Chapter 119
Passification Based Controlled Synchronization of Complex Networks

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Abstract In the paper an output synchronization problem for a networks of linear dynamical agents is examined based on passification method and recent results in graph theory. The static output feedback and adaptive control are proposed and sufficient conditions for synchronization are established ensuring synchronization of agents under incomplete measurements and incomplete control. The results are extended to the networks with sector bounded nonlinearities in the agent dynamics and information delays.

Keywords Complex networks · Passification · Synchronization

119.1 Introduction

Controlled synchronization of networks has a broad area of important applications: control of power networks, cooperative control of mobile robots, control of lattices, control of biochemical, ecological networks, etc. [1–5]. However most existing papers deal with control of the networks of dynamical systems (agents) with full state measurements and full control (vectors of agent input, output and state have equal dimensions). In the case of synchronization by output feedback additional dynamical systems (observers) are incorporated into network controllers.

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In this paper the synchronization problem for networks of linear agents with arbitrary numbers of inputs, outputs and states by static output neighbor-based feedback is solved based on passification method [6, 7] and recent results in graphs theory. The results are extended to the networks with sector bounded nonlinearities in the agent dynamics and information delays.

119.2 Problem Statement

Let the network S consist of d agents $S_i, i = 1, \dots, d$. Each agent S_i is modeled as a controlled system

$$\dot{x}_i = Ax_i + B_0 f(x_i) + Bu_i, \qquad y_i = C^{\mathrm{T}} x_i, \tag{119.1}$$

where $x_i \in \mathbb{R}^n$ is a state vector, $u_i \in \mathbb{R}^1$ is a controlling input (control), $y_i \in \mathbb{R}^l$ is a vector of measurements (output). Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be the digraph with the set of vertices $\mathcal{V}$ and the set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ such that for $i = 1, \dots, d$ the vertex v_i is associated with the agent S_i.

Let the control goal be:

$$\lim_{t\to\infty} \big(x_i(t) - x_j(t)\big) = 0, \quad i, j = 1, \dots, d. \tag{119.2}$$

119.3 Static Control

Let control law for S_i be

$$u_i(t) = K \sum_{j\in\mathcal{N}_i} \big(y_i(t-\tau) - y_j(t-\tau)\big) = KC^{\mathrm{T}} \sum_{j\in\mathcal{N}_i} \big(x_i(t-\tau) - x_j(t-\tau)\big), \tag{119.3}$$

where $K \in \mathbb{R}^{1\times l}$, $\mathcal{N}_i = \{k = 1, \dots, d | (v_i, v_k) \in \mathcal{E}\}$ is the set of neighbor vertices to v_i, $\tau \geq 0$ is communication delay.

The problem is to find K from (119.3) such that the goal (119.2) holds.

The problem is first analyzed for linear agent dynamics ($B_0 = 0$) without delays ($\tau = 0$) under the following assumptions:

(A1) *There exists a vector $g \in \mathbb{R}^l$ such that the function $g^{\mathrm{T}} W(s)$ is hyper-minimum-phase, where $W(s) = C^{\mathrm{T}}(sI - A)^{-1}B$.* (Recall that the rational function $\chi(s) = \beta(s)/\alpha(s)$ is called *hyper minimum phase*, if its numerator $\beta(s)$ is a Hurwitz polynomial and its highest coefficient β_{n-1} is positive [7].)

(A2) *The interconnection graph is undirected and connected.*

(A2D) *The interconnection graph is directed and has the directed spanning tree.*

Let $A(\mathcal{G})$ denote adjacency matrix of the graph $\mathcal{G}$. For digraph $\mathcal{G}$ consider the graph $\widehat{\mathcal{G}}$ such that $A(\widehat{\mathcal{G}}) = A(\mathcal{G}) + A(\mathcal{G})^{\mathrm{T}}$. Laplacian $L(\widehat{\mathcal{G}}) = D(\widehat{\mathcal{G}}) - A(\widehat{\mathcal{G}})$ of the graph $\widehat{\mathcal{G}}$ is symmetric and has the eigenvalues: $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_d$, [1, 3]. The main result is as follows.

Theorem 119.1 *Let assumptions* A1 *and either* A2 *or* A2D *hold and* $k \geq 2\kappa/\lambda_2$, *where*

$$\kappa = \sup_{\omega \in \mathbb{R}^1} \mathrm{Re}\big(g^{\mathrm{T}} W(i\omega)\big)^{-1}. \tag{119.4}$$

Then the control law (119.3) *with feedback gain* $K = -k \cdot g^{\mathrm{T}}, k \in \mathbb{R}^1$ *ensures the goal* (119.2).

Similar results are obtained for undirected and balanced directed communication graphs.

119.4 Adaptive Control

Let agent S_i be able to adjust its control gain, i. e. each local controller is adaptive. Let each controller have the following form:

$$u_i(t) = \theta_i(t) y_i(t), \tag{119.5}$$

where $\theta_i(t) \in \mathbb{R}^{1\times l}$—tunable parameter which is tuned based on the measurements from the neighbors of i-th agent.

Denote:

$$\overline{y}_i = \sum_{j \in \mathcal{N}_i} (y_i - y_j), \quad i = 1, \ldots, N$$

and consider the following adaptation algorithm:

$$\theta_i(t) = -g^{\mathrm{T}} \cdot k_i(t), \qquad \dot{k}_i(t) = y_i(t)^{\mathrm{T}} g g^{\mathrm{T}} \overline{y}_i. \tag{119.6}$$

Adaptive synchronization conditions are formulated as follows.

Theorem 119.2 *Let assumptions* A1, A2 *hold. Then adaptive controller* (119.5)–(119.6) *ensures achievement of the goal* (119.2).

The above results are extended to the networks with sector bounded nonlinearities in the agent dynamics and information delays.

119.5 Conclusions

The control algorithm for synchronization of networks based on static output feedback (119.3) to each agent from the neighbor agents is proposed. Since the number of inputs and outputs of the agents are less than the number of agent state variables, synchronization of agents is achieved under incomplete measurements and incomplete control. Synchronization conditions include passifiability (hyper-minimum-phase property) for each agent and some connectivity conditions for interconnection graph: existence of the directed spanning tree in case of directed graph and connectivity in case of undirected graph. Similar conditions are obtained for adaptive passification-based control of network with undirected interconnection graph, for sector bounded nonlinearities in the agent dynamics and information delays.

The proposed solution for output feedback synchronization unlike those of [4, 5] does not use observers. Compared to static output feedback result of [4, Theorem 4] the proposed synchronization conditions relax passivity condition for agents to their passifiability that allows for unstable agents. The paper [4], however, deals with time-varying network topology. The presented results extend our previous results [8–10].

Simulation results for the networks of double integrators and Chua circuits illustrate the theoretical results.

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