Use of Fuzzy Information for Heterogeneous Performance Evaluation

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Abstract. Personnel performance appraisals have been practiced in many organizations and institutions with the purpose for salary adjustments, promotions, training, and other decisions that affect employee status in the company. Human judgments, including preferences are often vague and cannot be estimated in exact numerical values. This paper uses a method under the linguistic framework for heterogeneous performance evaluation, which allocates different weights for assessor members to use linguistic terms in order to express their fuzzy preferences for candidate solutions and for individual judgments. The introduced method has been used in the empirical study, and the results have been analyzed.

Keywords: Performance evaluation, Group decision making, Fuzzy numbers.

1 Introduction

Fan and Zhang [4] and Chuu [5] stated that human beings are faced with issues of decision making that basically involves choosing the most-preferred alternatives from a limited set of alternatives to obtain certain-predefined objectives. Evaluating personnel is one of the most critical decisions that must be taken [6]. Most crucial and significant decisions in organizations are made by groups of managers or experts. There are two types of group decision making: (a) homogeneous and (b) heterogeneous. Contrary to homogeneous group decision making, the

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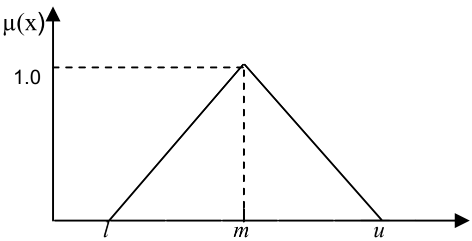
heterogeneous decision making considers opinions from decision makers that constitutes of different gender, age, education, functional specialization and expertise [7-9]. In such an environment, disagreement always happens in group decision making as members in a group generally do not come to the same decision [10]. To solve disagreements for one decision maker that involves multi criteria evaluation and ranking problems, the multi criteria decision making (MCDM) has been developed [11-16]. Crisp data are insufficient to simulate real life situations and managers' judgments normally include preferences are often vague and not precise estimates of the numerical value [17]. Chuu [5] stated that the traditional MCDM methods are random processes and deterministic, and unable to solve group decision making problems with inaccurate and vague information. Therefore, fuzzy MCDM methods were developed. The concept of fuzzy sets is one of the most important and significant instruments in the computational intelligence [18]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modelling in uncertain systems in industry, nature and humanity and act as facilitators for common-sense reasoning in decision making in the absence of complete and accurate information [19]. Therefore, designing applicable multi-dimensional appraisal systems for heterogeneous appraisers has been a main concern for scholars. This paper is trying to use heterogeneous group decision making model under fuzzy environment for personnel performance appraisal.

2 Preliminaries

Definition 2.1. *A* fuzzy set presents a boundary with a gradual contour, by contrast with classical sets, which present a discrete border. Let U be the universe of discourse and *u* a generic element of U, then $U = \{u\}$. A fuzzy subset Ã, defined in U, is: $\tilde{A} = \{(u, \mu_{\tilde{A}}(u)) \mid u \in U\}$, Where $\mu_{\tilde{A}}(u)$ is designated as membership function or membership grade of *u* in Ã[20].

Definition 2.2. A is a fuzzy number, if A is normal and convex [21]. A triangular fuzzy numbers can be expressed as M = (*l*, *m*, *u*), where $l \leq m \leq u$, in which $l \leq m \leq u$. In the fuzzy event, parameters (*l*, *m*, *u*) are the smallest, promising, and the largest possible value, correspondingly [22]. Equation (1) describes the triangular fuzzy number membership function M, when *l*=*m*=*u*, it is a non-fuzzy number by agreement as shown in Fig. 1 [23]



$$\mu_m(x) = \begin{cases} 0 & x < l \\ \dfrac{(x-l)}{(m-l)} & l \leq x \leq m \\ \dfrac{(u-x)}{(u-m)} & m \leq x \leq u \\ 0 & x > u \end{cases} \quad (1)$$

Fig. 1 The triangular fuzzy membership function [2]

Definition 2.3. For two triangular fuzzy numbers M_1 and M_2 of the main operational laws are as follows [24]:

$$M_1 + M_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2), \quad \omega \times M_1 = (\omega l_1, \omega m_1, \omega u_1),\ \omega>0,\ \omega \in R,$$
$$M_1 \times M_2 = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2), \quad M_1^{-1} = (1/u_1, 1/m_1, 1/l_1). \tag{2}$$

Definition 2.4. A variable with values of word or sentences in an artificial language is defined a linguistic variable [25]. After the cardinality of the linguistic terms set are recognized, linguistic terms and semantics that must be arranged as show in (Tables 1 & 2)..

Table 1 Linguistic variables for the ratings [1]

Very Poor	VP	(0, 0, 1)
Poor	P	(0, 1, 3)
Medium Poor	MP	(1, 3, 5)
Fair	F	(3, 5, 7)
Medium Good	MG	(5, 7, 9)
Good	G	(7, 9, 10)
Very Good	VG	(9, 10, 10)

Table 2 Linguistic variables for the importance weight of each criterion [3]

Very Low	VL	(0, 0, 0.1)
Low	L	(0, 0.1, 0.3)
Medium Low	ML	(0.1, 0.3, 0.5)
Medium	M	(0.3, 0.5, 0.7)
Medium High	MH	(0.5, 0.7, 0.9)
High	H	(0.7, 0.9, 1.0)
Very High	VH	(0.9, 1.0, 1.0)

3 The Fuzzy Heterogeneous Performance Evaluation Method

The purpose of this method is to enhance group agreement on the group decision making outcome based on Borda count. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, $P = \{P_1, P_2, \dots, P_k\}$ be the set of decision makers, and $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_p)$ be the weight vector of decision makers, where $\lambda_p \geq 0$, $P = 1,2,\dots, k$, and $\sum_{p=1}^{k} \lambda_p = 1$. Let $C = \{C_1, C_2, \dots, C_n\}$ be the set of attributes, and $w = (w_1, w_2, \dots, w_n)$ be the weight vector of attributes, where $w_n \geq 0$, $n = 1,2,\dots, j$, $\sum_{n=1}^{j} w_n = 1$ [26]. The fuzzy group decision problem can be concisely expressed as matrix format [27]:

$$\tilde{P}_t = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{3}$$

$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n]$ Where $\tilde{x}_{ij}^k$ and $\tilde{w}_j^k$ are linguistic variables that can be shown by fuzzy numbers as shown in (Tables 1, 2).

The proposed models are linearly described in the following 11 steps:

1- Identifying evaluation criteria. 2- Generating alternatives. 3- Identifying weights of criteria and weights of decision makers. 4- Presenting preferences on the part of each decision maker (every decision maker gives preferences to per alternative based on every attribute according to linguistic terms such as Table 2.

5- Construction of fuzzy decision matrix. In fuzzy decision matrix, we suppose that, each $\tilde{x}_{ij}^{k}$ is fuzzy number. 6- Construct the normalized fuzzy decision matrix that can be found in [28, 29]. If $(\tilde{x}_{ij}, i = 1,2,\ldots,m, j = 1,2,\ldots,n)$ are triangular fuzzy numbers, then the normalization process can be performed by [25]:

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right) \quad \tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) \qquad i = 1,2,\ldots,m, \qquad j \in C \tag{4}$$

Where B and C are the set of benefit criteria and cost criteria, respectively. 7- Construction of defuzzification decision matrix; the defuzzified value of fuzzy number can be obtained from Equation

$$BNP_i = [(U_i - L_i) + (M_i - L_i)]/3 + L_i, \ \forall i \tag{5}$$

8- Considering proper value (DM weights) of every decision making group member idea $N_{ij\lambda} = N_{ij} \times \lambda_p$ (6). N_{ij} is an element of defuzzification decision matrix for every DM, and λ_p is the weight of per DM idea. 9- Formation of R_j matrixes; while the rows of the matrix are alternatives and its columns are DMs opinions based on *j* criterion. So *n* matrixes in lieu of *j* attributes were established (R_j):

$$R_j = \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} r_{1,j}^1 & \cdots & r_{1,j}^p & \cdots & r_{1,j}^k \\ \vdots & & \vdots & & \vdots \\ r_{i,j}^1 & \cdots & r_{i,j}^p & \cdots & r_{i,j}^k \\ \vdots & & \vdots & & \vdots \\ r_{m,j}^1 & \cdots & r_{m,j}^p & \cdots & r_{m,j}^k \end{bmatrix} \quad \begin{matrix} , i = 1,2,\ldots,m \\ , j = 1,2,\ldots,n \\ , p = 1,2,\ldots,k \end{matrix} \tag{7}$$

Computing linear sum in lieu of P decision makers $\left(\sum_{p=1}^{k} r_{i,j}^p\right)$ and final grade of every alternative in lieu of *j* attributes would be calculated.

In this matrix the line with the highest mark is the first rank and the line with the lowest mark is *m* rank.

$$R_G = \begin{matrix} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{matrix} \begin{matrix} C_1 & \cdots & C_j & \cdots & C_n \end{matrix} \\ \begin{bmatrix} r'_{1,1} & \cdots & r'_{1,j} & \cdots & r'_{1,n} \\ \vdots & & \vdots & & \vdots \\ r'_{i,1} & \cdots & r'_{i,j} & \cdots & r'_{i,n} \\ \vdots & & \vdots & & \vdots \\ r'_{m,1} & \cdots & r'_{m,j} & \cdots & r'_{m,n} \end{bmatrix} \end{matrix} \tag{8}$$

10- Changing R_G matrix into Borda count, i.e. alternative with first rank based on per criterion would have *m-1* relative value on the basis of *m* alternatives. The same goes for, alternative with second rank (*m-2* relative value). Alternatives with *m* rank would receive zero relative values. We multiply the Borda count matrix with the corresponding weight vector of attributes function [2]

$$\begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m,1} & b_{m,2} & \cdots & b_{m,n} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} b_{1,1}.w_1 + & b_{1,2}.w_2 + & \cdots & b_{1,n}.w_n \\ b_{2,1}.w_1 + & b_{2,2}.w_2 + & \cdots & b_{2,n}.w_n \\ \vdots & \vdots & \ddots & \vdots \\ b_{m,1}.w_1 + & b_{m,2}.w_2 & \cdots & b_{m,n}.w_n \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \tag{9}$$

The alternative sum with the highest value would be considered as the first rank and the lowest represents the last rank.

4 Empirical Study

Table 3 Fuzzy performance measurement of assessor group done by professionals

Professional Team	Assessors			
	Manager	Colleagues	Inferior	Employee him/herself
kjkjlProf_1	VH	MH	MH	H
Prof_2	H	MH	M	ML
Prof_3	VH	H	MH	H
Prof_4	VH	H	VH	H
Prof_5	H	M	MH	L
Prof_6	H	M	M	MH
Prof_1	0.9,1.0,1.0	0.5,0.7,0.9	0.5,0.7,0.9	0.7,0.9,1.0
Prof_2	0.7,0.9,1.0	0.5,0.7,0.9	0.3,0.5,0.7	0.1,0.3,0.5
Prof_3	0.9,1.0,1.0	0.7,0.9,1.0	0.5,0.7,0.9	0.7,0.9,1.0
Prof_4	0.9,1.0,1.0	0.7,0.9,1.0	0.9,1.0,1.0	0.7,0.9,1.0
Prof_5	0.7,0.9,1.0	0.3,0.5,0.7	0.5,0.7,0.9	0.0,0.1,0.3
Prof_6	0.7,0.9,1.0	0.3,0.5,0.7	0.3,0.5,0.7	0.5,0.7,0.9
Total	5.500	4.133	4.067	3.733
Weight	0.32	0.24	0.23	0.21

This study was performed in Energy Efficiency Organization active in the field of energy consumption optimizing. Twenty-one persons were selected as a professional group according to the president of EEO organization opinion based on their experiences and education in order to determine the criteria, weights, and assessors' viewpoint weights as shown in Table. 3. In this paper the organization's middle managers from deputy of training and optimizing energy consumption were evaluated. So each manager was assessed by 4 assessors based on 33 quality and quantity attributes. A_1, A_2, A_3, A_4 was allocated to 4 managers. In step 4 of algorithm method, the assessors presented preferences for each alternative based on each attribute according to linguistic terms based on Table. 4. Next step constructed the normalized fuzzy decision matrix and converted the normalized fuzzy decision matrix to the defuzzification decision matrix by (Equation (6) as shown in (Table . 5). Step 8, 9: Considering proper value (DM weights) of every decision making group member idea by (Equation (7) and establishing n matrixes lieu of j attribute as (Table . 6). Step 10: In (Table . 6) Linear sum would be reached in lieu of P decision makers and final rank of every alternative in lieu of j attribute would be calculated. In these matrixes, the line with the highest mark is the first rank and the line with the lowest mark is m rank. Step 11: We change the R_G matrix into Borda count; multiply the Borda count matrix with the corresponding weight vector of attributes by (Equation (9). The alternative sum with the highest value would be considered as the first rank and the lowest represents the last rank. The ordinal ranks of four alternatives (Middle managers) are attained as follows: $A_3 >> A_4 >> A_1 >> A_2$. Therefore, A_3 is the optimal candidate.

Table 4 The ratings of four candidates by decision makers under all criteria

	C_1				C_2				C_3			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
P_1	MG	G	F	G	G	G	F	VG	G	G	VG	VG
P_2	G	G	G	VG	MG	G	MG	VG	VG	VG	G	VG
P_3	F	F	G	MG	F	F	G	G	G	G	VG	G
P_4	G	G	VG	MG	MG	G	VG	G	VG	VG	VG	VG

Table 5 The fuzzy normalized decision matrix and criteria weights

		C_1		C_2		C_3	
P_1	A_1	0.5,0.7,0.9	0.700	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867
	A_2	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867
	A_3	0.3,0.5,0.7	0.500	0.3,0.5,0.7	0.500	0.9,1.0,1.0	0.967
	A_4	0.7,0.9,1.0	0.867	0.9,1.0,1.0	0.967	0.9,1.0,1.0	0.967
P_2	A_1	0.7,0.9,1.0	0.867	0.5,0.7,0.9	0.700	0.9,1.0,1.0	0.967
	A_2	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867	0.9,1.0,1.0	0.967
	A_3	0.7,0.9,1.0	0.867	0.5,0.7,0.9	0.700	0.7,0.9,1.0	0.867
	A_4	0.9,1.0,1.0	0.967	0.9,1.0,1.0	0.967	0.9,1.0,1.0	0.967
P_3	A_1	0.3,0.5,0.7	0.500	0.3,0.5,0.7	0.500	0.7,0.9,1.0	0.867
	A_2	0.3,0.5,0.7	0.500	0.3,0.5,0.7	0.500	0.7,0.9,1.0	0.867
	A_3	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867	0.9,1.0,1.0	0.967
	A_4	0.5,0.7,0.9	0.700	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867
P_4	A_1	0.7,0.9,1.0	0.867	0.5,0.7,0.9	0.700	0.9,1.0,1.0	0.967
	A_2	0.7,0.9,1.0	0.867	0.7,0.9,1.0	0.867	0.9,1.0,1.0	0.967
	A_3	0.9,1.0,1.0	0.967	0.9,1.0,1.0	0.967	0.9,1.0,1.0	0.967
	A_4	0.5,0.7,0.9	0.700	0.7,0.9,1.0	0.867	0.9,1.0,1.0	0.967
Criteria weights		0.0335		0.0335		0.0299	

Table 6 Aggregation matrix (R_G) based on per criterion (Middle managers)

C_1	P_1	P_2	P_3	P_4	∑	R	C_2	P_1	P_2	P_3	P_4	∑	R
A_1	0.22	0.20	0.11	0.18	0.72	4	A_1	0.27	0.16	0.11	0.14	0.70	4
A_2	0.27	0.20	0.11	0.18	0.78	2	A_2	0.27	0.20	0.11	0.18	0.78	2
A_3	0.16	0.20	0.19	0.20	0.77	3	A_3	0.16	0.16	0.19	0.20	0.73	3
A_4	0.27	0.23	0.16	0.14	0.81	1	A_4	0.30	0.23	0.19	0.18	0.92	1

C_{33}	P_1	P_2	P_3	P_4	∑	R
A_1	0.27	0.23	0.19	0.20	0.91	3,4
A_2	0.27	0.23	0.19	0.20	0.91	3,4
A_3	0.30	0.20	0.22	0.20	0.94	2
A_4	0.30	0.23	0.19	0.20	0.94	1

$$\begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} 4 & 4 & \cdots & 3,4 \\ 2 & 2 & \cdots & 3,4 \\ 3 & 3 & \cdots & 2 \\ 1 & 1 & \cdots & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & \cdots & 0.5 \\ 2 & 2 & \cdots & 0.5 \\ 1 & 1 & \cdots & 2 \\ 3 & 3 & \cdots & 3 \end{bmatrix} \times \begin{bmatrix} 0.0335 \\ 0.0335 \\ \vdots \\ 0.0299 \end{bmatrix} = \begin{bmatrix} 1.259 \\ 0.663 \\ 2.055 \\ 2.023 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix}$$

Table 7 Comparison of proposed method and pervious personnel performance appraisal (Middle managers)

Candidates	A_1	A_2	A_3	A_4
Previous assessment grade	29.5	29	29.5	29.5
Fuzzy heterogeneous performance evaluation method	3	4	1	2

5 Conclusion

The personnel evaluation is one of the most important and complicated aspects of human resource management. A new proposed personnel performance appraisal model was used in this study, in which personnel are evaluated from different points of view and evaluation's errors are minimized. In multi criteria group decision making with linguistic variables, the assessors may have vague information, limited attention and different information processing capabilities. This paper uses a fuzzy group decision making method which allows group members to express their fuzzy preferences in linguistic terms for candidate selection and for individual judgments. The proposed method covered heterogeneous performance evaluation by considering the decision makers' viewpoint weights. The results of the mentioned models are compared with the pervious personnel performance appraisal model (the evaluation of the subordinate from manager point of view) Table . 7.

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