Mathematical Modelling of the Influence of Thermal Power Plant on the Aquatic Environment with Different Meteorological Condition by Using Parallel Technologies

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Abstract This paper presents the mathematical model of the thermal power plant in cooling pond under different meteorological conditions, which is solved by three dimensional Navier-Stokes equations and temperature equation for an incompressible fluid in a stratified medium. A numerical method based on the projection method, which divides the problem into three stages. At the first stage it is assumed that the transfer of momentum occurs only by convection and diffusion. Intermediate velocity field is solved by method of fractional steps. At the second stage, three-dimensional Poisson equation is solved by the Fourier method in combination with tridiagonal matrix method (Thomas algorithm). At the third stage it is expected that the transfer is only due to the pressure gradient. The compact scheme was used to increase the order of approximation. Then the basic laws of the hydrothermal processes depending on different hydrometeorological conditions were determined qualitatively and quantitatively approximate.

Keywords Stratified medium · Thermal power plant · Large eddy simulation · Parallel technologies

1 Introduction

Environment—the basis of human life, as mineral resources and energy are produced from them. Moreover they are the basis of modern civilization. However, the current generation of energy cause appreciable harm to the environment, worsening living conditions. The basis of the energy—are the various types of power plants. But power generation in thermal power plants (TPP), hydro power

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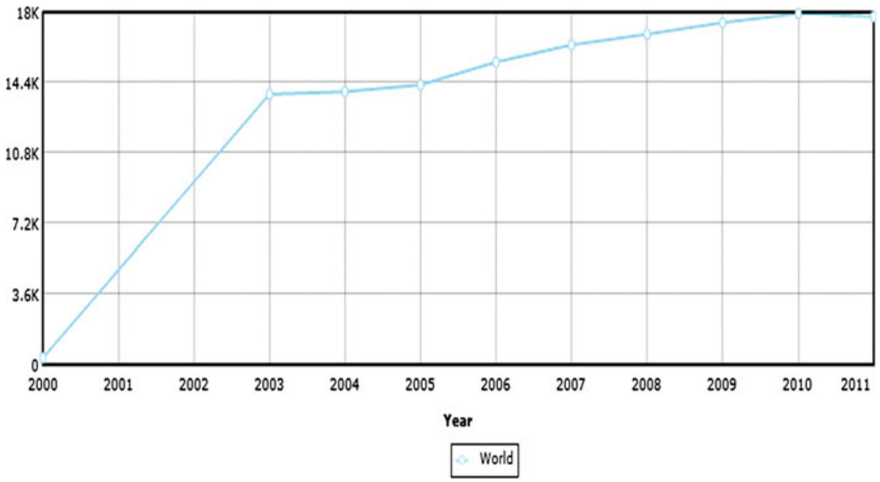

Fig. 1 Energy consumption in the world (in billion kWh)

plant (HPP) and nuclear power plants (NPP) are associated with adverse effects on the environment. The problem of the interaction of energy and the environment has taken on new features, extending the influence of the vast territory, most of the rivers and lakes, the huge volumes of the atmosphere and hydrosphere. Previously, the impact of TPP on the environment was not in first priority, as before to get electricity and heat had a higher priority. Technology of production of electrical energy from power plant is connected with a lot of waste heat released into the environment. Today the problem of influence of the nature by power is particularly acute because the pollution of the atmosphere and hydrosphere increases each year. Figure 1 shows that the energy consumption scale is increasing year by year, as a result the negative impact of energy on the environment increases too. In the terms of energy primarily guided feasibility in terms of economic were costs, but now the most important issue in the construction and operation of energy is their impact on the environment. Another problem, of TPP is thermal pollution of reservoirs or lakes. Dropping hot water—is a push chain reaction that begins reservoir overgrown with algae, it violates the oxygen balance, which in turn is a threat to the life of all its inhabitants. Thermal power plants with cooling water shed 4–7 kJ of heat for 1 kW/h electricity generation. Meanwhile, the Health Standards discharges of warm water with TPP should not raise the temperature higher than 3 °C in the summer and 5 °C in winter of the reservoir's initial temperature. As seen in Fig. 2, large proportion of electricity (81.3 %) in the world is produced by thermal power plants. Therefore, emissions of this type of power plants to the atmosphere and hydrosphere, provide the greatest amount of anthropogenic contaminants in it.

Spread of harmful emissions from TPP depends on several factors: the terrain, environmental temperature, wind speed, cloud cover, precipitation intensity. Speed

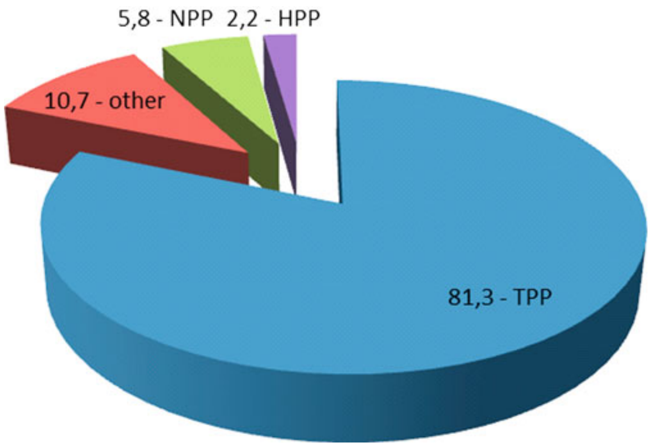

Fig. 2 Electricity production in the world by type of power plant (2010), in %

deployment and increases the thermal pollution area—are meteorology conditions. Thermal pollution of reservoirs' or lakes' water that cause multiple violations of their state is a one representation of environment danger. Thermal power plants generate energy through turbines, driven by hot steam and exhaust steam is cooled by water. Therefore, from the power plants the water flows with the temperature of 8–12 °C above the temperature of the reservoir is continuously transferred to the reservoirs or lakes itself. Large TPP shed till 90 m^3/s of heated water. For example, according to estimates of German and Swiss scientists, the possibility of rivers of Switzerland and the upper flows of the Rhine on the heating by stations heat relief have been exhausted. Hot water at any place of the river should not exceed more than 3 °C maximum temperature of the river water, which is assumed to be 28 °C. Following these conditions, the power station of Germany, constructed on the Rhine, Inna, Weser and Elbe, is limited by 35,000 MW.

Thermal pollution can lead to tragic consequences. Scientists predict changes in the characteristics of the environment in the next 100–200 years can cause large changes in the environment. Figure 3 shows the effect of TPP on the environment.

Let us consider hydrosphere pollution. Heat from TPP mainly is given to the environment from the water-cooled condenser steam turbines. The value of heat to the environment depends on the capacity of thermal power plants. Number of diverted energy to the environment is for condensing power plants from 40 % to 70 % of the thermal energy released by the combustion of fuel.

Thermal effects with through cooling water in and direct-flow-back scheme inflow and outflow water is limited by the local allowable increase in water temperature in the source water: river, lake and reservoir. Water supply system has a number of features of TPP. Almost all of the water to 95 % from total is applied to cool the condenser coils and auxiliary steam turbines. With up to 5 % of the

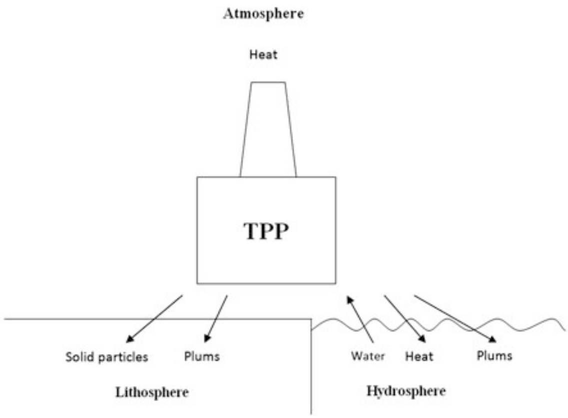

Fig. 3 TPP impact on the environment

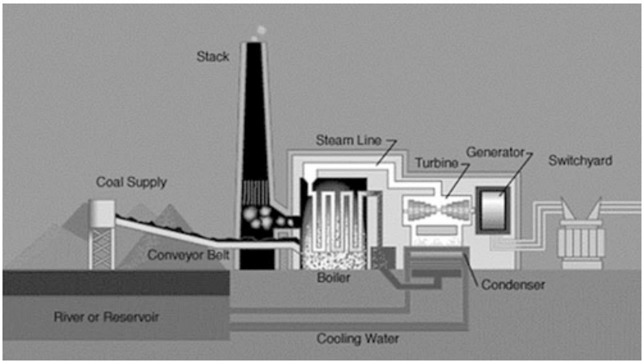

Fig. 4 Graphical scheme of cooling condensers of TPP

total value of the water supply to the thermal power plant equipment is generally irreversible consumption. As a rule, the main building of the condensing power plant is located directly at the shore line of the river, lake or reservoir-cooler. Water is supplied to the main unit of heat removal to the environment pumping stations. After heating it in condensers and heat exchangers, water is discharged to the surface water body. However, this amount of water is subject only to heating. Depending on the type of scheme water quantity of heat transfer in the once-through cooling water circuit for TPP will be minimal and some increase in the use of systems apply cooling towers. Figure 4 shows a graphical scheme of the cooling condensers of TPP with the cooling reservoir.

2 Background, Methodology and Literature Review

Often, industrial facilities, located on the shores of lakes and reservoirs disposed of warm water waste products in the form of impurities. If we consider that in the most developed countries in the cooling of thermal power plants and industrial facilities will be used by 10 % of water resources, the issues of efficient and effective use of the reservoir for cooling stations are of great importance. To solve these problems we need to be able to predict and control the temperature of the water and the spread of passive pollutants in reservoir. The distribution of temperature and passive scalar affect not only the processes of heat and mass, but also the process of density stratification. Stratification appears in connection with the difference between the density of water discharged from the density of the surrounding water in the reservoir, or the presence of impurities in the discharged water. For example, the heated water is easier, so it is in the form of a jet stretches a near the free surface. Stable density stratification of water reduces turbulent exchange between the vertical layers of fluids, especially in the area of the density jump. In general, hydrothermal regime of the reservoir is formed under the influence of uncontrollable environmental factors and factors amenable to regulation.

Strict restrictions associated with the protection of the environment were faced more often in recent years. According to the rules designer has not limit the limitation size of “zone transfer” of hot water so that it does not exceed half the width of the river and occupied no more than half of the total cross-sectional area and flow. Ignoring these rules may lead to short-term or long-term stopping of power plant, and therefore the accuracy requirements to structural analysis are very strict. In fact, emergance of the hydrodynamic problem can be described as a fully three-dimensional, with irregular boundaries, with the presence of the mass forces of buoyancy and the velocity of the main flow, which can vary by an order, sometimes it happens so fast that the important role is played by the effects of non-stationary. In addition, by certain combinations of conditions, for instance when the fault-heated water is almost drawn into the upstream region of cooling water, there are large areas of recycling. The result could be a significant loss of total operating efficiency of the system.

From the above it follows that the construction of a theoretical model relevant to real processes occurring in the water-cooler is quite big problem.

One of the first works of hydrodynamics and thermals of reservoirs—cooling was performed by Bernadskii and Proskuryakov [1]. Their method of calculation does not take into account the stratification of the environment, so hydrothermal problem was solved independently from heat. Sometimes the calculation of the hydrothermal processes reservoir—cooler part of the hydraulic calculation replace the physical simulation of flow in the reservoir. Reliable information about the reservoir, of course, give natural measurements in the field, but they are expensive, considerable difficult to hold them in the winter period. Therefore physical modeling are usually used. However, this method of study has its own difficulties, related primarily to the conservation of the similarity criteria. Reduction of the model to tens and sometimes to hundreds times from the natural scale of the fully simulated object imposes scales. Accordingly, the greater the degree of overlapping scale there are more difference in the results of the modelling. When the simulation of overlapping scales achieve similarity of the flow this breaks the similarity of turbulent mixing, heat transfer.

Now mathematical models are based on transport equations and their numerical solution by PC [2]. The flow in the reservoir is considered as stationary, the fluid density is constant. The process of heat transfer in the water body is not fixed and it is assumed that the inertial forces, the horizontal turbulent exchange and the change in temperature with depth are negligible. This model allows us to take into account the influence of the configuration of the reservoir, the bottom topography, the position of the jet direction levees and meteorological factors, including wind. The model was used by the authors to model the flow pattern of the temperature distribution on the area of one of the specific reservoir—cooling in the U.S.

Recently, the theoretical description of phenomena in hydrothermal reservoirs—coolers are increasingly used by various schemes, taking into account the stratification. This involves using two-layer model and the model with continuous stratification depth. Two-layer models are used when the value of the local density gradient in a layer called the thermocline is large. In this case, the real picture of the bundle with a higher degree of accuracy can be schematized as a two-layer flow profile of a gap at the interface. The caused by dumping heated water in the coastal zone of the sea was studied in work [3]. Two-layer model is used to smooth the hydrothermal solutions of the problem with heat transfer to the environment, as well as the effects of buoyancy and the involvement of fluid from the lower layer at the top.

The results of mathematical modeling of hydrothermal regime of reservoirs associated with hydro accumulating power station were discussed in work [3]. The water temperature is calculated consistently for the upper and lower layer of the water reservoir at the well-known one-dimensional model. The annual cycle of temperature change with homothermy spring as initial conditions was calculated according to this model. For each horizontal layer thickness of 0.5 m average temperature is defined, but the calculations were made with an hour time step. The mathematical model includes elements like: the calculation of evaporation through the surface, selective withdrawals and outfall, the levels of the free surface, the diffusion coefficients, wind mixing and calculation of rise and the melting of the ice cover.

A two-dimensional model of vertical mathematical model for the calculation of the flow and heat transfer in a stratified reservoir has been taken in work [2]. It is based on equations of continuity, motion and heat obtained from the corresponding three-dimensional equations using formal averaging the width of the reservoir. Turbulent exchange coefficients are determined as a function of the local velocity gradients and temperature.

The most successful mathematical model of the reservoir is proposed in [4], which deals with the three-dimensional flow. The model includes the equation of turbulent motion of a thermally stratified fluid in the Boussinesq approximation. As usual, it is assumed that the distribution by vertical direction is hydrostatic.

Horizontal sharing is not considered, and a two-parameter model which contains the equations for turbulence energy and its dissipation rate is used to determine the vertical turbulent exchange coefficients. A computational algorithm using the iterative method combined with the method of fractional steps is offered for the numerical implementation of this model. Merit of the work is that the model takes into account the real borders of the reservoir, bottom relief, as well as the effect of stratification on the averaged characteristics of hydrothermal regime of the reservoir.

3 Mathematical Models

Numerical simulation was carried out on the Ekibastuz SDPP-I reservoir, located in the Pavlodar region, 17 km to the north-east of Ekibastuz city, Kazakhstan. Technical water supply of SDPP-I was carried on the back of the circuit with cooling circulating water. The surface of the reservoir is at 158.5 m, the area is 19.6 km^2, the maximum size of 4×6 km, the average depth of 4.6 m and a maximum depth of 8.5 m at the intake, the volume of the reservoir is 80 million cubic meters. Selective intake and spillway combined type are used in the reservoir. Waste water enters the pre-channel mixer, then through a filtration dam uniformly enters the cooling reservoir. Water intake is at a distance of 40 m from the dam at a depth of 5 m. Design flow of water 120 m^3/s, and the actual flow rate varies depending on the mode of TPP within 80–100 m^3/s. A real coastal circuit of Ekibastuz SDPP-I reservoir was constructed for the numerical simulation of this problem.

In the reservoirs—cooling spatial temperature change is small. Therefore, stratified flow in the reservoirs—cooler can be described by equations in the Boussinesq approximation. The three dimensional equations of motion, the continuity equation and the equation for the temperature are considered for the mathematical modelling. Let us consider the development of spatial turbulent stratified reservoir—cooler [5–8]. Three dimensionally model is used for distribution of temperature modelling in a reservoir [9, 10, 19]

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_j u_i}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + v\frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u_i}}{\partial x_j}\right) + \beta g_i(T - T_0) - \frac{\partial \tau_{ij}}{\partial x_j} \tag{1}$$

$$\frac{\partial \overline{u_j}}{\partial x_j} = 0 \quad (i = 1, 2, 3). \tag{2}$$

$$\frac{\partial T}{\partial t} + \frac{\partial u_j T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\chi\frac{\partial T}{\partial x_j}\right) \tag{3}$$

where

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j \tag{4}$$

g_i—the gravity acceleration, β—the coefficient of volume expansion, u_i—velocity components, χ—thermal diffusivity coefficient, T_0—the equilibrium temperature, T—deviation of temperature from the balance.

We start with regular LES corresponding to a "bar-filter" of Δx width, an operator associating the function $\overline{f}(\overline{x}, t)$. Then we define a second "test filter" tilde of large $2\Delta x$ width associating $\tilde{f}(\overline{x}, t)$. Let us first apply this filter product to the Navier-Stokes equation. The subgrid-scale tensor of the field $\tilde{\overline{u}}_i$ is obtained from Eq. (4) with the replacement of the filter bar by the double filter and tilde filter [11]:

$$\tau_{ij} = \widetilde{\overline{u_i u_j}} - \tilde{\overline{u}}_i \tilde{\overline{u}}_j \tag{5}$$

$$l_{ij} = \widetilde{\overline{u}_i \overline{u}_j} - \tilde{\overline{u}}_i \tilde{\overline{u}}_j \tag{6}$$

Now we apply the tilde filter to Eq. (4), which leads to

$$\tilde{\tau}_{ij} = \widetilde{\overline{u_i u_j}} - \widetilde{\overline{u}_i \overline{u}_j} \tag{7}$$

Adding Eqs. (6) and (7) and using Eq. (5), we obtain

$$l_{ij} = \tau_{ij} - \tilde{\tau}_{ij}$$

We use Smagorinsky model expression for the subgrid stresses related to the bar filter and tilde-filter it to get

$$\tilde{\tau}_{ij} - \frac{1}{3}\delta_{ij}\tilde{\tau}_{kk} = -2C\tilde{A}_{ij} \tag{8}$$

where $A_{ij} = (\Delta x)^2 |\overline{S}| \overline{S}_{ij}$

Further on we have to determine τ_{ij}, the stress resulting from the filter product. This is again obtained using the Smagorinsky model, which yields to

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2CB_{ij} \tag{9}$$

where $B_{ij} = (2\Delta x)^2 \left|\tilde{\overline{S}}\right| \tilde{\overline{S}}_{ij}$

Subtraction of (8) from (9) with the aid of Germano's identity yields to

$$l_{ij} - \frac{1}{3}\delta_{ij} l_{kk} = 2CB_{ij} - 2C\tilde{A}_{ij}$$

$$l_{ij} - \frac{1}{3}\delta_{ij} l_{kk} = 2CM_{ij}$$

where

$$M_{ij} = B_{ij} - \tilde{A}_{ij} \tag{10}$$

All the terms of Eq. (10) may now be determined with the aid of $\overline{u}$. Unfortunately, there are five independent equations for only one variable C, and thus the problem is overdetermined. A first solution proposed by Germano is tensorially multiply (10) by $\overline{S}_{ij}$ to get

$$C = \frac{1}{2}\frac{l_{ij}\overline{S}_{ij}}{M_{ij}\overline{S}_{ij}}$$

This provides finally dynamical evaluation of C, which can be used in the LES of the bar field $\overline{u}$ [12, 13].

Initial and boundary conditions are defined for the non-stationary 3D equations of motion, continuity and temperature, which satisfy the equations.

4 Numerical Methods

Numerical solution of (1)–(3) is carried out on the staggered grid using the scheme against a stream of the second type and compact approximation for convective terms [14–17]. Projection method [7] is used to solve the problem in view of the above with the proposed model of turbulence. It is anticipated that at the first stage the transfer of momentum occurs only through convection and diffusion. The numerical solution of system is built on the staggered grid with usage of the scheme against a stream of the second type

$$\frac{\partial u\xi}{\partial x} = \frac{u_R\xi_R - u_L\xi_L}{\Delta x}$$

where ξ can be u,v,w

$$u_L = \frac{u_i + u_{i-1}}{2}, \quad u_R = \frac{u_{i+1} + u_i}{2}$$

$$\xi_L = \begin{cases} \xi_{i-1}, u_L > 0 \\ \xi_i, u_L < 0 \end{cases} \quad \xi_R = \begin{cases} \xi_i, u_R > 0 \\ \xi_{i+1}, u_R < 0 \end{cases}$$

and compact approximation for convective member.

$$f(x) = \frac{du}{dx}$$

$$\alpha f_{i-1} + \beta f_i + \gamma f_{i+1} = \frac{u_j - u_{j-1}}{h}$$

Factorizing the f(x) and u(x) to Taylor series we can determine α, β, γ.

For mathematical interpretation of projection method we can write like that:

1. $\frac{u^* - u^n}{\tau} = -(\nabla u^n u^* - v\Delta u^*)$
2. $\Delta p = \frac{\nabla u^*}{\tau}$
3. $\frac{u^{n+1} - u^*}{\tau} = -\nabla p.$

Intermediate field of speed is solved by using fractional steps method with the tridiagonal method (Thomas algorithm). The second stage is for pressure which is found by the intermediate field of speed from the first step. Three dimensional Poisson equations for pressure are solved by Fourier method for one coordinate in combination with the tridiagonal method (Thomas algorithm) that is applied to determine the Fourier coefficients. The numerical algorithm for three dimensional Poisson equation was parallelized on the high-performance system [18, 20].

$$p_{i,j,k} = \frac{2}{N_3} \sum_{l=0}^{N_3} \rho_l a_{i,j,k} cos \frac{\pi kl}{N_3}$$

$$-A_j \vec{a}_{j-1} + B_j \vec{a}_j - C_j \vec{a}_{j+1} = F_j$$

$$\alpha_{j+1} = (B_j - A_j \alpha_j)^{-1} C_j, \quad \alpha_1 = B_0^{-1} C_0$$

$$\beta_{j+1} = (B_j - Aj\alpha_j)^{-1} (F_j + A_j \beta_j), \quad \beta_1 = B_0^{-1} F_0$$

$$\vec{a}_j = \alpha_{j+1} \vec{a}_{j+1} + \beta_{j+1}, \quad \vec{a}_{N_2} = \beta_{N_2+1}$$

At the third stage, it is supposed that the transfer is carried out only by the pressure gradient.

5 Results of Numerical Modelling

Initial and boundary conditions were given respectively to solve these problems. The mesh of 100 × 100 × 100 size was used in the numerical simulation. Figures 5 and 6 show the solved spatial contours and isolines of the temperature distribution at different times after the launch of the Ekibastuz SDPP-1, on the surface, from different angles of view. Figures 7, 8, 9, 10 show the solved spatial contour, isoline of temperature and velocity vector field at different times in the different wind directions after the launch of the Ekibastuz SDPP-1, on the surface,

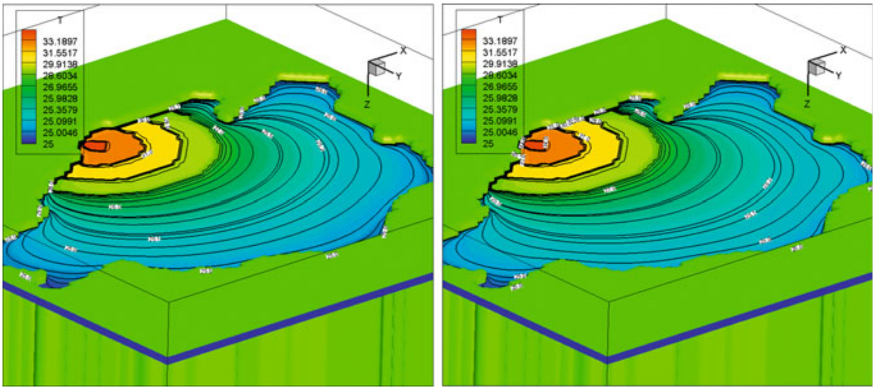

Fig. 5 Contours and isolines of the temperature distribution after 15 and 24 h after the launch of Ekibastuz SDPP-1, on the surface of the water, *the sides view*

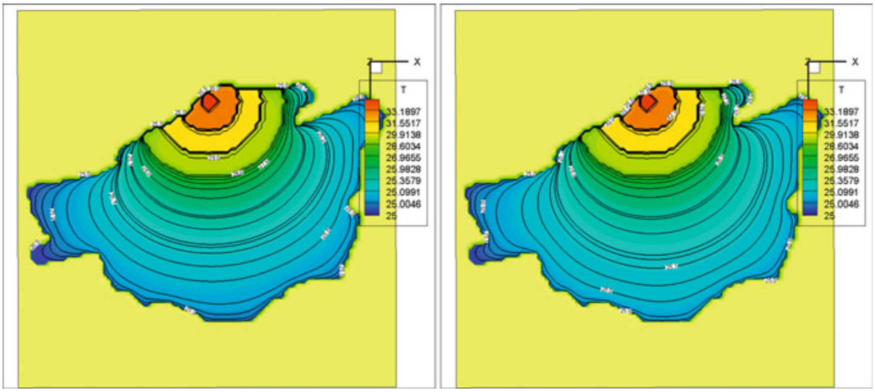

Fig. 6 Contours and isolines of the temperature distribution through 15 and 24 h after the launch of Ekibastuz SDPP-1, on the surface of the water, *the top view*

from different angles of view. In these figures we can see that temperature distribution strongly depends on wind direction and big part of heat follows in those directions.

In all figures we can see that the discharge of warm water creates a jet spreading in the longitudinal direction in the central part of the reservoir. There is a drift of recirculating flow on the surface, the lateral directions in the shallow part of the lake. The thickness of the layer of warm water and the temperature near the axis has changed slightly. At the bottom of the reservoir the most part is directed mainly towards the water intake. Current cold water is uniform in temperature. Thus, under a heavy load on the cooling reservoir and shallow enough to observe the development stratified flow. To achieve the optimal use of the reservoir for water supply power plants must have an impact on the formation of these currents.

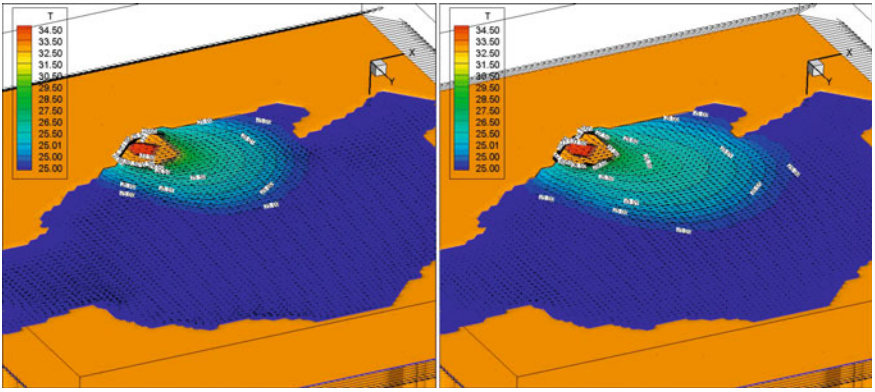

Fig. 7 Contours and isolines of temperature and velocity vector field at 15 and 24 h at the north-west wind after the launch of the Ekibastuz SDPP-1, on the surface, *the side view*

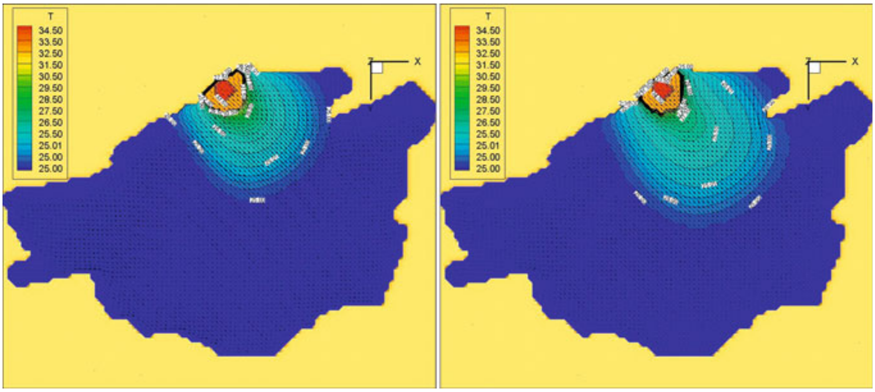

Fig. 8 Contours and isolines of temperature and velocity vector field at 15 and 24 h at the north-west wind after the launch of the Ekibastuz SDPP-1 on the surface of the water, *the top view*

6 Discussion

LES is a more universal approach to the construction of closure for large-eddy simulation models. A necessary condition for the performance of turbulent closures is the ability to "subgrid" model correctly describe the dissipation of the kinetic energy of smoothed velocity fluctuations—the ability to simulate the circuit direct energy cascade from large to small eddies. This stage is the primary mechanism for the redistribution of energy in the inertial range of three-dimensional homogeneous isotropic turbulence.

The principal advantage of the LES from RANS is that, due to the relative homogeneity and isotropy of the small-scale turbulence, plotting a subgrid model is much simpler than the construction of turbulence models for RANS, when it is

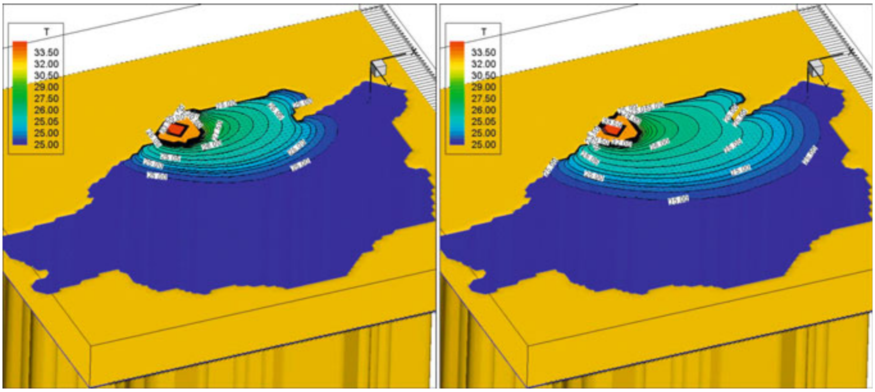

Fig. 9 Contours and isolines of temperature and velocity vector field at 15 and 24 h at the north wind after the launch of the Ekibastuz SDPP-1, on the surface, *the side view*

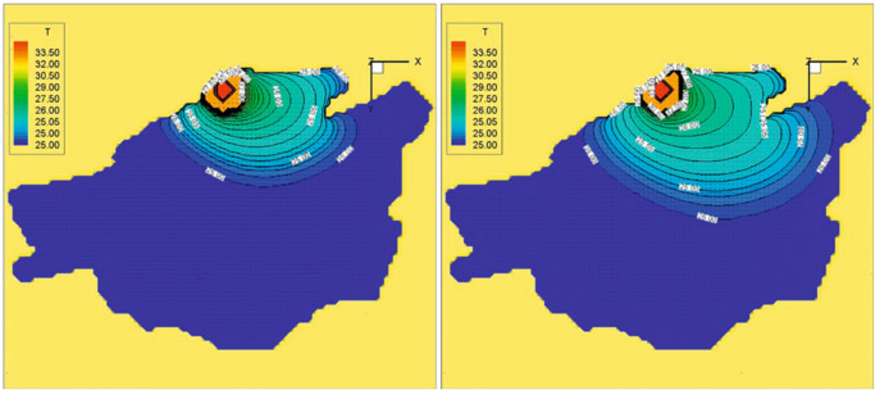

Fig. 10 Contours and isolines of temperature and velocity vector field at 15 and 24 h at the north wind after the launch of the Ekibastuz SDPP-1 on the surface of the water, *the top view*

necessary to model the full range of turbulence. For the same reason, the hope for a "universal" subgrid model for LES are much more reasonable than a similar model for RANS. Natural price to pay for these important benefits LES is a significant increase computational cost associated with the need (as in the case of DNS) of three-dimensional time-dependent calculations on sufficiently fine grids, even in cases where direct interest to the practice of the average flow is two-dimensional and stationary. On the other hand, for obvious reasons, the computational resources required to implement the LES, is much smaller than for the DNS.

The degree of influence of different processes governing the formation of stratified flows and hydrothermal conditions, the entire body of water can be divided into two zones: the first (near), directly adjacent to the water outlet structures, and the second is for the major part of the reservoir. In the near zone forming stratified flow is influenced by the processes of mixing water discharged from the water reservoir and its possible regulation by creating a specific hydraulic regime in the outfall. In the second zone of hydrothermal regime is formed primarily by the processes of heat transfer. The propagation of heat in this part of the reservoir is more dependent on the wind (direction and speed). When you reset the heated water density jump appears in a cold environment between the upper layer of warm water and cold bottom, where there is a compensation for the direction toward the spillway. This allows the use of a combined intake and outfall instead of building costly diversion canals to the spillway. This raises the problem of optimal choice of the geometrical and operational parameters of the cooling pond for efficient power plant.

7 Conclusions

All the figures show that the temperature distribution with distance from the flow approaches the isothermal distribution. The numerical results show that the temperature distribution is distributed over a larger area of the reservoir—cooler.

Thus, using a mathematical model of three-dimensional stratified turbulent flow can be determined qualitatively and quantitatively approximate the basic laws of the hydrothermal processes occurring in the reservoirs. And we can see that distribution of temperature more depend on meteorological conditions.

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