Chapter 51
Design and Perspective Construction: Why Is the *Chalice* the Shape It Is?

Richard Talbot

Introduction

The basis for this chapter is a study and examination of one of the most complex and well-known examples of early Renaissance perspective drawings, the *Chalice*. This drawing has become almost iconic within the history of perspective, although neither the author nor the exact date of its execution is certain. It is most likely to have been executed prior to 1460 and has been attributed variously to both Paolo Uccello and to Piero Della Francesca, based largely on references in Vasari's *Lives of the Artists*.[1] It is a drawing that I have long been fascinated and intrigued by because of its complexity, clarity and technical accomplishment.

I first attempted a reconstruction of the drawing over 20 years ago, primarily out of curiosity about the method of its perspective projection. I made a relatively simple version of the drawing, using only 16 facets (see Fig. 51.7 below), rather than the 32 in the original drawing. I quickly became aware, however, of the technical difficulties in making a similar drawing with such apparent accuracy and consistency at this scale the drawing is only 34 cm × 24 cm. I also realised that although the method behind the perspective projection is, in fact, very straightforward and is something that a good student could grasp in a few hours, many questions about the drawing remained.

First published as: Richard Talbot, "Design and perspective construction: Why is the Chalice the shape it is?", pp. 121–134 in *Nexus VI: Architecture and Mathematics,* Sylvie Duvernoy and Orietta Pedemonte, eds. Turin: Kim Williams Books, 2006.

[1] Vasari mentions *mazzocchi* and a faceted stellated sphere as being forms that Uccello attempted; he also mentions a faceted vase specifically in relation to Piero (Vasari 1996). Piero in *De Prospectiva Pingendi* constructs a *mazzocchio* (Fasola 1942: Tav. XXVII, Fig. LI).

R. Talbot (✉)
Fine Art, The Quadrangle, Newcastle University, Newcastle Upon Tyne NE1 7RU, UK
e-mail: richard.talbot@ncl.ac.uk

K. Williams and M.J. Ostwald (eds.), *Architecture and Mathematics from Antiquity to the Future*, DOI 10.1007/978-3-319-00143-2_4,

These questions partly concerned the practical aspects of making the drawing but primarily I felt that if someone had painstakingly constructed a drawing as complex as this then presumably the design of the *Chalice* was more than just something to hang a particularly clever and impressive technique on. The main question that arose, and which is the subject of this chapter, is 'why is the *Chalice* the shape it is?' The additional question of its attribution was not one that I had intended to engage during the course of this study, but I believe that my research has revealed aspects of the drawing that may have a bearing on it.[2]

The Subject Matter of the Drawing

The *Chalice* is a drawing of a completely self-contained object and does not appear to be a preparatory study for a painting or for an object that was intended to be set within a painting or some other kind of representation. It seems to fit into a category of drawings that could be considered autonomous, speculative or investigative. It is a 'wire frame' drawing a representation of an idealised three-dimensional form and as such does not imply any particular material or scale. However, the basic shape and specific forms and structures in the *Chalice* do reflect those of contemporary chalices and other ecclesiastic and secular objects; chalices often had hexagonal and octagonal elements in the base, a stem with a faceted widening and cut precious stones mounted around the surfaces. These all appear to be repeated, in some way, within the drawing, including the mixture of solid and transparent forms and the hexagonal and stellated octagonal sectioned *mazzocchi*. The *mazzocchio* is the semi-rigid frame within a particular fifteenth century head adornment but it has a form and structure that lends itself to being adapted, idealised and increased in complexity.[3] Whatever the source of the forms in the *Chalice*, they mostly appear to have simple geometric definitions. The forms themselves may have developed out of the process of geometric construction, rather than being developed from or referring to something already seen.

The object, therefore, could have been conceived as a pure crystalline structure and constructed on a purely theoretical and geometrical basis. This also means that, in theory, unravelling the procedures leading to the drawing should be more straightforward, as we are dealing, presumably, with the geometry of the square, circle and regular polygons and not with units, modules or possible wall thicknesses.[4] In addition, because of the way the *Chalice* is drawn, a vertical cross section parallel

[2] It forms part of a broader investigation on my part as an artist looking at how the physical layout of a drawing on the paper, the method and approach of the perspective construction, and the thinking processes relate to and influence each other.

[3] Kim Veltman (1986: 128–137) discusses the mazzocchio in relation to Leonardo and, in particular, his elaborate and playful transformations of it into spirals, wheels and cogs, etc.

[4] Given the size of the drawing and the degree of error inherent in the method of construction, I would expect high degrees of accuracy in the drawing. It seems to me the largest error that might legitimately arise is possibly 1.5 mm at the most.

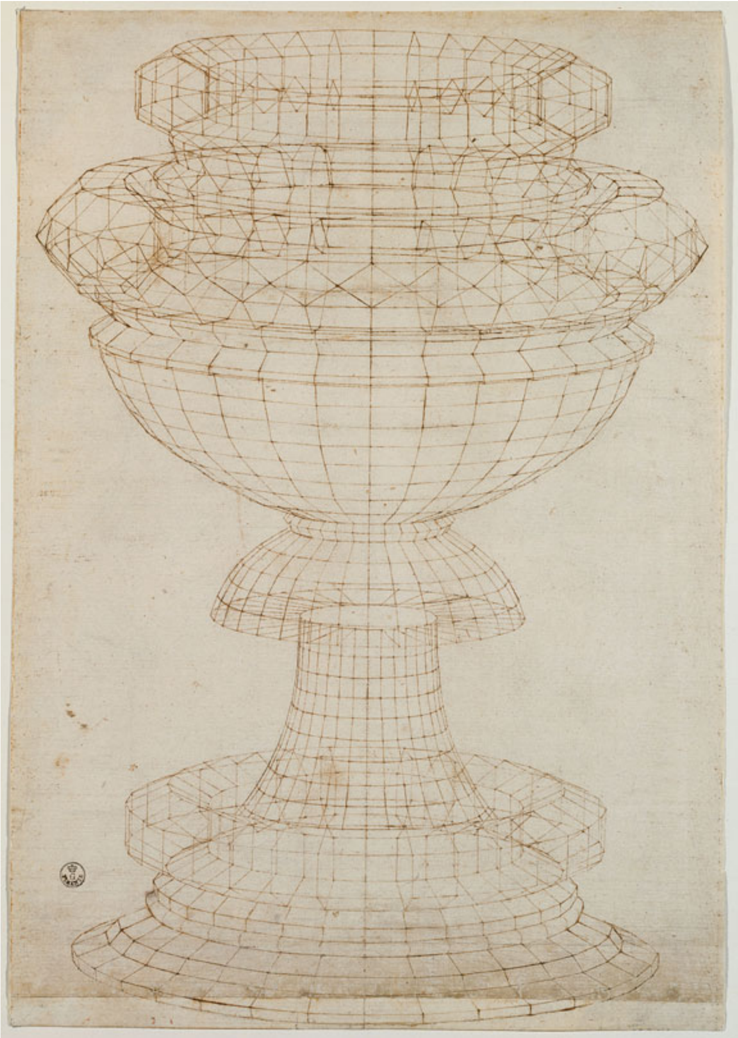

Fig. 51.1 Paolo Uccello (1397–1475), perspective study of the *Chalice*, 1430–1440 (pen and ink on paper). Gabinetto dei Disegni e Stampe, Uffizi, Florence, Italy. Image: ©Foto Scala Firenze. Reproduced by permission

to the picture-plane reveals its true elevation. Proportionally correct measurements relating to its elevation can therefore be taken directly from the drawing. A more direct relationship between the artist's intentions, the underlying construction and the final drawing should be apparent; however, in practise, many aspects remain very uncertain and any analysis will be qualified.

Physical Aspects of the Drawing

The visible lines that define the image are drawn in ink, but the overall impression of the actual drawing is of something more delicate than one would expect from seeing the various reproductions of it (Fig. 51.1).

The underlying construction lines are inscribed into the surface of the paper using a stylus and, where a specific point is plotted, the paper has often been pricked.[5] These various marks are almost invisible under normal lighting conditions and it is only in an early twentieth century photograph taken in a raking light that they are particularly evident. For this study, I have used a high-resolution scan and a much enlarged photographic print taken from this original photograph; I have also examined the actual drawing.

Building the Drawing

It is quite clear that the *Chalice* has been 'built' each point of every 32-sided polygon being located in space using information from a ground plan and an elevation. It is all constructed within a square-based, transparent rectangular block split into horizontal layers, each layer containing an individually drawn polygon. These make up separate but related interlinking geometric volumes, some of which are drawn as if transparent, others as if solid, and still others as a combination of both. The volumes that are drawn in their entirety are the self-contained forms, such as the mazzocchi. As everything that has been constructed has then been inked in, the decision as to what was to be visible was, I believe, taken while the drawing was being made.

The same procedures, and as much effort, would have been required to construct the smallest detail as would have been required to construct a larger or more visible form. In general, the method used throughout has been consistent, but there are exceptions. Two elements of the drawing appear to have been added by eye, without the aid of any construction lines. These are a tiny curved profile slightly below the lowest mazzocchio, and the vertical lip at the very base of the drawing, added, possibly, to confer a feeling of solidity. In addition, there is an inconsistency in the lowest constructed polygon, which could be the result of starting to use the coordinates from one circle and then inadvertently using those from another. It is here, at the bottom of the drawing that approximately 1 cm of extra paper has been attached and, although it is impossible to know at exactly what stage it was added, the scored lines on the surface appear to run seamlessly over the joint. However, even if this error in the lowest polygon had not been made, the drawing would still have fallen outside the bottom edge of the paper. As it is, the drawing appears to

[5] I do not believe that they are the result of the drawing being 'pricked through' as part of a copying process to enable the image to be used, possibly, in an intarsia design; see Kemp (1991: 241–242).

have been altered in order to fit into the available space. The topmost polygon, which is part of the upper hexagonal ring, is positioned as high as it could have been on paper of this size and it is therefore impossible to know how much the size of the paper ultimately dictated decisions regarding the design.

The top two mazzocchi are very accurately drawn, that is, they appear to contain true hexagonal and octagonal sections, but the lowest one, which at first sight appears to contain another regular hexagon, in fact, does not. Whether this was intentional or not we will never know, but as all the points of polygons of equal diameter should align vertically, it becomes apparent that this mazzocchio has been constructed using circles that are used elsewhere in the drawing. It transpires that several circles have been used more than once within the drawing and it appears that it is the circles from the upper regular forms that are used again lower down. This leads me to think that the drawing was, for the most part, constructed from the top down.

The section containing the uppermost mazzocchio apparently defies gravity it is not joined physically to the section below; this too supports the idea that it is an autonomous drawing and that it is not, and never was, intended to represent a 'real' chalice. It also suggests to me that the design was not fully resolved before the drawing was begun and that the progress of the drawing may have been relatively organic. The elevation, which in the case of the *Chalice* is what determines its design, originally may have been very simple and has grown in complexity, with decisions made and elements added during the process of its making.

The Perspective Construction: the Layout

The method used for the perspective projection is straightforward.[6] The preparatory stages of the drawing would have involved establishing a plan and elevation, although these could, to a certain extent, have evolved along with the drawing. One approach to the design would have been to draw the elevation first and then derive the plan from that elevation but, as I will demonstrate, I believe that the elevation has, in fact, been derived from geometry involved in drawing the plan.

There are good practical reasons for having a plan and an elevation laid out on the same sheet of paper as the main drawing, preferably positioned at the edges, as in Fig. 51.2.

Alternatively, they could occupy the same space, lying underneath the final drawing. However, examination of the *Chalice* suggests that the plan and elevation were not positioned at the edges, nor can they be found in the myriad of underlying

[6] Martin Kemp (1991: 241–242) suggests that the artist used some kind of orthographic projection, rather than perspective. Robin Evans also suggests that Piero's 'other method' was used for this drawing, but if this had been the case, different construction marks would have been present on the drawing; see Evans (1995: 173).

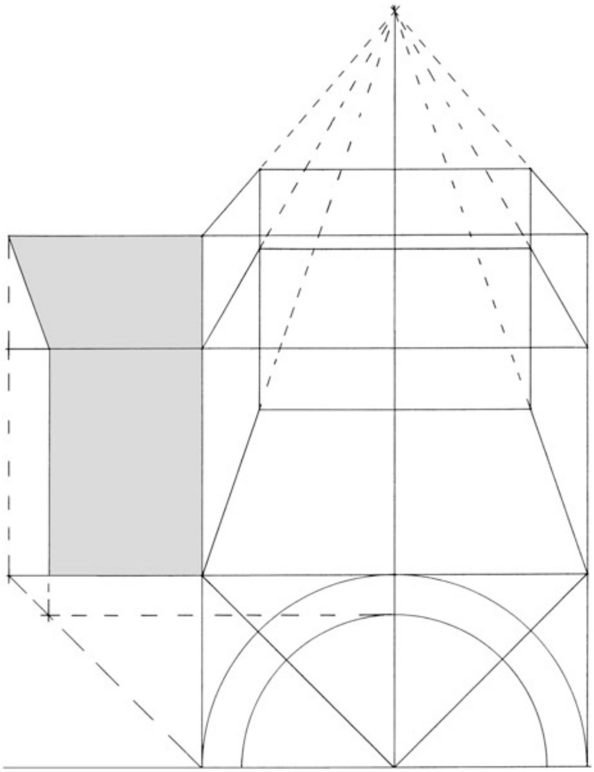

Fig. 51.2 The general principle of the perspective construction. Drawing: author

construction lines. This means that there must have been another drawing, or drawings, presumably now lost, which contained the plan and elevation from which the measurements required for the perspective construction were taken.

I propose that there were not two drawings, one each for the plan and the elevation, but a single drawing, containing a geometric construction that fulfilled both roles. This would certainly make sense on a practical level and, possibly, also on an aesthetic level. A simple example in support of this idea would be the preparatory drawings required to construct a faceted sphere; the elevation is constructed first and the plan is developed from it. The plan now effectively contains all the same information that is in the elevation, so the separate elevation can be discarded. The geometry and constructions in the plan and elevation become interchangeable the radiating lines required for establishing the elevation become the same radiating lines needed for the plan (Fig. 51.3).

The auxiliary drawing for the *Chalice* would have initially required the same construction procedures as those needed to create a 32-sided, faceted sphere a circle, square and octagon, sub-divided repeatedly in order to produce the 32 divisions around its surface. I therefore suggest that the elevation of the *Chalice* derived directly from these same constructions. The key to this is the construction of the octagon, which is clearly a prerequisite of drawing the largest

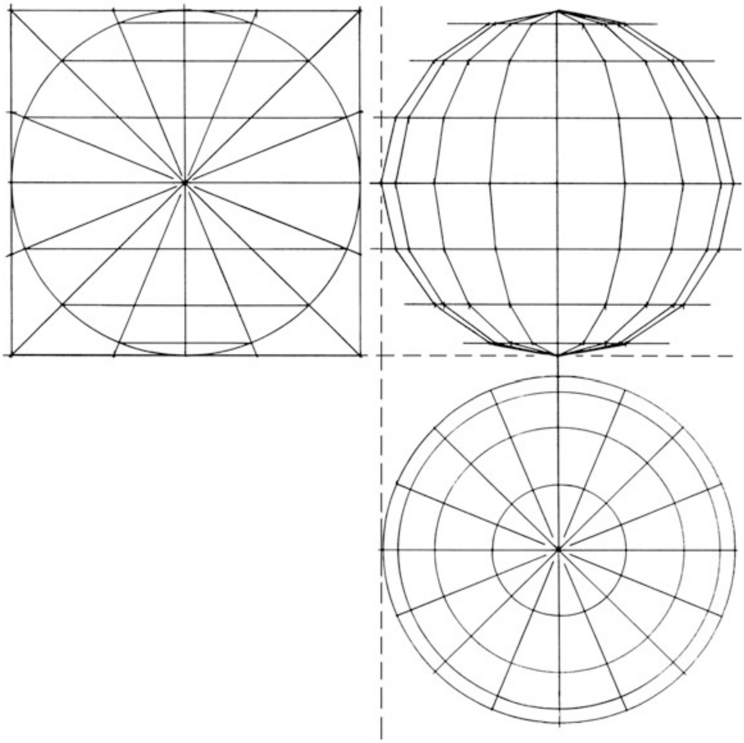

Fig. 51.3 Constructions in the plan and elevation are interchangeable. Drawing: author

mazzocchio but is also part of the construction of the radiating pattern that defines the points around the 32-sided polygon. The octagon is, therefore, required for the construction of elements within both the elevation and the plan (Fig. 51.4).

The Perspective Construction in Detail

The plan and elevation determined the dimensions of a square-based, transparent, rectangular block, which would be subsequently split into horizontal layers. Whatever the shape of the elevation, the plan would always be a set of concentric circles, split into 32 equal sections, equating to the 32 facets describing the surfaces around the object. The vanishing point of the *Chalice* has been placed centrally, at a level that is roughly twice the height of the drawing. The rear corners of the degraded square have been placed at a relatively low level, which means we are looking at the object from a distance of roughly nine times the height of the drawing. These measurements in themselves are not significant, but the relatively large viewing distance does reduce the possibility of uncomfortable distortions

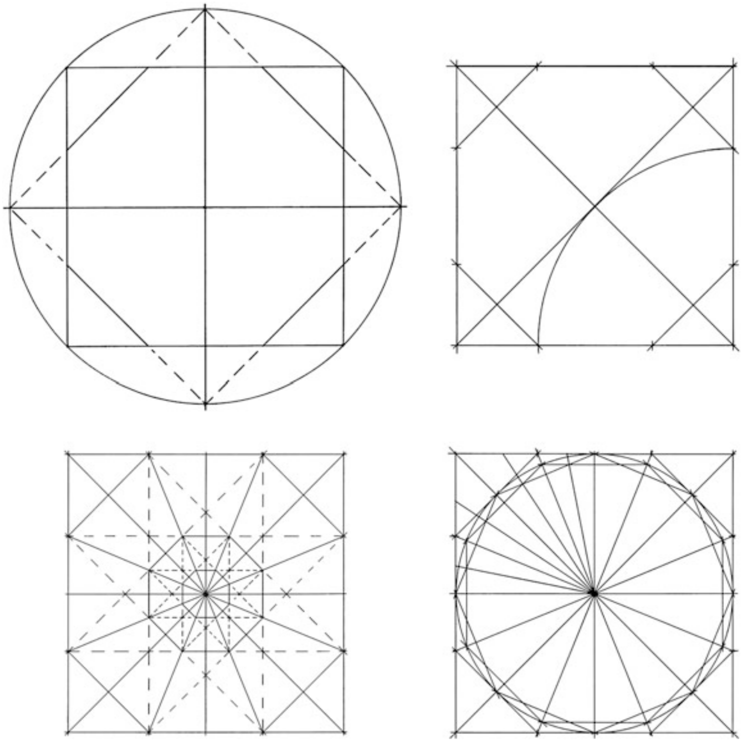

Fig. 51.4 Construction of the octagon and 32 divisions. Drawing: author

caused by being too close to the object. The construction has then taken place within this block, a separate polygon being constructed in each layer.

Each layer in the *Chalice* has been constructed within a few millimetres of the previous one, but it is only in those layers where a complete 32-sided polygon has been drawn, for instance in the mazzocchi, that a full set of radiating lines has been constructed. On the drawing the pricked marks indicating the ends of the radiating lines are visible. The radiating pattern may have been located by constructing it first in the base and then noting the points where the radiating lines touch the four edges of the base (Fig. 51.5).

The horizontal position of these points would then be the same for all the layers and so could be transferred using compasses, repeating their positions onto the four edges of each layer. Because of the very acute angle of the receding sides of each layer there is potential for inaccuracy if only the above method is used. Alternative simple geometric constructional routes, the kind described by Piero Della Francesca in *De Prospectiva Pingendi* and as demonstrated in Fig. 51.5, may have been used to locate, or at least double check, the positions of the points on the sides (Fasola 1942: Tav. VI, Fig. XIX).

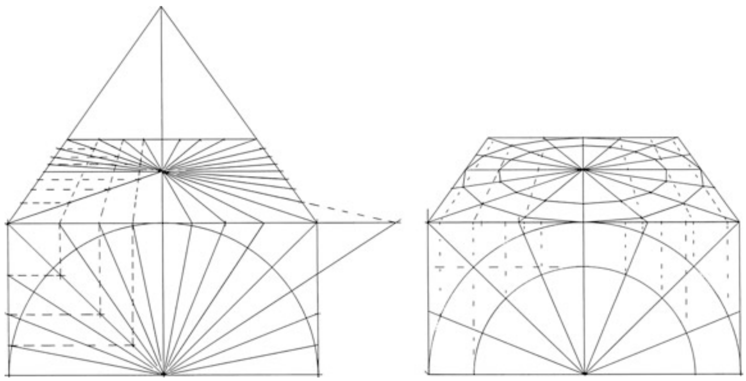

Fig. 51.5 Constructing the radiating lines and the polygons. Drawing: author

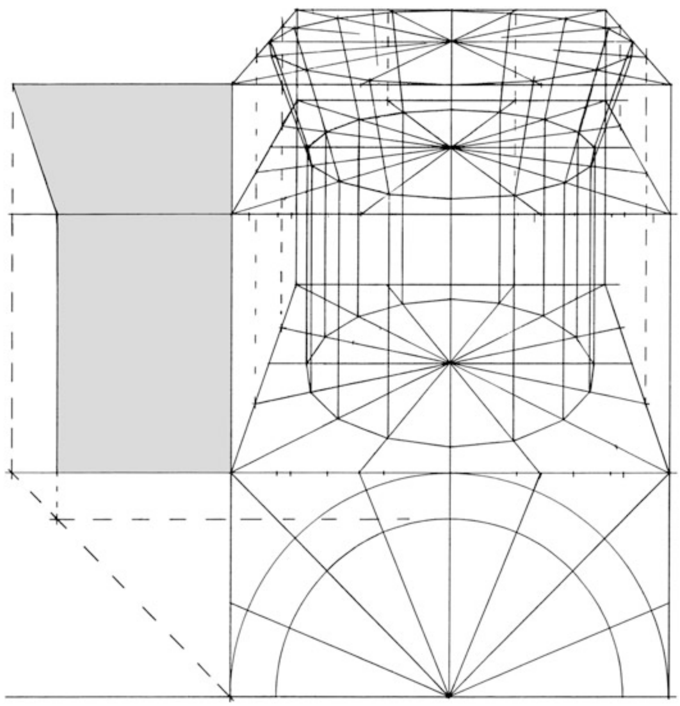

Fig. 51.6 The construction of polygons in each layer. Drawing: author

The points from the plan were then mapped onto the radiating pattern. The horizontal positions of the points of each 32-sided polygon have been measured using compasses and then transferred onto the front edge of the layer at which that polygon is drawn (Figs. 51.6 and 51.7).

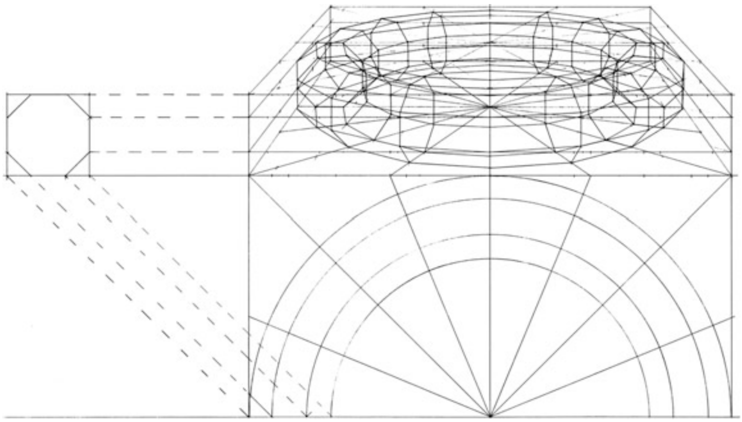

Fig. 51.7 The construction of a 16-sided octagonal sectioned mazzocchio. Drawing: author

In practise, this meant marking the same eight points on either side of the central vertical axis. These points in turn, were aligned with the vanishing point and their intersection with the appropriate radiating line located the points that form the polygon. Again, all the marks from this procedure are visible, either as scored lines from compasses on the leading edge, or as pricks in the paper where a point is established.

The Design: the Plan and the Elevation

Other evidence in support of my idea about the interrelation between the plan and the elevation comes directly from the drawing. For example, the object is symmetrical about a vertical axis but also appears to be symmetrical about a horizontal axis through the two concentric ellipses formed on the stem of the object. This suggests that the design was formed about a central point (Fig. 51.8).

Measurements from the drawing yield up simple relationships that, again, suggest that both the plan and the elevation were in some way, formed from the same geometric construction.[7] For instance, the radius **R** of the circle in Fig. 51.8, which appears to describe the curve of the stem in the vertical plane, is the same as the circle that defines the apexes of the pyramidal structures projecting from the uppermost mazzocchio in a horizontal plane. The size of this curve and its position may be defined by a construction that is itself part of the construction of an octagon. This circle also defines the dimensions of the block in which the whole construction has been made (Fig. 51.9).

[7] There is always a 'chicken and egg' situation in a geometric construction—what came first and what was derived from what?

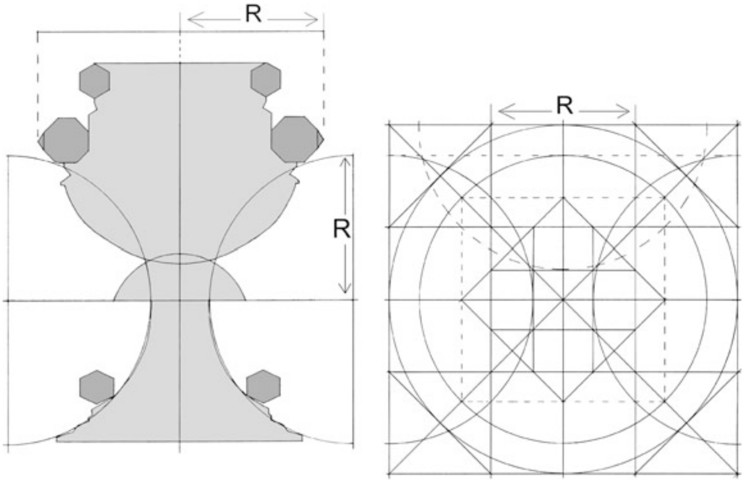

Fig. 51.8 Drawing: author

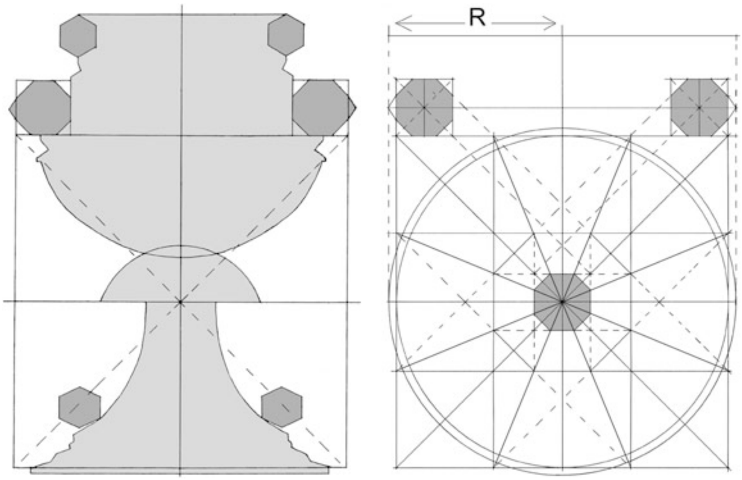

Fig. 51.9 Drawing: author

In addition, the height of the base of the octagonal mazzocchio above the central horizontal axis of the object corresponds with the outer radius of the mazzocchio. If the square is completed, as in Fig. 51.9, its base corresponds with the original level of the base of the object before the hand-drawn lip was added.

The dimensions of the octagon in the mazzocchio are very close to those of the small central octagon that is created as a consequence of repeatedly constructing octagons within the square, as shown in Fig. 51.9. The addition of the pyramidal structures on the outside of the mazzocchio then gives the radius **R**. The relationship between the outer radius of the mazzocchio and its octagonal section,

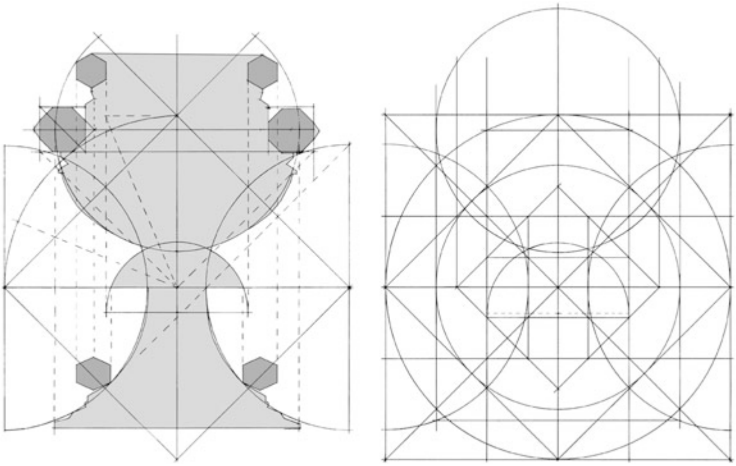

Fig. 51.10 Drawing: author

could of course, be coincidental but, as I will explain later, there are other examples of this. The main volumes of the object appear to be derived from, or are closely related to the construction in Fig. 51.10, although the curve of the bowl and its centre are particularly difficult to define.[8]

The Practicalities of Creating the Drawing

In the early stages of laying out the drawing the most crucial operations would have been to establish true perpendiculars and a method of maintaining horizontals. The accuracy and the logic of the drawing system rely on these and, as a consequence, any errors become more obvious. A draughtsman today would experience the same practical difficulties and use the same instruments as 600 years ago. While now they might not think of using a stylus to inscribe lines, overtly visible construction lines would quickly overwhelm the drawing process on an image of this scale and this complexity. To have successfully accomplished the drawing of the *Chalice* the use of a stylus was an absolute necessity. It is a much more accurate tool than a pencil, and I believe that there are other reasons why its use would have been important.

As there are no annotations on the drawing, it may seem that the main difficulty would be keeping track of all the information involved within a relatively small space. My experience, however, is that once the main "scaffolding" of the space is in place and visible, the logic of the process becomes clear and, in fact, annotations

[8] I am assuming that profiles of the curves are defined by circles, but I would not rule out them being parts of ellipses. Their construction would then have involved two concentric circles and a set of radiating lines.

would be a hindrance. In addition, the inscribed underlying structure, subtly emerging out of the paper by the action of light, would have played an essential role in helping the artist to follow the construction process. It would itself have been quite magical, but would also have been an aid to the imagination, enabling images and visual solutions to be found that otherwise might be inconceivable a process perhaps similar to Leonardo's use of indistinct images found within accidental marks on walls.

A separate sheet on which both the plan and elevation exist together may well have become an autonomous drawing a place for playful and creative thought, rather than simply being a means to an end. This drawing, which could have been relatively fluid and organic, would have provided measurements that were to be used in the other drawing, the one which was governed by the rational system of perspective. There would be a constant shift between the two drawings and elements in the drawings: between the plan, the elevation, the diagrammatic and the pictorial a quality that Martin Kemp has commented on specifically in relation to Piero Della Francesca's paintings (Kemp 1992: 20–40).

Thoughts on Its Attribution

There seems to be no conclusive proof of the *Chalice*'s attribution but its construction would have needed a good knowledge and understanding of geometry, including the concept of plan and elevation and the ability to rationalise a three-dimensional form, whether real or imaginary. Both Piero Della Francesca and Uccello are known to have drawn mazzocchi; Uccello uses them repeatedly as elements in his paintings, most notably in *The Deluge* and *The Battle of San Romano*. Their depiction was, however, to become a problem also tackled by many other artists, particularly those involved in intarsia.

There are drawings of a stellated sphere and a solid mazzocchio in the Paris Louvre, both attributed to Uccello, while catalogued alongside the *Chalice* in the Uffizi are two drawings of hollow mazzocchi, one hexagonal and one octagonal in section, (drawing 1756A), which are also usually attributed to him. These two drawings are on relatively rough paper, whereas that on which the *Chalice* is drawn is quite smooth. All three drawings in the Uffizi show evidence of the use of similar drawing instruments compasses, a straight edge, a fine sharp point and a stylus and they display, essentially, the same method of perspective construction.

In *De Prospectiva Pingendi* Piero demonstrates this same method, but he uses it only for relatively simple geometric objects. He also demonstrates the construction of a simple octagonal-sectioned mazzocchio, but using another more sophisticated method appropriate for any kind of object, in any orientation a method that is distinctly different from that displayed in all three drawings in the Uffizi. Nonetheless, it is clear that, as far as the method is concerned, Piero could also be the author of the drawing.

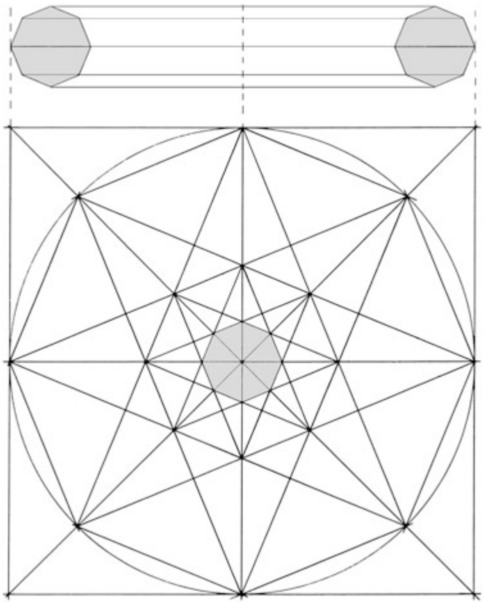

Fig. 51.11 The proportions of the octagonal mazzocchio (Uffizi 1756A). Drawing: author

The drawing in the Uffizi that has an octagonal section, drawing 1756A, is visibly of a much higher quality and accuracy than either of the other two mazzocchi drawings, which leads me to think that this drawing, in particular, relates directly to the *Chalice*. I base this partly on visual appearances, but also on the fact that the relationship between its octagonal section and its outer diameter and, therefore, its possible derivation, appears to match that of the octagonal mazzocchio in the *Chalice* (Fig. 51.11).

This relationship also appears to be the same as that of the octagonal-sectioned mazzocchio that Piero constructs in *De Prospectiva Pingendi*. He does not show how he arrived at the design of the particular mazzocchio that he uses, but he does show its plan and elevation, in which the same proportional relationship seems to be present. None of the mazzocchi in Uccello's paintings appear to show similar proportions. This evidence does not in itself prove that the *Chalice* is from Piero's hand, but it does favour him more than Uccello.

The Drawing in Context

Although the *Chalice* is, ostensibly, a drawing of a recognisable object, the methods and geometry used in its creation make it part, if not a forerunner, of a large genre of drawings of geometric objects. Some of Leonardo's drawings of churches show the same concern with the geometry of the octagon and its successive proportional division that seem to be demonstrated in the *Chalice*,[9] while a drawing by Nicéron, made at least 150 years later, shows an elevation being derived from a plan.[10] Unlike many methods of perspective projection, the method used in the *Chalice* positively lends itself to drawing individual, regular geometric forms. It is, in fact, akin to that of carving thinking and locating points and forms within the confines of a rectangular block. The concept of the transparent block, with its visible, internal three-dimensional logic, would facilitate thinking in three dimensions and, I believe, would have been a major factor in Piero's and, later, Leonardo's understanding and manipulation of the geometry of solid figures.

Biography Richard Talbot gained his BA in Fine Art at Goldsmiths' College, and his MA Sculpture at Chelsea School of Art. In 1980, he was awarded the Rome Scholarship in Sculpture and spent 2 years at the British School at Rome, travelling widely throughout Italy and Egypt. His drawings have been widely exhibited and he has recently completed a commission at Westminster Abbey. He is currently Head of Research and Head of Postgraduate Studies in Fine Art at the School of Arts and Cultures at Newcastle University.

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[9] See, for instance, his plan for a centralised temple conserved in the Bibliothèque Nationale, Institut de France (Ms 2184; BN2037, fol. 5v).

[10] See the drawing in Nicéron (1663), British Library, shelfmark L.35/40. (1).