Hidden Periodicity – Chaos Dependance on Numerical Precision

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Abstract. Deterministic chaos has been observed in many systems and seems to be random-like for external observer. Chaos, especially of discrete systems, has been used on numerous occasions in place of random number generators in so called evolutionary algorithms. When compared to random generators, chaotic systems generate values via so called map function that is deterministic and thus, the next value can be calculated, i.e. between elements of random series is no deterministic relation, while in the case of chaotic system it is. Despite this fact, the very often use of chaotic generators improves the performance of evolutionary algorithms. In this paper, we discuss the behavior of two selected chaotic system (logistic map and Lozi system) with dependance on numerical precision and show that numerical precision causes the appearance of many periodic orbits and explain reason why it is happens.

1 Introduction

The term "chaos" covers a rather broad class of phenomena, whose behavior may seem erratic, chaotic at first glance. Often, this term is used to denote

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phenomena, which are of a purely stochastic nature, such as the motion of molecules in a vessel with gas and the like. The discovery of the phenomenon of deterministic chaos brought about the need to identify manifestations of this phenomenon also in experimental data. Deterministically chaotic systems are necessarily nonlinear, and conventional statistical procedures, which are mostly linear, are insufficient for their analysis. If the output of a deterministically chaotic system is subjected to linear methods, such signal will appear as the result of a random process. Examples include the Fourier spectral analysis, which will disclose nonzero amplitudes at all frequencies in a chaotic system, and so chaos can be easily mistaken for random noise.

Deterministic chaos has been used during last decade as a pseudorandom number generator. For example in [8] is discussed possibility of generation of random or pseudorandom numbers by use of the ultra weak multidimensional coupling of p 1-dimensional dynamical systems. In the paper [1] is done deep investigation on logistic map as on possible pseudo-random number generator and is compared with contemporary pseudo-random number generators. A comparison of logistic map results is made with conventional methods of generating pseudorandom numbers. The approach used to determine the number, delay, and period of the orbits of the logistic map at varying degrees of precision (3 to 23 bits). Logistic map, we are using here, was also used in [2] like chaos-based true random number generator embedded in reconfigurable switched-capacitor hardware. Another paper [9] proposed an algorithm of generating pseudorandom number generator, which is called (couple map lattice based on discrete chaotic iteration) and combine the couple map lattice and chaotic iteration. Authors also tested this algorithm in NIST 800-22 statistical test suits and is used in image encryption. In [10] authors exploit interesting properties of chaotic systems to design a random bit generator, called CCCBG, in which two chaotic systems are cross-coupled with each other. For evaluation of the bit streams generated by the CCCBG, the four basic tests are performed: monobit test, serial test, auto-correlation, Poker test. Also the most stringent tests of randomness: the NIST suite tests have been used. Another research is done in [11]. A new binary stream-cipher algorithm based on dual one-dimensional chaotic maps is proposed in this paper with statistic proprieties showing that the sequence is of high randomness. Similar studies are done in [3] (discussing whether chaos work better than noise), [4] (proposes an experimental analysis on the convergence of evolutionary algorithms wit use of logistic map, Gauss map, Lozi map amongst the others), [5] (the Chen chaotic system is proposed as a pseudorandom sequence generator in this research article) and [6], (the effect of a weak random additive noise in a linear chain of N locally coupled logistic maps at the edge of chaos are investigated there. Maps tend to synchronize for a strong enough coupling, but if a weak noise is added, very intermittent fluctuations are observed and investigated with dependance on weak noise and map coupling). All mentioned papers less or more discuss randomness, chaos and its mutual intersections and influences.

More application oriented papers are for example [13] - [14] that shows how chaotic dynamics can be used instead of random number generator in evolutionary algorithms. It is numerically shown that algorithms powered by chaotic system has better performance than algorithms powered by random number generator. In fact, more comprehensive study of mutual intersections between chaos and evolution can be found in [7] and chaos and randomness in [8]. Chaos, especially of discrete systems, has been many times used instead of random number generator in so called evolutionary algorithms, [13] - [14], [7]. Comparing to random generators, chaotic systems generate values via so called map function that is deterministic and thus, next value can be calculated, i.e. between elements of random series there is no deterministic relation, while in the case of chaotic system i.e. despite this fact, the very often use of chaotic "random" generator, improves the performance of evolutionary algorithms. In this paper we discuss the behavior of selected chaotic system with dependance on numerical precision and show that numerical precision cause appearance of many periodic orbits.

In our experiments, the logistic map Eq. 1, (example of chaotic behavior is at Fig. 1) and Lozi system, Eq. 2, has been used. The main aim of this paper is to show how dependent the behavior of the two selected discrete deterministic chaotic systems on numerical precision and discuss the possibility as to whether we really need random number generators for evolutionary algorithms.

$$x_{n+1} = Ax_n\left(1 - x_n\right) \tag{1}$$

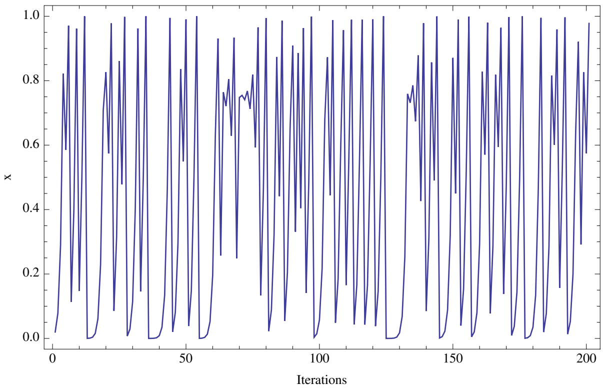

Fig. 1 Chaotic dynamics of logistic map, no restrictions has been given on numerical precision

2 Experiment Design

Similarly as in the [1] is the core object of this study also logistic map (Eq. 1) and Lozi system (Eq. 2) too. Comparing to [1] where period of the orbits of the logistic map were investigated at varying degrees of precision (3 to 23 bits), precision is in this paper is not used on binary level, but on decimal, i.e. the term precision means here how many numbers behind decimal point is used. For example precision 2 means *n.nn*, precision 3 *n.nnn* etc. Contrary to [1], both chaotic systems discussed here are not studied for their pseudo-randomness properties. Appearance of periodicity and its length is studied on the other side, with future use inside evolutionary algorithms instead of pseudo-random generators. Simply, both chaotic systems are used to generate n periodic series with different length.

For all experiments and results reported here has been used MacBook Pro with Intel Core 2 Duo 2.8 GHz and the Mathematica 9.0 software. Both chaotic systems has been used in a few levels of investigation, i.e.

- Identification of n periodic orbits with dependance on numerical precision.
- Global view on logistic map behavior with fixed control parameters.
- Global view on logistic map behavior for various parameter of $A \in [3.4-4]$, logistic map has been iterated for 1000, 10 000, 100 000 iterations.
- Analyze why and when is n periodic behavior generated for both chaotic systems.
- Detail analysis of logistic map behavior for parameter $A = 4$, numerical precision $\in [1, 20]$, initial conditions $x_{start} \in [0.01, 0.99]$ (x was incremented by increment $= 0.01$) and 1 000 000 iterations for each combination of this parameters.
- Detail analysis of Lozi system (Fig. 2. Eq. 2) behavior for parameter $A = 1.75, B = 0.5$, numerical precision $\in [1, 10]$, initial conditions $x_{start} \in [-0.1, 1]$, $y_{start} \in [-0.1, 1]$ (x was incremented by increment $= 0.1$) and 1 000 000 iterations for each combination of this parameters.

Identification of period length was done so that each dataset generated by each experiment was checked whether each value is in dataset only once (no periodicity in the range of used iterations was observed) or more time, that indicate period which certain length has been found. Results are discussed in the next section and summarized in the section Conclusion.

$$\begin{aligned} &x_{n+1} = 1 - A\,|x_n| + By_n \\ &y_{n+1} = x_n \\ &A = 1.7 \\ &B = 0.5 \end{aligned} \tag{2}$$

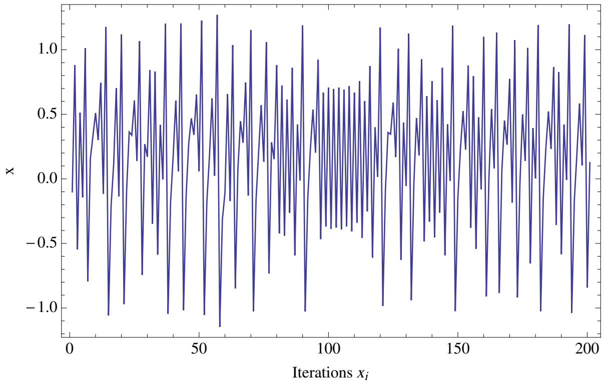

Fig. 2 Chaotic dynamics of Lozi system, no restrictions has been given on numerical precision

3 Results

In this section are described results from both chaotic systems. The first is discussed logistic map and then Lozi system.

3.1 Logistic Map

As described in the previous section, dynamics of the logistic map has been investigated and discussed when, how and what n periodical orbits can be observed in this dynamics and at the end of this paper is also discussed whether the random number generator is really so important for evolutionary algorithms.

Here a few simple demonstrations has been made on the logistic map. Periodic behavior of logistic map, with precision 2 and $x_{start} = 0.02$, (Fig. 3) and precision = 4 (Fig. 4) has been generated for example. As clearly visible, different precision caused different periodic data series. Also, the initial value x_{start} has an impact on the appearance of n periodic series, as was observed during experiments.In this part, it has been numerically demonstrated that different precision cause different n periodic series.

To get a more comprehensive picture, we have extend previous experiments so that the behavior of logistic map has been depicted in the Fig. 5. In this figure we can easily see, that for different precisions, the logistic map produces many periodical series. Color on this figure represent the amplitude of the logistic map under different setting.

The next step was focused on the simulation, where the behavior of logistic map has been investigated for different setting of parameter A and numerical precisions. Visualization results are on Fig. 6. Black squares represent settings of logistic map producing periodic behavior, while white are

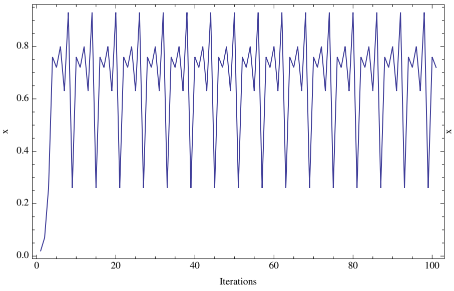

Fig. 3 Periodic behavior of logistic map with precision 2 and $x_{start} = 0.02$

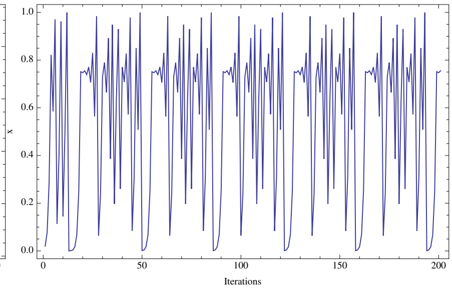

Fig. 4 Periodic behavior of logistic map with precision 4 and $x_{start} = 0.02$

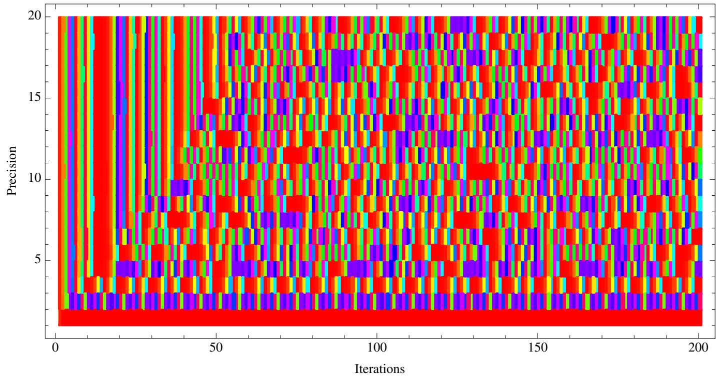

Fig. 5 Numerical demonstration: global view on behavior of logistic map with dependance on precision and iterations

settings for which, in the range of defined iterations, has not been observed periodicity. It is obvious that with increasing number of iterations, number of periodic series increase too and is logically evident, however on Fig. 7 - 8 is demonstrated why it happens.

On Fig. 7 is depicted the Cobweb diagram of the logistic map when precision is set to 1. This numerical precision of the map function is stepwise and thus projection from x_n to x_{n+1} is limited to finite number of possibilities, i.e. no chaotic series can be observed there. On Fig. 7, is depicted the time series with 50 iterations (see also Fig. 9). It is obviously visible when compared with Fig. 8, where for the same conditions, but with high precision, chaotic series has been generated.

Even in the case of higher precision, the mapping function is still discontinuous. The question is thus for what precision, x_{start} and parameter A, we can observe n periodic series. Extended version of previous experiments has been done. Parameter A has been set to 4, x_{start} changed in the interval [0, 1] by increments of 0.01. For each setting has been set precision for value from 1 to 20 (incremented by 1) and has been done 1 000 000 iterations. In the

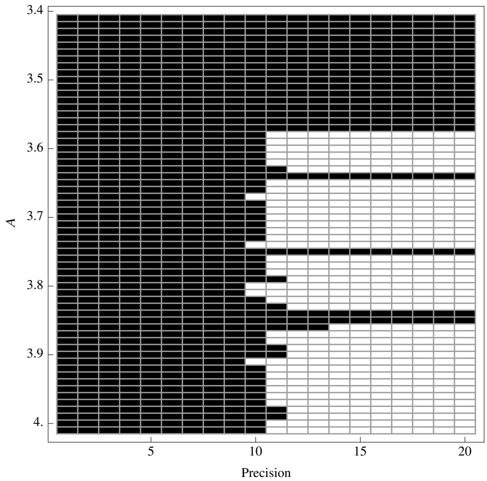

Fig. 6 Numerical demonstration: periodicity and "chaos". Logistic map has been iterated for 100 000 iterations, $x_{start} = .02$. Black squares represent settings of logistic map producing periodic behavior. With increasing number of iterations number of black squares increase.

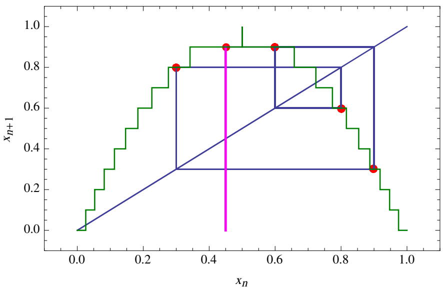

Fig. 7 Cobweb diagram: impact of low precision = 1 (one decimal behind zero) on shape of the mapping function (green stepwise curve). Four periodic orbit can be easily observed, see Fig. 9.

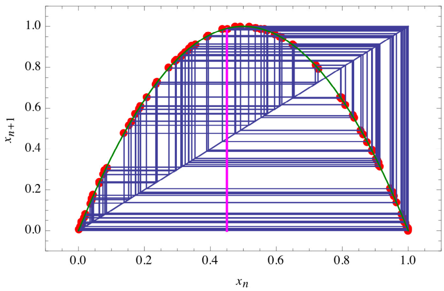

Fig. 8 Cobweb diagram: chaotic dynamics with of logistic map with high precision. Initial conditions of both Cobweb diagrams were the same. Different behavior is caused by different level of precision.

obtained results, a search for periodical series has been performed, see Fig. 10 - 11 and Table 1. The maximal period has been 922 744 and the shortest is 1 (i.e. stable state - constant value).

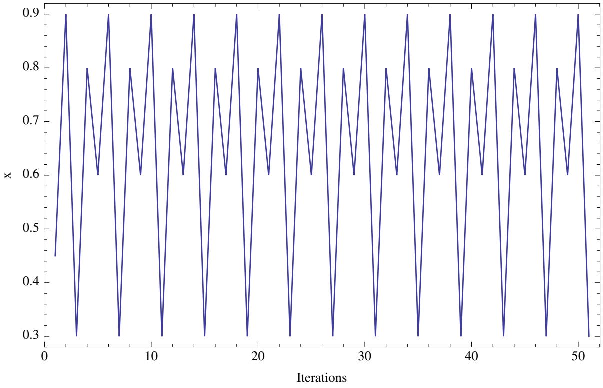

Fig. 9 Dynamic from Fig. 7 in time

Table 1 Periodical time series generated by logistic map. Value 0 means that no periodicity has been observed (in the range of used iterations).

Precision	Minimal Period	Maximal Period
1	4	4
2	2	10
3	10	29
4	15	36
5	67	170
6	143	481
7	421	758
8	1030	4514
9	2277	11227
10	2948	35200
11	9668	57639
12	65837	489154
13	518694	518694
14	75594	316645
15	1	0
16	1	0
17	264446	264336
18	18491	18491
19	46854	46854
20	70767	922744

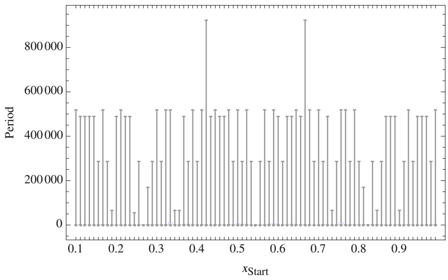

Fig. 10 Dependance of period on initial value x_{start}

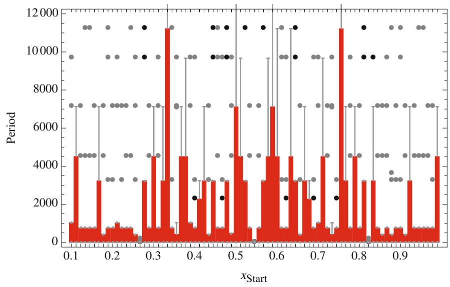

Fig. 11 Dependance of period on initial value x_{start} - detail view with outliers

3.2 Lozi System

Another system, that was used for demonstration of ideas proposed here is so called Lozi system, see Eq. 2. In the case of the Lozi system precision has been changed from 1 to 10, for each precision value was done 1 000 000 iterations and analyzed whether period can be observed under such experimental setting. In the Lozi system was done search for period under parameters $A = 1.75$ and $B = 0.5$.

Results are recorded in Tab. 2 and a few examples visualized on Fig. 12 and Fig. 13, compare with dynamics of Lozi system without any restrictions on numerical precision, see Fig.2.

When compare Tab. 1 and 2, it is visible, that in the case of Lozi system is sensitivity of appearance of periodical dynamics (with dependance on numerical precision) much bigger than in the case of Logistic map. Results from Tab. 2 are also visualized on Fig. 14.

Table 2 Periodical time series generated by Lozi system, see Fig. 14

Precision	Minimal Period	Maximal Period
1	6	11
2	22	50
3	701	919
4	191	3267
5	1351	17765
6	2806	91416
7	267868	482539
8	253954	989400
9	915344	970455
10	676598	885777

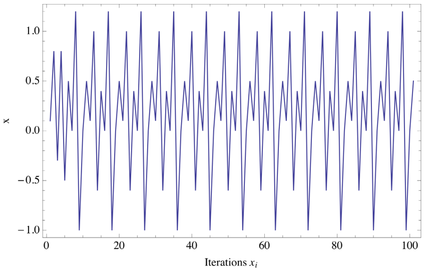

Fig. 12 Period of Lozi system, Eq. 2, for precision 1 and $x_{start} = 0.1$ and $y_{start} = 0.1$

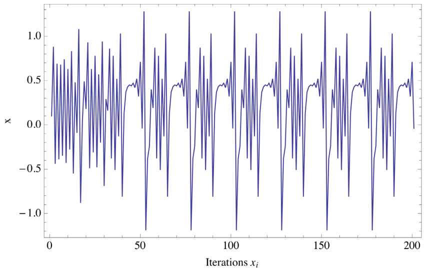

Fig. 13 Period of Lozi system, Eq. 2, for precision 2 and $x_{start} = 0.1$ and $y_{start} = 0.1$

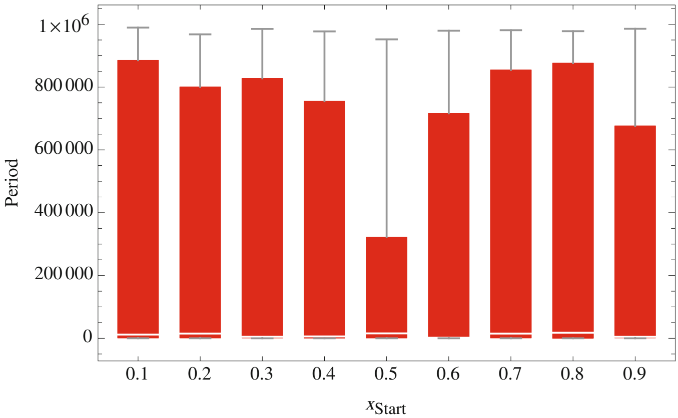

Fig. 14 Period of Lozi system generated by finite numerical precision,see Eq. 2, for different x_{start}, see Tab. 2

4 Conclusion

The main motivation of the research reported in this paper is whether it is possible to generate n periodic series by means of chaotic systems. All previous experiments has demonstrated an impact of numerical precision on logistic map and Lozi system dynamics.

During experiments, it has been observed that periodicity depends on initial conditions and precision (logistic map: Fig. 3 - 4, Tab. 1, Lozi system: Fig. 12 - 13, Tab. 2). It was also demonstrated on Fig. 6 for logistic map. Impact of precision on mapping function is explained on Fig. 7 - 8. Mapping function is due to numerical precision stepwise and dependance x_{n+1} on x_n is not continuous, like on Fig. 8, but discrete. This is the main reason why, when precision is low, we can observe low periodic time series.

At the end, the second set of experiments, using Lozi system (Eq. 2), was done. Results about observed period as well as its visualizations are in Tab.

2 and Fig. 12 - 14. It is clearly visible that numerical precision has also definitely clear impact on periodicity of chaotic systems.

Intensive experiments, focused on numerical investigation of under what conditions are n periodic orbits (series) generated and of what length have been conducted. It has been clearly demonstrated that on computers in fact, deterministic chaos does not exist at all, because computers work with finite numerical precision. A more important issue, that arises during this investigation was, that despite the fact that discrete chaotic systems contain deterministic map function (i.e. **no** pure randomness or pseudo-randomness is present) and its use inside evolutionary algorithms instead of pseudo-random number generators, improves very often its performance, see [13] - [14], [16] - [21].

Next research shall be focused on whether do exist causal relation between period length, precision and level of chaos, that can be measured by Lyapunov exponent. For those purposes can be used for example systems like already discussed logistic map (Eq. 1), Lozi system (Eq. 2) and other systems like Sine map amongst the others. Close relation between chaotic domains and Lyapunov exponent will be used to investigate impact of different level of chaos on evolutionary algorithms performance.

When summarize fact that chaos can improve performance of evolutionary algorithms, see for example [4] - [15], and is generated with finite precision (thus n periodical dynamics can be observed), it arise another question, whether evolutionary algorithms really require pseudo-random number generators or just deterministic periodical series. This is now under investigation.

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References

1. Persohn, K.J., Povinelli, R.J.: Analyzing logistic map pseudorandom number generators for periodicity induced by finite precision floating-point representation. Chaos, Solitons and Fractals 45, 238–245 (2012)
2. Drutarovsky, M., Galajda, P.: A robust chaos-based true random number generator embedded in reconfigurable switched-capacitor hardware. In: 17th International Conference Radioelektronika, Brno, Czech Republic, April 24-25, vol. 1, 2, pp. 29–34 (2007)
3. Bucolo, M., Caponetto, R., Fortuna, L., Frasca, M., Rizzo, A.: Does chaos work better than noise? IEEE Circuits and Systems Magazine 2(3), 4–19 (2002)

4. Caponetto, R., Fortuna, L., Fazzino, S., Xibilia, M.: Chaotic sequences to improve the performance of evolutionary algorithms. IEEE Trans. Evol. Comput. 7(3), 289–304 (2003)
5. Hu, H., Liu, L., Ding, N.: Pseudorandom sequence generator based on the Chen chaotic system. Computer Physics Communications 184(3), 765–768 (2013), doi:10.1016/j.cpc.2012.11.017
6. Pluchino, A., Rapisarda, A., Tsallis, C.: Noise, synchrony, and correlations at the edge of chaos. Physical Review E 87(2) (2013), doi:10.1103/PhysRevE.87.022910
7. Zelinka, I., Celikovsky, S., Richter, H., Chen, G.: Evolutionary Algorithms and Chaotic Systems, p. 550. Springer, Germany (2010)
8. Lozi, R.: Emergence Of Randomness From Chaos. International Journal of Bifurcation and Chaos 22(2), 1250021 (2012), doi:10.1142/S0218127412500216
9. Wang, X.-Y., Qin, X.: A new pseudo-random number generator based on CML and chaotic iteration. International Journal of Nonlinear Dynamics and Chaos in Engineering Systems 70(2), 1589–1592 (2012), doi:10.1007/s11071-012-0558-0
10. Pareek, N.K., Patidar, V., Sud, K.K.: A Random Bit Generator Using Chaotic Maps. International Journal of Network Security 10(1), 32–38 (2010)
11. Wang, X.-Y., Yang, L.: Design Of Pseudo-Random Bit Generator Based On Chaotic Maps. International Journal of Modern Physics B 26(32), 1250208 (9 pages) (2012), doi:10.1142/S0217979212502086
12. Davendra, D., Zelinka, I., Senkerik, R.: Chaos driven evolutionary algorithms for the task of PID control. Computers and Mathematics with Applications 60(4), 1088–1104 (2010)
13. Pluhacek, M., Senkerik, R., Davendra, D., Kominkova Oplatkova, Z.: On the Behaviour and Performance of Chaos Driven PSO Algorithm with Inertia Weight. Computers and Mathematics with Applications (in print), ISSN 0898-1221
14. Pluhacek, M., Budikova, V., Senkerik, R., Kominkova Oplatkova, Z., Zelinka, I.: On the Performance of Enhanced PSO Algorithm with Lozi Chaotic Map. In: Application of Modern Methods of Prediction, Modeling and Analysis of Nonlinear Systems. SCI, vol. 1, p. 18. Springer, Heidelberg (November 2012) (accepted for publication) ISSN: 1860-949X
15. Senkerik, R., Davendra, D., Zelinka, I., Oplatkova, Z., Pluhacek, M.: Optimization of the batch reactor by means of chaos driven differential evolution. In: Snasel, V., Abraham, A., Corchado, E.S. (eds.) SOCO Models in Industrial & Environmental Appl. AISC, vol. 188, pp. 93–102. Springer, Heidelberg (2013)
16. Yang, M., Guan, J., Cai, Z., Wang, L.: Self-adapting differential evolution algorithm with chaos random for global numerical optimization. In: Cai, Z., Hu, C., Kang, Z., Liu, Y. (eds.) ISICA 2010. LNCS, vol. 6382, pp. 112–122. Springer, Heidelberg (2010)
17. Coelho, L., Mariani, V.: Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect. IEEE Transactions on Power Systems 21(2), 989–996 (2006), doi:10.1109/TPWRS.2006.873410
18. Hu, G.-W.: Chaos-differential evolution for multiple sequence alignment. In: 3rd International Symposium on Intelligent Information Technology Application, Nanchang, Peoples R China, vol. 2, pp. 556–558., doi:10.1109/IITA.2009.511

19. Zhao, Q., Ren, J., Zhang, Z., Duan, F.: Immune co-evolution algorithm based on chaotic optimization. In: Workshop on Intelligent Information Technology Application (IITA 2007), Zhang Jiajie, Peoples R China, pp. 149–152.
20. Liua, B., Wanga, L., Jina, Y.-H., Tangb, F., Huanga, D.-X.: Improved particle swarm optimization combined with chaos. Chaos, Solitons & Fractals 25(5), 1261–1271 (2005)
21. Gandomi, A., Yun, G., Yang, X., Talatahari, S.: Chaos-enhanced accelerated particle swarm optimization. Communications In Nonlinear Science and Numerical Simulation 18(2), 327–340 (2013), doi:10.1016/j.cnsns.2012.07.017