Chapter 9 Quantum Vacuum Polarization Searches with High Power Lasers Below the Pair Production Regime

Daniele Tommasini, David Novoa, and Luis Roso

Abstract For high enough electromagnetic fields, such as those that can be achieved by ultra-intense laser pulses, light is expected to interact with light through the interchange of virtual particles. A rich phenomenology is then predicted to occur, such as the possible production of real electron-positron pairs for electromagnetic fields close enough to the Schwinger limit, or the polarization of the vacuum itself. These effects may be amplified by new physics, so that their search can also be used to test non-standard models involving axions or mini-charged particles. A recent work suggests that the diffraction of light by light in vacuum, in the absence of any material slit or obstacle, is most probably the first signature of the polarization of the vacuum that will be reachable in the near future. Surprisingly enough, this result could be achieved very soon in principle, either at a high repetition rate Petawatt facility such as VEGA, that is expected to be operative at the beginning of 2014 in Salamanca, Spain, or at other Multi-PW facilities, such as ELI-10 PW or PETAL. Calculations for a prospective 100-PW system are also included.

9.1 Introduction

Besides their multiple technological applications, extreme lasers are becoming part of a conceptually new experimental set-up to explore fundamental properties of the quantum world. Quantum Electrodynamics (QED) has been the most successful

D. Tommasini

D. Novoa e-mail: david.novoa@mpl.mpg.de

Department of Applied Physics, University of Vigo, 32004 Ourense, Spain e-mail: daniele@uvigo.es

D. Novoa \cdot L. Roso (\boxtimes) Centro de Láseres Pulsados, CLPU, Edificio M3—Parque Científico, Calle del Adaja, 37185 Villamayor, Spain e-mail: roso@usal.es

L. Roso (\boxtimes) e-mail: roso@clpu.es

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theory ever proposed, predicting properties at the atomic scale with an astonishing precision. Due to its triumphs, it has been used in Particle Physics as a guide for the construction of the theories of the strong and weak interactions as well.

However, QED effects implying the exchange of a very high number of particles are still controversial. In Classical Electrodynamics, Maxwell's equations in vacuum are linear in the fields and so do not allow for any interaction of light with light. In QED, however, two photons can couple each other by the exchange of a virtual particle-antiparticle pair (most likely an electron-positron virtual pair) [[29\]](#page-16-0). Quantum vacuum can thus be polarized by electromagnetic fields. For many years this idea has been considered a gedanken experiment [[33\]](#page-16-1), but we hope that in the near future some features of the exchange of such virtual pair of particles may become observable [[35\]](#page-16-2).

Todays femtosecond lasers can achieve the Petawatt level and beyond. When such lasers have a good quality wavefront—not easy at all at such extreme powers the laser pulse can be focused at intensities of the order of 10^{23} W/cm² or beyond, and probably in less than a decade the 10^{25} W/cm² intensity will be reached in the lab frame, and much more if we consider a Lorentz boosted frame. In such cases, the photon concentration is so extreme that many photons can interact with a single electron almost instantaneously. Photons are bosons and so they can be packed in a large number at the same quantum state at the same point. If they did not interact with each other, in principle their density could be arbitrarily high. In other words, there should be no limit for the maximum intensity of a laser pulse. However, QED predicts that photons interact, and it has been argued since Schwinger times that there is a theoretical limit for the field, beyond which vacuum would become unstable and spontaneous pair production would take place. This limit is now referred as the Schwinger limit [\[49](#page-16-3)], and corresponds to the critical value 1.3×10^{16} V/cm for the electric field. For a laser pulse, this corresponds to an intensity ∼10²⁹ W*/*cm2, which is six orders of magnitude beyond current possibilities, and four orders of magnitude above the reach of the new facilities that will be available in the next decade.

Nevertheless, we are not too far, and it has been suggested that real electronpositron pairs may be produced in significant amount via laser-induced electromagnetic cascades even below the Schwinger limit. In this process, charged electronpositron pairs are produced and accelerated within the laser pulse, subsequently emitting hard photons which in turn decay into further electron-positron pairs, which are accelerated by the laser pulse itself and so on. This cascade effect possibly leads to a saturation of the laser intensity at the level ∼10²⁶ W*/*cm2 [[23\]](#page-16-4), which is just an order of magnitude above the level that may be reached in a decade. It might be expected that more efficient configurations can be found leading to real pair production at a somewhat lower scale [[5,](#page-15-0) [22,](#page-16-5) [34](#page-16-6), [46,](#page-16-7) [47\]](#page-16-8). In any case, the production of real pairs is hardly expected to be observable below the 10^{24} W/cm² intensity scale [[5\]](#page-15-0).

Although we cannot expect to succeed in the production of real pairs, at least until the next decade, we may be closer to detect the first signatures of the polarization of vacuum induced by the production and annihilation of *virtual* pairs of particles. In QED, this effect is predicted to be generated by the same radiative corrections

that are responsible for photon-photon scattering. Although an experimental evidence of the latter has been obtained in the case of Delbruck scattering of a gamma ray of energy at the MeV scale from an atomic electric field [[43\]](#page-16-9), all the searches both for the scattering of *real* photons have only produced negative results, and no experimental evidence has been found so far of the interaction of light with light in the optical regime. This is due to the extremely tiny QED cross-section for the photon-photon scattering, in particular at optical wavelengths ($\sigma \sim 10^{-63}$ cm²), as shown in Fig. [9.1](#page-2-0).

Another possibility to gain experience with the nonlinear behavior of the quantum vacuum is to consider a gamma ray photon colliding with one, or a few, optical photons. This has been already done in the context of the SLAC-144 experiment. They observed collisions of a 46.6 GeV electron beam with a 527 nm Terawatt pulsed laser and subsequent positron production. Such positrons were arising from a combined process in which laser photons were backscattered by the electron beam first and then interact with several laser photons to produce an electron-positron pair. These results, in good agreement with QED predictions, were a clear experimental evidence for inelastic photon-photon scattering. In those experiments the peak laser intensity just arrived to 10^{18} W/cm² but the nonlinear QED effect was enhanced enough to be observed thanks to the backscattered photons [[12](#page-16-10), [13](#page-16-11)].

However, the process at the optical scale can have a very rich phenomenology, including the diffraction of light by light in the complete absence of matter. Moreover, it can be enhanced by additional contributions from new physics. In fact, while all Schwinger-type calculations assume that the lightest possible particle pair is the electron-positron, there are theories predicting the existence of axions and other mini-charged particles lighter than the electron. Extreme lasers can then represent a unique laboratory test for the existence, or not, of such particles. On the other hand, it has recently been argued that even the QED virtual electron-positron pair exchange can be tested at present PW facilities by searching for light by light diffraction in a head-on collision of two laser pulses [[50\]](#page-16-12). This result lowers by several orders of magnitude the requirement on the intensity for the polarization of the vacuum to become observable, as compared to previous works. As far as we know, the latter is the best candidate to provide the first signature of the virtual pair creation mediating the interaction of light with light in the optical regime.

Summarizing, to our knowledge there are two key questions where lasers can be relevant for our understanding of the quantum vacuum. The first question is the existence of the Schwinger limit itself, whose experimental study would require laser peak intensities beyond 10²⁹ W*/*cm2. Such intensities are beyond the possibilities of standard CPA technology, although conceptually new laser schemes that are under consideration, such as Backward Raman Amplification, might allow for reaching the Schwinger limit in the future. In any case, a deeper theoretical understanding and modeling on the laser-electron coupling at such extreme fields is needed, particularly in the context of radiation reaction that can be dominant beyond 10^{26} W/cm². While we wait for such developments we can try to solve the second key question: which can be the first effect to be observed at the lower laser intensities that will be available now or in the near future? The proposal that we present in this chapter tries to answer this question. As we shall see, the quantum interaction of light with light may be observed at laser facilities in the Petawatt or multi-Petawatt regime much before the Schwinger limit, and its search can also be used to address fundamental questions such as the existence of mini-charged or axion-like particles.

The chapter is organized as follows:

In Sect. [9.2](#page-4-0), we will briefly review the recent research in the field, that has opened the possibility of performing optical measurements of vacuum polarization effects using ultraintense lasers.

In Sect. [9.3](#page-5-0), we discuss the mathematical formalism and introduce the two parameters that drive the nonlinear vacuum effects. We review the predictions for the values of the relevant parameters both in QED and in non-standard models of particle physics, such as the Born-Infeld theory or scenarios involving new mini-charged or axion-like particles. We show that, in the case of detecting the effect of vacuum polarization above the QED level, the possible measurement of both the relevant parameters can be used to discriminate among the different types of new physics.

In Sect. [9.4,](#page-7-0) we discuss the current experimental constraints on the relevant parameters that have been obtained by the PVLAS collaboration [[9,](#page-15-1) [54\]](#page-16-13) from the negative search of birefringence of the vacuum in an external magnetic field. These limits constrain the cross-section for photon-photon scattering in the near infrared to be smaller than $\sim 10^{-58}$ cm², but this value, even if it is extremely tiny, is still \sim 5 × 10⁶ times higher than the prediction of QED, so that there is still much room for new physics to emerge.

In Sect. [9.5](#page-8-0), we discuss the main effect that is produced when two counterpropagating coherent light waves cross each other. In this case, the nonlinear quantum vacuum polarization makes each of the two beams to behave as a phase object for the other, thus producing a phase shift on its wavefront.

In Sect. [9.6](#page-9-0), we review one of the consequences of the phase shift that is produced in the collision of two laser pulses of different waists: the wider beam is diffracted by the more concentrated one. We discuss the possibility of searching for this effect by counting the number of diffracted photons on a ring detector, and show that this can be used to measure or constrain the parameters that describe quantum vacuum polarization.

In Sect. [9.7,](#page-11-0) we discuss the sensitivity for such measurements that, in principle, can be achieved at several ultraintense laser facilities that are scheduled to become operative in the near future.

In Sect. [9.8](#page-15-2), we draw our conclusions and future trends.

9.2 The Search for Quantum Vacuum Polarization

The interaction of light with light in complete absence of matter has still to be tested in the optical regime. In this regime a rich phenomenology of collective effects is expected to appear, due to the polarization of the quantum vacuum. In particular, in the last few years there has been an increasing interest in the phenomenological consequences of the quantum nonlinear corrections to the Maxwell equations due to the exchange of virtual particles and in proposing experimental tests that can also be used to search for new physics. Several different configurations have been proposed aimed at testing the nonlinear optical response of the vacuum, e.g. using harmonic generation in an inhomogeneous magnetic field [[18\]](#page-16-14), four-wave mixing [\[1](#page-15-3), [6](#page-15-4), [36,](#page-16-15) [44\]](#page-16-16), resonant interactions in microwave cavities [\[11](#page-15-5)], or vacuum birefringence [\[3](#page-15-6)] which can be probed by x-ray pulses $[19, 28]$ $[19, 28]$ $[19, 28]$ $[19, 28]$, among others $[38, 45]$ $[38, 45]$ $[38, 45]$ $[38, 45]$.

In particular, the possibility of purely optical tests is especially promising, since it can exploit the extraordinary advancements in laser technology that have been achieved in the last two decades. Indeed, an example of this kind has been performed recently by the PVLAS collaboration. In their experiment, a laser beam travels within a slowly-varying magnetic field. The nonlinear correction to the Maxwell equations induced by the external magnetic field would then imply the emergence of birefringence and dichroism that would induce a rotation of the laser polarization [\[9](#page-15-1)]. The non-observation of such effects was used to set the current limit on the photon-photon scattering cross-section at optical wavelengths. Although these limits are still 7 orders of magnitude above the prediction of QED, they can be used to set the best laboratory constraints on several kinds of new physics scenarios, such as Born-Infeld theory or models implying new mini-charged or axion-like particles in suitable mass ranges $[9, 20, 52]$ $[9, 20, 52]$ $[9, 20, 52]$ $[9, 20, 52]$ $[9, 20, 52]$ $[9, 20, 52]$ $[9, 20, 52]$. A second class of purely optical tests of the nonlinear corrections to the Maxwell equations is based on exploiting the field of the laser pulses themselves, instead of making the pulses to travel across external electric or magnetic fields. The main advantage of this approach is the fact that the electromagnetic intensities that can be achieved by focusing ultrashort laser pulses are nowadays 10 orders of magnitude larger than the intensities of stationary external fields that can be obtained in the laboratory, such as the magnetic field that is used in PVLAS experiment. This improvement compensates the much smaller interaction length, which is limited by the duration of the ultra-intense laser pulse. This kind of motivations suggested a configuration in which two counter-propagating laser pulses cross each other, inducing a phase shift onto each other due to the nonlinear quantum effects that can be directly measured either for parallel or for orthogonal polarizations [\[24](#page-16-23), [51,](#page-16-24) [52](#page-16-22)]. Very recently, this concept has been significantly improved by suggesting a configuration in which the crossing pulses have different waists and, instead of directly measuring the phase shifts, the number of diffracted photons is counted. On one hand, the diffraction of a probe laser by two additional counter-propagating ultra-intense pulses was proposed as a matter-less analogue to the Young double slit experiment [[32,](#page-16-25) [37\]](#page-16-26). On the other hand, a simpler case of crossing of just two counter-propagating laser pulses [[50\]](#page-16-12), analogue to a matter-less single slit experiment, has been found to provide a much more sensitive and promising configuration for the search of light-by-light diffraction in vacuum and the test of the nonlinear corrections to the Maxwell equations [[50](#page-16-12)]. This proposal will be reviewed in detail in Sect. [9.6](#page-9-0) below.

9.3 The Effective Lagrangian for the Electromagnetic Fields in QED and Non-standard Models

In this section, we will briefly review the QED and non-standard model predictions for the interaction of light with light at optical wavelengths, following Ref. [\[52](#page-16-22)].

Optical photons have energies well below the threshold for the production of real electron-positron pairs, so that we can assume an effective Lagrangian for the electromagnetic fields **E** and **B** of the form

$$
\mathcal{L} = \mathcal{L}_0 + \xi_L \mathcal{L}_0^2 + \frac{7}{4} \xi_T \mathcal{G}^2,\tag{9.1}
$$

being $\mathcal{L}_0 = \frac{\varepsilon_0}{2} (E^2 - c^2 B^2)$ the Lagrangian density of the linear theory, $\mathcal{G} =$ $\varepsilon_0 c(\mathbf{E} \cdot \mathbf{B})$ and ε_0 and *c* the dielectric constant and the speed of light in vacuum, respectively. \mathcal{L}_0^2 and \mathcal{G}^2 are the only two Lorentz-covariant terms that can be formed with the electromagnetic fields at the lowest order above \mathcal{L}_0 , thus they will describe the first correction to the linear evolution both in QED and non-standard models.

In QED, such terms arise due to the interchange of virtual charged particles running in loop box diagrams [[14\]](#page-16-27). The resulting Lagrangian density [[29\]](#page-16-0) coincides with [\(9.1\)](#page-5-1) with the identification $\xi_L^{QED} = \xi_T^{QED} \equiv \xi$, being

$$
\xi = \frac{8\alpha^2 \hbar^3}{45m_e^4 c^5} \simeq 6.7 \times 10^{-30} \frac{\text{m}^3}{\text{J}}.
$$
\n(9.2)

In non-standard models of Particle Physics, however, the two parameters *ξL* and *ξT* can acquire different values. In Born-Infeld theory [[7,](#page-15-7) [8\]](#page-15-8), that can derived from Superstring theory, one would obtain the relation $\xi_T^{BI} = 4\xi_L^{BI}/7$ [\[17](#page-16-28)], in general without a definitive prediction for the numerical value.

New mini-charged particles (MCPs) [\[2](#page-15-9), [15,](#page-16-29) [16](#page-16-30), [21,](#page-16-31) [25](#page-16-32), [26,](#page-16-33) [30](#page-16-34), [31\]](#page-16-35), that would appear naturally in a large class of gauge models, would provide an additional contribution analogous to that from the electron-positron box diagram. There are different possibilities for MCPs as we will discuss below.

If the new MCPs are spin $1/2$ fermions, and assuming that their mass m_{ε} is larger than the energy of the photons (the eV scale in optical experiments), we would obtain from Ref. [[52\]](#page-16-22)

$$
\Delta \xi_L^{\text{MCP}} = \Delta \xi_T^{\text{MCP}} = \left(\frac{\varepsilon m_e}{m_\varepsilon}\right)^4 \xi, \tag{9.3}
$$

where ε is the ratio of the charge of the particle with respect to the electron charge. The case of MCPs lighter than the photon energy would deserve a different treatment and in general would imply additional effects such as real MCPs production.

If the new MCP is a spinless boson of mass m_{ε} larger than the energy of the photons, from Ref. [[52\]](#page-16-22) we obtain

$$
\Delta \xi_L^{\text{MCP0}} = \frac{7}{16} \left(\frac{\varepsilon m_e}{m_\varepsilon} \right)^4 \xi, \qquad \Delta \xi_T^{\text{MCP0}} = \frac{1}{28} \left(\frac{\varepsilon m_e}{m_\varepsilon} \right)^4 \xi. \tag{9.4}
$$

On the other hand, if the MCP is a spin 1 boson, the result obtained from Ref. $[52]$ $[52]$ is

$$
\Delta \xi_L^{\text{MCP1}} = \frac{261}{16} \left(\frac{\varepsilon m_e}{m_\varepsilon} \right)^4 \xi, \qquad \Delta \xi_T^{\text{MCP1}} = \frac{243}{28} \left(\frac{\varepsilon m_e}{m_\varepsilon} \right)^4 \xi. \tag{9.5}
$$

Let us now discuss the case of an axion-like particle (ALP) [\[4](#page-15-10), [10,](#page-15-11) [27](#page-16-36), [39–](#page-16-37)[41\]](#page-16-38), such as the particle needed to solve the strong CP problem in Peccei-Quinn theory [\[48](#page-16-39)]. We can allow both for a Light Pseudoscalar Boson or a Light Scalar Boson, depending on the coupling with the photons, that is described in the Lagrangian son, depending on the coupling with the photons, that is described in the Lagrangian density by the terms $\mathcal{L}_P = -\sqrt{\hbar c g_P \Phi_P \mathcal{G}}$ and $\mathcal{L}_S = -\sqrt{\hbar c g_S \Phi_S \mathcal{L}}_0$, respectively. We can find the leading contribution to the effective Lagrangian when the photon energy is much smaller than the m_{ϕ} scale, that can be cast in the form of Eq. ([9.1](#page-5-1)) with an additional contribution given by

$$
\Delta \xi_T = \frac{2\hbar^3 g_P^2}{7cm_\phi^2}, \qquad \Delta \xi_L = 0,\tag{9.6}
$$

in the case of pseudoscalars, or

$$
\Delta \xi_L = \frac{\hbar^3 g_S^2}{2c m_\phi^2}, \qquad \Delta \xi_T = 0, \tag{9.7}
$$

in the case of scalars.

Figure [9.2](#page-7-1) shows the theoretical predictions for ξ_L and ξ_T including the contributions from these different new particle ensembles.

Note that for masses smaller than the uncertainty on the momenta of the colliding photons, the computation of $\Delta \xi_L$ is more complicated and the production of real axions has also to be taken into account [\[20](#page-16-21)].

The generalized Lagrangian of (9.1) implies a set of modified Maxwell's equations for the average values of the electromagnetic quantum fields similar to those that have been obtained in Ref. [[42\]](#page-16-40), the only difference being the distinction between ξ_L and ξ_T . In any case, as we can expect on dimensional grounds, the amount of the effect of these nonlinear corrections turns out to be driven by the product

Fig. 9.2 Theoretical predictions for the parameters ξ_L and ξ_T entering the effective Lagrangian Eq. ([9.1](#page-5-1)) in different new physics models. The *solid green line* describes the prediction including the contribution of a spinless MCP; the *dashed-dotted red line*dashed-dotted red line corresponds to adding a spin 1 MCP; the *dashed blue line* corresponds to adding a spin 1*/*2 MCP; the *solid orange line* (parallel to the vertical axis) to adding a pseudo-scalar ALP and the *solid black line* (parallel to the horizontal axis) to a scalar ALP. Although we only show the region close to the QED prediction, marked by the labeled starting point shared by all the lines displayed, each model line can be extended for higher values of *ξL* and *ξT*

 $\xi_{L,T}\rho$ of the relevant parameter with the energy density of the electromagnetic field.

9.4 Present Constraints

The current *laboratory* constraint on a combination of the parameters ξ_L and ξ_T that drive the nonlinear terms in the Lagrangian [\(9.1](#page-5-1)) have been obtained by the PVLAS collaboration [\[9](#page-15-1), [54](#page-16-13)] from the search for birefringence of the vacuum in a uniform magnetic field background. Such limit is more reliable, although much less stringent, than the model-dependent *cosmological* constraints that have been derived for the masses and coupling constants of MCPs and ALPs. With our notation, the 95 % C.L. limit from PVLAS reads

$$
\frac{|7\xi_T - 4\xi_L|}{3} < 1.5 \times 10^{-26} \frac{\text{m}^3}{\text{J}} = 2.2 \times 10^3 \xi. \tag{9.8}
$$

Assuming $\xi_L = \xi_T$ as in QED, this results implies constraint $\xi < \xi^{\text{PVLAS}} \equiv 1.5 \times$ 10−²⁶ m3*/*J. This can be translated into a limit for the photon-photon scattering cross-section, e.g. σ < 9.5 × 10⁻⁵⁹ cm² at λ = 1064 nm. Although such crosssection is extremely tiny, it is still \sim 5 × 10⁶ times higher than the prediction of QED for the same wavelength, so that there is still much room for new physics to emerge.

It is worth noting that the PVLAS experiment is only sensitive to the combination |7*ξT* −4*ξL*| and cannot be used to constrain the whole parameter space. In particular, a pure Born-Infeld theory, for which $\xi_T = \frac{4}{7}\xi_L$, cannot be constrained at all by the PVLAS results.

Finally, we note that, even for an external magnetic field as large as 10 T, the product that drives the quantum vacuum polarization effects is as small as $\xi \rho \sim$ 3×10^{-22} . It is then natural to explore the possibility of substituting the external magnetic field with the electromagnetic field of an ultraintense laser pulse, since in this case the product *ξρ* can already be improved by 10 orders of magnitude (*ξρ* \sim 4×10^{-12} at the current record intensity that has been reached by HERCULES [\[53](#page-16-41)]). Of course, this gain is partly compensated by the much shorter interaction region, that would be limited by the temporal duration of the laser pulse itself instead of the macroscopic propagation distances that are used in the PVLAS experiment.

9.5 Phase Shift of Crossing Polarized Beams

There are several geometries to observe vacuum polarization. In the present section we consider just two counter-propagating linearly-polarized plane waves. The effect of the nonlinear terms in Eq. (9.1) in this case can be computed as in Refs. [\[24](#page-16-23), [51](#page-16-24), [52](#page-16-22)]. Let us call A and B the two waves, and describe the electromagnetic fields with the usual four-vector potential A^{μ} . In the absence of the nonlinear interaction terms driven by ξ_L and ξ_T , the linear evolutions of A and B would simply sum each other, so that $A_{\text{lin}}^{\mu} = A_A^{\mu} + A_B^{\mu}$. Since the product $\xi \rho$ will be very small in all the experimental configurations that can be obtained in the near future, we can compute the solution of the full QED equations $\delta\Gamma/\delta A^{\mu} = 0$ by perturbing the linear propagation. The result which is relevant for our purposes is that each of the two waves acquires a phase-shift due to the crossing with the other [[51,](#page-16-24) [52](#page-16-22)]. The phase shift of the wave B is

$$
\Delta \phi_{L,T} = (a\xi)_{L,T} I_A k_B \tau_A,\tag{9.9}
$$

where $k_A = 2\pi/\lambda_A$, $k_B = 2\pi/\lambda_B$, $I_A = \rho_A c$ is the intensity of the wave A and τ_A is the temporal duration of the interaction. The indexes *L* and *T* refer to the two beams having parallel or orthogonal linear polarizations, respectively. Following Ref. [[50\]](#page-16-12), we have fixed $a_L = 4$ and $a_T = 7$.

As we shall discuss in the next section, this formula can be generalized to the case of a Gaussian laser pulse by using the transverse intensity distribution $I_A(r)$ instead of a constant *I* . In this case, we will also assume that the time variation can be approximated by a step function, which will be argued to be a reasonable approximation in the cases that will be considered in the next section.

As discussed in Refs. [[51,](#page-16-24) [52\]](#page-16-22), [\(9.9\)](#page-8-1) can be used to perform a set of experiments aimed at measuring the phase shifts resulting from the crossing of two pulses for parallel and for orthogonal polarizations. If $\xi_L = \xi_T$ as in QED or in theories involving mini-charged spin $1/2$ fermions, $\Delta \phi_T$ turns out to be more sensitive to the effect of the vacuum polarization by a factor 7/4 than $\Delta \phi_L$. More importantly, by performing the experiment with the two different configurations for the polarizations we will be able to test both parameters ξ_L , ξ_T appearing in the effective Lagrangian ([9.1](#page-5-1)), distinguishing between QED and other models such as Born-Infeld theories. Finally, we note that [\(9.9\)](#page-8-1) also implies that the high power pulse behaves like a birefringent medium, producing a relative phase shift $\Delta \phi_b = \Delta \phi_T - \Delta \phi_L = (7\xi_T - 4\xi_L)Ik\tau$ between the transverse an parallel polarizations of the low power beam.

However, instead of reviewing the proposal of searching for the direct phase shift or birefringence of a pulse due to the crossing with the other, that was discussed in Refs. [[51,](#page-16-24) [52](#page-16-22)], we will proceed to study the scenario of Ref. [[50\]](#page-16-12), which turns out to be more sensitive for the search of the vacuum polarization at future Petawatt of multi-Petawatt facilities.

9.6 Light by Light Diffraction in Vacuum: An Optimal Scenario

The possibility of using photon counting to search for vacuum polarization effects based on signatures of three-photon scattering was suggested in Refs. [\[6](#page-15-4), [36](#page-16-15), [44](#page-16-16)]. In Ref. [\[32](#page-16-25)], this idea was applied to a matter-less double slit configuration in which a laser pulse is diffracted by a pair of counter-propagating pulses. A simpler, more efficient and most probably optimal scenario was proposed in Ref. [\[50](#page-16-12)], involving only two counter-propagating laser pulses. Here, we will review in detail this configuration, following closely Ref. [[50\]](#page-16-12), and applying their results to several facilities that will be available in the near future.

In this scenario, illustrated in Fig. 9.3 , a polarized pulse A of waist w_A crosses an almost counter-propagating polarized laser pulse B of waist $w_B \gg w_A$, that will be used as the probe. We assume that the uncertainty in frequency $\Delta v_{A,B} \simeq 1/\tau_{A,B}$ of each beam is much smaller than the mean frequency $v_{A,B} = c/\lambda_{A,B}$, so that the pulses can be considered as monochromatic with a good accuracy. In fact, this approximation will be fully justified in all the practical cases that we will consider below. The uncertainties in the transverse components of the wavevector, $\sim 1/w_{A,B}$, are supposed to be negligible with respect to $k_{A,B}$ as well.

Let $A_A = A_A(0) \exp(-r^2/w_A^2)$ and $A_B = A_B(0) \exp(-r^2/w_B^2)$ describe the dependences of the non-vanishing components of the two waves on the radial coordinate $r \equiv \sqrt{x^2 + y^2}$, orthogonal to the propagation direction chosen in the *z*-axis. The intensity of the pulse A in the colliding region will then have the transverse distribution $I_A(r) = I_A(0) \exp(-2r^2/w_A^2)$. As a consequence, the space-dependent phase shift of the wave B just after the collision with the beam A is

$$
\phi(r) = \phi(0) \exp\left(-\frac{2r^2}{w_A^2}\right),\tag{9.10}
$$

where $\phi(0) = (a\xi)_{L,T} I_A k_B \tau_A$ and we understand one of the sub-indexes *L* or *T* in [\(9.10\)](#page-10-0).

As a consequence, after the crossing the shape of the pulse B becomes $A_B =$ $A_B(0) \exp[-(r^2/w_B^2) + i\phi(r)]$. Taking into account that ϕ is expected to be very small at all the facilities that will be available in the near future, we can make the approximation $exp[i\phi(r)] \approx 1 + i\phi(r)$ and obtain

$$
A_B = A_B(0) \left[\exp\left(-\frac{r^2}{w_B^2}\right) + i\phi(0) \exp\left(-\frac{r^2}{w_0^2}\right) \right],\tag{9.11}
$$

where we have defined $w_0 \equiv (2/w_A^2 + 1/w_B^2)^{-1/2}$.

Since the field A_B propagates linearly after the collision, we can sum the free evolution of each term in (9.11) that can be computed within the paraxial approximation $\omega = c \sqrt{k^2 + k_\perp^2} \simeq c(k + k_\perp^2/2k)$ for the angular frequency, where $\mathbf{k}_{\perp} = (k_x, k_y, 0)$, assuming that $\Delta k_{x,y} = 1/w \ll k$. To be conservative, we require that the number of non-diffracted photons on the ring detector of inner radius r_0 and outer radius R is 100 times smaller than the value of the diffracted photons. This requirement can then be used to compute a safe value for the inner radius [\[50](#page-16-12)]:

$$
r_0 = w_D w_U \sqrt{\frac{\log(\frac{10w_B}{\phi(0)w_0})}{w_D^2 - w_U^2}},
$$
\n(9.12)

where $w_U \equiv w_B \sqrt{1 + (2d/k_B w_B^2)^2}$ and $w_D \equiv w_0 \sqrt{1 + (2d/k_B w_0^2)^2}$ are the widths of the non-diffracted and diffracted patterns at the distance *d* from the collision point.

The result for the number of diffracted photons hitting the ring detector, as computed in Ref. [\[50](#page-16-12)], is then

$$
N_D^{\mathcal{N}} = \frac{8f\mathcal{N}}{\pi\hbar c} \frac{E_A^2 E_B w_0^2}{\lambda_B w_A^4 w_B^2} \left(e^{-\frac{2r_0^2}{w_D^2}} - e^{-\frac{2R^2}{w_D^2}}\right) (a\xi)_{L,T}^2, \tag{9.13}
$$

where *f* is the efficiency of the detector, and $E_A = P_A \tau_A$ and $E_B = P_B \tau_B$ are the total energies of the two pulses.

The number of scattered photons is then proportional to $E_A^2 E_B$. This requirement can be obtained economically by producing both beams simultaneously, e.g. by dividing a single pulse of energy $E = E_A + E_B$ before the last focalizations. The maximum value for N_D can then be achieved for $E_A = 2E/3$ and $E_B = E/3$. Of course, this choice is not necessary in the case of facilities such as VEGA [\[55](#page-16-42)], that will provide two different high power pulses simultaneously.

The other parameters that can be adjusted in order to maximize N_D are the widths w_A and w_B of the two colliding beams. Their choice is constrained by the following requirements:

- 1. w_B should be much larger than w_A ;
- 2. the pulse A must not spread in a significant way during the crossing;
- 3. the center of pulse A has to remain close to the central part of beam B during the interaction (we will allow that it can deviate at most by a 10 $\%$);
- 4. the scattering angle θ should be close to π , but at the same time it should be large enough that the trajectories of the two beams out of the collision point are separated by a distance sufficiently larger than their width. We assume that such an angular distance is 6 times the divergence of the beam A, which is $\sim \lambda/\pi w_A$.

As shown in Ref. [[50\]](#page-16-12), these requirements can be fulfilled by the safe and optimal choices

$$
w_A = \sqrt{60c\tau_B\lambda/\pi}; \qquad \pi - \theta \simeq 6\lambda/\pi w_A. \tag{9.14}
$$

On the other hand, the best choice for the value of $w_B > w_A$ will be computed numerically by maximizing N_D as given by ([9.13](#page-10-2)), Finally, the outer radius R will be chosen slightly larger than $\sqrt{2w_D} \sim 2\lambda d/\pi w_A$, by requiring that only a few percent of the diffracted wave is lost.

N_D can then be used to determine the values of the parameters ξ_L and ξ_T . To evaluate the best possible sensitivity, we will suppose that the background noise can be kept below the signal level, which may not be a trivial requirement. In this case, the best sensitivity would correspond to the detection of 10 diffracted photons, so that the zero result could be excluded within three standard deviations. The minimum values of ξ_L and ξ_T that in principle could be measured would then be given by ([9.13\)](#page-10-2), taking $N_D^{\mathcal{N}} = 10$ and all the optimization choices reviewed above. (In the numerical computations that we will present in the next section, we also include a small correction $sin^4(\theta/2)$ that appears in the expression of $\phi(0)$ as shown in Ref. [\[52](#page-16-22)].)

9.7 Sensitivity at Selected Ultraintense Laser Facilities

Let us now study the possibility of searching for light by light diffraction as described in the previous section at different ultraintense laser facilities that will be available in the next few years.

Facility	P (PW)	τ (fs)	λ (nm)	ξ_r^{lim}/ξ	$\xi_T^{\rm lim}/\xi$	$\xi_T^{\text{lim}}/\xi^{\text{PVLAS}}$
VEGA	$1 + 0.2$	30	800	4.0×10^{2}	2.3×10^{2}	1.0×10^{-1}
PETAL		500	1053	24	14	6.3×10^{-3}
ELI 10 PW	10	30	800	14	8.0	3.6×10^{-3}
100 PW	100	30	800	0.42	0.24	1.1×10^{-4}

Table 9.1 Limiting values of the parameters ξ_L and ξ_T that can be measured at different facilities

One of the systems we proposed for our calculations is the VEGA laser at the Spanish Pulsed Laser Centre (CLPU, Centro de Láseres Pulsados) at Salamanca [\[55](#page-16-42)]. VEGA laser is a CPA system working at 30 femtoseconds after compression. It will have, by the beginning of 2014, a PW line, 30 J in 30 fs at one Hz, synchronized with a 200 TW line, 6 J in 30 fs. The VEGA system is based on standard CPA technology using a Ti:sapphire amplifier. The laser is going to be very relevant because it is going to be running at one Hz (one shot per second) and has the possibility to be upgraded to 5 Hz. VEGA laser will be quite unique in using as probe a 200 TW laser. Of course the conclusions for VEGA can be easily adapted to any other system at short pulse PW level.

The next candidate for the search of quantum vacuum features will be ELI (the Extreme Light Infrastructure). As indicated in the ELI whitebook [\[56](#page-16-43)], ELI will be the first infrastructure devoted to the fundamental study of laser-matter interaction in the ultra-relativistic regime ($I > 10^{24}$ W/cm²). In its first stage, ELI plans to arrive to the 10 PW level and in a second stage expects to pass over the 100 PW barrier.

Such systems correspond to high field lasers, with pulse durations close to 30 fs, or less. Nevertheless, there are other systems with longer pulses, the high energy lasers. For comparison, we have selected the PETAL system as the most representative in this category [\[57](#page-16-44)].

By performing the optimized computation discussed in the previous section, we can compute the limiting value of the parameters ξ_L and ξ_T that can be measured within 3σ for a single shot experiment, depending on the parameters of the laser pulses (power *P*, duration *τ* and wavelength $λ$). In Table [9.1](#page-12-0), we list few facilities that will be available in the near future, and compute the minimal value of the parameters ξ_L^{lim} and ξ_T^{lim} that can be measured for a single shot experiment, as compared either to the QED prediction *ξ* , or to the current PVLAS limit *ξ* PVLAS. We see that all the facilities under consideration are potentially able either to detect signals of new physics, such as axion-like or mini-charged particles, or to significantly improve the PVLAS limits. A 100 PW laser such as that considered in the last line of the table would be able to measure the QED effect. These results are also shown in graphical form in Fig. [9.4](#page-13-0) (that can be compared with Fig. [9.2\)](#page-7-1), where we plot the regions in the $\xi_L - \xi_T$ plane that will be testable in a single-shot experiment with either orthogonal or parallel polarization at the same VEGA, ELI 10, and 100 PW facilities that appear in Table [9.1](#page-12-0).

Let us first discuss the results of the computations for VEGA that appear in Fig. [9.4](#page-13-0) and in the first line of Table [9.1](#page-12-0). As we have mentioned above, VEGA

Fig. 9.4 Predicted sensitivity in the $\xi_L - \xi_T$ parameter space (in logarithmic scale) for a single shot experiment searching for light by light diffraction at different (multi) PW laser facilities, corresponding to Table [9.1](#page-12-0). The result labeled "1 PW" corresponds to VEGA. The outer region bounded by the *red solid lines* is excluded by the current PVLAS data. We also plot the prediction of the pure Born-Infeld theory (*green dashed line*). We see that the single shot experiment at a 100 PW facility will explore a region including the QED prediction, highlighted with the *blue point*. Observe that both scales have been normalized by the QED parameter ξ , so QED falls at the (1*,* 1) point in the figure, only accessible with a 100 PW laser at single shot regime. Note that the sensitivity can be systematically improved by increasing the number of shots, as discussed in the text, so that in principle even VEGA could detect the QED effect, if the noise could be reduced below the signal level

will provide two synchronized laser pulses, one at 0.2 PW—which can be used as the probe pulse B—and the other at 1 PW, suitable for playing the role of pulse A. The limiting sensitivity for a single shot given in Table [9.1](#page-12-0) and Fig. [9.4](#page-13-0) is obtained with the following optimal choices for the experimental parameters:

- 1. the waist of the two pulses at the focus (coinciding with the crossing point) are $w_A = 12 \mu m$ and $w_B = 59 \mu m$, so that the focused intensities of the two pulses are $I_A = 4.6 \times 10^{20}$ W/cm² and $I_B = 3.7 \times 10^{18}$ W/cm²;
- 2. the angle between the two beam directions is $\pi \theta = 7.4^{\circ}$;
- 3. the inner and outer radius of the detector are $r_0 = 0.23$ cm, $R = 0.52$ cm (for $d = 10$ cm);
- 4. the divergence of the diffracted wave that hits the detector is approximately 7 times larger than the divergence of the non-diffracted wave;
- 5. the efficiency of the detector is chosen to be 0.5, which is a realistic value in the near IR with present technology.

The choice of the parameters for ELI 10 and the possible 100 PW laser considered in Table [9.1](#page-12-0) and in Fig. [9.2\)](#page-7-1) is quite similar [[58\]](#page-16-45). On the other hand, in the case of PETAL the optimization requires much larger focused waists, namely $w_A = 48$ µm and $w_B = 0.14$ mm, that may be a technological challenge.

Apart from VEGA, all the other facilities belong to the upcoming multi-PW lasers generation. There are two main reasons for choosing VEGA, among the several PW lasers available worldwide. The first is the fact that it automatically provides two pulses, which is of course an advantage for our experiment. The second reason is its high repetition rate, that is planed to be 1 Hz, and possibly even 5 Hz, as we have mentioned above. From ([9.13](#page-10-2)), we see that the expected number of events is proportional to the number of repetitions N , so that the limiting sensitivity $\xi_{L,T}^{\text{lim}} \propto \mathcal{N}^{-1/2}$ improves as $\mathcal{N}^{-1/2}$ with respect to the single-shot values given in Table [9.1.](#page-12-0) In particular, after $N \approx 5.2 \times 10^4$ shots (less than one day run) VEGA will reach a sensitivity at the QED level for the measurement of ξ_T , since in this case $\xi_T^{\text{lim}} = \xi$. The number of shots needed to measure also ξ_L at the QED level would be $\mathcal{N} \simeq 1.6 \times 10^5$. Of course, the measurement would only be reliable if the noise level, including all possible sources of background, can be kept below the level of the signal, which may not be a trivial requirement. A preliminary analysis of the possible sources of background indicates that VEGA may only reach sensitivity at the QED level if it operates in such an extreme vacuum that may be a challenge for the present technology. For more reasonable values of the pressure of the tube, it will be eventually able to find signals of new physics, or to improve the current limits of ξ_L and ξ_T from PVLAS by several orders of magnitude. On the other hand, the upcoming generation of the 10 PW facilities listed in Table [9.1](#page-12-0) will most probably be able to detect the QED effect. A 100 PW facility would be able to detect the QED effect even in a single shot experiment.

As a result, this kind of experiment might provide the first signature of quantum vacuum polarization in the near future. It should be noted that the four-wave mixing configuration discussed in Ref. [[36\]](#page-16-15) has also been argued to be potentially sensitive to the QED effect (under extreme vacuum requirements) using PW lasers such as Astra Gemini. In such a proposal, three ultra-intense laser pulses are focused close to the diffraction limit and forced to collide at the same space-time point, possibly producing a signal of ultraviolet photons. However, the production and alignment of three ultra-concentrated and ultra-short laser pulses represent a greater technological challenge, as compared to our minimal scenario based on the crossing of only two pulses. Moreover, in the proposal discussed in Ref. [\[36](#page-16-15)] two of the incoming pulses are chosen to double the frequency of the laser source, which is not a trivial requirement from a technical point of view. In any case, even though our proposal is simpler and probably easier to perform, we think that both experiments deserve to be carried out since they can provide two independent tests of the quantum vacuum.

9.8 Conclusions

Ultra-intense laser pulses constitute a unique system in which an enormous amount of photons are packed in the same quantum state in the same microscopic volume. This allows for the possibility of studying the coherent interaction of light with light in vacuum due to the interchange of virtual particles, which is a prediction of the quantum theory. This kind of effects strongly depend on the particle content of the theory, so that any experiment searching for them can also be used to search for new physics, such as axion-like or mini-charged particles, that have been introduced to solve theoretical problems but are still wanted for observation. The first possible signal that may be detectable of such an interaction is the polarization of the quantum vacuum by the laser pulses itself. As we have seen, in principle this effect can be measurable even at the rate predicted by Quantum Electrodynamics (QED) with only the known particle content at a high repetition rate PW laser such as VEGA. We have described an experiment searching for light by light diffraction in vacuum in which two almost counter-propagating pulses cross each other. The optimal choice for the focused intensity of the most concentrated of the two pulses has been argued to be 'just' 5×10^{20} W/cm², a value which is several orders of magnitude lower than the intensities at which another important basic process may become observable, the production of real electron-positron pairs from the vacuum. The study of the background noise, which is in progress, seems to indicate that in practice we will have to wait for the next generation of 10 PW laser for the process of light by light diffraction in vacuum to be observable at the QED rate. In any case, VEGA will already be able either to demonstrate the quantum vacuum polarization due to new physics, or to improve the current limits on photon-photon scattering in vacuum at optical wavelengths by several orders of magnitude. We believe that these results provide strong support to the use of ultra-intense lasers in fundamental research.

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