Maximality-Based Labeled Transition Systems Normal Form

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Abstract. This paper proposes an algorithm (functional method) for reducing Maximality-based Labeled Transition Systems (MLTS) modulo a maximality bisimulation relation. For this purpose, we define a partial order relation on MLTS states according to a given maximality bisimulation relation. We prove that a reduced MLTS is unique. In other word, it provides a normal form.

Keywords: Formal concurrency semantics, Maximality semantics, Maximality-based labeled transition systems, Bisimulation relation, Complete partial order.

1 Introduction

Action refinement has been deeply studied for characterising true concurrency semantics. For this purpose, several authors have proposed new semantics, and in the same context new equivalence relations proven to be preserved under action refinement and supporting action duration (See [6] for survey). The ST-semantics is one of the most studied of these propositions, originally defined in [8] over Petri nets, in which semantics, non atomic actions are split into start and end sub actions. The ST-semantics has been applied in the literature to process algebras [1,9].

The interleaving ST-bisimulation (ST-bisimulation in short) without silent moves has been defined on Petri nets [8], and further on prime event structures [7]. In [13], an alternative definition of the ST-bisimulation has been proposed for prime event structures with silent moves; the main point of this definition is that it does not require any-more to split actions as previously, the partial order relation among events being used instead for determining the set of maximal events in each configuration. The same idea has been used in [5] for defining the maximality preserving bisimulation on labeled P/T nets, and with the hypothesis that all visible actions are non atomics, the maximality preserving bisimulation coincides with the ST-bisimulation.

For implementing the ST-Bisimulation relation, in [3] an algorithm for a particular process algebra has been proposed. This approach consists in verifying the ST-bisimulation relation between process algebra terms; actions are split

A. Amine et al. (Eds.): *Modeling Approaches and Algorithms*, SCI 488, pp. 337–346.
DOI: 10.1007/978-3-319-00560-7_37

into start and end sub actions. Then, the proposed solution is ad hoc to the considered process algebra. Dealing with non dependability between concurrency semantic model and specification models, another concurrency semantic model, named maximality-based labeled transition system, has been defined in the literature and used for expressing the semantics of process algebras and P/T Petri net with the hypothesis that actions are not necessary atomic [4,11,12], i.e. actions are abstractions of finite processes and elapse in time. The main interest of maximality-based labeled transition system model is that it can be implemented and used in verification without splinting actions into starts and ends sub actions. In this paper, given a maximality-based labeled transition system *mlts* and a maximality bisimulation relation $\mathfrak{R}$ on *mlts* nodes, we propose an algorithm like a recursive function (functional implementation) for reducing *mlts* w.r.t $\mathfrak{R}$.

Let us take the example in [11]. Consider the Petri net of Fig.1.(a). By applying the approach of [12], the corresponding maximality-based labeled transition system of this Petri net is given by Fig.1.(b).

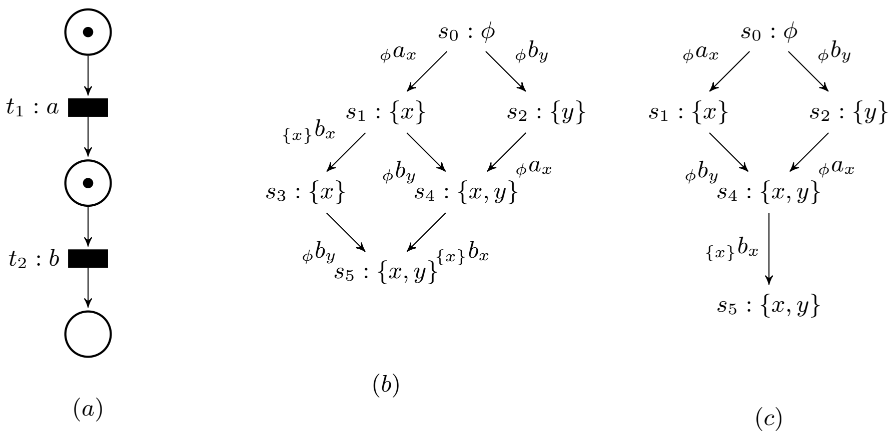

Fig. 1. Operational semantics of a Petri net in terms of MLTS

At first, recall that a maximality-based labeled transition system is given by a graph labeled on both states and transitions. Each state is labeled by a set of event names. Each event name identifies the start of execution of an action (eventually under execution) which occurred before this state. This action is said to be potentially under execution in this state. A transition between two states s_i and s_j is labeled by a 3-uple (M, a, x) (denoted ${}_M a_x$) where x is the event name identifying the start of execution of the action a and M denotes the set of event names representing some causes of the action a. Elements of M belong to state s. Occurrence of this transition terminates actions identified by M, thus, the set of event names corresponding to state s_j is that of s_i from which we subtract the set M and add the event name x. Formal definition of a maximality-based labeled transition system will be given in Sect.2.1.

In the initial state (state s_0) of the maximality-based labeled transition system of Fig.1.(b), no action is running, from where the association of the empty set with this state. From state s_0, actions a and b can start their execution independently, their starts are respectively identified by event names x and y. a and b can be launched in any order. The set $\{x\}$ (resp. $\{y\}$) in state s_1 (resp. s_2) stipulates that the action a (resp. b) are potentially under execution in this state. The set $\{x, y\}$ in s_4 shows that actions a and b can be executed simultaneously.

Note that when the system is in state s_1, while the action a has not been terminated yet, the only evolution concerns the start of b. However, when a terminates, we can start the action b caused by a or the action b which is independent from the end of a. Resulting states are respectively s_3 and s_4. We can observe that from state s_3, the start of b is always possible. However, the same ending constraint of a is imposed for the execution of b at the level of state s_4. Note that causal dependence between execution of b across from the action a is captured by the consumption of the produced token coming from the transition t_1 during the firing of t_2 in the Petri net.

Notice that from state s_1, transitions leading respectively to states s_3 and s_4 are due to the firing of the same transition t_2. In the first firing, the token of the initial marking is used whereas in the second firing, the used token is that produced by the firing of t_1. On the other hand, as noted above, the derivation by b leading to state s_4 is not conditioned by the end of the action a, while the derivation leading to state s_3 is conditioned by the end of a.

By maximality bisimulation relation, we can omit the derivations $s_1 \longrightarrow s_3 \longrightarrow s_5$ in the maximality-based labeled transition system of Fig.1.(b). In other words, the maximality-based labeled transition system of Fig.1.(c) is the reduced system modulo maximality bisimulation relation which preserve action refinement.

The paper is organized as follows. In Sect.2, we give the definition of a maximality-based labeled transition system and maximality bisimulation relation. In Sect.3, we define a partial order relation on a maximality bisimulation relation by witch an algorithm for reducting of maximality-based labeled transition systems modulo maximality bisimulation relation is described as a functional implementation. This paper is ended by some conclusions of this work. Proofs can be found in [2].

2 Preliminaries [4,10]

2.1 Maximality-Based Labeled Transition Systems

Definition 1. *Let $\mathcal{M}$ be a countable set of event names, a maximality-based labeled transition system of support $\mathcal{M}$ is a tuple $(\Omega, \lambda, \mu, \xi, \psi)$ with:*

- $\Omega = (S, T, \alpha, \beta, s_0)$ *is a transition system such that:*
  - S *is the set of states in which the system can be found, this set can be finite or infinite.*

  - *T is the set of transitions indicating state switch that the system can achieve, this set can be finite or infinite.*
  - *α and β are two applications of T in S such that for all transition t we have: $\alpha(t)$ is the origin of the transition and $\beta(t)$ its goal.*
  - *s_0 is the initial state of the transition system Ω.*
- *(Ω, λ) is a transition system labeled by the function λ on an alphabet Act called support of (Ω, λ). In the other word $\lambda : T \to Act$.*
- *$\psi : S \to 2^{\mathcal{M}}$ is a function which associates to each state the finite set of maximal event names present in this state.*
- *$\mu : T \to 2^{\mathcal{M}}$ is a function which associates to each transition the finite set of event names corresponding to actions that have already begun their execution and the end of their executions enables this transition.*
- *$\xi : T \to \mathcal{M}$ is a function which associates to each transition the event name identifying its occurrence.*

such that $\psi(s_0) = \phi$ and for all transition t, $\mu(t) \subseteq \psi(\alpha(t))$, $\xi(t) \notin \psi(\alpha(t)) - \mu(t)$ and $\psi(\beta(t)) = (\psi(\alpha(t)) - \mu(t)) \cup \xi(t)$

Note 1. In what follows, we use the following assumptions:

- In this present paper we suppose the uniqueness of event name.
- Let $mlts = (\Omega, \lambda, \mu, \xi, \psi)$ a maximality-based labeled transition system such that $\Omega = \langle S, T, \alpha, \beta, s_0 \rangle$. $t \in T$ is a transition for which $\alpha(t) = s$, $\beta(t) = s'$, $\lambda(t) = a$, $\mu(t) = E$ and $\xi(t) = x$. The transition t will be noted $s \xrightarrow{E a_x} s'$.
- Let $f : E \to F$ be a function (bijection) such that domain $Dom(f) = E$ and codomain $Cod(f) = F$, and let D (respectively C) be a subset of E (respectively of F). Restrictions of f with respect to its domain and codomain are defined by:
  - $f \lceil D = \{(x, y) \in f | x \in D\}$
  - $f \lfloor C = \{(x, y) \in f | y \in C\}$
- $\mathfrak{F} \subseteq 2^{\mathcal{M} \times \mathcal{M}}$ is the set of all bijective functions between subsets of $\mathcal{M}$.
- Id_A is the identity function on elements of a set A.
- For $s \in S$ and $Y \subseteq S$: $T_{a_x}[s] = \{s' | (s,_M a_x, s') \in T\}$ and $T_{a_x}[Y] = \cup\{T_{a_x}[s] | s \in Y\}$.
- The set of Maximality-based labeled transition systems is noted $\mathfrak{Mlts}$.

2.2 Maximality Bisimulation Relation

Definition 2. *Let $mlts_1 = (\Omega_1, \lambda_1, \mu_1, \xi_1, \psi_1)$ and $mlts_2 = (\Omega_2, \lambda_2, \mu_2, \xi_2, \psi_2)$ be two maximality-based labeled transition systems such that $\Omega_1 = \langle S_1, T_1, \alpha_1, \beta_1, s_0^1 \rangle$ and $\Omega_2 = \langle S_2, T_2, \alpha_2, \beta_2, s_0^2 \rangle$. $mlts_1$ and $mlts_2$ are said to be* maximally bisimilar*, noted $mlts_1 \approx_m mlts_2$, if there is a relation $\mathfrak{R} \subseteq S_1 \times S_2 \times \mathfrak{F}$ with*

1. *$(s_0^1, s_0^2, \emptyset) \in \mathfrak{R}$. Initial states of $mlts_1$ and $mlts_2$ are related by the relation. Since the sets of maximal events in initial states are empty, the function relating these two sets is empty.*

2. If $(s_1, s_2, f) \in \mathfrak{R}$ *then*
   (a) $Dom(f) \subseteq \psi(s_1)$ *and* $Cod(f) \subseteq \psi(s_2)$.
   (b) If $s_1 \xrightarrow{E a_x} s_1'$ *then there is* $s_2 \xrightarrow{F a_y} s_2'$ *such that*
      i. $\forall (u, v) \in f$, *if* $u \notin E$ *then* $v \notin F$
      ii. $(s_1', s_2', f') \in \mathfrak{R}$ *with* $f' = (f\lceil(\psi(s_1') - \{x\}))\lfloor(\psi(s_2') - \{y\}) \cup \{(x, y)\}$
   (c) If $s_2 \xrightarrow{F a_y} s_2'$ *then there is* $s_1 \xrightarrow{E a_x} s_1'$ *such that*
      i. $\forall (u, v) \in f$, *if* $v \notin F$ *then* $u \notin E$
      ii. $(s_1', s_2', f') \in \mathfrak{R}$ *with* $f' = (f\lceil(\psi(s_1') - \{x\}))\lfloor(\psi(s_2') - \{y\}) \cup \{(x, y)\}$.

3 Partial Order on a Maximality Bisimulation Relation

In this section, we assume a given $mlts = (\Omega, \lambda, \mu, \xi, \psi)$ to be a maximality-based labeled transition system such that $\Omega = \langle S, T, \alpha, \beta, s_0 \rangle$ and $\mathfrak{R}$ a maximality bisimulation relation on *mlts* states. We define two partial order relations. The first relation is over a set of states of mlts. The second relation is over the set of maximality-based labeled transition systems. This last relation will be used for computing the normal form of a maximality-based labeled transition system. We prove that both relations are complete partial orders. These partial order relations will be used to define a recursive function[1] of reduction of maximality-based labeled transition systems modulo maximality bisimulation relation. The reduced maximality-based labeled transition system constitutes its normal form.

3.1 Partial Order Over a Set of States

Definition 3. *Let* $(s, s', f) \in \mathfrak{R}$, $s \leq s'$ *if and only if:*

$$\forall x \in \psi(s) : \exists y \in \psi(s') \text{ and } (x, y) \in f.$$

Proposition 1. *Given* $(s, s', f) \in \mathfrak{R}$*:*

1. *We have* $s \leq s'$ *or* $s' \leq s$.
2. *The relation* $\leq$ *is a partial order.*
3. $(S, \leq)$ *is a Complete Partial Order (CPO).*

Example 1. In $mlts_1$ of Fig.2, we have $(s_2, s_3, f_1) \in \mathfrak{R}$, $(s_4, s_5, f_2) \in \mathfrak{R}$ and $(s_5, s_6, f_3) \in \mathfrak{R}$ such that $f_1 = \{(z, y)\}$, $f_2 = \{(z, y), (t, u)\}$ and $f_3 = \{(u, v), (x, x)\}$. In other words, we have $s_2 \leq s_3$, $s_4 \leq s_5$ and $s_6 \leq s_5$.

In this CPO, the chains which have the same least upper bound forms a partition on S. This partition is formally defined by Definition.4.

Definition 4. *Let* $D = (S, \leq)$ *be a CPO, we can define a partition* Υ_D *as follow:* $\Upsilon_D = \{B_i \subseteq S |$ *for any* $X \subseteq B_i$, *if* $\exists Y \subseteq S$ *with* $\bigsqcup X = \bigsqcup Y$ *then* $Y \subseteq B_i\}$. *Such that* X *and* Y *be two chains over* D.

Example 2. Given $mlts_1$ of Fig.2, we have $\Upsilon_D = \{\{s_0\}, \{s_1\}, \{s_2, s_3\}, \{s_4, s_5, s_6\}\}$, its stems from the fact that $\sqcup\{s_0\} = s_0$, $\sqcup\{s_1\} = s_1$,$\sqcup\{s_2, s_3\} = s_3$ and $\sqcup\{s_4, s_5\} = \sqcup\{s_5, s_6\} = s_5$.

[1] It is straightforward to propose an imperative algorithm

3.2 Normal Form of a Maximality-Based Labeled Transition System

Definition 5. *Let $mlts = (\Omega, \lambda, \mu, \xi, \psi)$ and $mlts' = (\Omega', \lambda, \mu', \xi', \psi')$ be two maximality-based labeled transition systems such that $\Omega = \langle S, T, \alpha, \beta, s_0 \rangle$ and $\Omega' = \langle S', T', \alpha', \beta', s'_0 \rangle$, we define a relation $\leq$ on $\mathfrak{Mlts} \times \mathfrak{Mlts}$ as follows: $mlts \leq mlts'$ if and only if : $\forall s \in S. \exists s' \in S'$ such that $s \leq s'$.*

Proposition 2. *$(\mathfrak{Mlts}, \leq)$ is a complete partial order.*

Consider the example in Fig.2, we have in $mlts_1$, $(s_2, s_3, f_1) \in \mathfrak{R}$, $(s_4, s_5, f_2) \in \mathfrak{R}$ and $(s_5, s_6, f_3) \in \mathfrak{R}$ such that $f_1 = \{(z, y)\}$, $f_2 = \{(z, y), (t, u)\}$ and $f_3 = \{(u, v), (x, x)\}$. In the other words, we have $s_2 \leq s_3$, $s_4 \leq s_5$ and $s_6 \leq s_5$. We can deduce easily that $mlts_1 \leq mlts_2 \leq mlts_3 \leq mlts_4$, so $Y = \{mlts_1, mlts_2, mlts_3, mlts_4\}$ is a chain with $\sqcup Y = mlts_4$. From this example, we can remark that $mlts_2$ is the $mlts_1$ after suppression the state s_6, because $s_6 \leq s_5$. From the fact that $s_2 \leq s_3$, we obtain $mlts_3$ from $mlts_2$ by suppression the state s_2. Since $s_4 \leq s_5$, we obtain $mlts_4$ by suppression the state s_4 of $mlts_3$.

In other words, we can obtain $mlts_4$ (seen as a normal form) from $mlts_1$ by the suppression of the states s_6, s_2 and s_4. We signal here, that the suppression of the states is not in the arbitrary order (see Definition.9). Also, the suppression order is not unique (Property.4 of Proposition.4). The reader may remark that the graph obtained from $mlts_1$ by suppression of s_4 is not a maximality-based labeled transition system ($\psi(s_5)$ is not respected).

To have a normal form, we propose a recursive function Γ (Definition.11) which is a continuous function on CPO $(\mathfrak{Mlts}, \leq)$. The function Γ is constructed by the following definitions.

Definition 6. *Let s_1 and s_2 be two states of mlts with $s_1 \leq_f s_2$. The state s_1 is an eliminated state if and only: $\exists s'_1, s'_2 \in S : s'_1 =_{f'} s'_2$ such that s'_1 predecessor of s_1 and s'_2 predecessor of s'_2.*

Definition 7. *Let $(s, s', f) \in \mathfrak{R}$, we define a substitution function as follows:*

- *$\sigma_{\phi,mlts} = \iota$ (identity substitution),*
- *$\sigma_{f,mlts} = [z/x][z/y]\sigma_{f-\{(x,y)\},mlts[z/x][z/y]}$ such that z doesn't appear in mlts (new event name).*

Proposition 3. *$\mathfrak{R}' = (\mathfrak{R} - \{(s, s', f)\} \cup \{(s, s', f\sigma_{f,mlts})\}$ is a maximality bisimulation relation on $mlts\sigma_{f,mlts}$.*

Definition 8. *Let $s_1, s_2 \in S$ with $(s_1, s_2, f) \in \mathfrak{R}$, and let $mlts' = (mlts)\sigma_{f,mlts} = (\Omega, \lambda, \mu', \xi', \psi')$ such that $\mu' = (\mu)\sigma_{f,mlts}$, $\xi' = (\xi)\sigma_{f,mlts}$ and $\psi' = (\psi)\sigma_{f,mlts}$. Let $\Gamma : S \times S \times \mathfrak{Mlts} \to \mathfrak{Mlts}$ be a function such that $Dom(\Gamma) = \{(s, s', mlts)$ such that $(s, s', f') \in \mathfrak{R}$ and verifies Definition.6 for mlts$\}$. $\Gamma_{s_1,s_2}(mlts) = (\Omega', \lambda, \mu'', \xi'', \psi'')$ is a maximality-based labeled transitions system in which s_1 is removed such that $\Omega' = \langle S', T', \alpha', \beta', s_0 \rangle$ with:*

1. *$S' = S - \{s_1\}$,*
2. *$T' = T - \{In(s_1) \cup Out(s_1)\} \cup Set\_Out(s_1, s_2) \cup Set\_In(s_1, s_2)$,*

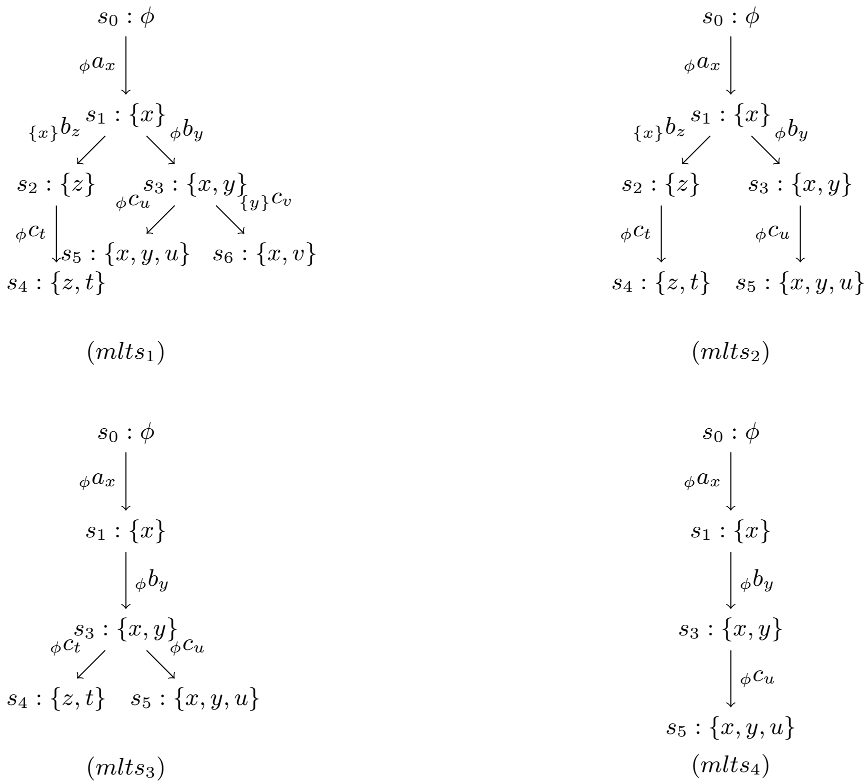

Fig. 2. Partial Order on $\mathfrak{Mlts}$

3. $\mu'' = \mu' - (\{((s_i, s_1), X_i)\} \cup \{((s_1, s_j), X_j)\})$,
4. $\xi'' = \xi' - (\{((s_i, s_1), x_i)\} \cup \{((s_1, s_j), x_j)\})$,
5. $\psi'' = \psi' - (s_1, X)$, $\alpha' = \alpha \lceil T'$ *and* $\beta' = \beta \lceil T'$.

with: $In(s) = \{t | \forall t \in T : \beta(t) = s\}$, $Out(s) = \{t | \forall t \in T : \alpha(t) = s\}$, $set\_In(s_1, s_2) = \{(s_{J,M}\, a_x, s_2) | \forall (s_{j,N}\, a_x, s_1) \in T \wedge M = \psi(s_j) \setminus \psi(s_2)\}$ *and* $set\_Out(s_1, s_2) = \{(s_{2,M}\, a_x, s_j) | \forall (s_{1,N}\, a_x, s_j) \in T \wedge M = \psi(s_2) \setminus \psi(s_j)\}$.

Example 3. In Fig.2, we can affirm that: $\Gamma_{s_6,s_5}(mlts_1) = mlts_2$, $\Gamma_{s_2,s_3}(mlts_2) = mlts_3$ and $\Gamma_{s_4,s_5}(mlts_3) = mlts_4$.

Proposition 4. *Let mlts be a maximality-based labeled transition system:*

1. *For any* s_1 *and* s_2 *two states of mlts, if* $s_1 \leq_f s_2$ *then* $mlts \leq \Gamma_{s_1,s_2}(mlts)$,
2. Γ *is monotone : for any* s_1 *and* s_2 *two states of mlts, if* $s_1 \leq_f s_2$ *and* $mlts \leq mlts'$ *then* $\Gamma_{s_1,s_2}(mlts) \leq \Gamma_{s_1,s_2}(mlts')$,
3. Γ *is continuous :* $\sqcup\{\Gamma_{s_1,s_2}(mlts) | mlts \in Y\} = \Gamma_{s_1,s_2}(\sqcup Y)$,
4. *For any states* s_1, s_2, s_3 *and* s_4 *of mlts, if* $\Gamma_{s_3,s_4}(mlts)$ *and* $\Gamma_{s_1,s_2}(mlts)$ *are both defined then* $\Gamma_{s_1,s_2} \circ \Gamma_{s_3,s_4}(mlts) \cong \Gamma_{s_3,s_4} \circ \Gamma_{s_1,s_2}(mlts)$.

Given *mlts* a maximality-based labeled transition system and subset $R \subseteq \mathfrak{R}$. $\overrightarrow{R}$ denotes a set of suppression sequences w.r.t R (Definition.9) ranged over $\overrightarrow{R}_i$. $\Gamma_{\overrightarrow{R}_i}$: a function suppression a states w.r.t a sequence $\overrightarrow{R}_i$ is defined by Definition.10.

Definition 9. *Let $R \subseteq \mathfrak{R}$, $\overrightarrow{R}_i$ is called a suppression sequence of mlts w.r.t R if and only if:*

- *either $\overrightarrow{R_0} = \epsilon$ if $R_0 = \phi$ (empty sequence),*
- *$\overrightarrow{R_n} = (s_n, s'_n).\overrightarrow{R_{n-1}}$ such that $(s_n, s'_n, f_n) \in R$ and $\overrightarrow{R_{n-1}}$ is a suppression sequence of $\Gamma_{s_n,s'_n}(mlts)$ under a maximality bisimulation relation $R_{n-1} = R_n - \{(s_n, s'_n, f_n)\}$ (with the hypothesis that $R = R_n$).*

Example 4. In $mlts_1$ of Fig.2, we have $R = \{(s_2, s_3, f_1); (s_4, s_5, f_2); (s_6, s_5, f_3)\}$. We can obtain from R, three suppressions sequences:
$\overrightarrow{R}_1 = (s_2, s_3).(s_4, s_5).(s_6, s_5)$, $\overrightarrow{R}_2 = (s_2, s_3).(s_6, s_5).(s_4, s_5)$ and
$\overrightarrow{R}_3 = (s_6, s_5).(s_2, s_3).(s_4, s_5)$. The sequence $(s_4, s_5).(s_2, s_3).(s_6, s_5)$ is not a suppression sequence, therefore, Definition.6 is not verified for (s_4, s_5).

Definition 10. *Let $\overrightarrow{R}_i$ be a suppression sequence, we define $\Gamma_{\overrightarrow{R_i}}$ as follows:*

- *either $\Gamma_{\overrightarrow{R_i}}(mlts) = mlts$ if $\overrightarrow{R}_i = \epsilon$,*
- *$\Gamma_{\overrightarrow{R_n}}(mlts) = \Gamma_{\overrightarrow{R_{n-1}}} \circ \Gamma_{(s_n,s'_n)}(mlts)$ such that $\overrightarrow{R_n} = (s_n, s'_n).\overrightarrow{R_{n-1}}$.*

Given *mlts* a maximality-based labeled transition system and $D = (S, \leq)$ a CPO over $\mathfrak{R}$, we can have from Υ_D a suppression sequence with respect to $\mathfrak{R}$ (Proposition.5). We define an operator Γ which eliminates a set of states w.r.t $\mathfrak{R}$.

Proposition 5. *Let $mlts = (\Omega, \lambda, \mu, \xi, \psi)$ be a maximality-based labeled transition system such that $\Omega = \langle S, T, \alpha, \beta, s_0\rangle$, and $D = (S, \leq_f)$ a CPO. Given $\Upsilon_D = \{Y_i | i = 0..n\}$ with $Y_0 = \{s_0\}$ and $\exists a \in Act : (T_{a_{f(x)}}[Y_i] = Y_j) \wedge i < j$.*

1. *We can obtain, over $\Gamma_{\overrightarrow{Y_i}}$, the suppression sequence $\overrightarrow{Y_{i+1}} = (s_1, \sqcup Y_{i+1})...(s_j, \sqcup Y_{i+1})...(s_n, \sqcup Y_{i+1})$ with $s_j \in Y_{i+1}$ and $s_j \neq \sqcup Y_{i+1}$,*
2. *The composition in $\overrightarrow{Y_{i+1}}$ is commutative.*

Example 5. Given $\Upsilon_D = \{\{s_0\}, \{s_1\}, \{s_2, s_3\}, \{s_4, s_5, s_6\}\}$ of $mlts_1$ of Fig.2, we have $\Gamma_{\Upsilon_D}(mlts_1) = \Gamma_{(s_4,s_5).(s_6,s_5)} \circ \Gamma_{(s_2,s_3)}(mlts_1)$

Definition 11. *Let $mlts = (\Omega, \lambda, \mu, \xi, \psi)$ be a maximality-based labeled transition system such that $\Omega = \langle S, T, \alpha, \beta, s_0\rangle$, and $D = (S, \leq)$ a CPO. Given $\Upsilon_D = \{Y_i | i = 0..n\}$ with $Y_0 = \{s_0\}$ and $\exists a \in Act : (T_{a_{f(x)}}[Y_i] = Y_j) \wedge i < j$. We can define the function Γ as follows: $\Gamma(mlts) = \Gamma_{\overrightarrow{Y_n}} \circ .. \circ \Gamma_{\overrightarrow{Y_1}}(mlts)$.*

Proposition 6. *Let mlts be a maximality-based labeled transition system: $mlts \leq \Gamma(mlts)$ and $\Gamma(mlts)$ is unique.*

Definition 12. *Given mlts* $= (\Omega, \lambda, \mu, \xi, \psi)$ *a maximality-based labeled transition system such that* $\Omega = \langle S, T, \alpha, \beta, s_0 \rangle$*, mlts is in normal form if and only if: for any maximality bisimulation relation* $\mathfrak{R}$ *and for any* $s, s' \in S$*:* $((s, s', f) \in \mathfrak{R} \Rightarrow s = s')$.

Theorem 1. *Given mlts a maximality-based labeled transition system,* $\Gamma(mlts)$ *is the normal form of mlts.*

4 Conclusions

This paper proposes a functional method for reducing maximality-based labeled transition systems. The choice of this model is motivated by its independence from any concurrency specification model. For this purpose, we define a complete partial order on both the set of maximality-based labeled transition systems and the set of maximality-based labeled transition system states. These relations allow us to define an algorithm reducing a maximality-based labeled transition system according to maximality bisimulation relation. As a perspective of this work, it remain the definition of an algorithm computing a maximality bisimulation relation for any maximality-based labeled transition system and by the way analysing realistic systems specified in Petri nets and processes algebras using results of [11,12].

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