, a shear-lag phenomenon and a deplanation of cross-sections.

The chapter which discusses the finite element method presents both some theoretical as well as practical aspects that have been applied in resolving the issues of stability and dynamic buckling of thin-walled structures with flat walls. The ANSYS software [206], like other commercial programs based on the finite element method, is a closed code. It is almost impossible or very difficult to include own procedures in it. However, the ANSYS software allows one to prepare and include some user's routines, but generally this type of software is used for numerical experiments confirming the theoretical investigations or the results obtained using own software. It should be noted that a good interpretation of results of the analysis requires wide knowledge of the theoretical and analytical solution. In this publication, this is highlighted in chaps 6 to 9 by comparing the calculation results obtained with an application of both the aforementioned methods.

The thin-walled structure is a structure which consists of one or several thin plates or shells connected together at their common edges. Among thin-walled structures, plates, girders, beams, columns and shells are included. It is almost impossible to draw precisely the borderline between thin-walled elements and elements with average thickness. In the literature, one can find the information that the thin-walled rod is the one in which the wall thickness is at least 10 times smaller than the smallest cross-sectional dimension. This monograph presents computational examples of structures fulfilling the above definition of thin-walled structures.

Thin-walled structures have an ability to form freely the cross-section and, thus, to maximize mechanical properties of the material. Therefore, they have been more and more often used in many industries. Thin plates or thin-walled structures are used in sport and automotive industry, aerospace and civil engineering. As an example of such structural elements, a snowboard, a ski or poles can be mentioned, as well as all kinds of crane girders, structural components of automobiles (a car body sheathing and all longitudinal members), aircraft fuselages and wings, supporting structures of walls and roofs of large halls and warehouses.

As it is apparent from the above-mentioned applications, this type of structures can be made of isotropic materials (for example: steel, aluminium), as well as anisotropic and orthotropic ones (different kinds of composite materials, for example: a multilayered fibre composite, a sandwich composite, a functionally graded material, etc.). thin-walled structural elements have several advantages, such as:

- high dimensional accuracy;
- ease of installation;
- dimensional diversity;
- dimensional stability;
- relatively simple manufacturing technology;

- optimal distribution of the material in the cross-section due to the freedom of its formation;
- use of the mechanical properties of the material - lightweight and material savings;
- aesthetic appearance.

All the above structures, as well as many others which can be regarded as the thin-walled ones, exhaust their carrying capacity not by exceeding allowable stresses but by a stability loss. Therefore, not only the critical load but also the postbuckling behaviour of thin-walled structures subjected to static and dynamic load provides essential knowledge for designers. The use of more accurate mathematical models allows them to explore the phenomena occurring after a loss of stability and to describe more precisely their quick and easy software to be used to analyse the behaviour of thin-walled structures. Therefore, it has been decided to explore this issue, propose a mathematical model and a method of analysis of orthotropic thin-walled structures subjected to static and dynamic load.

1.1 Static Buckling and Postbuckling Behaviour

A stability loss or buckling is a system transition from one equilibrium state to another (the bifurcation point), or a jump from the stable to unstable equilibrium path (the limit point). The load resulting in a loss of stability is referred to as the critical load. The behaviour of the structure subjected to load higher than the critical one can be described by a stable (the growth of displacement is caused by increased load - see Fig. 1.1a) or unstable (displacements grow with a decrease in load) postbuckling equilibrium path (Fig. 1.1 b). Typical postbuckling equilibrium paths for such structures like columns (rods), plates, girders and shells are presented in Fig. 1.1.

The postbuckling behaviour of structures depends on their type. For example, cylindrical shells subjected to axial compression change their equilibrium stage (buckling) by the unstable bifurcation point or the limit point (Fig. 1.1e). Long rods or columns subjected to axial compression have usually sudden global buckling (the bifurcation point of the passage to a new postbuckling equilibrium path - Fig. 1.1d). Thin plates supported on all edges lose their stability having the local buckling mode and the stable postbuckling equilibrium path (Fig. 1.1d). The recalled types of buckling and postbuckling behaviour for given thin-walled structures are the same for ideal structures as for structures with geometrical imperfections. Columns made of thin prismatic plates can have the local buckling mode, the global (flexural, torsional or distortional) or coupled one.

In order to acquaint the reader with an application scope of the solution method and the results of calculations, a typical graph presenting the behaviour of thin plates or plated structures with a stable postbuckling equilibrium path is shown in Fig. 1.2.

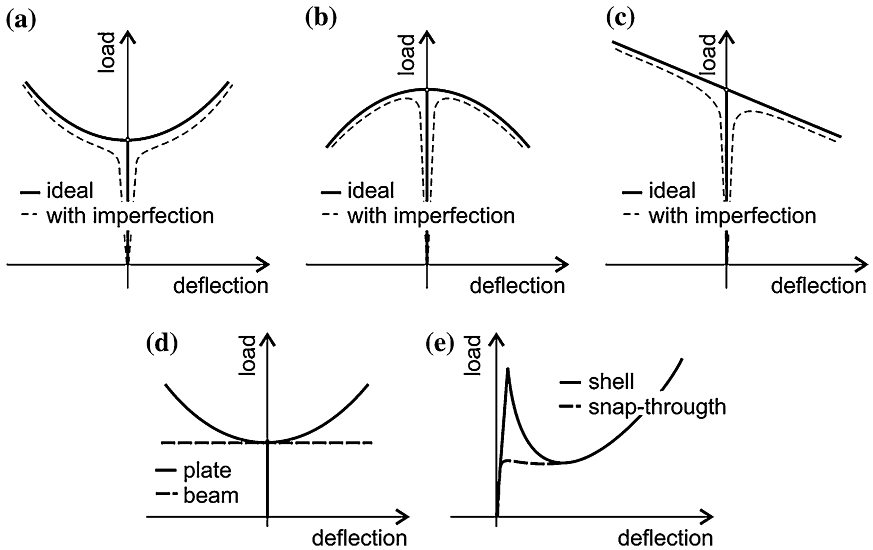
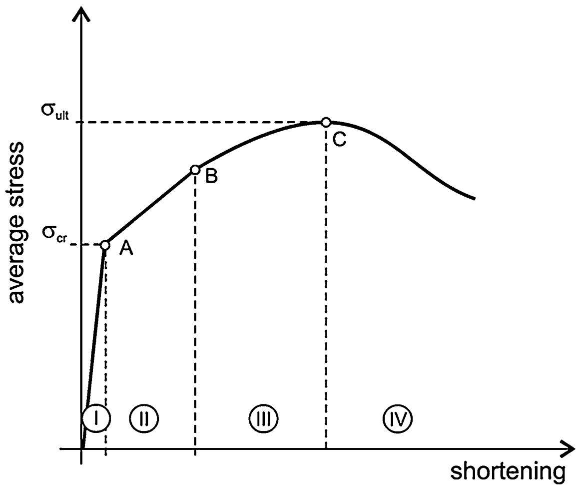


Fig. 1.1 Typical postbuckling equilibrium paths

Fig. 1.2 Diagram of behaviour of a thin-walled structure with a flat wall



The first range (Fig. 1.2) is the prebuckling state. The structures after local buckling (point A – Fig. 1.2) are able to sustain further load (range II – Fig. 1.2) because the displacement increase is only possible by increasing the load value (stable postbuckling equilibrium path). A further increase in load leads to plasticity (point B – Fig. 1.2) or reaching a new, this time unstable, bifurcation point (global buckling). The range III is a postbuckling phase in the elastic–plastic range. The maximal load (point C – Fig. 1.2) is referred to as load carrying capacity after which the failure phase begins (range IV in Fig. 1.2).

The most dangerous form of a stability loss is the interactive buckling (coupled buckling) which usually causes the structure transition to the unstable equilibrium path, which leads to destruction of the structure with the load lower than the critical load corresponding to each mode separately. An interaction of different buckling modes occurs when the critical loads corresponding to different buckling modes are close to each other.

Generally, the buckling mode depends on the slenderness of compressed columns. It also depends on the type of structures, initial geometrical imperfections and also a type of load. Taking into account the length (parameter of the slenderness) of the compressed column, the buckling load can be divided into a local and global mode in the elastic and elastic–plastic range. Fig. 1.3 presents exemplary relations between buckling load and length for the compressed rod (Fig. 1.3 a) and the compressed thin-walled column (Fig. 1.3 b).

For a simple rod, the global buckling mode may be only taken into consideration. For a long and thin column, buckling takes place in the elastic range (slenderness of the rod is higher than the slenderness limit) and the buckling load is determined from the Euler formulae [22]. In the case when the rod is short, i.e., the rod slenderness is less than the limit of slenderness, buckling occurs in the elastic–plastic range and the buckling load may be determined using the well-known Jonson-Ostenfeld, Tetmajer-Jasinski or Rankine-Gordon formulae. For thin-walled columns, the situation is slightly more complicated – depending on length of the column, a different buckling mode may occur (Fig. 1.3b). For very short columns, the local buckling mode in the elastic–plastic range occurs (range I - Fig. 1.3b). For relatively short columns, different local buckling in the elastic range takes place (range II - Fig. 1.3b). For a long compressed column, a different global buckling mode (flexural, torsional, distortional) and their interaction with the local mode may occur (range III - Fig. 1.3b). Finally, for a very long column, the global mode takes place (range IV - Fig. 1.3b).

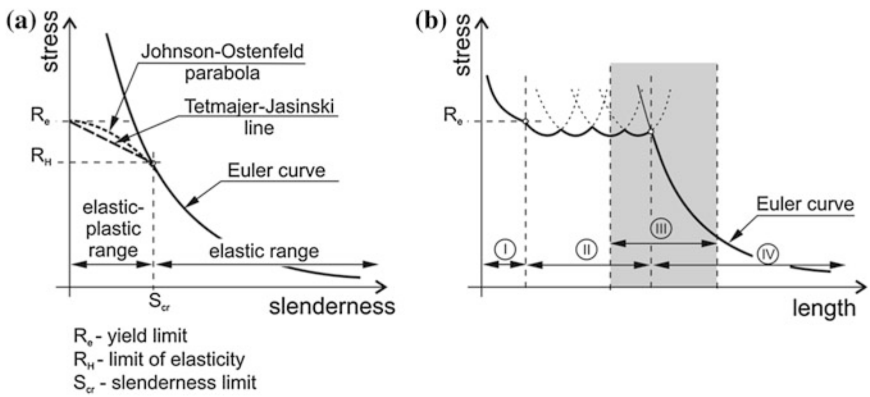


Fig. 1.3 Buckling load vs. slenderness or length of the compressed rod (a) and columns (b)

1.2 Dynamic Buckling

Real structures are not only subjected to static load but also to the dynamic one. The character of the phenomena that occur in the case of dynamic loads is determined by the duration of pulse load and its amplitude. In the literature, a quantity of ‘‘pulse intensity’’ [8] or ‘‘pulse velocity’’ [37] is introduced. In the case when the pulse duration is very short and the magnitude of load amplitude is relatively high, an impact phenomenon is observed. If the pulse duration corresponds to the period of natural vibrations and the magnitude of amplitude has an average value, the dynamic buckling occurs, whereas for a long period of the pulse duration, load is quasi-static.

It should be noted that during a short pulse duration, the dynamic critical load can be higher than the static buckling load.

In the literature, the dynamic buckling problem is analysed mainly for structures subjected to compression loads acting along the axis of the structure. Therefore, the present study has been limited only to the analysis of structures loaded in the plane of plates (walls of the beam-column under analysis), completely without the transverse load. Pulse loads are variable in time and act on the structure immediately, may be of a finite or infinite duration. A time diagram of dynamic loads with a finite duration (pulse loading) may take a parabolic, sinusoidal, rectangular, triangular, exponential or irregular shape [61]. There is a variety of pulses due to attempts to model the real load of the dynamic character. For example, pressure from a wave of the sea hitting a side of the ship or the boat is a sinusoidal pulse. The rectangular pulse models a hitting of bottom of the high-speed motor boat over the surface of water or a mass hitting the structure and then rebounded. Sudden and abrupt manoeuvres of flying objects generate dynamic loads of a trapezoidal shape. The exponential course describes the load caused by explosion, whereas the pulse with a triangular-step shape load describes load changes during a nuclear explosion. The above-mentioned examples have been confirmed by the experimental results which are presented, for example, in [63, 72, 196].

As mentioned above, the dynamic buckling occurs when the loading process is of intermediate amplitude and the pulse duration is close to the period of fundamental natural flexural vibrations (in a range of milliseconds) with a mode corresponding to the static buckling mode. In such a case, the effects of dumping can be neglected according to [96] – an influence of the damping effect on the dynamic response is not greater than 1 %. As shown in [133], the damping effect can be neglected only in the case when the problem is solved in the elastic range.

It should be noted that a dynamic stability loss may occur only for structures with initial geometric imperfections; therefore, the dynamic bifurcation load does not exist. For ideal structures (without geometrical imperfections), the critical buckling amplitude of pulse loading tends to infinity [25]. The dynamic buckling can be considered as strengthening the imperfections - the initial displacement of the structure. The critical dynamic buckling load should be defined on the basis of the assumed criterion.

The precise mathematical criteria were formulated for structures having the unstable postcritical equilibrium path or having the limit point [25, 168]. But for the structures having the stable postbuckling equilibrium path (thin plates, thin-walled beam-columns with the minimal critical load corresponding to local buckling), the precise mathematical criteria have not been defined till now. Therefore, Simitses [168] suggested not defining the dynamic buckling for structures with the stable postbuckling behaviour but rather defining it as a dynamic response to pulse loads. However, many scientists want to have a “tool” allowing them to define the critical amplitude of pulse load causing a loss of stability. Therefore, many authors adopted the criterion formulated for thin shells by Budiansky and Hutchinson [70]. Other scientists solving the dynamic buckling problem of thin plates proposed their own criteria. The oldest and probably the easiest to use is the dynamic stability criterion formulated by Volmir [195]. Other popular criteria are Ari-Gur and Simonetta criteria 8 and the failure criterion proposed by Petry and Fahlbush [147]. With the arrival of new papers [181] dealing with dynamic buckling, new criteria allowing to determine the critical amplitude of the pulse load have appeared - including a new criterion proposed by the author [115] of this publication (see Sect. 5.5).

In the analysis of the behaviour of the structure subjected to pulse loads, the concept of the dynamic load factor *DLF* defined as a ratio of the pulse amplitude to the static buckling load for the perfect structure is introduced. The critical value of the dynamic load factor DLF_{cr}, according to the above-mentioned reason, is determined on the basis of established criteria. The value of DLF_{cr} determines the ability of a structure to sustain dynamic loads. The author of this publication has proposed a new approach for determining the dynamic load factor [119] (see Sect. 5.8).

1.3 Literature Review

The literature overview presented below is focused only on main papers dealing with the elastic static buckling, the postbuckling behaviour and the dynamic buckling of thin-walled plates or structures composed of flat plate (walls). However, some of the most important publications dealing with other thin-walled structures, especially in the case of pulse load, are mentioned as well.

1.3.1 Buckling and Postbuckling Behaviour

Buckling and postbuckling of thin-walled structures subjected to static load have been investigated by many authors for more than one hundred years. The following scientists: Bernoulli and Euler [22], Timoshenko [186] and Volmir [194] should be included in the group of precursors of the investigations on stability of the thin-walled structure problem.

In the worldwide literature, there are numerous papers dealing with linear and nonlinear stability of thin-walled structures subjected to load of different kinds. Nowadays, numerous software packages usually based on the FEM, allowing to one calculate the critical load for most practical structures subjected to any type of load and to analyse their postbuckling behaviour, are available.

The widest development of research on stability of thin-walled isotropic structures took place in the 1970s and the 1980s. The exemplary papers dealing with local buckling of thin-walled structures are papers written by Davis and Hancock [44], Graves-Smith [58] or Mulligan and Pekoz [141]. Buckling and the postbuckling behaviour of isotropic thin-walled structures were analysed, for example, by Graves-Smith [59], Grimaldi and Pignataro [60], Koiter [78] Krolak [102]. Many works are devoted to multimodal (interactive) buckling and some of them are mentioned below. Koiter and Pignataro [80] presented a theoretical basis for the interaction of local and global buckling. Byskov and Hutchinson [29] dealt with the interactive buckling of cylindrical shells. An analysis of the interaction between the global mode and two local buckling modes was proposed by Koiter and van der Neut [81]. A more comprehensive review of the literature concerning the interactive buckling analysis of an isotropic structure can be found, for example, in Ali and Sridharan [5], Benito and Sridharan [21], Byskov [30], Koiter and Pignataro [79], Kolakowski [82–84], Manevich [132], Moellmann and Goltermann [139], Pignataro et al. [148], Pignataro and Luongo [149, 150], Sridharan and Ali [174, 175]. The interactive buckling of orthotropic structures has been discussed, for instance, in [89, 92].

In the wide world literature, works dealing with nonlinear problems of stability of thin-walled structures made of orthotropic materials can be found easily. The oldest work on this subject was published almost 80 years ago. Seydel [160], Smith [169] and Chang [34] dealt with orthotropic plate buckling. Reissner and Stavsky [152] published a study on the critical stress for anisotropic laminated plates with arbitrarily stacked layers. The theoretical background for buckling of composite and anisotropic plates was published by Lekhnitskii [120], Ambartsumyan [6], Ashton and Whitney [9] or Vinson and Chou [192]. In the literature, there are many works on anisotropic plates - among them, March's [137] and Thielmann's [185] works are worth mentioning. Fraser and Miller [53] established the critical load for orthotropic plates using the Ritz method. Mandell [131] presented the results of experimental studies on buckling of anisotropic rectangular plates with simply supported or clamped edges. Chailleux et al. [32] delivered the results of experimental studies on the stability of columns and square laminate plates. Noor [143] in his work presented a comparison between the classical theory of plates, the theory of linear shear and a 3-D theory for elastic stability of orthotropic laminated plates. Chandra and Raju [33] published a study on the postbuckling behaviour of orthotropic rectangular plates with simply supported edges. They analysed plates subjected to load causing uniform shortening of edges. They compared the results of their study with the previously published works.

A similar problem was solved by Prahakara and Chia [151]. They carried out a theoretical analysis of the postbuckling behaviour of orthotropic, rectangular plates with simply supported edges and subjected to biaxial compression. To describe the deformation, a double Fourier series was used. Massey [138] and Brunelle and Oyibo [24] looked for areas of instability for orthotropic plates subjected to pure shear. Libove [123] and Ting with co-authors [187] analysed the unstable behaviour of orthotropic plates under biaxial load.

In the 1980's and the 1990's, scientists widely used analytical–numerical and numerical methods to solve the stability problem. The finite strip method as well as the finite element method started to be used. Dawe and others [45, 46] analysed the postbuckling behaviour of geometrically nonlinear elastic plates and thin-walled prismatic layered composite laminates subjected to load causing uniform edges close-up. They used the finite strip method based on the classical plate theory and the first order deformation theory. Kasagi and Sridharan [75] used the finite strip method to study the stability and the postbuckling behaviour of multilayered composite plates subjected to shear. The authors employed a trigonometric function to describe deflection along the plate and assumed very long plates to decrease the boundary conditions influence. The finite element method was used by Hu and Tzeng [65] to analyse the stability of rectangular plates with elastic fibrous composite laminates with different arrangement of layers. The simply supported or clamped plates subjected to eccentric compressive load were analysed. They employed the commercial software – ABAQUS. Bao et al. [14] used the FEM to analyse the critical stress for flat rectangular orthotropic thin plates with different boundary conditions. Not only plates but also beam-columns made of anisotropic materials were investigated. Barbero and Tomblin [15] dealt with a loss of global stability of thin-walled I-section beam-columns made of various fibre composites. To determine the critical stress, the Southwell method was used. They compared the experimental results with the theoretical ones receiving a very good agreement – the percentage difference between both the methods was less than 6.2 %. Gupta and Rao [62] studied the stability of a thin cantilever beam with a Z-cross-section made of two (45/-45) or three- (0/45/0) layered laminates. The authors employed the finite element method and used two-node beam elements with three degrees of freedom at each node to build a discrete model of the beam under analysis.

In the last decade, Awrejcewicz and co-authors have published monographs [10–13] devoted to dynamics and statics of plates and shells made of iso- and orthotropic materials. They have presented a broad spectrum of analytical and numerical methods applied to solve problems of static stability and vibration of thin-walled structures.

Despite the fact that since the first work on stability of the compressed rod [22] and then other thin-walled structures, more than a century has passed, stability and load carrying capacity of thin-walled structures is still a current topic. Below there are quoted papers published during the last five years and dealing with stability and load carrying capacity of thin-walled structures.

Among the work of Polish authors on this subject, the papers published by Szymczak and Chroscielewski from the Gdansk University of Technology, Tomski and others from the Czestochowa University of Technology, Teter from the Lublin University of Technology, Garstecki, Magnucki and Zielnica from the Poznan University of Technology should be mentioned.

Szymczak [176] deals with stability of the construction of halls modelled as thin-walled frames. It has been proven that the obtained bifurcation point is unsymmetrical and unstable, which can lead to a reduction of critical loads due to some geometrical and loading imperfections.

Chroscielewski et al. in [36] discusses the effect of initial deflection on torsional buckling load of the thin-walled I-beam column. The numerical results obtained using the theory of thin-walled members and the non-linear 6-parameter theory of shells are compared. The authors have observed and analysed the localisation of local buckling modes.

Tomski et al. in their papers [188–190] present results of theoretical and numerical studies on the slender geometrically nonlinear system subjected to non-conservative loading.

Thin-walled beam-columns with intermediate stiffeners have been investigated by Teter and Kolakowski [182, 183]. They have analysed an interaction between global and local buckling and an influence of this interaction on buckling load.

The linear and non-linear stability analyses of double sigma members in the elastic range are analysed by Rzeszut and Garstecki [156, 157]. They use the finite element method to illustrate the importance of proper modelling of structures with slotted connections accounting for initial imperfections. The authors have also modelled the imperfections measured in situ [156] and have analysed their influence on the postbuckling behaviour of structures.

Magnucki with his team have published a few papers [128–130] devoted to global and local stability of cold-formed thin-walled channel beams with open or closed flanges. Papers [128, 130] present a simple analytical description and calculations, the numerical FEM analysis and the laboratory tests of selected beams. The numerical investigations of the optimization problem have been carried out by Magnucki and Paczos [129]. The authors have defined the optimization criterion and the dimensionless objective function as a quality measure.

The main area of Zielnica's interest are sandwich conical and cylindrical shells. In the latest papers, Zielnica et al. present a derivation of the stability equation and the method of solution for an elastic–plastic open conical shell made of orthotropic materials [145]. They take into consideration a bilayered open conical shell subjected to longitudinal force and lateral pressure. The solutions for a freely supported sandwich cylindrical shell with unsymmetrical faces, loaded by longitudinal forces, transversal pressure and shear, can be found in [73]. Paper [205] presents a buckling analysis and equilibrium stability paths of the sandwich conical shell with unsymmetrical faces subjected to combined load.

Kolakowski et al. [88, 90] have analysed the interactive buckling and the postbuckling behaviour of thin-walled columns with different cross-sections using the asymptotic Koiter theory for conservative systems. In [88], Kolakowski and

Kowal-Michalska analyse an influence of the axial extension mode on the interactive buckling of a thin-walled channel subjected to uniform compression.

Krolak with co-workers have performed experimental investigations and a numerical analysis of stability and load carrying capacity of multi-cell thin-walled columns of triangular and rectangular cross-sections [103–105]. The results of FEM calculations have been compared to the theoretical and experimental investigations.

Thin-walled beam-columns with open and closed cross-sections subjected to compression or pure bending have been analysed across the whole world.

Loughlan et al. have conducted a numerical analysis and experimental tests on lipped cross-section [125], I-section and box-section [124] struts. In the numerical analysis, the ANSYS software based on the finite element method has been employed. They have examined buckling, a postbuckled response and the failure mode of thin-walled struts assuming the elastic–plastic material behaviour. They have proposed FEM models and procedures for the determination of the coupled local-distortional interactive response of thin-walled lipped channel sections [125]. Ovesy et al. have employed the finite strip method [144] to carry out a numerical analysis and have compared the obtained results with the FEM and the experiments.

The postbuckling behaviour, the load carrying capacity estimation and the failure mode of stainless steel stub columns [126] and multilayered plate structures [94] have been analysed by Kotelko, Kowal-Michalska, Rhodes and others. Kotelko in cooperation with Dubina and others [191] has made an inventory and classified geometrical and analytical models for local-plastic mechanisms aiming to characterize the ultimate capacity of cold-formed steel sections.

A summary of the recent research on stability, postbuckling behaviour and load carrying capacity of cold-formed steel members and structures is presented in papers written by Rhodes and Macdonald [127, 153]. In [127] they discuss an influence of various aspects on the behaviour of thin-walled members under various loadings. The paper presents the investigation results carried out by students during their short duration programmes. Rhodes in his paper [153] presents the effects of end fixity on the plain channel column behaviour, the effects of transverse impact on struts and the damaged strut capacity and the large deflection behaviour of slender rings under diametrically opposed point loads. Paper [127] deals with cold-formed steel members and discusses the particular characteristics affecting their design.

The progress in computational methods has allowed for development and improvement of original programs useful for the buckling and postbuckling analysis of thin-walled structures. In recent years, two competitive software codes allowing the determination of critical load for uncoupled and coupled buckling have been developed. They enable also analysing the postbuckling behaviour of thin-walled beam-columns. One of them, called GBT [56], has been developed at the Technical University of Lisbon and is based on the generalized beam theory. The second one is called cFSM and is based on the constrained finite strip method. The cFSM software has been developed by Schafer [161] from Johns Hopkins University. The authors of the mentioned software in collaboration with each other

[2] and with other scientists have issued numerous publications explaining the behaviour of thin-walled beam-columns [3, 31, 48, 50, 57, 158, 166]. The developed software has been validated by a comparison of the calculation results with the experimental tests [51, 202] and other analyses using the commercial software based on the finite element method [3, 4, 31, 49, 50, 140, 166, 177].

For many years, the University of Sydney has been a recognized centre in which the experimental studies of long thin-walled beam-columns were carried out. Rasmussen et al. published the results of their experimental investigations on steel lipped channels [20, 155] and I-section columns [18] under axial compression. The interactions of local and overall buckling were considered. On the basis of the obtained test results, the FEM model incorporating the non-linear stress-strain behaviour, anisotropy, enhanced corner properties and initial imperfections was prepared and used in further investigations [17, 19]. The ABAQUS software was employed.

Rasmussen used also the isoparametric spline finite strip method to carry out an elastic buckling analysis of perforated thin-walled structures [52] and inelastic buckling of perforated plates [201]. In the case of study in the inelastic range, several material models were included in the analysis - elastic perfectly-plastic, linear hardening models and models with nonlinear yielding and isotropic hardening.

1.3.2 Dynamic Buckling

In the world literature, the topic of dynamic buckling has been known since the 1930s. The first known papers [93, 178] dealt with Euler buckling of columns subjected to compressive pulse with a finite duration. Those works did not take into account axial inertia forces. The authors noted that the rod took the dynamic load with amplitude higher than the static critical load. A further work carried out by Sevin [159] showed that the effect of axial inertia forces could be neglected as long as the column was in the elastic range.

A faster development of research and analysis of dynamic buckling dates at the 1960s and the early 1970s and is associated with the papers written by Budiansky [25–28, 70] dealing with shells and Volmir's publications [194, 195] devoted to the dynamic stability of thin plates and shells. In contrary to the articles published in the 1930s, the aforementioned works formulates the criteria allowing one to determine the critical value of amplitude of the dynamic load. The criterion allowing for determining the critical dynamic pressure was given by Budiansky and Roth [28]. They studied the response segment of a spherical shell loaded with pulse pressure. The critical pressure is defined as the one that causes a rapid change in strain. For pulse compression, a shell dynamic stability criterion was formulated by Hutchinson and Budiansky [70]. This work is often cited in the literature and concerns the dynamic stability of cylindrical shells with initial geometrical imperfections subjected to rectangular and triangular shape pulse load of a finite duration. The critical value of the amplitude of the pulse is based on the analysis of the system response to dynamic load, saying that the amplitude of pulse load causing unlimited growth of deflection is accepted as the critical load. Budiansky-Hutchinson criterion is adapted by many authors to determine critical loads of thin plates. It has been widely discussed in Sect. 5.2.

Volmir [194, 195] analysed simply supported rectangular plates and shells subjected to compressive pulse and shear pulse load. He proposed a very simple but time consuming method for dynamic buckling load determination (see Sect. 5.1).

The largest group of papers dealing with dynamic buckling is dedicated to shells. The most important are papers written by Budianky et al. [25–28, 70], Schokker, Sridharan and Kasagi [162], Huyan and Simitses [69], Sofiyev [170] Yaffe and Abramovich [200], Bisagni [23] and Virelli, Godoy and Suarez [193].

Another relatively large group of publications are papers related to thin columns with full- or thin-walled cross-sections, but taking into account only buckling in the Euler sense. Cui et al. [37–41, 64] were involved in experimental investigations. They tested columns with different dimensions of rectangular cross-sections and of various lengths. They assumed that the dynamic buckling occurred when the initial deflection and/or the initial speed of deflection were amplified to such a level that it began to grow to infinity. Zhanga and Taheri [204] analysed a dynamic response of composite beam-columns subjected to compression pulse load. The results of theoretical investigations and a comparison with experimental tests of rods impacted with high velocity are presented by Karagiozova and Jones [74]. Anwen and Wenying [7] used the finite strip method to analyse the dynamic buckling of thin rods in the elastic–plastic range. Kenny, Pegg and Taheri [76] used the finite difference method and the finite element method to analyse a dynamic response of rods subjected to pulse load with relative high pulse amplitude and short time of its duration. The results of studies on the influence of accidental geometrical imperfections on dynamic buckling are verified with the experimental tests [77].

The non-linear dynamic buckling analysis of the compressed rod modelled as a series of weightless rods connected one by one by a spring-damper system are presented by Kounadis, Gantes and Simitses [96]. For the assumed discrete model, the Lagrange equation was determined and solved. They defined the critical load as the one that leads to unrestricted movement or large jumps to remote stable equilibrium paths. The use of a damper in the model allowed for analysing the damping influence. The presented example showed that the difference in the results for the structure with viscous Rayleigh damping and without damping was only 0.2 %.

An isolated plate with different boundary conditions can be treated as a single wall of thin-walled structures. Knowing the behaviour of the plate, the local static and dynamic buckling load for beam-columns can be determined. Therefore, numerous publications dealing with the dynamic buckling of thin plates appear in the world literature. The most important are papers written by Volmir [194, 195], Weller, Abramovich and Yaffe [197], Abramovich and Grunwald [1], Ari-Gur and Simonetta [8], Petry and Fahlbush [147]. In the above-mentioned papers, the results of research and analysis of thin plates subjected to dynamic load are presented. An influence of the initial imperfection, different pulse shapes (sine, rectangle) and the pulse duration on a dynamic response of plates were presented. It was noted that dynamic buckling loads were usually greater than the static one but that relationship might be reversed for relatively large initial geometrical imperfections. Abramovich and Grunwald [1] conducted experimental investigations of composite plates with simply supported or clamped edges. Ari-Gur and Simonetta [8] used the finite difference method to solve the problem. Taking into account the inertial forces in the plane of the plate, they analysed the behaviour of isotropic and composite plates subjected to pulse load. On the basis of the results, four criteria of dynamic buckling were formulated (see Sect. 5.3). Taking into account the inertia forces in the plane of the plate, they analysed the behaviour of isotropic and composite plates subjected to pulse load. Petry and Fahlbusch [147] conducted a very comprehensive study examining an impact of the pulse duration to the maximum deflection, comparing different types of pulses – triangular, rectangular and sinusoidal. They also analyzed an influence of the size and the distribution (different number of halfwaves) of the geometry of the imperfection on the value of the dynamic critical load. On the basis of the results, a failure criterion was proposed, stating that the dynamic load is critical when at any point of the structure the equivalent stress was equal to the yield limit (see Sect. 5.7).

Cui, Cheong and Hao in their papers [35, 39] presented the results of experimental results for 15 rectangular plates clamped on loaded edges and with different boundary conditions (clamped - clamped, clamped - simply support, clamped - free edge) on longitudinal edges. All the above-mentioned plates were subjected to pulse load. The authors analysed different failure modes and proposed their own dynamic plastic failure criterion. They also performed a numerical analysis [40, 41] using the ABAQUS commercial software based on the finite element method. For the numerical analysis, they assumed an elastic–plastic model of the material with different coefficients of strengthening, noting that the dynamic critical load in the elastic–plastic range increased with an increasing material strengthening curve in the elastic–plastic state. Cui et al. studied an impact of the damping effect on the dynamic critical load, stating that the damping effect depended on the duration of the pulse and could be omitted when the pulse duration was close or less than the period of natural vibration of the structure under analysis.

The results of the dynamic buckling of the thin composite plate can be found in [43, 54, 97, 107, 108, 110, 146, 154].

In papers written by Papazouglou and Tsouvalis [146] and also Kowal-Michalska, Kolakowski and Czechowski [43, 98], the Galerkin method was used to derive the equation of motion, which was solved using the Runge–Kutta method.

Among the publications on dynamic buckling, there are papers which present an analysis of structures subjected to the simultaneous mechanical and thermal pulse load [55, 67, 68].

One- and bi-axial compression of rectangular plates was considered by Batra and Wei [16]. They studied the dynamic stability of orthotropic rectangular plates with elastic–plastic material models assuming the Hill's criterion of plasticity.

Plates with longitudinal stiffeners were the subject of paper [203] in which the authors present a theoretical analysis of the plates subjected to sinusoidal compression pulse with a duration equal to the period of fundamental natural vibrations. The problem was solved by an analytical–numerical method. To determine the dynamic buckling load, the Budiansky-Roth criterion was applied.

An extensive list of works dealing with dynamic buckling can be found, for example, in the book edited by Kowal-Michalska [99], written by Simitses [167] or Grybos [61]. Grybos' book [61] is the first synthetic scientific description dealing with the dynamic buckling of thin-walled structures subjected to pulse loading. The author presents the investigation results for bars, plates and shells.

Simitses in his work [168] presented the dynamic stability problems depending on the type of structure and its response to static loads. For the structures with "sudden" buckling (called snap-through buckling), he formulated concepts and methodologies to determine dynamic critical conditions. In the case of the structure with stable postbuckling equilibrium path, Simitses proposed that there was not any precise the critical dynamic buckling criterion. For the pulse load, we should rather speak about a dynamic response of the structure than about the dynamic buckling.

Among the wide literature dealing with the dynamic interactive buckling of columns (Euler buckling), only the papers written by Sridharan and Benito [173] and Kolakowski [85] have been found. Sridharan and Benito draw attention to the possibility of an interaction of local and global buckling. Kolakowski focuses on an interaction of various forms of global buckling. Other works on interactive buckling of thin-walled columns have been published by the author of this monograph [112, 113] and in cooperation with Kolakowski [91, 92].

The case of dynamic buckling in the elastic–plastic range, including the viscoplastic effect, has been investigated by Mania and Kowal-Michalska [133, 135, 136]. In [134] Mania has proven a significant impact of the strain rate effect on the dynamic buckling load of short columns. In the world literature, it is also possible to find a paper dealing with dynamic buckling of thin-walled structures subjected to combined load [106]. Czechowski [42] modelled a girder subjected to twisting and bending, considering only one plate subjected to shear and compression. A general summary showing which parameters have an influence on dynamic buckling of plated structures can be found in [100, 101].

Since the time a problem of dynamic stability started to be considered in the world literature, the majority of works has been devoted to shell structures and this trend is still present. During the last five years, numerous publications dealing with dynamic buckling of shells structures have been issued. Some of them to be mentioned are the papers written by Sofiyev with different co-authors [47, 142, 171, 172], Huang and Han [66] and Xu et al. [198, 199]. The latest papers written by Sofiyev are devoted to nonlinear dynamic buckling of cylindrical [172] and truncated conical shells [47, 142, 171] made of functionally graded materials. A similar problem, i.e., nonlinear dynamic buckling of composite cylindrical shells, made of ceramic–metal functionally graded materials, was discussed by Huang and Han [66]. Xu et al. in their papers present the local buckling problem of the cylindrical shell under torsion [199] and the local and global buckling problem of cylindrical shells under axial compression [198].

The theory allowing one to analyse dynamic axial-torsion buckling of structural frames was proposed by Leung [121, 122]. A nonlinear dynamic response of a sandwich plates was analyzed by Shariyat et al. [163–165]. They take into consideration the sandwich beam with SMA hybrid composite face sheets and a flexible core. Shariyat also analysed dynamic buckling of thin plates, paying attention to sandwich or composite structures. He used a finite element formulation based on a higher-order shear deformation theory of vibration and dynamic buckling of the FGM rectangular plate investigation. Paper [163] deals with plates with piezoelectric sensors and actuators subjected to thermo-electro-mechanical loads. The latest papers by Shariyat [164, 165] deal with dynamic buckling of imperfect viscoelastic sandwich plates. He analyses the nonlinear dynamic thermo-mechanical buckling and the postbuckling problem taking into consideration imperfect viscoelastic composite laminated/sandwich plates.

Currently two centres (i.e., the Lodz University of Technology and the Lublin University of Technology) are involved in the dynamic buckling problem of thin-walled structures with flat walls – with the plate model of beam-columns. Among the papers which have not been mentioned yet but have been published in the latest five years, there are publications by Teter [179, 180, 184], Kolakowski [86, 87], Kotelko and Mania [95] and Jankowski [71].

Kolakowski and Teter [86, 179, 180, 184] have investigated the interactive static and dynamic buckling of thin-walled columns with stiffeners subjected to axial compression. They have checked different dynamic buckling criteria, among them one proposed by Teter [179], which is based on phase-plane portraits.

Kotelko and Mania [95] have focused their attention on top hat section and plain channel section columns subjected to uniaxial uniform compression. They have made numerical and experimental investigations of an influence of loading velocity on the structural behaviour of TWCF members. The analysis has been performed using the FEM and the analytical solution based on the plastic yield-line analysis.

Jankowski [71] has used the finite element method software to analyse the dynamic buckling problem of thin-walled girders subjected to pulse loading. He has considered short columns (girders) with open (channel) and closed (rectangular and trapezoidal) cross-sections.

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