Chapter 54 The Mathematics of Palladio's Villas

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 $By \ldots$ showing to what extent [Palladio] was a natural geometer, we do not make him less the great architect; on the contrary, we show, in a way that gives more than mere lip service to the proposition, how great architecture may flow from geometry

(Hersey and Freedman [1992:](#page-13-0) 12).

Introduction

Much has been written about the mathematical qualities of Andrea Palladio's architecture, including his own I quattro libri dell'architettura. Often this has been analysed within the context of a larger collection of architectural treatises, including Vitruvius' De architectura and Alberti's De re *aedificatoria*, as well as works by contemporaries of Palladio, such as Daniele Barbaro, Cesare Cesariano, Sebastiano Serlio, and Giacomo Barozzi da Vignola. These Cinquecento writings underscore the importance of proportion, symmetry and geometry in Renaissance Italy; for example, Barbaro maintains that "some arts have more of science and others less," and the "more worthy (are) those wherein the art of numeracy, geometry, and mathematics is required" (Puppi [1975:](#page-13-0) 18). Lionello Puppi concludes, "Architecture obviously came into this category. ... Palladio ... bring [s] to the concrete stage of his planning operation a single-minded scientific approach, arrived at through 'lofty speculation' into number and proportion" (Puppi [1975:](#page-13-0) 18). Rudolph Wittkower asserts, "[t]he conviction that architecture is a science, and that each part of a building, inside as well as outside, has to be

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integrated into one and the same system of mathematical ratios, may be called the basic axiom of Renaissance architects" (Wittkower [1952](#page-13-0): 89). Many modern authors have analysed Wittkower's thesis that harmonic proportions derived from musical scales played a central role in the minds and designs of Renaissance theorists and architects. Central to this debate is Palladio's oeuvre—his architecture and his Quattro Libri.

This chapter provides a review of the mathematical aspects of Palladio's work as it has been discussed in the literature and offers a novel perspective on his mathematical approach to architectural design. The argument is made that, whether or not harmonic proportions played a major role in the beauty of Palladio's architecture, it is now time to search further for other mathematical facets of his design philosophy. For convenience the body of analysis is arranged in three sections, based on the categories of geometry, proportion and symmetry.

Geometry

One of Palladio's great gifts was his ability to analyse the ancient and contemporary architecture of Rome, visualize the key elements of plan, section and elevation, and extract the forms that were appropriate to his own design needs. His interest in geometrical form and the process of the extraction of that form from classical elements of architecture developed throughout his career and may be traced in the evolution of his villas.

The Villa Godi at Lonedo di Lugo (1540) has a façade devoid of classical orders.¹ "The central spine ... is simply inserted into a rectangular block rather than being integrated into it by interlocking parts or by the proportions of the plan or elevation" (Ackerman [1966:](#page-13-0) 164). The "ornamentation" of the facade is simply the pure form of the geometry. The Villa Valmanara at Vigardolo (1541) takes this a step further. The simple façade is articulated at the entrance with a Serlian arch, accented by two oculi flanking the arch and square window and a third oculus placed well above it. Two drawings for possible villas from this period also demonstrate Palladio's early recognition of the natural elegance of simple geometric forms. The first is a plan and elevation study for a villa featuring a square perimeter, a biapsidal loggia, and a cruciform, cross-vaulted salone (Fig. [54.1\)](#page-2-0). This study anticipates the Villa Malcontenta and the Villa Rotonda. The second drawing is the penultimate plan and elevation for the Villa Pisani, Bagnolo (Fig. [54.2](#page-3-0)).

The major difference between paper and building is the hemicyclical portico of the former. The Villa Poiana at Poiana Maggiore (c. 1548) is an illustration of Palladio's geometric interpretation of Roman elements. "The familiar Roman columns and tabernacles were transformed into cubic blocks in a composition that depends wholly on geometric form for its effect" (Ackerman [1967](#page-13-0)).

¹ For a detailed discussion of Palladio's architecture, see Boucher [\(1994](#page-13-0)).

Fig. 54.1 Study of a ground plan and elevation for Villa Valmarana, Vigardolo. Image: RIBA31775, by permission of the RIBA Library Drawings & Archives Collections

Although his mature villas are not abstracted to the point where architectural forms give way to purely geometric ones, their treatment is governed by a notable rigour often mathematical in nature. In the Villa Pisani at Montagnana (1552), the

Fig. 54.2 Study of a ground plan and elevation for the Villa Pisani, Bagnolo. Image: RIBA31796, by permission of the RIBA Library Drawings & Archives Collections

salone is square in plan, four free-standing columns marking the central area, with eight engaged columns and four corner pilasters flanking apses in the corners. The vaulting system is rich in geometrical intricacy. Several techniques are used by Palladio to help integrate the entire design in the Villa Cornaro at Piombino Dese (1552): the squarish salone has a flat beamed ceiling supported by four free-standing Ionic columns which align with the second and fifth columns of the

Fig. 54.3 Plan and Elevation of the Villa Barbaro from I Quattro Libri. Image: Palladio ([1570](#page-13-0): II, xiii, 51)

loggias; the Corinthian columns in the upper loggia are 1/5 thinner than the Ionic columns below, lending them a strong verticality; as with the Villa Pisani, the entablature of the first-storey loggia is continued around the entire elevation. At the Villa Badoer at Fratta Polesine (1556) the most purely geometrical of the experiments with the classical elements is the use of colonnades in the form of quadrants (quarter circles) to integrate the agricultural outbuildings within the design of the villa. The rear of the complex of the Villa Barbaro at Maser (late 1550's) opens onto a hemicyclical nymphaeum, answered by a sweeping semicircular exedra facing the road at the front of the villa (Fig. 54.3).

The plan of the Villa Malcontenta (also known as Villa Foscari) at Gambarare di Mira (1560) is dominated by a Latin cross *salone* with a "semicircular cross vault," the impost of which is "as high above the ground as the hall is broad" (Palladio [1997:](#page-13-0) II, xiv, 128). Of the rear facade, Rowe writes, "it is by vertical extension into arch and vault, diagonal of roof line and pediment that Palladio modifies the geometrical asperities of his cube; and this use of the circular and pyramidal elements with the square seems both to conceal and to amplify the intrinsic severity of the volumes" (Rowe [1976](#page-13-0): 11).

In summary, the main focus of the analysis of plan is the *salone*, since this is often the most geometrically powerful room: it is given the form of the Latin cross in the Villa Malcontenta and the Villa Pisani at Bagnolo, the Greek cross in one of his theoretical villas (Fig. [54.1](#page-2-0)), the highly articulated square in the Villa Pisani at Montagnana, and the pure circle in the Villa Rotonda and the Villa Trissino, which will be discussed further below.

In Palladio's mature architectural vocabulary, the elements he chose to extract, taken from classical Roman architecture, were often geometric in form, in plan and section but also largely in elevation. In a sense, Palladio developed a geometric toolkit that included linear, planar and spatial tools, from mouldings and rustication to arches and oculi to vaults and pediments. As we shall see in the next sections, Palladio treated most of his classical elements in mathematical ways through the measured use of proportion and symmetry.

Proportion

Beauty will derive from a graceful shape and the relationship of the whole to the parts, and of the parts among themselves and to the whole, because buildings must appear to be like complete and well-defined bodies, of which one member matches another and all the members are necessary for what is required (Palladio [1997](#page-13-0): I, I, 7; similar statements are found in II, i-ii, 77–78).

These words of Palladio essentially restate principles that Vitruvius and Alberti had embraced (Vitruvius [1960](#page-13-0): I, ii, 13–14 and VI, ii, 174; (Alberti [1986\)](#page-13-0): I, I, 1; VI, ii, 113 ; and IX, v, 195). They suggest (at least) two criteria: (1) the parts of a building must relate among themselves, and (2) the parts of a building must relate to the whole (and vice versa). However, depending on the interpretation of the first criterion, the two criteria may be in conflict! Thus it is necessary to consider what the first criterion meant for Renaissance architects, Palladio in particular. "Parts" may be defined in different ways. First, consider "parts" as the components of the orders. These were governed by precise mathematical rules, but different theories were developed to satisfy different requirements. Palladio had two sources for the rules he considered legitimate: the authority of Rome, and mathematics.² He omits

 2^2 For a complete discussion of Palladio's extensive rules governing the proportion of the orders, Palladio ([1997:](#page-13-0) xiii–xix, 18–55).

the anthropomorphic origins of the column types discussed by Vitruvius and Alberti; he also sides with the direct teachings of antiquity over Vitruvius when they differ. The aspect of Palladio's ornamentation that at first seems the least mathematical is in fact rich in proportional content.

Architectural historians have focused principally on two related aspects of "parts": on individual rooms and on their dimensions. The relationships between the length, width, and height of a room were highly important to Renaissance theorists. The underlying reason for this is the focus of the debate over harmonic proportions mentioned in the Introduction. The idea that harmonic proportions are beautiful to the ear because they are part of a higher universal design and thus should be equally beautiful to the eye is traced by Wittkower to the Pythagoreans via Plato, who explained in Timaeus that "cosmic order and harmony are contained in certain numbers" (Wittkower [1952:](#page-13-0) 91). Alberti draws on Pythagoras when he "conclude [s] that the same Numbers, by means of which the Agreement of Sounds affects our Ears with Delight, are the very same which please our Eyes and our Mind" (Alberti [1986](#page-13-0): IX, v, 196–197). Palladio seems somewhat ambivalent on the subject:

He subscribed to the ancient topoi that the macrocosm of the world was reflected in the microcosm of man and that the rules of architecture refer to the rules of nature, but there is very little evidence that Palladio treated such concepts as more than metaphors. Indeed he once remarked appositely that "just as the proportions of voices are harmony to the ears, so those of measurement are to the eyes, which according to their habit delights [in them] to a great degree, without it being known why, save by those who study to know the reasons of things" (Boucher [1994:](#page-13-0) 239).

Although Palladio makes no specific mention of analogies to music in his Quattro Libri, there is substantial use of harmonic proportions in that treatise. One conclusion reached by Deborah Howard and Malcolm Longair ([1982:](#page-13-0)121ff) in their study of all 44 plans of Book II in order to measure statistically Palladio's use of "harmonic numbers" is that about 2/3 of the dimensions followed harmonic proportions, whereas only 45 % would be harmonic had Palladio picked dimensions at random. Branko Mitrović contrived an explanation of how $\sqrt{2}$:1 can be viewed as a musical ratio using the augmented fourth of a tempered scale. After taking into account heights as well as lengths and widths, he concludes that Wittkower's thesis was more consistent than it seemed (Mitrović [1990:](#page-13-0) 281–285).

If harmonic proportions really are at work in Palladio's architecture, does it not imply that some mathematical proportions are inherently more beautiful than others? If so, does this not admit the possibility of additional mathematical components of beauty? But if harmonic proportions are not at work, the search for the operative factor must need to be expanded! One such operative factor may lie in pure mathematics.

An examination of the plans of Book II for Palladio's seven preferred room shapes found significant evidence that room shapes were more important to him than harmonic ratios. Howard and Longair suggest that either "Palladio used a system of musical harmonies ...; or ... that he adhered to his own simpler recommendations concerning room shapes; or ... that he recognized the practical

Fig. 54.4 Illustration of Alberti's construction of a cube exhibiting the proportions $\sqrt{3}$ and $\sqrt{2}$: "...we may consider the Line drawn from one Angle of the Cube to that which is directly opposite to it, so as to divide the Area of the Square into two equal Parts, and this is called the Diagonal. What this amounts to in Numbers is not know: Only it appears to be the Root of an Area, which is as Eight on every Side; besides which it is the Diagonal of a Cube which is on every Side, as twelve" (Alberti [1986:](#page-13-0) IX, vi, 199). Image: author, after (Alberti [1986:](#page-13-0) Pl. 64)

advantages of using simple, easily divisible numbers."³ Mitrović is more provocative. He finds that six unexplained ratios are close to $\sqrt{3}$:1, with the closest being the four large corner rooms of the Villa Rotonda. Each corner room has dimensions 26 by 15, which differs from $\sqrt{3}$:1 by only 0.07 %, a deviation smaller than the allowable error used in many rigorous scientific experiments! (Mitrović [1990:](#page-13-0) 285–286).

The ratio $\sqrt{3:1}$ is referred to as *triangulature* since it can be derived as the ratio of the height of an equilateral triangle to half of the base. Mitrovic^{$\dot{\text{}}$} informs us that this method was well known in Renaissance times. In fact, Alberti (1955: IX, vi, 199) describes precisely this construction, but immediately before this, he describes a construction based on "some other natural Proportions for the Use of Structures, which are not borrowed from Numbers, but from the Roots and Powers of Squares" (Fig. 54.4). This construction simultaneously produces the ratios $\sqrt{3}$:l and $\sqrt{2}$:1 by way of a perfect cube, and this may have appealed greatly to Palladio.

Regarding room proportions, many writers have argued that the difference between a 30 \times 30 room and a 29-1/2 \times 30-1/2 room might be imperceptible and that the concept of a proportional system (harmonic or otherwise) is thus meaningless. This is not the point, however, with Palladio, who governed himself

³ Howard and Longair [\(1982](#page-13-0): 136). For the seven preferred room shapes, see Palladio ([1997:](#page-13-0) I, xxi, 57). He recommends circles, squares and rectangles of proportions √2:l, 4:3, 3:2, 5:3 and 2:1. The last four are harmonic proportions; all are consistent with Vitruvius and/or Alberti, though circles are discussed only in terms of temples; see Vitruvius [\(1960](#page-13-0): IV, viii, 122–124 and VI, iii, 177–179); Alberti (1955: VII, iv 138–139 and IX, v–vi, 197–199).

by principles, many of which were mathematical in nature. While it was impossible for him to capture the ratio $\sqrt{3}$:l with integers, it was still possible to capture the principle of the perfect cube, in a sense, by using an extremely close approximation to this ratio.

Proceeding now with the other interpretation of the first criterion, we may take whole rooms as the 'parts' of a building and consider ways in which multiple rooms can relate among themselves. This was of explicit concern to Palladio:

But the large rooms should be distributed with the medium-sized, and the latter with the small rooms in such a way that ... one part of the building corresponds to the other so that the whole body of the building would have an inherently suitable distribution of its members, making the whole beautiful and graceful (Palladio [1997:](#page-13-0) II, ii, 78).

Wittkower maintains Palladio's

systematic linking of one room to the other by harmonic proportions was the fundamental novelty. ... Those proportional relationships which other architects had harnessed for the two dimensions of a façade or the three dimensions of a single room were employed by him to integrate a whole structure (Wittkower [1952:](#page-13-0) 113).

What, then, was Palladio's method? A simple answer lay in the restriction of the dimensions of individual rooms to the "harmonic numbers," thus the rooms would relate to each other via harmonic proportions. Another, more ingenious approach is found in Palladio's rules for determining the heights of rooms. For flat ceilings, the height is taken to be equal to the width, $h = w$. For vaulted ceilings in square rooms, Palladio's rule is simply $h_s = (4/3)w = (4/3)l$. For vaulted ceilings in rectangular rooms, the height is determined in three possible ways, corresponding to the arithmetic, geometric, and harmonic means: using the arithmetic mean, $h_a = (w + l)/2$; using the geometric mean, $h_g = \sqrt{wl}$; using the harmonic mean, $h_h = 2wl/(w + l)$. Of course, Palladio uses neither these names nor this modern notation; his definitions are purely numerical, and he supplies examples with numbers aligned in particular ways for ease of comprehension. (Alberti defines the three means in his treatise, naming them arithmetical, geometrical and musical (Alberti [1986](#page-13-0): IX, vi, 199–200). Why Palladio does not use these names is a good question.) More importantly, Palladio supplements the numerical methods with illustrations of geometric constructions for each mean, employing a geometric approach to ensure correct proportional relationships, both within rooms and between rooms (Fig. [54.5](#page-9-0)).

Of the use of the three means for the heights of vaulted rooms, Palladio writes,

[W]e should make use of each of these heights depending on which one will turn out well to ensure that most of the rooms of different sizes have vaults of an equal height and those vaults will still be in proportion to them, so that they turn out to be beautiful to the eye and practical for the floor or pavement which will go above them (Palladio [1997](#page-13-0): I, xxiii, 59).

This can be done with a (limited) number of Palladio's preferred proportions in such a way that the height/width ratios are also among the Mitrovic^c smaller rectangular rooms of the Villa Rotonda are related to the large corner rooms. Recall that each corner room has length/width ratio of 26:15, approximately equal

Fig. 54.5 Palladio's constructions for the three means (equations 3–5) from the *Quattro Libri*. For those wishing to verify Palladio's definitions, use the Pythagorean theorem on the second and similar triangles on the third. Image: Palladio ([1570:](#page-13-0) I, xxiii, 53–54)

to $\sqrt{3}$. The height of the corner room is determined using the arithmetic mean, $h_a = (26 + 15)/2 = 20$ 1/2, so that the height/width ratio is 20 1/2: 15 = 1.3666. Each smaller room has dimension 15×11 (they share a dimension with the large room), a length/width ratio of $15/11 = 1.3636$.⁴ The closeness of 1.3666 and 1.3636 suggests that Palladio was very careful about the proportional relationships of his most celebrated villa, and it appears that he relied on pure mathematics as opposed to harmonic proportions.

Finally, let us turn to the criterion that the parts of the building must relate to the whole, where we still consider "parts" to be rooms and their dimensions. The "additive problem," choosing rooms from a small set of ratios such that they add to produce another one of these ratios, is not easily solved, especially if the ratios must be commensurate.⁵ Therein lies the inherent conflict between the two criteria. Order may be introduced through the use of a square grid, as, for example, a 3×3 square grid containing only the ratios 1:1, 3:2, 2:1, and 3:1 (as opposed to a generic 3×3 grid, which can have as many as 36 different ratios embedded in it⁶) but this may not always be either practical or aesthetically pleasing. Often the most interesting solutions involve incommensurate ratios based either on $\sqrt{2}$ or $\sqrt{5}$. Though solutions may be found with commensurate ratios, Scholfield notes, "Palladio omits the overall dimensions from his plans, and so avoids the problem of adding the separate dimensions together ... His system of proportion integrates the whole structure in the sense that it links the parts, or separate rooms, to each other, but it still fails to relate them to the whole" (Scholfield [1958](#page-13-0): 64). Howard and Longair addressed this while trying to discover whether or not Palladio used wall thicknesses to help develop additive solutions, (Howard and Longair [1982:](#page-13-0)

⁴ Mitrovic´ ([1990:](#page-13-0) 289–291). Both decimal figures are close approximations of $(1 + \sqrt{3})/2$; for those interested in pure trigonometry, this equals $\sin 30^\circ + \cos 30^\circ = \sqrt{1 + \cos 30^\circ} = \sqrt{\frac{1 + \sqrt{3}}{2}}$ $\sqrt{\frac{1+\sqrt{3}}{2}}$.

⁵ Although Palladio did allow himself the use of approximations of the incommensurate ratio $\sqrt{2:1}$, he did not use it very often; see Howard and Longair [\(1982](#page-13-0): Appendix, Table A4, 141–143), where this ratio is found only four times out of over one hundred entries.

 6 For more on the additive problem, see Scholfield [\(1958](#page-13-0): 132–134).

128–129) concluding that in some cases he did use wall thicknesses, while in others he missed opportunities to use them.

In fact, Palladio recognized the occasional need to bend the rules, as his instructions on correct vault heights indicate: "There are other heights for vaults which do not come under any rule, and the architect will make use of these according to his judgment and practical circumstances" (Palladio 1997: I, xxiii, 59). Many of his rules for dimensions of doors and windows are practical rather than being based on abstract mathematics or harmonics. "Palladio's intelligence and experience would not have allowed him to suggest that a single proportional theory alone would enable one to design a beautiful building, any more than a musician could compose a great symphony merely with a knowledge of harmony and counterpoint" (Howard and Longair [1982:](#page-13-0) 137). Indeed, Palladio's toolkit contained many tools, including one that was especially effective in solving the problem of relating the parts to the whole.

Symmetry

The discussion of proportion often focuses on the rooms flanking the salone, especially with consideration to the relationships of sequences of rooms to each other. Palladio's commitment to symmetry, simply yet forcefully expressed, ties these elements together into a cohesive whole. His main paradigm is reflective symmetry, the type of symmetry found in the bodies of so many of Earth's creatures. Palladio usually employs "a triadic composition with a central block built around the axis of the entranceway, and two symmetrical flanking blocks. ... The design was thus tightly knit as an organism" (Ackerman $1966: 160-161$ $1966: 160-161$).⁷ The major events of his designs occur on axis, both in plan and in elevation, relating the two to achieve a more integrated whole. Further, no walls are aligned on top of the axis, and there are almost invariably doors at the perpendicular intersection of walls with the axis, so that one has the pleasure of experiencing the design from anywhere on the axis.

Palladio uses an especially rich symmetry in the Villa Rotonda and the Villa Trissino at Meledo di Sarego.⁸ Both are sites on hilltops with excellent views in all directions. Palladio uses the sacred circle for the shape of the salone and provides loggias on all four sides, creating two perpendicular axes of symmetry that result in 180° rotational symmetry.⁹ The clarity of geometry and depth of symmetry make these villas two of Palladio's most influential designs.

⁷ See also Ackerman and James $(1967: 11-12)$ $(1967: 11-12)$.

⁸ The latter was designed ca. 1567 but never completed, see Palladio [\(1997:](#page-13-0) II, iii, 94–95 and II, xv, 138); Puppi (1975: 384–388).

⁹ To be precise, the rotational symmetry is broken in the Villa Trissino by the forecourt, the arcades of which project from the central block in quadrants as with the Villa Badoer. The Villa Rotonda,

The symmetry concept underwent a rapid evolution during the Renaissance (Hersey and Freedman [1992](#page-13-0): 15–37). Symmetry's original meaning was closer to our concept of commensuration or correspondence in measure, and related more to proportion than to our modern concept of symmetry. Vitruvius often employed the term along with the concept of proportion, as in the phrase "symmetrical proportions" (Vitruvius [1960:](#page-13-0) VI, ii–iii, 174–180). Hermann Weyl writes:

[i]n the one sense symmetric means something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole. Beauty is bound up with symmetry. ... [T]he second sense in which the word symmetry is used in modern times [is] bilateral symmetry" (Weyl [1952:](#page-13-0) 3-4).

When symmetry took on its current meaning "the word's ancient association with 'beautiful' probably strengthened the idea that a design with two identical halves was more beautiful than one without" (Hersey and Freedman [1992:](#page-13-0) 16). In addition to bilateral or reflective symmetry, translatory and rotational symmetry were also regarded as common denotations of symmetry. Wittkower points out that Alberti, Leonardo, Francesco di Giorgio, and Serlio were quite intrigued by central plans for churches (Wittkower [1952](#page-13-0): 1–28, plates 1–13), their drawings showing an explicit interest in rotational symmetries.

Because Vitruvius prescribed symmetry only for public buildings, the use of symmetry for house plans in the Quattrocento "required vigorous reinterpretation" (Hersey and Freedman [1992:](#page-13-0) 31). To this end, Cesare Cesariano was more than willing to "clarify and extend" Vitruvius' notion of symmetry so that it applied to domestic as well as public architecture (Hersey and Freedman [1992:](#page-13-0) 33). Daniele Barbaro insisted that private houses should be equipped with all the refinements of public buildings, including the rigours of proportion and symmetry. Palladio's illustrations for Barbara's edition of Vitruvius include a plan, section and elevation exhibiting the hybrid design.

For his part, Palladio justified symmetry on structural grounds:

Rooms must be distributed at either side of the entrance hall, and one must ensure that those on the right correspond and are equal to those on the left so that the building will be the same on one side as on the other and the walls will take the weight of the roof equally $[...]$ if the rooms on one side are made large and those on the other side small, the former will be more capable of resisting the load because of the thickness of their walls, while the latter will be weaker, causing grave problems. (Palladio Palladio, Andrea [1997](#page-13-0): I, xxi, 57).

Renaissance theorists, including Palladio, had thoroughly convinced themselves that symmetry, at very least reflective symmetry, was the only correct design choice. Though he did not give an anthropomorphic rationale for symmetry, he did use the analogy of the human body in explaining the proper placement of rooms. There are, in fact, many rationales for and interpretations of symmetry besides the anthropomorphic and economic. The kinaesthetic rationale is related to the experience of architectural space:

on the other hand, has essentially 90° rotational symmetry, except that the rectangular rooms do not quite align in 90° rotation.

[a] single axis of symmetry in a space impels the spectator smoothly along it, whereas two precisely balancing cross-axes ... as in ... the Villa Rotonda, impart a sense of static serenity (Tabor [1982:](#page-13-0) 21).

For Palladio, despite his structural claims, a combination of the political and aesthetic arguments seems to be at work. On the one hand, the link through Vitruvius to Rome provided legitimacy; on the other, the use of symmetry went a long way towards solving the aesthetic problem of relating the parts to the whole.

Conclusion

Palladio exhibits a strong interest in geometry, both in the crafting of architectural spaces and in geometric constructions for the correct design of architectural elements and their interrelationships. We have explored Palladio's concern with the proportional relationships of parts among themselves and to the whole, as in the elements of the orders and in the dimensions within and between rooms. Concerning the latter, the whole numbers and occasional simple fractions used are chosen for a number of reasons: to employ proportions suggested by Vitruvius and Alberti, possibly informed by analogies to musical theory; to reference proportions derived from simple, pure geometry; and to provide practical solutions to the problems of a particular design. Finally, Palladio's consistent use of symmetry was an aesthetically pleasing and seemingly correct way to link plan and elevation into a cohesive whole.

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