Planning Sensors with Cost-Restricted Subprocess Calls: A Rare-Event Simulation Approach

Frédéric Dambreville*

Lab-STICC UMR CNRS 6285, ENSTA Bretagne
2 rue François Verny, 29806 Brest Cedex 9, France
submit@fredericdambreville.com

Abstract. This paper deals with optimal sensor planning in the context of an observation mission. In order to accomplish this mission, the observer may request some intelligence teams for preliminary prior information. Since team requests are expensive and resources are bound, the entire process results in a two-level optimization, the first level being an experiment devoted to enhance the criterion modelling. The paper proposes a solve of this problem by rare-event simulation, and a mission scenario is addressed.

1 Introduction

The main background of this paper is the optimal planning of sensors in the context of an acquisition mission. Typically, the acquisition mission may result in the localisation of a target, with the final purpose of intercepting this target. In this work, we focus especially on dealing with the modelling errors of the sensor planning problem. Then, the question of interest is: *how to spend resources optimally in order to reduce the model errors, and how does that affect the sensor planning problem?*

Sensor planning, especially in order to localize a target, has been thoroughly studied in the literature. First works in this domain track back to the works of Koopman during World War II [1,2]. This seminal works has been extended in various manner, so as to take into account motion models [3,4], or reactive behaviours of the target [5,6]. Sensor planning now deals with the general domain of search and surveillance [7,8]. The combination of multiple sensors with their constraints is addressed by some works and in various application contexts: optimizing the performance of a sensor network [9,10]; optimizing the tasks-to-sensors affectation in the context of an intelligence collection process [11,12,13,14]. Another major issue in sensor planning is also to maximize the positive effect of subsequent data processing in regards to mission objectives. In [15], entropic-based criterion are used in order to take into account optimal post-processing of the collected information (typically data fusion). A more direct approach has also been addressed by means of Partially Observable Markov Decision Processes [16,17]. From this last point of view, sensor planning is clearly related to the domain of robotic.

* Corresponding author.

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Thus, a variety of approach have been investigated for many contexts of the sensor planning. Nevertheless, there is not as much works dedicated to the question of modelling the sensor planning. In the inspiring work[18,19], Koopman addressed initially this formalisation, priorly to sensor planning problem. In [20], Le Cadre studied various practical case of use of the model of Koopman, and deduced related parametrization of the models. Whatever, it appears that a minimal effort is necessary for acquiring a good estimation of the parameters modelling our sensor planning. In the case of a reproducible scenario, it is possible to learn such parameters. However, there are cases where a prior learning of the parameters is clearly impossible. Such cases hold typically when the planning team has a limited control on the sensors, and relies on sub-processes or on sub-teams in order to implement the sensors or compute their performance parameters. Learning the parameters is generally not possible in such case, since any experiment on the sensors is a request to a sub-process, which is generally done at the expense of limited resources.

The main purpose of this paper is to handle the sensor planning as a bi-level optimization, involving:

- The improvement of the prior knowledge on the mission. This is done by planning probing experiments, which result in requests to sub-processes,
- The optimal sensor planning on the basis of the enhanced prior.

This problem is related to some issues in optimal experiment planning. Especially in [21,22], approaches (inspired from kriging) are proposed in order to plan experiments when the model of measure is known imperfectly. In such approach, the experiments are optimized in order to both enhance the measure model and the measure plan. The problem considered in this paper is somewhat different: the resources allocable for enhancing the models are distinct to the resources allocable for performing the mission.

In the first section 2 of this paper, we propose a general formalisation of the sensor planning with experiment sub-processes. In section 3, a rare-event simulation approach is proposed for solving this bi-level sensor planning. Section 4 presents a scenario and numerical results. Section 5 concludes.

2 Sensor Planning with Experiment Sub-processes

We are interested in the general problem of optimizing the planning of a sensors so as to accomplish an observation mission. A main and first issue in such optimization problem is the modelling of the formal optimization criterion and constraints. This is prerequisite to any practical sensor planning process, and it appears that the models are known with significant model noise. Two consequences are implied. First, it is not necessary to obtain an accurate optimum for a function when it is known to be noisy; smoothed criteria, derived from the expectation of the model, are much more relevant. Second, it is interesting to harvest additional information , so as to reduce this model noise. This is obtained by probing experiments, which are resources expensive. A balance has to

be decided between the experiment expense and the final accuracy of the optimal sensor planning.

Our approach to this problem is formalized in section 2.2. In this paper, we model the mission criterion to be optimized by means of the function $(d, \epsilon) \mapsto f(d, \epsilon)$, which is dependent on both a decision parameter d to be optimized and on a noise parameter ϵ which encompass the uncertainty about the model. It is interesting to present as an introduction the well known *Efficient Global Optimization*, which is a reference method applicable to a sub-case of this problem.

2.1 Efficient Global Optimization

The EGO is a method for optimizing an unknown function by planning efficiently the point-evaluations of this function: a point evaluation is seen as an experiment which will enhance a modelling of the actual but unknown criterion function. EGO as introduced in [21], is based on a kriging interpolation model, with a spatial Gaussian noise, of the criterion function, which takes the form of the following functional prior:

$$f(\mathbf{d}) = \mathbf{p}(\mathbf{d})^T \mathbf{b} + Z(\mathbf{d}) \ ,$$

where $\mathbf{b} = b_{1:N}$ is a model parameter (typically known with a flat prior), $\mathbf{p} = p_{1:N}$ is a predefined functional basis by which the function f is interpolated, and Z is a model spatial Gaussian law with zero-mean and a covariance $\mathrm{Cov}(Z(\mathbf{d}), Z(\mathbf{d}')) = K(\mathbf{d} - \mathbf{d}')$, which is typically dependant on a distance between the decision parameters. Being given this prior model, the estimate of the function (and of its minimizer) is computed with increasing accuracy by evaluating the real criterion function on a sequence of experimental decisions $\{\mathbf{d}_k\}$. Of course, each experiment implies a cost, and the sequence of experiment has to be optimized. Jones and al [21] proposed to optimize each step of experiment by maximizing a criterion based on the *estimated function* $\mathbf{d} \mapsto \widehat{f}(\mathbf{d})$ and the *variance of the prediction error*, $\mathbf{d} \mapsto \widehat{\sigma}(\mathbf{d})$ computed from the model and previous experimental measures. A common criterion for choosing a new experimental decision $\mathbf{d}_{k+1}$ is to maximize the *Expected Improvement* (EI), which is given by:

$$\mathbf{d}_{k+1} \in \arg\max_{\mathbf{d}} EI(\mathbf{d}) \ , \quad \text{with} \quad EI(\mathbf{d}) = \widehat{\sigma}(\mathbf{d}_{k+1})(u\Phi(u) + \phi(u)) \ ,$$

where:

$$u = \frac{\min_{i=1:k} f(\mathbf{d}_i) - \widehat{f}(\mathbf{d})}{\widehat{\sigma}(\mathbf{d})} \ .$$

There has been many successful applications and extensions of the EGO algorithm during the last years [22].

In this paper, however, we will consider a different optimization scheme, in the sense that the experiment processes and the functional evaluation will not work on the same variables: we will not be able to probe the decision $\mathbf{d}_{k+1}$ directly; instead, we will request and experiment $\mathbf{r}_{k+1}$ which is not in the same

space than $\mathbf{d}_{k+1}$. For solving this problem, a direct simulation-based approach will be proposed. Notice that it is probably possible to consider extensions of the EGO algorithm to the problem formalized subsequently, for example by handling variables $\mathbf{r}, \mathbf{d}$ as a joint variable, and defining a function prior on this joint variable. This approach is not considered in this paper.

2.2 Formalisation of a Direct Approach

From now on, we are studying a bi-level sensor planning, involving a first stage of model improvement by means of experiment request and a second stage of sensor planning on the basis of the corrected model. This problem is characterized by:

variables: process variables; noise variables; control variable, including decisions and experiment requests,
Known functions and parameters: noise-dependant objective function; cost function; cumulative cost bound,
Prior probabilistic laws: model noise; measure law.

Criterion and Constraints

Definition of the variables

- $d \in D$ is a variable describing the decision of the sensor planner. This variable is intended to be optimized. The set D encompasses all the possible control decision of the planner,
- $\epsilon \in E$ is a variable describing the error of the model. The value ϵ is obtained from a known random process, and the planner cannot control this value.
- $r_{1:N} \in R$ are variables describing a sequence of N experiments requested by the planner. These variables are intended to be optimized, but N is assumed as a known parameter of the problem. The set R encompasses all possible experiments likely to be required by the planner. The planner does these requests before deciding for a control of the sensor. These experiments are intended to reduce the uncertainty about the noise ϵ,
- $m_{1:N} \in M$ are variables describing a sequence of N measures resulting from the requested experiments $r_{1:N}$. The set M encompasses all possible measures.

Definition of the parameters and functions

- $(d, \epsilon) \mapsto f(d, \epsilon)$ is the objective function to be maximized. It depends both on the decision variable and on the model error,
- $r \mapsto \gamma(r)$ is a *positively* valued cost function. This function evaluates the cost of the experiments,
- Γ is the cumulative cost bound. The sum of all experiment costs cannot exceed this value.

Definition of the laws

- $\epsilon \mapsto p(\epsilon)$ is the law of the error of model.
- $(m, r, \epsilon) \mapsto p(m|r, \epsilon)$ is the law of measure conditionally to the request and model error. It is assumed that the measures are obtained independently.

Criterion and Constraints

Criterion. The success of the mission is evaluated by means of the criterion function f. The purpose is to optimize the decision d so as to maximize the expected success; the expectation is computed according to the law of the model error, conditionally to the requested experiments and resulting measures:

$$\max_{d \in D} \int_{\epsilon \in E} p(\epsilon|m_{1:N}, r_{1:N}) f(d, \epsilon)\, d\epsilon \ .$$

The entire bi-level planning also involves the choice of a sequence of experiments, priorly to the mission:

$$\max_{r_{1:N} \in R} \int_{m_{1:N} \in M} p(m_{1:N}|r_{1:N}) \max_{d \in D} \int_{\epsilon \in E} p(\epsilon|m_{1:N}, r_{1:N}) f(d, \epsilon)\, d\epsilon\, dm_{1:N} \ .$$

Combining the model and measure law in a same joint law, the entire criterion is equivalently rewritten:

$$\max_{r_{1:N} \in R} \int_{m_{1:N} \in M} \max_{d \in D} \int_{\epsilon \in E} p(\epsilon, m_{1:N}|r_{1:N}) f(d, \epsilon)\, d\epsilon\, dm_{1:N} \ , \tag{1}$$

where:

$$p(\epsilon, m_{1:N}|r_{1:N}) = p(\epsilon) \prod_{n=1:N} p(m_n|r_n, \epsilon) \ . \tag{2}$$

The optimization (1) may as well be rewritten:

$$\max_{r_{1:N}} \max_{m_{1:N} \mapsto d} \int_{m_{1:N} \in M, \epsilon \in E} p(\epsilon, m_{1:N}|r_{1:N}) f\big(d(m_{1:N}), \epsilon\big)\, d\epsilon\, dm_{1:N} \ , \tag{3}$$

Constraints. The only constraint is resulting form the cumulative cost bound for the experiments:

$$\sum_{n=1:N} \gamma(r_n) \leq \Gamma \ . \tag{4}$$

Bi-Level Optimization Problems. Summing up both (1) and (4), the optimization problem comes as follows:

$$\text{Solve } \arg\max_{r_{1:N}} \max_{m_{1:N} \mapsto d} \int_{m_{1:N} \in M, \epsilon \in E} p(\epsilon, m_{1:N}|r_{1:N}) f\big(d(m_{1:N}), \epsilon\big)\, d\epsilon\, dm_{1:N} \ , \tag{5}$$

$$\text{Under constraint } \sum_{n=1:N} \gamma(r_n) \leq \Gamma \ . \tag{6}$$

Sometime, it is useful to reformulate this problem as an optimization on parametric laws :

$$\text{Solve } \arg\max_{\pi\in\Pi} \int_{m_{1:N}\in M,\epsilon\in E} p(\epsilon, m_{1:N}|r_{1:N})\pi(d, r_{1:N}|m_{1:N})f(d,\epsilon)\, d\epsilon\, dm_{1:N} \ , \tag{7}$$

where Π is a family of conditional laws $\pi(d, r_{1:N}|m_{1:N})$ which are compliant with constraint (4). As shown in [17], reformulations based on parametric laws are efficiently used for approximating such optimization problem. These questions are outside the scope of this paper however.

Sub-case of interest. In section 4, two scenarios are proposed where the measures are reduced to a *detection/non detection paradigm*. For convenience, it is also assumed that:

$$\Gamma = N \text{ and } \gamma = 1 \ , \tag{8}$$

so that:

$$\text{Constraint (6) is removed.} \tag{9}$$

All measures are assumed independent, so that multiple experiment will multiplicatively decrease the probability of non detection. There are two way to handle this, depending whether the measure processes are discrete or continuous.

Discrete case: In this case, each experiment $r \in R$ is related to a predicate $X_r(\epsilon)$ which may be true or false depending on the value of ϵ. Conditionally to the hypothesis that X_r is true, it is assumed that each request to experiment r will result in a positive confirmation (*i.e.* detection **d**) with probability $\theta(r) \in [0,1]$. Otherwise, the confirmation is negative (*i.e.* non detection **nd**). The measure set and the measure probability are then defined as follows:

$$M = \{\mathbf{d}, \mathbf{nd}\} \text{ and } \begin{cases} p(\mathbf{d}|r,\epsilon) = \theta(r) \text{ if } X_r(\epsilon) = \mathbf{true} \ . \\ p(\mathbf{d}|r,\epsilon) = 0 \text{ if } X_r(\epsilon) = \mathbf{false} \ . \end{cases} \ . \tag{10}$$

Continuous case: In this case, it is considered that the requests are implemented continuously, so that *a request takes the form of a ratio of time dedicated to an experiment*. It is defined the set K of experiments (in this case, the experiments $k \in K$ are distinguished from the requests). Instead of making N sequential requests, we will do a single, but *vectorial*, request. The (single) measure is a vector of confirmation for all possible experiment. As a consequence, N, M and R are defined as follows:

$$N = 1 \ , \tag{11}$$

$$M = \{\mathbf{d}, \mathbf{nd}\}^K \ , \tag{12}$$

$$R = \left\{ \rho_K \in \mathbb{R}^{+K} \Big/ \sum_{k\in K} c_k\rho_k = C \right\} \ , \tag{13}$$

where c_k is a cost rate for request k and C is a cumulative cost bound.

Now, an experiment $k \in K$ is related to a predicate $X_k(\epsilon)$ which may be true or false depending on the value of ϵ. Conditionally to the hypothesis that X_k is true, it is assumed that each request ρ_k to an experiment k will result in a positive confirmation (*i.e.* detection **d**) with the exponential probability $1 - \exp(-\omega_k \rho_k)$, where ω_k is a detection rate characterizing the infinitesimal probability of detection. Otherwise, the confirmation is negative (*i.e.* non detection **nd**). The measure probability is then defined as follows:

$$\text{For any } m_K \in M,\ p(m_K|\rho_K, \epsilon) = \prod_{k \in K} p_k(m_k|\rho_k, \epsilon) \tag{14}$$

$$\text{where } \begin{cases} p_k(\mathbf{d}|\rho_k, \epsilon) = 1 - \exp(-\omega_k \rho_k) \text{ if } X_k(\epsilon) = \textbf{true} \,. \\ p_k(\mathbf{d}|\rho_k, \epsilon) = 0 \text{ if } X_r(\epsilon) = \textbf{false} \,. \end{cases} \,. \tag{15}$$

These cases of interest will be implemented in the scenarios of section 4.

3 A Rare-Event Simulation-Based Implementation

A mathematical approach for solving problem (5) and corollaries is not straightforward, and would need more refinement on the model. On the other hand, this problem is well suited to simulation approaches, especially as the optimization criterion is obtained by means of an expectation. Especially, we are interested in model-based simulation approaches, which encompass the cross-entropy method (CE) created by Rubinstein [23], or the model reference adaptive search method (MRAS) [24]. In the current stage of this work, the cross-entropy method (CE) is implemented. The MRAS method seems promising but is not considered for this paper.

3.1 The Cross-Entropy Method

It is assumed a $\mathbb{R}$-valued function $y \mapsto \varphi(y)$ to be optimized for $y \in Y$. The domain Y is probabilized. The purpose is to optimize y so as to maximize $\varphi(y)$:

$$\max_{y \in Y} \varphi(y) \,.$$

For solving this optimization, model-based simulation approaches have been proposed, based on the following general synopsis:

- Generate samples by means of a parametric distribution,
- Evaluate the quality of the samples in accordance with the criterion function,
- Update the parametrized distribution by learning from the samples graded with their quality.

Especially, the implementation of the *Cross-Entropy* method will involve the following elements:

- A sampling distributions family, $\pi(\cdot|\lambda)$ with $\lambda \in \Lambda$, which applies on variable y,
- An increasing selection function, $\sigma : \mathbb{R} \to [0,1]$,
- A smoothing prameter $\theta \in]0,1]$.

The CE algorithm for maximizing $\varphi(y)$ on the basis of π is derived as follows:

1. Initialize $\lambda \in \Lambda$,
2. Generate S samples $y_{1:S}$ by means of $\pi(\cdot|\lambda)$,
3. Compute the weighting parameters $\sigma_s = \sigma\big(\varphi(y_s)\big)$ for all samples y_s,
4. Learn $\widetilde{\lambda}$, by minimizing the Kullback-Leibler divergence with the weighted samples:
$$\widetilde{\lambda} \in \arg\max_{\lambda \in \Lambda} \sum_{s=1:S} \sigma_s \ln \pi(y_s|\lambda) \ , \tag{16}$$
5. Set $\lambda = \theta\lambda + (1-\theta)\widetilde{\lambda}$,
(it is assumed that this operation makes sense in Λ)
6. Repeat from step 2 until convergence.

It is noticed that the selection function may evolve with the iteration step and the samples statistic. In the classical implementation of the CE for example, the sample selection is based on the quantiles: being given the selection rate $\rho \in]0,1[$, the $\lfloor \rho S \rfloor$ best samples are selected. In this case, the selection function is computed as follows:

- Build $\gamma \in \mathbb{R}$ and $\Sigma \subset [\![1, S]\!]$ such that:
$$\mathrm{card}\,(\Sigma) = \lfloor \rho S \rfloor \ , \quad \text{and} \quad \varphi(y) \leq \gamma \leq \varphi(z) \text{ for any } y \in [\![1, S]\!] \setminus \Sigma \text{ et } z \in \Sigma \ ,$$
- Define $\sigma(\varphi) = I[\varphi \geq \gamma]$, where:
$$I[\mathbf{true}] = 1 \text{ and } I[\mathbf{false}] = 0 \ . \tag{17}$$

This selection principle will be used in this work.

3.2 Implementation of the Sub-cases of Interest

The point here is to define the evaluation function, the sampling family and the learning step for the discrete case and the continuous case. The choice of the selection rate and of the smoothing parameter is not difficult in practice.

In the scenario of section 4, the decisions d are same-dimension real vectors.

Subcase of Interest: Discrete Case

Evaluation function. The evaluation function is defined by:

$$\varphi((m_{1:N} \mapsto d), r_{1:N}) = \int_{m_{1:N} \in M, \epsilon \in E} p(\epsilon, m_{1:N}|r_{1:N}) f\big(d(m_{1:N}), \epsilon\big)\, d\epsilon\, dm_{1:N} \ . \tag{18}$$

By defining explicitly a function of measure $m = \mu(r, \epsilon, \nu)$, where ν is a noise of law $p(\nu)$, the function φ is equivalently rewritten:

$$\varphi((m_{1:N} \mapsto d), r_{1:N}) = \int_{\nu,\epsilon} p(\epsilon)p(\nu) f\Big(d\big(\mu(r_1, \epsilon, \nu), \cdots, \mu(r_N, \epsilon, \nu)\big), \epsilon\Big)\, d\epsilon\, d\nu \;, \tag{19}$$

which is computed by means of a Monte-Carlo simulation on the variables (ϵ, ν).

Sampling family. In our examples, the variable $\big((m_{1:N} \mapsto d), r_{1:N}\big)$ is sampled by the means of the family:

$$(d, r; m) \mapsto N_d(d|\mu_m, \Sigma_m) \times \pi_r(r_{1:N}) \;, \tag{20}$$

where N_d is any multivariate Gaussian law on d (defined conditionally to m) and π_r is any discrete law defined on $r_{1:N}$. The family parameter is $\lambda = \big((\mu_m, \Sigma_m)_m, \pi_r\big)$.

Distribution update. The optimisation (16) is easy and implies an empirical estimation of the law parameters:

$$\widetilde{\pi}_r(r) = \sum_s \sigma_s I[r_s = r] \Big/ \sum_s \sigma_s \;, \tag{21}$$

$$\widetilde{\mu}_m = \sum_s \sigma_s I[m_s = m] d_s \Big/ \sum_s \sigma_s I[m_s = m] \;, \tag{22}$$

$$\widetilde{\Sigma}_m = \sum_s \sigma_s I[m_s = m](d_s - \widetilde{\mu}_m)(d_s - \widetilde{\mu}_m)^T \Big/ \sum_s \sigma_s I[m_s = m] \;. \tag{23}$$

The values (d_s, m_s, r_s) are issued from sample s.

Subcase of Interest: Continuous Case

Evaluation function. The evaluation function is defined by:

$$\varphi((m_K \mapsto d), \rho_K) = \int_{m_K \in M, \epsilon \in E} p(\epsilon, m_K|\rho_K) f\big(d(m_K), \epsilon\big)\, d\epsilon\, dm_K \;. \tag{24}$$

By defining explicitly a function of measure $m_k = \mu_k(\rho_k, \epsilon, \nu)$, where ν is a noise of law $p(\nu)$, the function φ is equivalently rewritten:

$$\varphi((m_K \mapsto d), \rho_K) = \int_{\nu,\epsilon} p(\epsilon)p(\nu) f\Big(d\big(\mu_k(\rho_k, \epsilon, \nu)\big|k \in K\big), \epsilon\Big)\, d\epsilon\, d\nu \;, \tag{25}$$

which is computed by means of a Monte-Carlo simulation on the variables (ϵ, ν).

Sampling family. In our examples, the variable ρ_K is derived by a bijective transform from a real vector ϱ_K of dimension $\mathrm{card}\,(K)-1$. The variable $\big((m_K \mapsto d), \varrho_K\big)$ is sampled by the means of the family:

$$(d, r; m) \mapsto N_d(d|\mu_m, \Sigma_m) \times N_\varrho(\varrho|\mu_\varrho, \Sigma_\varrho)\ , \tag{26}$$

where N_d is any multivariate Gaussian law on d (defined conditionally to m) and N_ϱ is any multivariate Gaussian law on $r_{1:N}$. The family parameter is $\lambda = \big((\mu_m, \Sigma_m)_m, \mu_\varrho, \Sigma_\varrho\big)$.

Distribution update. The optimisation (16) is easy and implies an empirical estimation of the law parameters:

$$\widetilde{\mu}_\varrho = \sum_s \sigma_s \varrho_s \Big/ \sum_s \sigma_s\ , \tag{27}$$

$$\widetilde{\Sigma}_\varrho = \sum_s \sigma_s (\varrho_s - \widetilde{\mu}_\varrho)(\varrho_s - \widetilde{\mu}_\varrho)^T \Big/ \sum_s \sigma_s\ , \tag{28}$$

$$\widetilde{\mu}_m = \sum_s \sigma_s I[m_s = m] d_s \Big/ \sum_s \sigma_s I[m_s = m]\ , \tag{29}$$

$$\widetilde{\Sigma}_m = \sum_s \sigma_s I[m_s = m](d_s - \widetilde{\mu}_m)(d_s - \widetilde{\mu}_m)^T \Big/ \sum_s \sigma_s I[m_s = m]\ . \tag{30}$$

The values (d_s, m_s, ϱ_s) are issued from sample s.

4 Scenario and Numerical Results

4.1 Scenario

The mission is to intercept a target (symbolized by a smiley on picture 4), which is hidden within the theatre. In order intercept this target, the planer has to position a patrol as close as possible to the target. Then, this patrol will proceed to the search of the target and to its interception.

At the early beginning of the mission, the position of the target is known with uncertainty, and this uncertainty is characterized by means of a Gaussian distribution. In order to enhance this prior knowledge, the planner may request some teams, which will collect information in the neighbourhood about the target, and if it is in the neighbourhood, it will assert the presence of the target with a given probability.

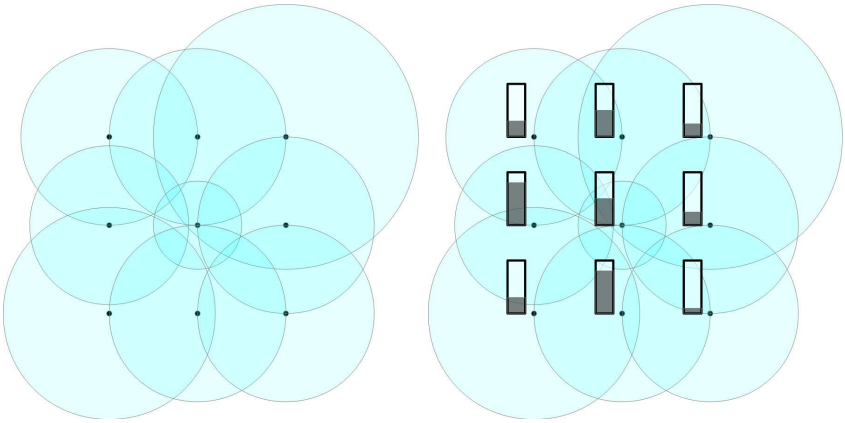
The problem is then to:

1. Select the teams to request,
2. Plan the patrol in regards to the earned information.

Sensors are positioned regularly on a grid. The position, range and detection probabilities of the team are indicated in picture 1 and 2. These pictures respect

the relative dimension of these parameters. These parameters, position, range (R), detection probability, are given subsequently in this order:

$(-1,1)$,	$R=1$,	0.3	$(0,1)$,	$R=1$,	0.5	$(1,1)$,	$R=1.5$,	0.25
$(-1,0)$,	$R=0.9$,	0.8	$(0,0)$,	$R=0.5$,	0.5	$(1,0)$,	$R=1$,	0.25
$(-1,-1)$,	$R=1.2$,	0.3	$(0,-1)$,	$R=1$,	0.8	$(1,-1)$,	$R=1$,	0.1

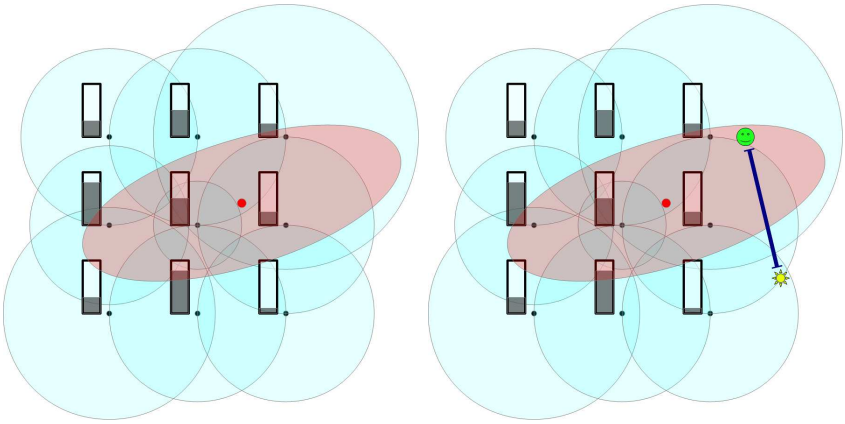


Sensors grid with range / detection probability

The target is known with a Gaussian uncertainty with mean μ_T and covariance Σ_T:

$$\mu_T = \begin{pmatrix} 0.5 \\ 0.25 \end{pmatrix} \quad \text{and} \quad \Sigma_T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

The target uncertainty is indicated in picture 3. The evaluation criterion of the mission is the *estimated distance* between the patrol and the target, as indicated in picture 4.



Target uncertainty / criterion: distance(target,sensor)

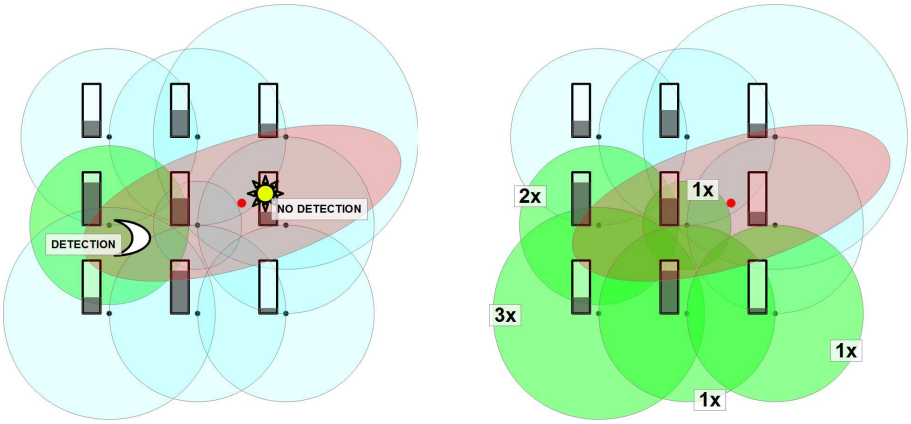
The parameters for the CE optimization are $\rho = \alpha = 0.15$ and the number of samples $S = 100$. The Monte-Carlo expectation is computed by means of 1000 particles. For the subsequent examples, the convergence is considered achieved after 100 to 200 iterations.

4.2 Results

Test 1 and test 2. Test 1 and test 2 are both about discrete requests of experiment. In test 1, however, only 1 request is done, while 8 are done in test 2.

Picture 5 indicate the result of the planning for test 1. Team 4 (in green) is requested and it is shown the decided patrol positioning: this position depends on detection (moon) or non-detection (sun). These results are compliant with the setting of the problem.

Picture 6 indicate the result of the planning for test 2. Teams 4 (2×), 5 (1×), 7 (3×), 8 (1×) and 9 (1×) (in green) are requested. It is not possible to give here the patrol positioning, since there are actually 32 possible cases. Again, these results are compliant with the setting of the problem.



Planning with 1 request / Planning with 8 request

Test 3. This test implements the subcase with continuous requests of experiments. The scenario parameters are identical, with 9 possible requests, $K = \{1 : 9\}$. In addition, the cumulative cost bound is $C = 10$, and the cost rates c_K and detection rates ω_K are given by the subsequent table:

$c_1 = 5,$	$\omega_1 = 0.36$	$c_2 = 25,$	$\omega_2 = 0.69$	$c_3 = 10,$	$\omega_3 = 0.29$
$c_4 = 10,$	$\omega_4 = 1.61$	$c_5 = 30,$	$\omega_5 = 0.69$	$c_6 = 2,$	$\omega_6 = 0.29$
$c_7 = 10,$	$\omega_7 = 0.36$	$c_8 = 20,$	$\omega_8 = 1.61$	$c_9 = 5,$	$\omega_9 = 0.11$

As a result, the following table indicates the optimized efforts ρ_K :

$\rho_1 = 0.24$	$\rho_2 = 0.03$	$\rho_3 = 0.09$
$\rho_4 = 0.58$	$\rho_5 = 0.01$	$\rho_6 = 0.10$
$\rho_7 = 0.02$	$\rho_8 = 0.03$	$\rho_9 = 0.03$

Interpretation of these last results is not so easy, although it is noticed that the optimization does the balance between the cost and detection rates.

5 Conclusion

In this paper we considered a bi-level optimization problem consisting in a first experiment request stage and in a final mission optimization stage. The first stage is dedicated to the improvement of the prior model, which is known with parameter uncertainty and condition the main objective. This problem is related to the domain of experiment plan optimization. We propose an original formalization and optimization method for this problem. Our solving approach is based on simulation methods. Our algorithm has been tested on a target search and interception scenario. The result is promising.

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