Chapter 33
Study of Stability Matter Problem in Micropolar Generalised Thermoelastic

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Abstract The theory of micropolar thermoelasticity has many applications. One form of the recent years concerning the problem of propagation of thermal waves at finite speed and the possibility of "second sound" effects established a new thermo mechanical theory of deformable media that uses a general entropy balance as postulated and the theory is illustrated in detail in the context of flow of heat in a rigid solid, with particular reference to the propagation of thermal waves at finite speed. Then theory of thermoelasticity for non-polar bodies, based on the new procedures, was discussed and employed the eigen value approach to study the effect of rotation and relaxation time in two dimensional problem of generalized thermoelasticity. Recently investigation shows the dynamic response of a homogeneous, isotropic, generalized thermoelastic half-space with voids subjected to normal, tangential force and thermal stress. In this paper we introduce the eigen value approach, following Laplace and Fourier transformation has been employed to find the general solution of the field equation in a micropolar generalized thermoelastic medium for plane strain problem. An application of an infinite space with an impulsive mechanical source has been taken to illustrate the utility of the approach. The integral transformation has been inverted by using a numerical inversion technique to get result in physical domain. The result in the form of normal displacement, normal force stress, tangential force stress, tangential couple stress and temperature field components have been obtained numerically and illustrated graphically. Special case of a thermoelastic solid has also been deduced.

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33.1 Introduction

The theory of micropolarthermoelasticity has been a subject of intensive study. A comprehensive review of works on the subject was given by [4] and [19]. There has been very much written in recent years concerning the problem of propagation of thermal waves at finite speed. A generalized theory of linear micropolarthermoelasticity that admits the possibility of “second sound” effects was established by [1]. Recently, [9] established a new thermomechanical theory of deformable media that uses a general entropy balance as postulated by [8]. The theory is illustrated in detail in the context of flow of heat in a rigid solid, with particular reference to the propagation of thermal waves at finite speed. A theory of thermoelasticity for non-polar bodies, based on the new procedures, was discussed by [10]. Bahshi et al. [2] employed the eigen value approach to study the effect of rotation and relaxation time in two dimensional problem of generalized thermoelasticity. Kumar and Rani [15] studied the deformation due to mechanical and thermal sources in generalized orthorhomtic-thermoelastic material. Kumar and Rani [16] investigated the dynamic response of a homogeneous, isotropic, generalized thermoelastic half-space with voids subjected to normal, tangential force and thermal stress. The micropolar theory was extended to include thermal effects by [4] and [19]. Kumar and Chadha [13] derived the expressions for displacements, microrotation, force stress, couple stress and first moment for a half - space subjected to an arbitrary temperature field and a particular case of line heat source has been discussed in detail. The uniqueness of the solution of some boundary value problems of the linear micropolarthermoelasticity was investigated by [3]. Passarella [21] solved the initial-boundary value problem for micropolarthermoelasticity and proved a uniqueness theorem for the problem. Mahalanabis and Manna [17] discussed eigen value approach to linear micropolarthermoelasticity by arranging basic equations of elasticity in the form of matrix deferential equation in the Hankel transform and extended the approach to linear thermoelasticity. Marin and Lupu [18] investigated harmonic vibrations in thermoelasticity of micropolar bodies. Kumar and Deswal [14] discussed the disturbance due to mechanical and thermal sources in homogeneous isotropic micropolar generalized thermoelastic half-space.

33.2 Formulation and Solution of the Problem

We consider a homogeneous, isotropic, micropolar generalized thermoelastic solid in an undisturbed state and initially at uniform temperature. We take a cartesian system (x, y, z) and z-axis pointing vertically into the medium.

Following [6], [12] and [11], the field equations and the constitutive relations in micropolar generalized thermoelastic solid without body forces, body couples and heat sources can be written as

$$(\lambda + 2\mu + K)\,\nabla(\nabla.\mathbf{u}) - (\mu + K)\,\nabla \times \nabla \times \mathbf{u} + K\nabla \times \boldsymbol{\phi} - \nu\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla \mathrm{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

$$(\alpha + \beta + \gamma)\,\nabla(\nabla.\boldsymbol{\phi}) - \gamma\nabla \times \nabla \times \boldsymbol{\phi} + K\nabla \times \mathbf{u} - 2\mathrm{K}\boldsymbol{\phi} = \rho\, j \frac{\partial^2 \boldsymbol{\phi}}{\partial t^2}$$

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 \mathrm{T}}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\,\tau_0 \frac{\partial^2}{\partial t^2}\right)\nabla \cdot \mathbf{u}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i},$$

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \varepsilon_{ijr}\varphi_r\right) - \nu\left(\mathrm{T} + \tau_1 \frac{\partial T}{\partial t}\right)\delta_{\mathrm{ij}},$$

For the L-S (Lord Shulman) theory $\tau_1 = 0$, $\Xi = 1$ and for G - L (Green Lindsay) theory $\tau_1 = 0$, $\Xi = 0$,

The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G-L theory only. However, it has been proved by [22] that the inequalities are not mandatory for τ_0 and τ_1 to follow.

For two dimensional plane strain problem parallel to *xz*-plane, we assume

$$\mathbf{u} = (u_1, 0, u_3)\,, \quad \boldsymbol{\phi} = (0, \phi_2, 0)$$

The displacement components u_1, $u_3$and microrotation component depend upon x, z and t and are independent of co-ordinate y, so that $\frac{\partial}{\partial y} \equiv 0$. With these considerations and using (2.6) and introducing the non-dimensional quantities as

$$x' = \frac{\omega^* x}{C_1}, \qquad z' = \frac{\omega^* z}{C_1},$$

$$T' = \frac{T}{T_0}, \qquad u_1' = \frac{\rho\omega^* C_1 u_1}{\nu T_0},$$

$$m_{32}' = \frac{\omega^*}{C_1 \nu T_0} m_{32},$$

where

$$\omega^* = \frac{C^*(\lambda + 2\mu)}{K^*}, \qquad C_1^2 = \frac{\lambda + 2\mu}{\rho}.$$

Now applying Laplace and Fourier transform defined by

$$\overline{f}(x, z, p) = \int_0^\infty f(x, z, t) exp(-pt) dt,$$

$$\tilde{f}(\xi, z, p) = \int_{-\infty}^\infty \overline{f}(x, z, p) \mathrm{e}(-\iota\xi x) dx,$$

on the set of Eq. (2.1)–(2.3), after suppressing primes, we get

$$\frac{d^2\tilde{u}_1}{dz^2} = \frac{1}{\mathrm{m}_3}\left[\left(\mathrm{m}_1\xi^2 + \mathrm{p}^2\right)\tilde{u}_1 - \iota\xi\mathrm{m}_2\frac{d\tilde{u}_3}{dz} + \mathrm{m}_4\frac{d\tilde{\phi}_2}{dz} + \iota\xi\left(1+\tau_1 p\right)\tilde{T}\right]$$

$$\frac{d^2\tilde{u}_3}{dz^2} = \frac{1}{\mathrm{m}_1}\left[-\iota\mathrm{m}_2\xi\frac{d\tilde{u}_1}{dz} + \left(\mathrm{m}_3\xi^2 + \mathrm{p}^2\right)\tilde{u}_3 - \mathrm{m}_4\iota\xi\tilde{\phi}_2 + \frac{d\tilde{T}_1}{dz}\right]$$

$$\frac{d^2\tilde{\phi}_2}{dz^2} = -\mathrm{m}_5\frac{d\tilde{u}_1}{dz} + \iota\xi\mathrm{m}_5\tilde{u}_3 + \left(2\mathrm{m}_5 + \xi^2 + \mathrm{m}_6\mathrm{p}^2\right)\tilde{\phi}_2$$

$$\frac{d^2\tilde{T}}{dz^2} = \varepsilon p\left(1+\tau_0 p\,\Xi\right)\left\{\iota\xi\tilde{u}_1 + \frac{d\tilde{u}_3}{dz}\right\} + \left\{\xi^2 + p\left(1+\tau_0 p\right)\tilde{T}\right\}$$

where

$$m_1 = \frac{\lambda+2\mu+K}{\rho C_1^2}, \qquad m_2 = \frac{\lambda+\mu}{\rho C_1^2},$$

$$m_4 = \frac{K}{\rho C_1^2}, \qquad m_5 = \frac{K C_1^2}{\rho\omega^{*2}},$$

$$m_7 = \frac{\mu}{\rho C_1^2}, \qquad m_8 = \frac{\lambda}{\rho C_1^2},$$

$$\varepsilon = \frac{\mathrm{T}_0\beta_1^2}{\rho K^*\omega^*}.$$

Equations (2.9)– (2.12) can be written in the vector matrix differential equation form as

$$\frac{d}{dz}W\left(\xi, z, p\right) = \mathrm{A}\left(\xi, p\right)\mathrm{W}\left(\xi, z, p\right)$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \qquad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & f_{12} & f_{13} & 0 \\ f_{21} & 0 & 0 & f_{24} \\ f_{31} & 0 & 0 & 0 \\ 0 & f_{42} & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} g_{11} & 0 & 0 & g_{14} \\ 0 & g_{22} & g_{23} & 0 \\ 0 & g_{32} & g_{33} & 0 \\ g_{41} & 0 & 0 & g_{44} \end{bmatrix}$$

and O is the Null matrix of order 4 with

$$f_{12} = \frac{-l\xi m_2}{m_3}, \qquad f_{13} = \frac{m_4}{m_3},$$

$$f_{31} = -m_5, \qquad f_{42} = \varepsilon p\left(1+\tau_0 p\,\Xi\right),$$

$$g_{22} = \frac{\left(\mathrm{m}_3\xi^2 + \mathrm{p}^2\right)}{m_1}, \qquad g_{22} = \frac{-\mathrm{lm}_4\xi}{m_1},$$

$$g_{41} = l\varepsilon\xi p\,(1 + \tau_0 p\,\Xi)\,, \qquad g_{44} = \xi^2 + p\,(1 + \tau_0 p)\,,$$

To solve the Eq. (2.14), we take $W(\xi, z, p) = X(\xi, p)\mathrm{e}^{\mathrm{qz}}$ for some q, So we obtain

$$A(\xi, p)W(\xi, z, p) = qW(\xi, z, p),$$

This leads to Eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$det\,(A - q\mathrm{I}) = 0$$

which on expansion provides us

$$q^8 - \sigma_1 q^6 + \sigma_2 q^4 - \sigma_3 q^2 + \sigma_4 = 0$$

where

$$\begin{aligned}
\sigma_1 &= g_{11} + g_{22} + g_{33} + g_{44} + f_{24}f_{42} + f_{12}f_{21} + f_{13}f_{31}\,,\\
\sigma_2 &= g_{11}g_{22} + g_{22}g_{33} + g_{33}g_{11} + g_{44}g_{11} + g_{44}g_{22} + g_{44}g_{33} + f_{12}f_{24}g_{41}\\
&\quad + f_{24}f_{42}g_{11} + f_{24}f_{42}g_{33} + f_{24}f_{42}f_{13}f_{31} - g_{32}g_{23} + f_{12}f_{21}g_{33}\\
&\quad + f_{12}f_{21}g_{44} - f_{12}f_{31}g_{23} + f_{13}f_{31}g_{22} + f_{13}f_{31}g_{44} - g_{14}g_{41} - g_{14}f_{21}f_{42},\\
\sigma_3 &= g_{11}g_{22}g_{33} + g_{22}g_{33}g_{44} + g_{33}g_{44}g_{11} + g_{44}g_{11}g_{22} - g_{11}g_{23}g_{32} - g_{44}g_{23}g_{32}\\
&\quad + f_{24}f_{42}g_{11}g_{33} + f_{13}f_{24}g_{41}g_{32} + f_{31}f_{42}g_{14}g_{23} + f_{12}f_{21}g_{33}g_{44}\\
&\quad - f_{13}f_{21}g_{32}g_{44} - f_{12}f_{31}g_{23}g_{44} - f_{42}f_{21}g_{14}g_{33} + f_{13}f_{31}g_{22}g_{44}\\
&\quad - g_{14}g_{41}g_{22} - g_{14}g_{41}g_{33},\\
\sigma_4 &= g_{11}g_{22}g_{33}g_{44} - g_{23}g_{32}g_{11}g_{44} + g_{14}g_{41}g_{22}g_{33} + g_{23}g_{32}g_{14}g_{41}.
\end{aligned}$$

The eigen values of the matrix A are the characteristic roots of the Eq. (2.19). The vectors X (ξ, p) corresponding to the eigen values $q_s$can be determined by solving the homogeneous equations

$$[A - qI]\,\mathrm{X_s}\,(\xi, \mathrm{p}) = 0.$$

The set of eigen vectors $\mathrm{X_s}\,(\xi, \mathrm{p})$; s = 1, 2, 3, ..., 8 may be defined as

$$\mathrm{X_s}(\xi, \mathrm{p}) = \begin{bmatrix} \mathrm{X_{s1}}(\xi, \mathrm{p}) \\ \mathrm{X_{s2}}(\xi, \mathrm{p}) \end{bmatrix}$$

where

$$\mathrm{X}_{s1}(\xi, \mathrm{p}) = \begin{bmatrix} \mathrm{a_s q_s} \\ \mathrm{b_s} \\ -\xi \\ \mathrm{c_s} \end{bmatrix}, \qquad \mathrm{X_{s2}}(\xi, \mathrm{p}) = \begin{bmatrix} \mathrm{a_s q_s^2} \\ \mathrm{b_s q_s} \\ -\xi\,\mathrm{q_s} \\ \mathrm{c_s q_s} \end{bmatrix},$$

$$\mathrm{X}_{l1}(\xi, \mathrm{p}) = \begin{bmatrix} -\mathrm{a_s q_s} \\ \mathrm{b_s} \\ -\xi \\ \mathrm{c_s} \end{bmatrix}, \qquad \mathrm{X}_{l2}(\xi, \mathrm{p}) = \begin{bmatrix} \mathrm{a_s q_s^2} \\ -\mathrm{b_s q_s} \\ \xi \mathrm{q_s} \\ -\mathrm{c_s q_s} \end{bmatrix},$$

$$\begin{aligned}
a_s &= \frac{-\xi}{\mathrm{m}_3 \Delta_\mathrm{s}} \left[\left\{ \xi^2 + p\,(1+\tau_0 p) - q_s^2 \right\} \left\{ m_4 m_5 + m_2 \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2 \right) \right.\right. \\
&\quad \left.\left. + \varepsilon p \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2 \right) (1+\tau_1 p)\,(1+\tau_0 p)\, \Xi \right\} \right] \\
b_s &= \frac{\iota}{\mathrm{m}_3 \Delta_\mathrm{s}} \left[\left\{ \xi^2 + p\,(1+\tau_0 p) - q_s^2 \right\} \left\{ m_4 m_5 q_s^2 + \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2 \right) \right.\right. \\
&\quad \left.\left. \times \left(m_1 \xi^2 + p^2 - m_3 q_s^2 \right) \right\} + \varepsilon p \xi^2 \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2 \right) (1+\tau_1 p)\,(1+\tau_0 p)\, \Xi \right] \\
c_s &= \frac{\varepsilon p q_s\,(1+\tau_0 p\,\Xi)\,(\iota \xi a_s + b_s)}{\left[q_s^2 - \left\{ \xi^2 + p\,(1+\tau_0 p) \right\} \right]} \\
\Delta_s &= \frac{\mathrm{m}_5}{\mathrm{m}_3} [\left\{ \xi^2 + p\,(1+\tau_0 p) - q_s^2 \right\} \{ m_2 q_s^2 - \left(m_1 \xi^2 + p^2 - q_s^2 m_3 \right) \} \\
&\quad + \varepsilon p\,(1+\tau_0 p\,\Xi)\,(1+\tau_1 p) \left(q_s^2 - \xi^2 \right)]
\end{aligned}$$

Thus solution of Eq. (2.14) is as given by [23]

$$\mathrm{W}(\xi, z, p) = \sum_{s=1}^{4} \left[E_s X_s\,(\xi, p)\, \mathrm{e}^{\mathrm{q_s} z} + E_{s+4} X_{s+4}\,(\xi, p)\, \mathrm{e}^{-\mathrm{q_s} z} \right]$$

$\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \mathrm{E}_4, \mathrm{E}_5, \mathrm{E}_6, \mathrm{E}_7$ and E_8 are eight arbitrary constants. The Eq. (2.32) represents a general solution of the plane strain problem for isotropic, micropolar generalized thermoelastic solid and gives the displacement, microrotation and temperature field in the transformed domain.

33.3 Applications

Mechanical Source

We consider an infinite micropolar generalized thermoelastic space in which a concentrated force where F_0 is the magnitude of the force, $\mathrm{F} = -F_0 \delta(\mathrm{x}) \delta(\mathrm{t})$ acting in the direction of the z-axis at the origin of the Cartesian co-ordinate system as shown in Fig. 33.1. The boundary condition for present problem on the plane $\mathrm{z} = 0$ are

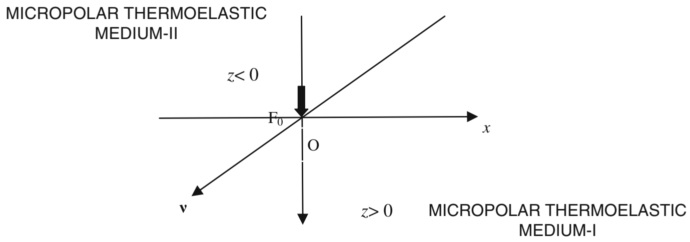

Fig. 33.1 .

$$
\begin{aligned}
&\mathrm{u}_1(\mathrm{x}, 0^+, \mathrm{t}) - \mathrm{u}_1(\mathrm{x}, 0^-, \mathrm{t}) = 0, \mathrm{u}_3(\mathrm{x}, 0^+, \mathrm{t}) - \mathrm{u}_3(\mathrm{x}, 0^-, \mathrm{t}) = 0,\\
&\phi_2(\mathrm{x}, 0^+, \mathrm{t}) - \phi_2(\mathrm{x}, 0^-, \mathrm{t}) = 0, T(\mathrm{x}, 0^+, \mathrm{t}) - T(\mathrm{x}, 0^-, \mathrm{t}) = 0,\\
&\frac{\partial T}{\partial z}\left(\mathrm{x}, 0^+, \mathrm{t}\right) - \frac{\partial T}{\partial z}\left(\mathrm{x}, 0^-, \mathrm{t}\right) = 0, t_{31}\left(\mathrm{x}, 0^+, \mathrm{t}\right) - t_{31}\left(\mathrm{x}, 0^-, \mathrm{t}\right) = 0,\\
&t_{33}\left(\mathrm{x}, 0^+, \mathrm{t}\right) - t_{33}\left(\mathrm{x}, 0^-, \mathrm{t}\right) = -F_0\delta(\mathrm{x})\delta(\mathrm{t}), m_{32}\left(\mathrm{x}, 0^+, \mathrm{t}\right) - m_{32}\left(\mathrm{x}, 0^-, \mathrm{t}\right) = 0
\end{aligned}
$$

Making use of Eq. (2.6)–(2.7) and $F_0' = \frac{F_0}{K}$ in Eq. (2.4)–(2.5), we get the stresses in the non-dimensional form with primes. After suppressing the primes, we apply Laplace and Fourier transforms defined by Eq. (2.8) on the resulting equations and from Eq. (3.1), we get transformed components of displacement, microrotation, temperature field, tangential force stress, normal force stress and tangential couple stress for $z > 0$ are given by

$$
\begin{aligned}
\tilde{\mathrm{u}}_1(\xi, \mathrm{z}, \mathrm{p}) &= -\left\{\mathrm{a}_1\mathrm{q}_1\mathrm{E}_5e^{-\mathrm{q}_1z} + \mathrm{a}_2\mathrm{q}_2\mathrm{E}_6e^{-\mathrm{q}_2z} + \mathrm{a}_3\mathrm{q}_3\mathrm{E}_7e^{-\mathrm{q}_3z} + \mathrm{a}_4\mathrm{q}_4\mathrm{E}_8e^{-\mathrm{q}_4z}\right\},\\
\tilde{\mathrm{u}}_3(\xi, \mathrm{z}, \mathrm{p}) &= \mathrm{b}_1\mathrm{E}_5e^{-\mathrm{q}_1z} + \mathrm{b}_2\mathrm{E}_6e^{-\mathrm{q}_2z} + \mathrm{b}_3\mathrm{E}_7e^{-\mathrm{q}_3z} + \mathrm{b}_4\mathrm{E}_8e^{-\mathrm{q}_4z},\\
\tilde{\phi}_2(\xi, \mathrm{z}, \mathrm{p}) &= -\xi\left\{\mathrm{E}_5e^{-\mathrm{q}_1z} + \mathrm{E}_6e^{-\mathrm{q}_2z} + \mathrm{E}_7e^{-\mathrm{q}_3z} + \mathrm{E}_8e^{-\mathrm{q}_4z}\right\},\\
\tilde{T}(\xi, \mathrm{z}, \mathrm{p}) &= \mathrm{c}_1\mathrm{E}_5e^{-\mathrm{q}_1z} + \mathrm{c}_2\mathrm{E}_6e^{-\mathrm{q}_2z} + \mathrm{c}_3\mathrm{E}_7e^{-\mathrm{q}_3z} + \mathrm{c}_4\mathrm{E}_8e^{-\mathrm{q}_4z},\\
\tilde{\mathrm{t}}_{31}(\xi, \mathrm{z}, \mathrm{p}) &= \left(\mathrm{m}_3\mathrm{a}_1\mathrm{q}_1^2 + \iota\xi\mathrm{b}_1\mathrm{s}_{10} + \xi\mathrm{m}_4\right)\mathrm{E}_5e^{-\mathrm{q}_1z} + \left(\mathrm{m}_3\mathrm{a}_2\mathrm{q}_2^2 + \iota\xi\mathrm{b}_2\mathrm{m}_7 + \xi\mathrm{m}_4\right)\mathrm{E}_6e^{-\mathrm{q}_2z} +\\
&\quad \left(\mathrm{m}_3\mathrm{a}_3\mathrm{q}_3^2 + \iota\xi\mathrm{b}_3\mathrm{m}_7 + \xi\mathrm{m}_4\right)\mathrm{E}_7e^{-\mathrm{q}_3z} + \left(\mathrm{m}_3\mathrm{a}_4\mathrm{q}_4^2 + \iota\xi rmb_4\mathrm{m}_7 + \xi\mathrm{m}_7\right)\mathrm{E}_8e^{-\mathrm{q}_4z},\\
\tilde{\mathrm{t}}_{33}(\xi, \mathrm{z}, \mathrm{p}) &= -[(\iota\xi\mathrm{m}_8\mathrm{a}_1\mathrm{q}_1 + \mathrm{m}_1\mathrm{b}_1\mathrm{q}_1 + \mathrm{c}_1(1 + \tau_1 p))\mathrm{E}_5e^{-\mathrm{q}_1z}\\
&\quad + (\iota\xi\mathrm{m}_8a_2\mathrm{q}_2 + \mathrm{m}_1\mathrm{b}_2\mathrm{q}_2 + \mathrm{c}_2(1 + \tau_1 p))\mathrm{E}_6e^{-\mathrm{q}_2z}\\
&\quad + (\iota\xi\mathrm{m}_8a_3\mathrm{q}_3 + \mathrm{m}_1\mathrm{b}_3\mathrm{q}_3 + \mathrm{c}_3(1 + \tau_1 p))\mathrm{E}_7e^{-\mathrm{q}_3z}\\
&\quad + (\iota\xi\mathrm{m}_8a_4\mathrm{q}_4 + \mathrm{m}_1\mathrm{b}_4\mathrm{q}_4 + \mathrm{c}_4(1 + \tau_1 p))\mathrm{E}_8e^{-\mathrm{q}_4z},\\
\tilde{m}_{32}(\xi, \mathrm{z}, \mathrm{p}) &= \xi\mathrm{s}_8\left\{\mathrm{q}_1\mathrm{E}_5e^{-\mathrm{q}_1z} + \mathrm{q}_2\mathrm{E}_6e^{-\mathrm{q}_2z} + \mathrm{q}_3\mathrm{E}_7e^{-\mathrm{q}_3z} + \mathrm{q}_4\mathrm{E}_8e^{-\mathrm{q}_4z}\right\},
\end{aligned}
$$

for $z<0$, the above expressions get suitably modified, e.g.

$$
\tilde{\mathrm{u}}_1(\xi, \mathrm{z}, \mathrm{p}) = \mathrm{a}_1\mathrm{q}_1\mathrm{E}_1e^{\mathrm{q}_1z} + \mathrm{a}_2\mathrm{q}_2\mathrm{E}_2e^{\mathrm{q}_2z} + \mathrm{a}_3\mathrm{q}_3\mathrm{E}_3e^{\mathrm{q}_3z} + \mathrm{a}_4\mathrm{q}_4\mathrm{E}_4e^{\mathrm{q}_4z},
$$

Making use of the transformed displacements, microrotation, microstretch and stresses given by (3.6)–(3.12) in the transformed boundary conditions, we obtain eight linear relations between the E_i's, which on solving gives

$$
\begin{aligned}
E_1 = E_5 &= \frac{F_0}{2q_1\Delta_1}\left[c_2\,(a_3 - a_4) + c_3\,(a_4 - a_2) + c_4\,(a_2 - a_3)\right], \\
E_2 = E_6 &= \frac{F_0}{2q_2\Delta_1}\left[c_1\,(a_4 - a_3) + c_3\,(a_1 - a_4) + c_4\,(a_3 - a_1)\right], \\
E_3 = E_7 &= \frac{F_0}{2q_3\Delta_1}\left[c_1\,(a_2 - a_4) + c_2\,(a_4 - a_1) + c_4\,(a_1 - a_2)\right], \\
E_4 = E_8 &= \frac{F_0}{2q_4\Delta_1}\left[c_1\,(a_3 - a_2) + c_2\,(a_1 - a_3) + c_3\,(a_2 - a_1)\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_1 = m_1[c_1\,&\{(a_2b_3 - a_3b_2) + (a_3b_4 - a_4b_3) + (a_4b_2 - a_2b_4)\} \\
&+ c_2\,\{(a_3b_1 - a_1b_3) + (a_1b_4 - a_4b_1) + (a_4b_3 - a_3b_4)\} \\
&+ c_3\,\{(a_1b_2 - a_2b_1) + (a_4b_1 - a_1b_4) + (a_2b_4 - a_4b_2)\} \\
&+ c_4\,\{(a_2b_1 - a_1b_2) + (a_1b_3 - a_3b_1) + (a_3b_2 - a_2b_3)\}],
\end{aligned}
$$

Thus functions $\tilde{u}_1, \tilde{u}_3, \tilde{\varphi}_2, \tilde{T}, \tilde{t}_{31}, \tilde{t}_{33}$ and $\tilde{m}_{32}$ have been determined in the transformed domain and these enable us to find the displacements, microrotation, temperature field and stresses.

Case I : For L-S theory, a_s, b_s and c_s in the expressions (3.5)–(3.12) take the form

$$
\begin{aligned}
a_s &= \frac{-\xi}{\mathrm{m}_3\Delta_\mathrm{s}}[\left\{\xi^2 + p\,(1 + \tau_0 p) - q_s^2\right\}\{m_4m_5 + m_2\left(2m_5 + \xi^2 + p^2m_6 - q_s^2\right) \\
&\quad + \varepsilon p\left(2m_5 + \xi^2 + p^2m_6 - q_s^2\right)(1 + \tau_0 p)\}], \\
b_s &= \frac{\iota}{\mathrm{m}_3\Delta_\mathrm{s}}[\left\{\xi^2 + p\,(1 + \tau_0 p) - q_s^2\right\}\{m_4m_5q_s^2 + \left(2m_5 + \xi^2 + p^2m_6 - q_s^2\right) \\
&\quad \times\left(m_1\xi^2 + p^2 - m_3q_s^2\right)\} + \varepsilon p\xi^2\left(2m_5 + \xi^2 + p^2m_6 - q_s^2\right)(1 + \tau_0 p)], \\
c_s &= \frac{\varepsilon pq_s\,(\iota\xi a_s + b_s)}{\left[q_s^2 - \left\{\xi^2 + p\,(1 + \tau_0 p)\right\}\right]},
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_s = \frac{\mathrm{m}_5}{\mathrm{m}_3}&[\left\{\xi^2 + p\,(1 + \tau_0 p) - q_s^2\right\}\{m_2q_s^2 - \left(m_1\xi^2 + p^2 - q_s^2m_3\right)\} \\
&+ \varepsilon p\,(1 + \tau_1 p)\left(q_s^2 - \xi^2\right)]\ ;\ \mathrm{s} = 1,2,3,4
\end{aligned}
$$

and $\pm q_s$ (s = 1, 2, 3, 4) are roots of the Eq. (2.19) in which $\sigma_1, \sigma_2, \sigma_3$ *and* σ_4 are obtained respectively from expressions(2.20)–(2.23) by taking $\tau_1 = 0$, $\Xi = 1$.

Case II : For G-L theory, as b's and c's in the expressions (3.5)–(3.12) take the form

$$a_s = \frac{-\xi}{\mathrm{m}_3\Delta_\mathrm{s}}[\left\{\xi^2 + p\,(1+\tau_0 p) - q_s^2\right\}\{m_4 m_5 + m_2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right) + \varepsilon p\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)(1+\tau_1 p)\}],$$

$$b_s = \frac{\iota}{\mathrm{m}_3\Delta_\mathrm{s}}[\left\{\xi^2 + p\,(1+\tau_0 p) - q_s^2\right\}\{m_4 m_5 q_s^2 + \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(m_1\xi^2 + p^2 - m_3 q_s^2\right)\} + \varepsilon p\xi^2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)(1+\tau_1 p)],$$

$$c_s = \frac{\varepsilon p q_s\,(\iota\xi a_s + b_s)}{\left[q_s^2 - \left\{\xi^2 + p\,(1+\tau_0 p)\right\}\right]},$$

where

$$\Delta_s = \frac{\mathrm{m}_5}{\mathrm{m}_3}[\left\{\xi^2 + p\,(1+\tau_0 p) - q_s^2\right\}\{m_2 q_s^2 - \left(m_1\xi^2 + p^2 - q_s^2 m_3\right)\} + \varepsilon p\,(1+\tau_1 p)\left(q_s^2 - \xi^2\right)]\ ;\ \mathrm{s} = 1,2,3,4$$

and $\pm q_s$ (s =1, 2, 3, 4) are roots of the equation (2.19) in which $\sigma_1, \sigma_2, \sigma_3 and \sigma_4$are obtained respectively from expressions (2.20)–(2.23) by taking $\Xi = 0$

Case III : For Green and Naghdi theory (G-N), Eq. (2.1), (2.3) and (2.4) can be written as

$$(\lambda + 2\mu + K)\,\nabla\,(\nabla.\mathbf{u}) - (\mu + K)\,\nabla\times\nabla\times u + K\nabla\times\boldsymbol{\phi} - \nu\nabla \mathrm{T} = \rho\frac{\partial^2\mathbf{u}}{\partial \mathrm{t}^2}$$

$$\mathrm{K}^*\nabla^2\mathrm{T} = \rho \mathrm{C}^*\frac{\partial^2\mathrm{T}}{\partial \mathrm{t}^2} + \nu T_0\frac{\partial^2\,(\nabla.\mathbf{u})}{\partial \mathrm{t}^2}$$

$$t_{\mathrm{ij}} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{\mathrm{i,j}} + u_{\mathrm{j,i}}\right) + K\left(u_{\mathrm{j,i}} - \varepsilon_{ij,\mathrm{r}}\phi_r\right) - \nu T\delta_{ij}$$

and K^* is not the usual thermal conductivity but a material characteristics constant in G - N & theory and is given $K^*\left(= \frac{C^*(\lambda+2\mu)}{4}\right)$

With the help of Eq. (3.26)–(3.28) and following the procedure of the previous sections, we get the expressions for displacements, microrotation, temperature, field, force stresses and couple stress by taking in Eq. (3.5)-(3.11).

$$(1+\tau_1 p) = 1, \qquad (1+\tau_0 p) = 4p,$$

Particular Case I : Neglecting micropolarity effect i.e. $\alpha = \beta = \gamma = K = j = 0$ in Eq. (3.5)–(3.12), the expressions for displacement components, force stresses and temperature field are obtained in a thermoelastic medium as

$$\tilde{u}_1(\xi, z, p) = -\left\{a_1^* q_1 E_4^* e^{-q_1^* z} + a_2^* q_2^* E_5^* e^{-q_2^* z} + a_3^* q_3^* E_6^* e^{-q_3^* z}\right\},$$

$$\tilde{u}_3(\xi, z, p) = b_1^* E_4^* e^{-q_1^* z} + b_2^* E_5^* e^{-q_2^* z} + b_3^* E_6^* e^{-q_3^* z},$$

$$\tilde{T}(\xi, z, p) = -\xi\left\{E_4^* e^{-q_1^* z} + E_5^* e^{-q_2^* z} + c_3 E_6^* e^{-q_3^* z}\right\},$$

$$\begin{aligned}\tilde{t}_{31}(\xi, z, p) = m_7\{&\left(a_1^* q_1^{*^2} + \iota\xi b_1^*\right) E_4^* e^{-q_1^* z} + \left(a_2^* q_2^{*^2} + \iota\xi b_2^*\right) E_5^* e^{-q_2^* z} \\ &+ \left(a_3^* q_3^{*^2} + \iota\xi b_3^*\right) E_6^* e^{-q_3^* z}\},\end{aligned}$$

$$\begin{aligned}\tilde{t}_{33}(\xi, z, p) = -[&\left\{m_1^* q_1^* b_1^* + \iota\xi m_8 a_1^* q_1^* - \xi\left(1 + \tau_1 p\right)\right\} E_4^* e^{-q_1^* z} \\ &+ \left\{m_1^* q_2^* b_2^* + \iota\xi m_8 a_2^* q_2^* - \xi\left(1 + \tau_1 p\right)\right\} E_5^* e^{-q_2^* z} \\ &+ \left\{m_1^* q_3^* b_3^* + \iota\xi m_8 a_3^* q_3^* - \xi\left(1 + \tau_1 p\right)\right\} E_6^* e^{-q_3^* z}],\end{aligned}$$

where

$$E_1^* = E_4^* = \frac{F_0\left(a_3^* - a_2^*\right)}{2q_1^* \Delta_1^*},$$

$$E_2^* = E_5^* = \frac{F_0\left(a_1^* - a_3^*\right)}{2q_2^* \Delta_1^*},$$

$$E_3^* = E_6^* = \frac{F_0\left(a_2^* - a_1^*\right)}{2q_3^* \Delta_1^*},$$

$$\Delta_1^* = m_1^*\left\{\left(a_2^* b_3^* - a_3^* b_2^*\right) + \left(a_3^* b_1^* - a_1^* b_3^*\right) + \left(a_1^* b_2 - a_2^* b_1^*\right)\right\},$$

$$a_s^* = \frac{-\xi}{m_3^* \Delta_s^*}[m_2\left\{\xi^2 + p\left(1 + \tau_0 p\right) - q_s^{*2}\right\} + \varepsilon p\left(1 + \tau_0 p \Xi\right)\left(1 + \tau_1 p\right)\}],$$

$$\begin{aligned}b_s^* = \frac{\iota}{m_3^* \Delta_s^*}[&\left\{\xi^2 + p\left(1 + \tau_0 p\right) - q_s^{*2}\right\}\left(m_1^* \xi^2 + p^2 - m_3^* q_s^{*2}\right)\} \\ &+ \varepsilon p \xi^2\left(1 + \tau_0 p \Xi\right)\left(1 + \tau_1 p\right)],\end{aligned}$$

$$\Delta_s^* = \frac{\varepsilon p q_s^{*2}\left(1 + \tau_0 p \Xi\right)}{\xi m_3^*}\left\{\left(m_1^* \xi^2 + p^2 - q_s^{*2} m_3^*\right) - \xi^2 m_2\right\},$$

and $\pm q_s^*$ $(s = 1, 2, 3)$ are the roots of the equation

$$q^{*^6} - \sigma_1^* q^{0^4} + \sigma_2^* q^{*^2} - \sigma_3^* = 0$$

$$\sigma_1^* = \left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^*\xi^2 + p^2}{m_1^*}\right) + \left\{\xi^2 + p\,(1 + \tau_0 p)\right\} + \frac{\varepsilon p}{m_1^*}\,(1 + \tau_0 p\,\Xi)\,(1 + \tau_1 p) - \frac{\xi^2 m_2^2}{m_1^* m_3^*},$$

$$\sigma_2^* = \left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^*\xi^2 + p^2}{m_1^*}\right) + \left(\frac{m_3^*\xi^2 + p^2}{m_1^*}\right)\left\{\xi^2 + p\,(1 + \tau_0 p)\right\}$$

$$+ \left\{\xi^2 + p\,(1 + \tau_0 p)\right\}\left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) + \frac{\varepsilon p}{m_1^*}\,(1 + \tau_0 p\,\Xi)\,(1 + \tau_1 p)\left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right)$$

$$- \frac{\xi^2 m_2^2}{m_1^* m_3^*}\left\{\xi^2 + p\,(1 + \tau_0 p)\right\} - \frac{2\varepsilon p \xi^2 m2}{m_1^* m_3^*}\,(1 + \tau_0 p\,\Xi)\,(1 + \tau_1 p) + \frac{\varepsilon p \xi^2}{m_3^*}\,(1 + \tau_0 p\,\Xi)\,(1 + \tau_1 p)\,,$$

with

$$m_1^* = \frac{\lambda + 2\mu}{\rho C_1^2}, \qquad m_3^* = \frac{\mu}{\rho C_1^2}$$

i. For L-S theory : Taking $\tau_1 = 0,\ \Xi = 1$in expression given by (3.29)–(3.33) of particular case I, we obtain expressions for displacement components, temperature field and force stresses.

ii. For G-L theory : Taking $\Xi = 0$ in expressions given by (3.29)–(3.33)of particular case I, we obtain expressions for displacement components, temperature field and force stresses

iii. For G-N theory : Neglecting micropolarity effect i.e. ($\alpha = \beta = \gamma = K = j = 0$) in subcase III of case I, we get the expressions for displacement components, temperature field and force stresses are obtained in a thermoelastic medium by taking

$$1 + \tau_1 p = 1, \quad 1 + \tau_0 p = 4p, \quad 1 + \tau_0 p\,\Xi = p, \quad \varepsilon = \frac{\varepsilon_1}{4}, \quad \omega^* = \frac{C_1}{h}, \quad \varepsilon_1 = \frac{T_0 \nu^2}{\rho K^*}$$

in Eq. (3.29)–(3.44) as

$$\tilde{u}_1(\xi, z, p) = -\left\{a_1^0 q_1 E_4^0 e^{-q_1^0 z} + a_2^0 q_2^0 E_5^0 e^{-q_2^0 z} + a_3^0 q_3^0 E_6^0 e^{-q_3^0 z}\right\},$$

$$\tilde{u}_3(\xi, z, p) = b_1^0 E_4^0 e^{-q_1^0 z} + b_2^0 E_5^0 e^{-q_2^0 z} + b_3^0 E_6^0 e^{-q_3^0 z},$$

$$\tilde{T}(\xi, z, p) = -\,\xi\left\{E_4^0 e^{-q_1^0 z} + E_5^0 e^{-q_2^0 z} + c_3 E_6^0 e^{-q_3^0 z}\right\},$$

$$\tilde{t}_{31}(\xi, z, p) = m_7\{\left(a_1^0 q_1^{02} + \iota\xi b_1^0\right) E_4^0 e^{-q_1^0 z} + \left(a_2^0 q_2^{02} + \iota\xi b_2^0\right) E_5^0 e^{-q_2^0 z}$$
$$+ \left(a_3^0 q_3^{02} + \iota\xi b_3^0\right) E_6^0 e^{-q_3^0 z}\},$$

$$\tilde{t}_{33}(\xi, z, p) = -\,[\left\{m_1^0 q_1^0 b_1^0 + \iota\xi m_8 a_1^0 q_1^0 - \xi\,(1 + \tau_1 p)\right\} E_4^0 e^{-q_1^0 z}$$
$$+ \left\{m_1^0 q_2^0 b_2^0 + \iota\xi m_8 a_2^0 q_2^0 - \xi\,(1 + \tau_1 p)\right\} E_5^0 e^{-q_2^0 z}$$

$$+\left\{m_1^0 q_3^0 b_3^0 + \iota\xi m_8 a_3^0 q_3^0 - \xi\left(1+\tau_1 p\right)\right\} E_6^0 e^{-q_3^0 z}],$$

where

$$\begin{aligned}
E_1^0 &= E_4^0 = \frac{F_0\left(a_3^0 - a_2^0\right)}{2q_1^0 \Delta_1^0},\\
E_2^0 &= E_5^0 = \frac{F_0\left(a_1^0 - a_3^0\right)}{2q_2^0 \Delta_1^0},\\
E_3^0 &= E_6^0 = \frac{F_0\left(a_2^0 - a_1^0\right)}{2q_3^0 \Delta_1^0},\\
\Delta_1^0 &= m_1^0\left\{\left(a_2^0 b_3^0 - a_3^0 b_2^0\right) + \left(a_3^0 b_1^0 - a_1^0 b_3^0\right) + \left(a_1^0 b_2 - a_2^0 b_1^0\right)\right\},\\
a_s^0 &= \frac{-\xi}{m_3^* \Delta_s^0}[m_2\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^{02}\right\} + \varepsilon_1 p^2,\\
b_s^* &= \frac{\iota}{m_3^* \Delta_s^0}[\left\{\varepsilon_1 p^2 \xi^2 + \left(\xi^2 + 4p^2 - q_s^{02}\right)\right\}\left(m_1^* \xi^2 + p^2 - m_3^* q_s^{02}\right)\}\\
\Delta_s^0 &= \frac{\varepsilon_1 p^2 q_s^0}{\xi m_3^*}\left\{\left(m_1^* \xi^2 + p^2 - m_3^* q_s^0 2\right) - \xi^2 m_2\right\}, \varepsilon_1 = 4\varepsilon,
\end{aligned}$$

and $\pm q_s^0$ $(s = 1, 2, 3)$ equation

$$\begin{aligned}
&q^{06} - \sigma_1^0 q^{0^4} + \sigma_2^0 q^{02} - \sigma_3^0 = 0,\\
&\sigma_1^0 = \left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right) + \left(\xi^2 + 4p^2\right) + \frac{\varepsilon_1 p^2}{m_1^*} - \frac{\xi^2 m_2^2}{m_1^* m_3^*},\\
&\sigma_2^0 = \left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right) + \left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right)\left(\xi^2 + 4p^2\right)\\
&\quad + \left(\xi^2 + 4p^2\right)\left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right) + \frac{\varepsilon_1 p^2}{m_1^*}\left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right)\\
&\quad - \frac{\xi^2 m_2^2}{m_1^* m_3^*}\left(\xi^2 + 4p^2\right) - \frac{2\varepsilon_1 p^2 \xi^2 m_2}{m_1^* m_3^*} + \frac{\varepsilon_1 p^2 \xi^2}{m_3^*},
\end{aligned}$$

where $\tilde{f}_e$ and $\tilde{f}_0$ are even and odd parts of the functions $\tilde{f}(\xi, z, p)$ respectively. Thus, expression (37.1) gives us the Laplace transform $\bar{f}(x, z, p)$ of the function f (x, z, t). Following [11], the Laplace transform function $\bar{f}(x, z, p)$ can be inverted to f (x, z, t).

$$\sigma_3^0 = \left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right)\left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right)\left(\xi^2 + 4p^2\right) + \frac{\varepsilon_1 p^2 \xi^2}{m_3^*}\left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right),$$

Thus, the expressions given by equations (3.5)–(3.12) with the help of (3.13)–(3.16) and (3.17) represent the solution of plane strain problem under consideration in the transformed domain using eigen value approach.

33.4 Inversion of the Transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms for three theories that is L-S, G-L and G-N. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ respectively and hence are of the form $\bar{f}(x, z, p)$. To get the function $f(x, z, t)$in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x, z, p) = \int_{-\infty}^{\infty} \exp(i\xi x)\tilde{f}(\xi, z, p)d\xi = \frac{1}{\pi}\int_{0}^{\infty}\left\{\cos(\xi x)\tilde{f}_e + i\sin(\xi x)\tilde{f}_0\right\}d\xi$$

The last step in the inversion process is to evaluate the integral in Eq.(37.1). This was done using Romberg's integration with adaptive step size. This method uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero. The details can be found in [20].

33.5 Numerical Results and Discussion

Following [5], we take the following values of relevant parameters for the case of Magnesium crystal as

$\rho = 1.74\,\mathrm{gm/cm^3}$,	$j = 0.2 \times 10^{-15}\,\mathrm{cm^2}$,	$\lambda = 9.4 \times 10^{11}\,\mathrm{dyne/cm^2}$,
$\mu = 4.0 \times 10^{11}$ dyne,	$K = 1.0 \times 10^{11}\,\mathrm{dyne/cm^2}$,	$C^* = 0.23\,\mathrm{Call/gm}^0 C$,
$\gamma = 0.779 \times 10^{-4}$ dyne,	$\varepsilon = 0.073$.	$K^* = 0.6 \times 10^{-2}$ cal/cmsec,
$T_0 = 23°\mathrm{C}$,	$\tau_0 = 6.131 \times 10^{-13}$ sec,	$\tau_1 = 8.765 \times 10^{-13}$ sec
$h = 1$ cm,	$z = 1$	

33.6 Discussion

The variations of normal displacement U_3 with distance x for three different theories (L-S, G-L and G-N) in both media after multiplying the original values for G-N theory in MTE medium by 10 are shown in Fig. 33.2 The values of normal displacement due to microrotation effect are less in MTE medium in comparison to TE medium in the $0 \leq x \leq 0.5$for all three theories, whereas the values of U_3 oscillate as x increases further in the rest of the range for both media. It is also evident that normal displacement decreases for both media for L-S and G-L theories, increases gradually in MTE medium for G-N theory and oscillate in TE medium for G-N theory.

The values of normal force stress T_{33}in magnitude are more for three different theories in MTE medium in comparison to TE medium. It is also noticed that the values of normal force stress oscillate for L-S and G-L theories in MTE and TE media. The values of normal force stress also oscillate for G-N theory in TE medium, whereas

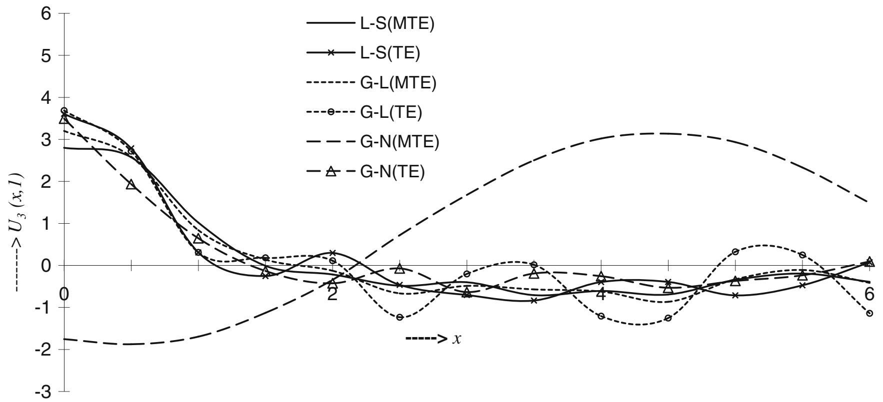

Fig. 33.2 Variation of normal displacement $U_3(x, l)$

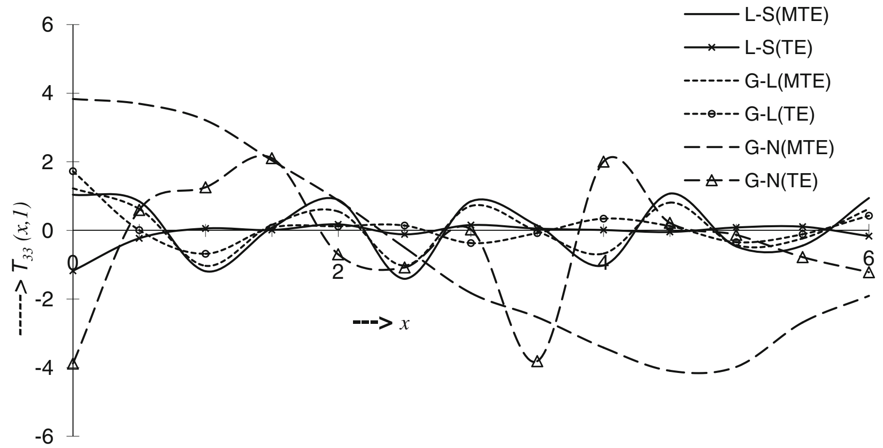

Fig. 33.3 Variation of normal force stress $T_{33}(x, l)$

these decrease gradually with increasing value of x in MTE medium. These variations of normal force stress have been shown in Fig. 33.3 after dividing the original values by 10 in case of G-N theory in MTE medium.

Figure 33.4 depicts the variations of tangential couple stress M_{32} for three different theories in MTE medium after dividing the original values for G-N theory by 10. The behaviour of tangential couple stress is oscillatory for three theories. It is noticed that the value of tangential couple stress for G-N theory are large in comparison to L-S and G-L theories in the range $0 \leq x \leq 2.5$ and the values are small for the rest of the range.

The range of values of temperature field in magnitude is large in case of three theories in MTE medium in comparison to TE medium. It is also observed that temperature field oscillate in TE medium for three different theories but in MTE medium for L-S and G-L theories, the temperature field oscillate. The values of temperature field for

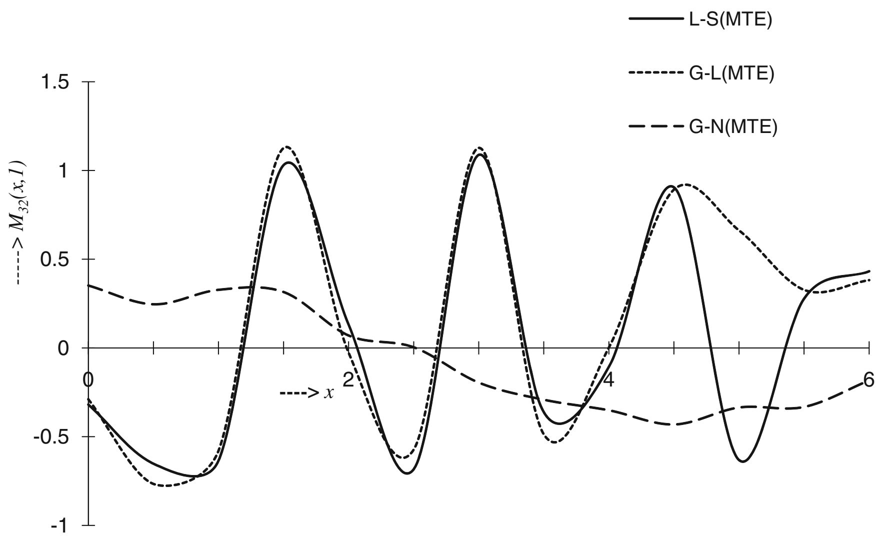

Fig. 33.4 Variation of tangential couple stress $M_{32}(x, l)$

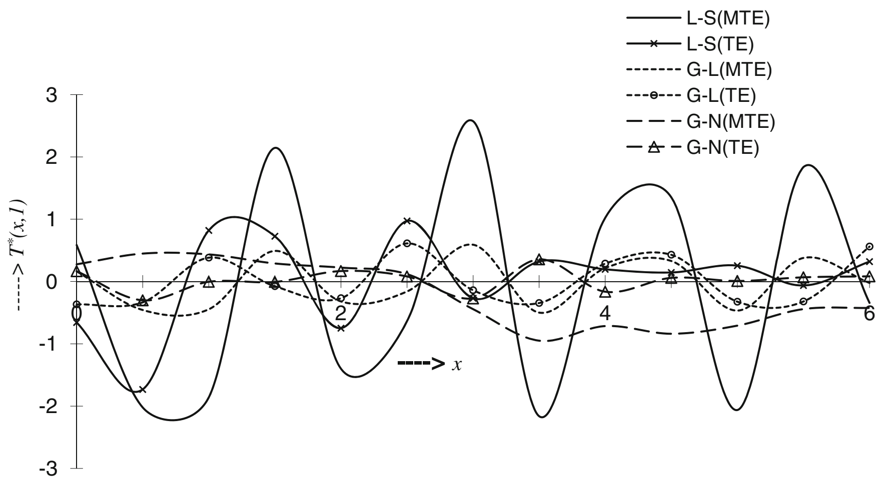

Fig. 33.5 variation of temprature field $T^*(x, l)$

G-N theory decrease gradually with increasing value of x in MTE medium. These variations shown in Fig. 33.5 after multiplying the original values in case of L-S and G-N theories by 10^2 and 10^2 respectively in MTE medium; the original values in case of G-N theory (TE medium) and also magnified by multiplying 10^2.

33.7 Conclusion

From the above numerical results, we conclude that micropolarity has a significant effect on normal displacement, normal force stress and temperature field mechanical-source for three theories.Mcropolar effect is more appreciable for normal displacement and temperature field in, comparison to normal force stress. Application of the present paper may also be found in the field of steel and oil industries. The present Problem is also useful in the field of geomechanics, where, the interest is about the various phenomenon occurring in the earthquakes and measuring of displacements, stresses and temperature field due to the presence of certain sources.

33.8 Nomenclature

λ, μ = Lame's constants
α, β, γ, K = Micropolar material constants
$\alpha_0, \lambda_0, \lambda_1$ = Material constants due to the presence of stretch.
$\lambda_I, \mu_I, K_I, \alpha_I, \nu, \gamma_I, \alpha_{0I}, \lambda_{0I}, \lambda_{1I}$ = Microstretch viscoelastic constants
ρ = Density j = Micro-inertia $\mathbf{u}$ = Displacement vector $\boldsymbol{\phi}$ = Microrotation vector
$\boldsymbol{\phi}^*$ = Scalar microstretch
t_{ij} = Force stress tensor
m_{ij} = Couple stress tensor
λ_l = Microstress tensor
δ_{ij} = Kronecker delta
ε_{ijr} = Alternating tensor
Δ = Gradient operator
ι = Iota
And dot denotes the partial derivative w.r.t. time.

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