Chapter 102
Modeling Urban Patterns Across Geographical Scales by a Fractal Diffusion-Aggregation Approach

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Abstract An integral urban growth model is introduced. We developed this model to capture the different spatial morphologies and urban dynamics observed when more than one city are interacting on a specific region creating large metropolitan areas at different geographical scales. For small scales (1:1500000) our model is based on two well-known fractal growth processes, diffusion and percolation, in order to represent two of the main urban growth drivers: people migration and economics of agglomeration respectively. Morphology at large scales (1:50000) is derived from a Self-Organized Criticality (SOC) model, adapted to urban interactions to explore the possible relations between "avalanches" and city redensification processes.

102.1 Assumptions for the Model

The approach presented here is based on three previously reported urban models:

- Colored diffusion-limited aggregation for urban migration [1],
- Modeling large Mexican urban metropolitan areas by a Vicsek-Szalay approach [2], and
- Modeling Mexican urban metropolitan areas by a self-organized criticality approach [3].

The model is based on the following assumptions:

1. Cities and systems of cities, at certain scales, reflect fractal growth patterns.
2. Cities and systems of cities exhibit Self-Organization capabilities.
3. Cities and systems of cities can be thought of as complex systems with certain Self-Organization Criticality characteristics.

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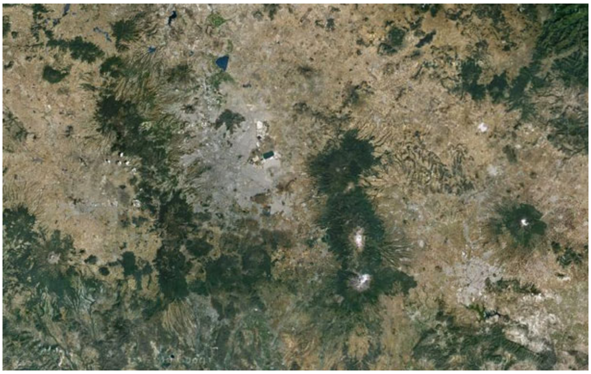

Fig. 102.1 Great metropolitan area of central Mexico 12044.8 km^2 (7528 square miles) 22 million people

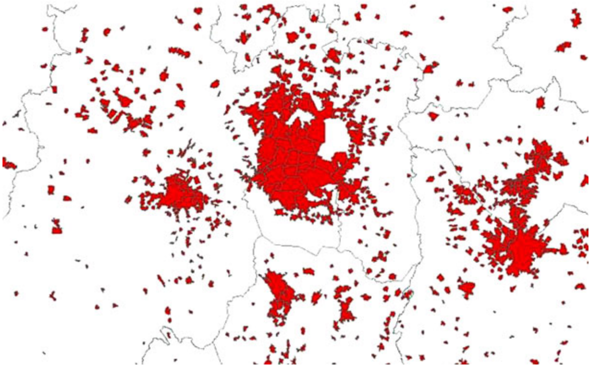

Fig. 102.2 Vector image of the actual central México metropolitan areas

Figures 102.1 and 102.2 depict two different views of the Central Mexico Metropolitan Area (CMMA). Notice that it is composed primarily of three large metropolitan areas, México, Puebla and Toluca.

Our ambition is to model metropolitan areas from first principles, at two different geographical scales, 1:1000000 and 1:200000. To accomplish this, different theoretical models will be used:

(i) Diffusion Limited Aggregation,
(ii) Percolation, and
(iii) Self-Organized Criticality.

102.2 Urban Migration—Diffusion Limited Aggregation

Why people move? Is there a reason why a group of persons would change its way of life in one geographical place to begin a new one in another? Economics, land and government changes changes are never simple, whether in type or intensity. New opportunities emerge in different places. Things could just not have been working any more at home, or, maybe, are getting better somewhere else. It is the difference

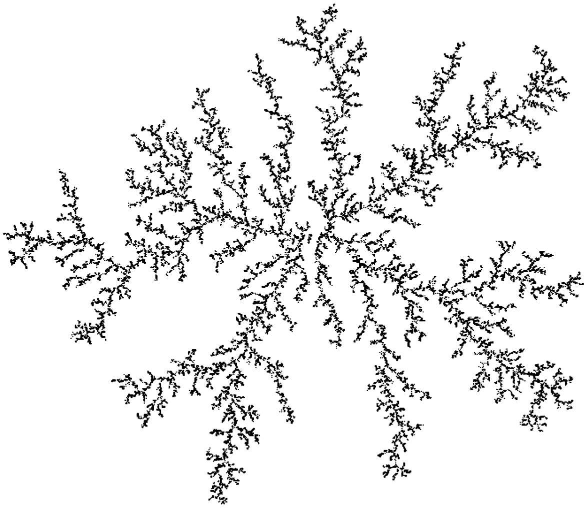

Fig. 102.3 Classic configuration obtained with the DLA urban model. This model can be consulted in Colored diffusion-limited aggregation for urban migration can be consulted in Ref. [1]

between the present and the feeling of other opportunities which push people's decisions in motion. When this feeling grows bigger and bigger and there are no barriers in the way, a migration takes place. For migration to occur we need three important features:

- Complementarity—In order to migrate between places, an offer must exist in one place and a need in another
- Intervention opportunity—Complementarity between places can generate exchange just in the absence of intervention opportunities
- Migration Cost—Is a weight on real time and cost. If the time and cost of moving a distance is too big, movement will not take place, regardless of perfect complementarity and the absence of intervention opportunities

Figure 102.3 depicts a typical Diffusion Limited Agreggation growth pattern [4], observed under those conditions [1].

102.3 Urban Growth Through Economics of Agglomeration—Vicsek-Szalay-Batty

Tamás Vicsek and Alexander S. Szalay [5] proposed a cellular automaton model (VS) to study the fractal distribution of galaxies. In their lattice model, a cell i, which represents a mass element, would become part of a galaxy based on two parameters; the potential of belonging to a galaxy at position i, and a given threshold that regulates such potential.

Economic agglomerations theory states that

> "Just as matter in the solar system is concentrated in a small number of bodies (the planets and their satellites); economic life is concentrated in a fairly limited number of human settlements (cities and clusters). Furthermore, paralleling large and small planets, there are large and small settlements with very different combinations of firms and households [6]."

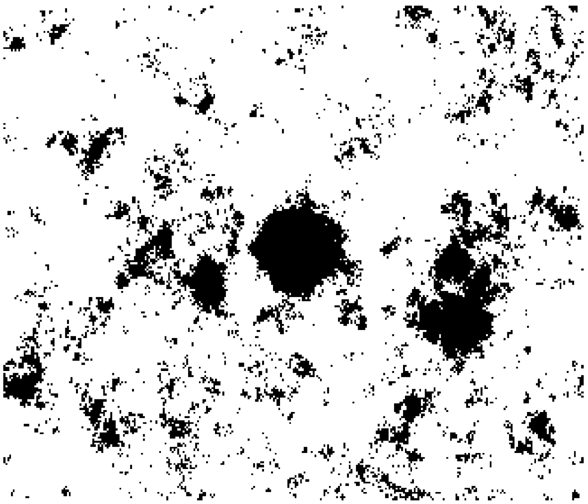

Fig. 102.4 Example of a configuration obtained by our model set to ($\Phi = 4.5$, $t = 500$). The three main clusters from the actual CMMA are clearly present, along with several satellite clusters. This model can be consulted in Modeling large Mexican urban metropolitan areas by a Vicsek Szalay approach can be consulted in Ref. [2]

The use of a model developed for galaxies to understand urban settlements is thus a reasonable approach. Batty [7] took VS and adapted it to an urban context, the VSB model. Letting $P_i(t)$ denote the potential of population growth at site i and time t, we have the following evolution:

$$P_i(t+1) = \sum_{j=\Omega_i} \frac{P_j(t)}{5} + \varepsilon_i(t) \tag{102.1}$$

$$P_i(t) > \Phi \quad \text{and} \quad D_i(t-1) = 0 \quad \text{then } D_i(t) = 1, \quad \text{otherwise } D_i(t) = 0 \tag{102.2}$$

In this system, Ω_i denotes the four cells neighboring i on a two-dimensional square grid, $\varepsilon_i(t)$ is a random variable which take values ± 1 with equal probabilities, Φ is a threshold parameter, and $D_i(t)$ is an index indicating whether or not cell i has undergone development.

Using this approach, we found important and, to some extent, surprising similarities with the actual digitized area. These include Box Counting dimension, Zipf's law, morphology, and possible 2nd-order phase transitions between sprawl and compact configurations depending on the threshold parameter. These configurations are Independent of "time".

Figure 102.4 illustrates the type of configurations derived by the model [2].

102.4 Urban Redensification—Self-organized Criticality

Like a pile of sand, cities change, it is an undeniable fact. Cities change from one state to another all the time due the aggregation of new activities, such as births, migration, new real estate developments, etc. Most of the activities change through

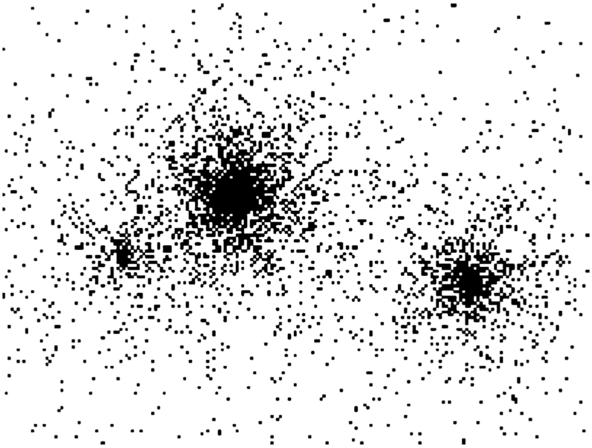

Fig. 102.5 Example of a configuration obtained by our model at $t = 5000$. The three main clusters from the actual CMMA are clearly present, along with many isolated points, each one representing an urban town. This model can be consulted in Ref. [3]

processes of redistribution. Each time an activity changes its location, it triggers a chain reaction in which other activities are motivated to move, because economic agents that make up these activities readjust to the new circumstances. Empirical evidence tell us that the city continues to exist with basically the same morphology while these chain reactions occurs, and that such reactions do not continue indefinitely. These processes can be modeled using self-organized criticality [8–10]. In first instance, assume that a population is distributed in an urban area according to a empirical power law that relates population density $\rho(r)$ at a distance r from the city's Central Business District (CBD) as follows:

$$\rho(r) \sim r^{-\alpha} \tag{102.3}$$

where α is a density gradient parameter, which in Ref. [7] is held constant over time, even though it appears to decrease gradually as the city grows. This parameter affects the density of the population, not the SOC process itself, so for practical purposes, we follow this recommendation in this paper, and α will remain constant through all simulations. In terms of the sand pile model, the critical slope of the distribution of the population over a city is controlled by α. Certainly avalanches occur, but do not follow a power law.

Figure 102.5 illustrates a configuration obtained by this model.

102.5 Integrated Model

We combine the initial parameters of Vicsek Szalay Batty and SOC in the same layout

- Initial Potential—1:1000000
- Maximum Capacity MC—1:200000
- Current Capacity CC—1:200000

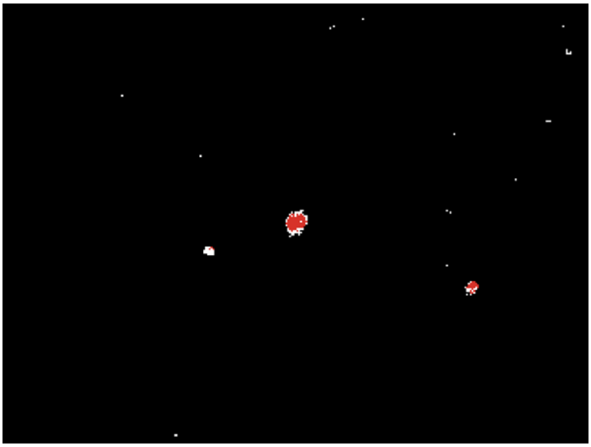

Fig. 102.6 Urban configuration at $t < 50$. Threshold equals to 4.5. *The red portions* represent the avalanches

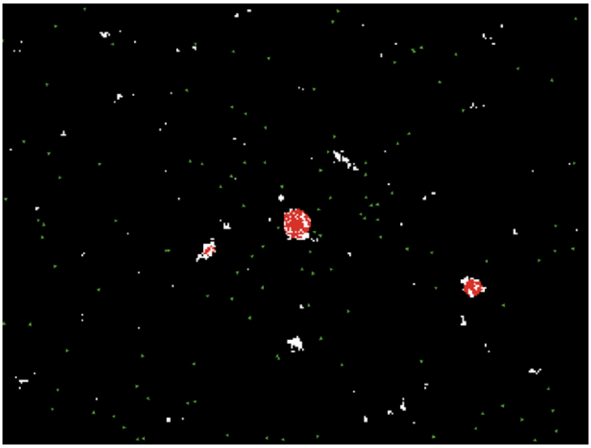

Fig. 102.7 Urban configuration at $t = 150$. Threshold equals to 4.5

The local interactions provoke local avalanches, which increase CC, which, at some point, affects the global potential of a zone. The potential is acting at 1:1000000, a scale at which we can actually observe urban growth.

Figures 102.6, 102.7 and 102.8 summarize the configuration observed in the course of time.

At $t = 0$ the process is initiated.

At $t = 50$ a series of migration waves begins to take place. These migrating units (agents) follow a direct walk towards the central cluster, as seen in Figs. 102.7 and 102.8.

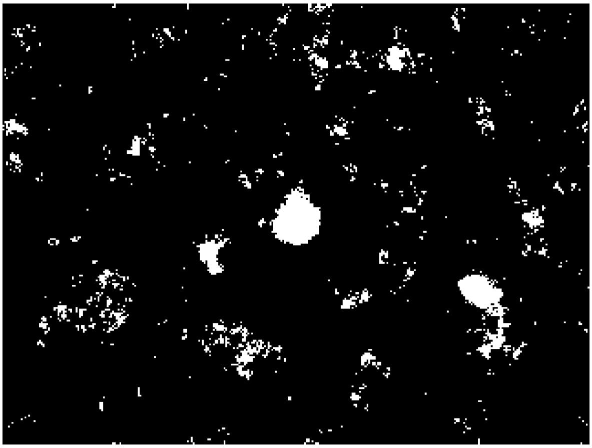

Fig. 102.8 Urban configuration at t = 300. Threshold equals to 4.5

102.6 Comments

- At the 1:1000000 scale we have not obtained significantly different morphologies than with VSB.
- Avalanches do not follow a power law distribution at a Metropolitan Area scale
- But Zipf's law is still fulfilled—Fuzzy Clustering
- Emerging patters obtained are consistent with the actual urban patterns.
- The influence of the Current Capacity parameter needs further investigation.
- With only VSB and Migration, we have obtained a possible second order transition.
- This situation will probably would repeat with the inclusion of the SOCs parameters.

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