Chapter 48
The Revolutionary, The Reactionary and The Revivalist: Architecture and Mathematics After 1500

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revolutionary |ˌrevəˈloō sh əˌnerē| *adjective*
Involving or causing a complete or dramatic change.

reactionary |rēˈak sh əˌnerē| *adjective*
Opposing a person, a set of views, or political or social practice.

revivalist |riˈvīvəˌlist| *adjective*
A tendency or desire to resurrect a former custom or practice.

Premise

A common conceit in historical analysis is to view the world and its history as following an essentially forward, progressive or evolutionary trajectory (Foucault 1972; Turchin 2003). This way of conceptualizing the past permits the construction of a narrative sequence that commences with a time dominated by superstitions and primitive practices and then moves towards an era of reason and civilization. Even the naming of historic periods—from the "Dark Ages" to the age of "Enlightenment"—serves to reinforce the message that advances in science, philosophy and art illuminate and thereby clarify the world around us (Adorno 1998). As a corollary to this idea, if history is following an essentially forward trajectory towards a more advanced or aware state, then movements or events that

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either retard or undermine this progress are regarded as following an inferior or retrograde path. One example of this view is found in Edward Gibbon's argument, in *The History of the Decline and Fall of the Roman Empire* (1996) that a loss of civic virtue, coupled with wanton immorality and decadence, set a once great civilisation on a regressive path, culminating in its collapse and the consequent spread of barbarism across Europe (Womersley 1994).

A variation of this way of thinking about history as a progressive trajectory is found in the work of the German philosopher Theodore Adorno. Adorno developed a school of thought, known as Critical Theory, which used techniques from the social sciences and humanities to interpret or analyse history (Benzer 2011). One feature of Adorno's critique of the twentieth century is his division of various cultural, artistic and scientific theories into two progressive types: the reactionary and the revolutionary (Adorno 1981). Reactionary works are those motivated by failures of the recent past to argue for the introduction of changes to improve the situation. Adorno is mildly critical of such works because, through their process of rejection, they inadvertently reinforce and sustain the continued presence of some trace of a past, flawed, system. In contrast, revolutionary works are motivated by a desire for something completely new. They are not intellectually framed in opposition to the present or the past. Instead, the revolutionary agenda is relentlessly forward-looking and both unfettered and uncontaminated by the past (Adorno 2003). While both reactionary and revolutionary developments continue the forward trajectory of civilization, there is also a category of behaviour which theorists of the Frankfurt school, like Adorno, suggest might reverse this progress. These regressive behaviours could be described as revivalist or revisionary practices; they either stall or reverse progress, no matter which field they occur in. This way of thinking about history as a trajectory towards (or away from) enlightenment is useful for interpreting political, social, and aesthetic movements, but there are also parallel theories for examining developments in science and mathematics.

Thomas Kuhn (1962), Karl Popper (2002) and Bruno Latour (1987) have all observed that knowledge in science and mathematics is developed through a series of revolutionary stages (paradigm shifts), which are then followed by successive reactionary sub-stages, each of which refine or test knowledge. Such cyclical revolutionary and reactionary processes are progressive, in the sense that they are motivated by the will to develop new knowledge, even if they accept past knowledge as part of a larger evolutionary trajectory. Although this way of thinking might describe the majority of recent research in science and mathematics, throughout history there have been moments when revivalist notions have deterred this forward progress. For example, prior to the fifteenth century scientists and mathematicians were frequently involved in debates about esoteric and hermetic principles as well as being engaged in alchemical and occult practices. In the sixteenth century, the research of mathematician John Dee (1527–1608) was focused on advancements in both algebra and divine magic. In the seventeenth century, the pioneering experiments in chemistry of scientist Robert Boyle (1627–1691) recorded details not only about the materials and techniques he was using, but also about astronomical events that occurred during his experiments. The research of Dee and Boyle, both of whom worked *after* the scientific revolution, could now be interpreted as following a partially reactive (scientific) and partially regressive (superstitious) trajectory.

These three ways of thinking about the history of ideas—the revolutionary, the reactionary and the revivalist—are useful for conceptualizing patterns in the development and application of knowledge. They are, of necessity, only available to us in hindsight and are only ever true in a limited sense. Furthermore, this categorization of movements or trends is also, as Adorno suggests, more relevant to the history of ideas and aesthetics than to, for example, the history of technology. However, when examining connections and relationships between architecture and mathematics—fields that are integral to both the history of ideas and aesthetics—these categories can be useful. In particular, from the start of the sixteenth century, the field of mathematics could be regarded as pursuing a more consistently progressive path, while architecture's trajectory over the same period is marked by both forward and backward tendencies. This divergence between the paths taken by architecture and mathematics begins to explain the separation between these disciplines, which became apparent in the 1500s and widened in the eighteenth and nineteenth centuries, before showing more recent signs of realignment.

In the present chapter these three tendencies—the revolutionary, the reactionary and the revivalist—are used to frame changing relationships between architecture and mathematics. However, rather than explicitly adopting the Frankfurt school's or Kuhn's criticisms of regressive tendencies, here we take a more reconciliatory stance. This is because, throughout history, certain instances of revivalism provide moments of insight into the relationship between architecture and mathematics. Furthermore, while revolutionaries, by their very nature, outline brave or radical propositions, this does not necessarily make them more interesting than their reactionary counterparts. Thus, while we shall use these three concepts to frame an overview of the changing relationship between architecture and mathematics since the 1500s, for the most part they are simply used to suggest patterns of behaviour or attitudes towards knowledge.

Another reason for introducing these ways of conceptualizing the history of ideas and aesthetics is because they help to explain why, despite hundreds of scholarly works that identify connections between architecture and mathematics, there are historic moments and movements that are barely considered. This is because many of the most fertile connections between architecture and mathematics have occurred when architects have sought inspiration for their work through appeals to science or geometry (Evans 1995; Ostwald 1999; Di Cristina 2001). These moments are more likely to occur when architecture is pursued in a revolutionary or reactionary way, or when both architecture and mathematics have parallel, progressive trajectories. Examples of this type are especially prevalent in Renaissance, Baroque and Modern architecture. This is also potentially why few examples exist of innovative connections between architecture and mathematics during the Gothic Revival, Egyptian Revival or Post Modern Historicist movements. At such times architecture has sought inspiration primarily from within, and the only—if any—mathematics being used in the design process is adapted from a previous era.

Tracing Trajectories

The choice of the year 1500 as the starting point for the present volume was suggested more by changes in science, technology and mathematics than by specific developments in architecture. In the closing years of the fifteenth century the invention of the printing press by Johannes Gutenberg (1395–1468) gradually led to many changes in society, not the least of which was finally achieving a widespread acceptance of Arabic, rather than Roman or Indian, numeric systems. Furthermore, from the early years of the sixteenth century a growing number of people would view the world in an empirical manner rather than accepting explanations of faith or magic. Research was also undertaken from both an introspective (inquiry-based) and a more overtly global (discovery-based) frame of mind. In particular, the Age of Discovery, the beginning of which is conventionally dated by the fall of Constantinople in 1453, led to a remapping of the world based on exploration, with Ferdinand Magellan (1480–1521) circumnavigating the globe in 1522. Nicholas Copernicus (1473–1543) first considered the prospect that the earth might revolve around the sun in 1514, and expressed this in his *De revolutionibus orbium* in 1543. That same year saw the publication of *De Humani Corporis Fabrica* by Andreas Vesalius (1514–1564). This work provided the first detailed, illustrated and comprehensive description of anatomical dissection, but failed to identify the location of the human soul. The works of Copernicus and Vesalius, along with those of Tycho Brahe (1546–1601), Gerardus Mercator (1512–1594) and Galileo Galilei (1564–1642), are considered as having triggered the scientific revolution. While these events did not precipitate the type of social upheaval set in motion by the Protestant or English Reformations, they nevertheless signalled the first stage of a substantial philosophical paradigm shift. In architecture, awareness of these discoveries manifested themselves in a vocabulary of form that grew from being based on circles and squares to embrace the ellipse, although the language of that form remained classical. To be sure, mysticism and symbolism continued to shape architecture's formal and stylistic language, just as they continued to interest some mathematicians. But in the aftermath of the scientific revolution the influence of mysticism gradually waned (Shapin 1998; Henry 2008).

Technical advances that aided empiricism included the invention of optical microscopes (Hans Lippershey 1590), refracting telescopes (Hans Lippershey 1608, and Galileo 1609) and thermometers (Roger Fludd 1638). The sixteenth and seventeenth centuries also saw many important advances in mathematics which helped people to understand what the new instruments made it possible to observe. The age abounded in genius: John Napier (1550–1617), Henry Briggs (1561–1630), Johann Bernoulli (1667–1748), René Descartes (1596–1650), Pierre de Fermat (1601?–1665), and Blaise Pascal (1623–1662) all developed major insights in mathematics. Isaac Newton (1642–1727) and Christopher Wren (1632–1723) each proposed new applications of geometry and considered their use in architecture. Notwithstanding continued and often clandestine interests in alchemy (Isaac Newton is a case in point), the cabala and other hermetic traditions, mathematicians and scientists of the era were typically committed to the advancement of ideas through rigorous and empirical testing, rather than through the more symbolic or metaphysical properties of numbers. Thus, it is not surprising that the first scientific journals began to be published in the seventeenth century (the English *Philosophical Transactions of the Royal Society* was established in 1665) and the first practices of scholarly and scientific review can also be traced to this era, affirming the progressive (reactionary and revolutionary) agenda of science.

Architecture was slower to advance. Because of the time taken to design and construct buildings, architecture in the sixteenth and seventeenth centuries continued to use materials and methods similar to those employed in previous epochs. The works of Guarino Guarini (1624–1683), such as the church of San Lorenzo and the Chapel of the Holy Shroud, both in Torino, feature a Baroque mantel of architectural forms cloaking the Gothic masonry structure that holds it up. Moreover, many of the major works of architectural theory remained dominated by ideas and principles derived from classical Rome that had been rediscovered and refined two centuries earlier during the Renaissance (Kruft 1994). Because architecture serves both a cultural and social function, and as buildings can last for many generations, developments in architecture were also less dramatic than those in science or mathematics. For example, in Italy Mannerism, the name given to the style adopted by Michelangelo and others from about 1520 to about 1580, whose emphasis on tension and instability contrasted with the composed, static equilibrium of the Early Renaissance, could be regarded as a variation of Renaissance architecture, just as in England Jacobean architecture grew from the Elizabethan style. Both of these developments were reactionary in their trajectory and, for the layperson, neither of these had an especially immediate or obvious impact.

The most important stylistic movement of the seventeenth century in Europe, the Baroque, a development of late Renaissance architecture, started in central Europe (Italy, Germany and France) and spread to England, Spain and Portugal (and through them, to their colonies) and Russia. Baroque architecture was characterized by new explorations of form, light and shadow and dramatic intensity to produce new spatial effects such as dilatation and contraction. Baroque architecture was designed to provide extravagant evidence of the predominance and centrality of the Catholic Church during the Counter-Reformation, and was thus, by intent at least, not progressive. Another movement that flourished in the seventeenth century is what is today known as Palladian architecture, another reactionary descendent of the late Renaissance, characterised by symmetrical facades and plans and an appreciation of the impact of proportion and perspective. However, while Palladian architecture took advantage of some new ways of thinking about space, this was tempered with a desire to reference the seemingly timeless or transcendent forms of ancient Greek and Roman temples.

Such approaches underline the tension that existed between the progressive trajectory of the scientific and mathematical communities, and the more resistant views of the clergy, philosophers and the wider community. Architecture, which was informed by mathematics, but served a more general communal and cultural function by expressing social values, was frequently at the centre of such strained relations and sought ways to reconcile the two. This tension was to worsen across Europe in the eighteenth century, an era which is now seen as being responsible for reviving or revising various older styles, theories and beliefs. Neo-classical architecture and Gothic Revival architecture were amongst the largest stylistic trends in the eighteenth and nineteenth centuries. The neo-classicists included Karl Friedrich Schinkel (1781–1841) and John Soane (1753–1837) while the works of French neo-classical architects Claude Nicolas Ledoux (1736–1806) and Étienne-Louis Boullée (1728–1799) displayed a classical vocabulary of elements draped over their otherwise more Rationalist works. Conversely, in the hands of John Ruskin (1819–1900) and Augustus Pugin (1812–1852), the stark, Platonic forms of neo-classicism were rejected in favour of the allegedly more spiritual and phenomenally significant impact of a revived Gothic tradition. By the mid-nineteenth century there were relatively short-lived revivals of Romanesque, Egyptian, Greek and Renaissance (Italianate) styles. Despite some notable if isolated successes, architecture at this time must have looked increasingly moribund and self-referential to the younger generation of graduates. It is not surprising then that, from amongst the young architects of the late nineteenth century, and especially those who had been trained in these various revivalist traditions, the roots of the Modernist rebellion can be traced.

In the early years of the twentieth century the Futurist and Constructivist manifestos, each with their overt revolutionary agendas, strongly rejected the revivalist and retrograde tendencies of the previous generation. Even the more reactionary Arts and Crafts movement demonstrated an interest in themes which prefigured Modernist preoccupations with labour, materiality and emancipation. Organic and Functionalist architecture sought different inspirations, respectively nature and industrialization, for producing a contemporary architectural style (Le Corbusier 1927; Wright 1995). Both of these variations of Modernism, despite later criticism (Blake 1974; Brolin 1976), reunited the progressive trajectories of architecture and mathematics, which had been separated for 200 years. In the 1940s and 1950s theories of non-Euclidian geometry were swiftly adopted by architects and artists (Henderson 1983) and advances in complex surfaces and shells soon found their way into architectural design and then returned to mathematics by way of textbooks.

In the later years of the twentieth century (and following a different type of historical revivalism in the 1980s) a major revolution in geometric thinking—fractal geometry—was almost immediately appropriated by architects, leading to a series of works, unbuilt and built, which tried to capture the essence of fractal form (Ostwald 2001; Harris 2012). After that time, architects began to use topographic tiling, including aperiodic tiles, often less than a decade after particular sets (like Penrose or Conroy tiles) had been published in mathematical journals (Di Cristina 2001). By the late 1990s the disciplines of architecture and mathematics had become so specialized that few people could truly be considered to make advances in both fields, while the progressive, revolutionary and reactive trajectories of each had returned to a relatively close alignment, with new computer-aided design tools guaranteeing their continued interaction.

This Present Work in Context

The shifting relationships between the revolutionary, reactionary and revivalist trajectories of architecture and mathematics after 1500 are traced in different ways in the present work. Volume II continues the pattern developed in Volume I, of interweaving the theory and history of architecture and mathematics. It is divided into five parts, two of which are structured chronologically to cover the years between 1500 and 1800 (Part VIII) and from 1800 to the present day (Part IX). The 24 chapters in these 2 sections provide a broad coverage of the works of iconic architects from these eras along with analysis of major built and unbuilt works. Part IX is also the only section in these two volumes that is dominated by developments in North America; prior sections have tended to emphasise people, concepts and buildings in Europe, Asia, South America and the Middle East. The remaining three parts in Volume II are all about theories or practices which connect architecture and mathematics. The first of these, Part VII, is concerned with the modes of representation that played a significant role in shaping European architecture in the sixteenth and seventeenth centuries. The second of these, Part X, examines contemporary approaches to the use of mathematics in design and analysis. The final section, Part XI, concludes with eight chapters about computational, parametric and algorithmic approaches to architecture. Parts IX and X also feature examples of architecture from the Oceania region.

These final two parts of Volume II are the only ones which break from the alternating structure adopted across both volumes, which interweaves a historical chronology of buildings and architects with theories that were of relevance to the era. These last two parts are both classified as primarily concerned with theory, but they also feature research into specific buildings, designs or ideas. This reflects the fact that the closer we come to the present day, the less historical the tone of the research. Thus, even if a chapter is about a specific building or design, it is not generally considered through a historiographical framing, but rather as an example of the development or testing of a theory. The fact that several of the chapters in the last section are about computational design, an approach that often directly extrapolates or evolves a theory into the visualisation of a design, further reinforces this difference. Finally, the chapters in the last section include both forward-looking or projective research, and philosophical musings about the entire relationship between architecture and mathematics enabled by the computer. In a sense, Part XI, is neither purely historical nor purely theoretical, but rather comprises an extended conclusion where the two come together. In the sections that follow we will describe the content and themes in each of the five parts of this second volume.

Part VII: Theories of Representation

Part VII is about theories and practices of representation, focussing on perspective projections in art and architecture. The content of several of these chapters is based on works or advances which occurred prior to 1500 but which were to remain influential for several centuries. Prior to the Renaissance, artists used various techniques and practices to suggest depth in a representation. One of the most common involved the layering of objects in an image—so-called register perspective—such that those deeper in a scene (or further away from the viewer) were represented as being positioned partially behind those which were closer to the front of the scene. Despite such practices, the origins of perspectival approaches to constructing representational depth were only formalised in the fourteenth century (Andersen 2007). For example, historians commonly trace the origins of mathematical models of linear perspective to Filippo Brunelleschi's (1377–1446) paintings and panels (including the famous 'peep-hole' and mirror demonstration) of the Baptistery of Florence (Argan 1946; Grave 2010). *De Pictura*, the mid-fifteenth century treatise on painting by Leon Battista Alberti (1404–1472), further defined the geometric construction techniques used by Brunelleschi, making them more accessible. The contribution of Piero della Francesca (1415–1492), which followed within two decades of Alberti's work, was to refine the method used for depicting Phileban solids; a crucial step for architectural representation (Damisch 1995). Despite such progress, perspectival projections remained a major point of both fascination and contention throughout the Renaissance. During this era many different alternative construction techniques were proposed and tested, with some being closely protected secrets of particular artists or schools, while others were more widely disseminated by academicians (Damisch 1995).

The reason for the Renaissance world's fascination with perspective is itself a complex topic that has been extensively researched in the past. Perspective representations appeared to offer a geometry-based system that defined the relationship between the human body and the world. During the Renaissance this relationship was still regarded as being the province of theology, and thus perspectival constructions were of both practical and symbolic importance (Panofsky 1996). Indeed, as has often been the case throughout history, developments in mathematics and representation were frequently used to sustain arguments about metaphysics, social reform and political power. However, in a more subtle way, the rise of perspective techniques also brought into question the purpose of representation. These themes are considered in the four chapters in Part VII, all of which are about theories and techniques of representation, their geometric construction and significance.

In 'Architecture, Mathematics and Theology in Raphael's Paintings' (Chap. 49), David Speiser examines the geometric construction of perspective in two famous works by Raphael, *L'incoronazione della Madonna* (The Incoronation of the Madonna) and *Lo Sposalizio* (The Wedding of the Virgin), both dated 1504. Speiser argues that these present an early instance of multi-point perspective construction. Prior to this time, artists and architects used a type of artificial one-point perspective involving a single vanishing point to which all lines, which are not parallel to the viewing plane, converge. Through an analysis of the two paintings Speiser demonstrates the existence of a non-frontally-constructed perspective, which requires more than one vanishing point. Thereafter, he offers an explanation for how Raphael constructed these representations and why they were so revolutionary at the time. Kristina Luce's chapter 'Raphael and the Pantheon's Interior: A Pivotal Moment in Architectural Representation' (Chap. 50) is also about Raphael, but this time specifically concerning architecture. The subject is a much-copied sketch by Raphael of the interior of the Pantheon in which the artist attempted to represent the Pantheon simultaneously in both section and perspective. Luce argues that this signals a critical attempt to reconcile the conflicting geometries of architecture (the orthographic section) and of representation (the interior perspective). While such techniques are commonplace today, Raphael's simple sketch suggests a major conceptual advance in Renaissance representation.

One of the landmark images in the history of perspective is a simple line drawing of a chalice. Completed in the mid-fifteenth century, and inconclusively attributed to both Paolo Uccello and Piero Della Francesca, this drawing depicts an intricate, geometrically-faceted footed goblet. What is so striking about the image is its rigorous and transparent construction; in much of the drawing, lines that would otherwise be hidden are displayed. Richard Talbot's chapter, 'Design and perspective construction: Why is the Chalice the shape it is?' (Chap. 51) suggests that the elevation of the chalice is critical to the construction of its representation. This is important because, unlike many perspective drawings of this era, whose construction relied on the human eye and a rudimentary knowledge of vanishing points, there is evidence in the chalice drawing to suggest that it was directly projected from a designed object. Thus, this drawing is a true visualisation of intent, rather than a mediated representation of experience. While being completed several decades prior to Raphael's sketch of the Pantheon interior, both of these examples dramatize the subtle difference between representations of an object's physical characteristics (its dimensionality) and its visual ones (its phenomenological properties).

The final chapter in this section is about António Rodrigues's late sixteenth century treatise on architectural perspective. Rodrigues, a Portuguese architect and educator, was well-versed in geometry and like many of his contemporaries, worked on the design of fortresses and other military structures. In 'Perspective in António Rodrigues's *Tratado de Arquitectura*' (Chap. 52) João Pedro Xavier examines the book entitled *Liuro de Perspectiva*, considering both geometric and political properties of the work. As mentioned earlier, methods of perspectival representation were often regarded as having important theological or symbolic significance. Equally, they could be viewed as representing a rejection of previous practices. For both of these reasons, treatises like that of Rodrigues are important for understanding the techniques of architectural depiction, as well as the politics of representation.

Part VIII: Architecture from 1500 AD to 1800 AD

The chapters in Part VIII span 300 years of architectural history. The first group is focussed on the sixteenth century and includes research into the works of Mimar Sinan (1489?–1588), Andrea Palladio (1508–1580), Michelangelo Buonarroti (1475–1564) and Francesco Borromini (1599–1667). Thereafter two chapters consider the architecture of António Rodrigues (1520–1590) and Juan Bautista Villalpando (1552–1608) along with the latter's impact in the seventeenth and eighteenth centuries. The remaining chapters offer readings of the designs and theories of Inigo Jones (1573–1652), Christopher Wren (1632–1723), Robert Hooke (1635–1703) and Claude Perrault (1613–1688). Throughout Part VIII some of the chapters question the canonical interpretation of various architects, while others reinforce the significance of their buildings and writings. However, it is the influence of Vitruvius's *De Architectura*—re-discovered and disseminated across Europe at this time—that is the thread that binds many of these works together (Kruft 1994).

The first chapter in Part VIII is about the great Ottoman architect Mimar Sinan who was born in the late fifteenth century in Turkey. Serving three Sultans (including Süleyman the Magnificent), Sinan completed several hundred buildings in his career, the most famous of which are the Selimiye Mosque in Edirne and the Süleymaniye Mosque in Istanbul (Kuban 1987). A contemporary of Michelangelo, Sinan's influence in the sixteenth and seventeenth centuries was extensive, with several of his former students completing major works in Europe, the Middle East and Asia (Necipoğlu 2005). Indebted to his training as a military engineer, and inspired by the structure of the Hagia Sophia, Sinan's architecture has been repeatedly praised for both its geometric and structural properties and its careful layering of form and ornament. Zafer Sağdiç's chapter, 'Ottoman Architecture: Relationships Between Architectural Design and Mathematics in Architect Sinan's Work' (Chap. 53) stresses the importance of Euclidean geometric forms in Sinan's architecture, recognising a range of recurring proportional systems. Sagdic also describes the role Sinan's knowledge of geometry and construction played in his later works.

The second and third chapters in this section are dedicated to Andrea Palladio. Born in Padua (Italy), Palladio's work acknowledges a clear conceptual lineage to the ideas of the Roman architect Vitruvius (Ackerman 1974). Supported by wealthy patrons, Palladio was able to design and construct an influential series of houses and public buildings during his lifetime (Giaconi and Williams 2003). Palladio's classically-inspired forms and ideals were promulgated through his writings, notably including his *I Quattro Libri dell'Architettura* (The Four Books of Architecture). This work, illustrated with woodcuts of Palladio's own designs, was published in Venice in 1570 and sets out a series of detailed rules for design and construction (Tavernor 1991). Palladio's architecture and writings have long fascinated architectural historians and mathematicians for the way in which they depict the use of harmonic proportions in plan, elevation and section (Wittkower

1971; Rowe 1982). This interpretation of Palladio's work, made famous by German art historian Rudolph Wittkower in the early years of the twentieth century, is still accepted today, though not unconditionally (Mitrović 1990); we now know that it is not the only possible mathematical influence on Palladio's work.

In 'The Mathematics of Palladio's Villas' (Chap. 54), American mathematician Stephen Wassell argues that Wittkower's (1971) canonical reading of Palladio's villas tends to privilege just one of the major mathematical trends of the era, harmonic proportion. Wassell suggests that the significance of other, equally important theories of proportions, symmetry and geometry, has tended to be understated in Wittkower's work. For example, Wassell notes the significance of several geometric constructions in Palladio's designs including ratios derived from $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$. The following chapter, 'Golden Proportions in a Great House: Palladio's Villa Emo' (Chap. 55) by Rachel Fletcher, has a similar focus on alternative proportional systems. Constructed in the 1550s, the plan of Villa Emo was published almost unchanged in *I Quattro Libri dell'Architettura*, whereas many of the other plans in Palladio's treatise were presented in a more idealised manner. For this reason the Villa Emo is regarded as one of Palladio's most successful attempts to reconcile the practicalities of construction with the desire to reflect a perfect mathematical premise. However, as Fletcher makes clear, several features of the villa are proportioned in response to the golden section, even though it is the harmonic proportions of the plan that are typically praised. Fletcher, through a detailed review of the Villa, uncovers the appearance of hitherto unrecognised golden proportions in both the elevation and plan, as well as in the placement of individual doors and windows. Like Wassell, Fletcher's purpose is not to disprove Wittkower, but rather to uncover the existence of other significant, geometric systems in Palladio's oeuvre.

Trained in Florence and steeped in the humanist tradition, Michelangelo Buonarroti undertook early commissions in both Venice and Bologna. Over the next 50 years he created works of art for wealthy patrons from the Medici rulers to popes, cardinals and ambassadors in both Florence and Rome (Condivi 2007). Today Michelangelo's name is synonymous with the High Renaissance of the sixteenth century and he is revered as an artist, sculptor, poet and architect. Michelangelo is credited as initiating the Mannerist tradition in architecture; an intellectual, artificial (neither animistic nor naturalistic) and highly coded extension of the Renaissance tradition (Ackerman 1986). Two chapters here are about his architecture.

Constructed within the cloister of the Basilica di San Lorenzo, the Laurentian Library has long been regarded as the site of the genesis of Mannerism (Shearman 1991). While the Library itself has been the subject of many interpretations, it is the design of its red and white terracotta pavement that is the focus of 'The Hidden Pavement Designs of the Laurentian Library' (Chap. 56) by Ben Nicholson, Jay Kappraff and Saori Hisano. Concealed for much of the last two centuries by the wooden library desks placed on top of it, this elaborate pavement consists of 15 pairs of panels, each comprising a different geometric design. In this chapter, which is written in such a way as to reflect the differing views of its authors, a series of interpretations of the many unusual patterns and measures present in the pavement is offered. One such explanation is that the designs reflect the content of the works which were catalogued on shelves above them, thereby using geometry and mathematics as a type of ordering device or commentary. The authors admit that the original purpose of the panels will remain a mystery, but they underline the significance of intellectual and geometric tropes in Mannerist design.

In 'Measuring up to Michelangelo: A Methodology' (Chap. 57), Paul Calter and Kim Williams (also co-editor of the present volume) describe the method used to survey the Medici Chapel (the New Sacristy of the Basilica of San Lorenzo) in Florence. Using a theodolite to record specific points and trigonometry to triangulate their position in space, Calter and Williams outline a procedure which can be applied to the measurement of other historic structures, while also providing evidence that the Medici Chapel possesses a set of recurring proportional relations derived from a $\sqrt{2}$ rectangle. Calter and Williams emphasise the importance of accurate measured structures for the validation of theories both historic and new that seek to relate mathematics to architecture.

The next pair of chapters directs the reader's attention away from Italy and towards Portugal and Spain. In 'António Rodrigues, a Portuguese Architect with a Scientific Inclination' (Chap. 58), João Pedro Xavier returns to the topic of Rodrigues, featured previously in this volume for his work on perspective representation. The Onze Mil Virgens Chapel at Alcácer do Sal is the starting point for Xavier's second chapter and through a review of this building and Rodrigues's Santa Maria da Graça Church at Setúbal, Xavier identifies a particular way of working with geometry. Tracing the presence of a range of geometric construction and proportion systems—from *ad quadratum* geometry to the use of 5:4 and 6:7 proportions—Xavier offers an explanation of the role of mathematics in Rodrigues's work.

Born in Córdoba, Spain three decades after Rodrigues, Juan Bautista Villalpando was a renowned architect, mathematician and theologian. Villalpando designed several significant buildings for the Jesuit order, including the San Hermenegildo Church in Seville. However, his most enduring contribution to architecture is found in his elaborate reconstruction of the Temple of Solomon (Kravtsov 2005). Published in 1604 in *Ezechielem Explanationes*, Villalpando's Temple of Solomon, was designed around the principles of Platonic harmonies and ancient measurements. Tessa Morrison, who has also produced the first extensive English translation of part of *Ezechielem Explanationes* (Morrison 2009), provides the next chapter in this section. In 'Villalpando's Sacred Architecture in the Light of Isaac Newton's Commentary' (Chap. 59) Morrison notes that Villalpando imagined the temple as a perfect demonstration of the formal grammar of classical architecture, and sought to reconcile theological and architectural arguments in a single work. From this beginning Morrison considers the impact of Villalpando's design on later scholars and architects, in particular Isaac Newton, who undertook a detailed analysis of Villalpando's Temple of Solomon, arguing that it was one of the most important attempts to imagine an ideal architecture supported by perfect mathematical principles.

British architect Inigo Jones was influenced by both the classical tradition (by way of Vitruvius) and Palladio's interpretation of these principles. Jones, Palladio and Villalpando, each in different ways, sought to develop Vitruvian values through the practical and symbolic application of geometry in architecture. Completing his better-known works in the seventeenth century, Jones also held the post of the Royal Surveyor; it was while he was engaged in this role that he was asked by King James I to examine the origins of Stonehenge. In 'Coelum Brittanicum: Inigo Jones and Symbolic Geometry' (Chap. 60) Rumiko Handa describes how Jones interpreted Stonehenge, through the geometry of its plan, as a Roman temple of the Tuscan order. Handa demonstrates how Jones's reading of Stonehenge was part of his larger vision of *Coelum Brittanicum*, a poetic conceit popularised by writer Thomas Carew to reimagine Great Britain and its monarchy through a classical Roman allegorical lens (Sharpe 1987). Handa draws on an analysis of Jones's architecture and masques, informed by the mathematics of Robert Recorde and John Dee, to develop this thesis, which is as much about politics and symbolism as it is architecture and mathematics.

Typical characteristics of the seventeenth-century Baroque architectural tradition include the presence of complex curvilinear forms, elaborate spatial interpenetrations and a fascination with surface and naturalistic decoration (Wölfflin 1964; Norberg-Schulz 1971). The Italian Baroque is commonly regarded as arising from the works of Gian Lorenzo Bernini (1598–1680) and Francesco Borromini in central Italy and from the architecture of Guarino Guarini and Filippo Juvarra (1638–1736) in northern Italy. In Germany and across the Austro-Hungarian Empire, Johann Bernhard Fischer von Erlach (1656–1723), Johann Balthasar Neumann (1687–1753) and Christoph Dientzenhofer (1655–1722) all developed particular, geometrically-rich variations of the Baroque (Wölfflin 1964; Hubala 1989). Thereafter it spread across Europe, retaining some of its Central European characteristics, but also being modified to accommodate local traditions and materials in Spain and Portugal to the west, Poland and Russia to the east, and England and the Netherlands to the north (Hempel 1965).

Born near Lugano in Switzerland in 1599, Francesco Borromini was responsible for several great works of the Baroque style (Millon 1961). Educated in Milan, Borromini's first serious commissions were in Rome and it was there that he produced his most revered designs including the exquisite San Carlo alle Quattro Fontane, the church of Sant'Agnese in Agone and Sant' Ivo alla Sapienza (Wölfflin 1964). Unlike many architects of the preceding era, who published detailed treatises promoting their ideas, Borromini's sources are poorly documented. In 'The Science Behind Francesco Borromini's Divine Geometry' (Chap. 61), John Hatch suggests that one reason for Borromini's seemingly hermetic approach to design is that he was as much influenced by science as he was by architecture. From this premise Hatch traces a connection between the work of astronomer Johannes Kepler and Borromini's architecture, suggesting that features of S. Carlo alle Quattro Fontane and Sant' Ivo alla Sapienza are indebted to Kepler's interpretation of the cosmos. John Clagett's chapter, 'Transformational Geometry and the Central European Baroque Church' (Chap. 62) is also set against a backdrop of developments in science and mathematics although Clagett focuses on the work of Desargues, Newton, Leibniz and Descartes. From this foundation Clagett demonstrates the impact of mathematics on the design of space and form, transforming the ideally proportioned but often static architecture of the Renaissance into a more dynamic, transformative and challenging Baroque.

The next pair of chapters, both by American mathematician Maria Zack, are about English Baroque architecture and its mathematical and scientific properties and aspirations. In the aftermath of the Great Fire of London in 1666, Christopher Wren was appointed King's Surveyor of Works and Robert Hooke was named Surveyor to the City of London. They were both responsible for rebuilding large parts of the city as well as for the design of many major structures. Wren, trained as an astronomer, geometer and physicist, had at that time just returned from Paris, where he had viewed the plans of Bernini (Jardine 2003). Like Inigo Jones and Villalpando, Wren had studied Vitruvius and was interested in the classical Roman and Greek traditions, but the new Baroque buildings of Central Europe also shaped his architecture. Although English Baroque was rarely as extravagant as its Italian relative, it retained many of the latter's proclivities, although tempered by a more classical, geometric *parti* in plan and section (Downes 1966). In 'Are There Connections Between the Mathematical Thought and Architecture of Sir Christopher Wren' (Chap. 63) Zack investigates possible connections between Wren's building designs and his earlier mathematical theories, while in 'Robert Hooke's Fire Monument: Architecture as a Scientific Instrument' (Chap. 64), this time about Wren's colleague, scientist and architect Robert Hooke, she analyses the latter's Fire Monument (Stevenson 2005). This structure, consisting of a column on an orthogonal prism base, contained a zenith telescope, an instrument which could be used to measure gravitational affects or the position of stars.

The final chapter in Part VIII is about Claude Perrault's Observatoire de Paris. Designed around the same time as the Great Fire of London, this building was originally intended to house the Paris Academy of Sciences. In 'Practical and Theoretical Applications of Geometry in Perrault's Observatoire' (Chap. 65) Randy Swanson provides an overview of the building, focussing on the conceptual themes that shaped the design. Swanson stresses two particular achievements in the Observatoire, the first being in stereotomy—and associated with the design of a cantilevered, elliptical-vaulted semi-helical stairwell—and the second a series of previously unexplained dimensional eccentricities in the building's form. Swanson maintains that the Observatoire de Paris, presented Perrault with a unique opportunity to test his own theories of proportionality and geometry.

Perrault's work is also a fitting conclusion to Part VIII because not only was his architecture influenced by the theories of Vitruvius, but he also embraced a revived classicism more strongly than many of his contemporaries (Herrmann 1973). This is significant because, starting in the mid-eighteenth century, a series of stylistic revivals—led by neo-classicism but also including the Gothic revival—began to dominate the design of major public buildings in Europe and America.

Part IX: Architecture from 1800 to 2000

Many of the architects who are featured in Part IX were born in the closing years of the nineteenth century and came to prominence in the twentieth century. Trained in classical, traditional or arts-and-crafts-style architecture, these designers—including Frank Lloyd Wright, Le Corbusier and Louis Kahn—went on to define Organic, Functionalist and Brutalist variations of Modernism, as well as twentieth-century Rationalism (Jencks 1973; Frampton 2007). Whereas in previous centuries the symbolic potential of geometry was often tied to religion, in the twentieth century it was more commonly used to suggest a connection to nature (Portoghesi 2000; Steadman 2008). Moreover, in this era various structural and technical advances allowed architects to experiment with new formal and spatial compositions which were not easily available to previous generations. This same era is also marked by a gradual shift in focus away from a consideration of churches and palaces as key purveyors of architectural style and towards more profane, domestic and commercial structures (Walden 2011).

The first chapter in Part IX is about Claude Fayette Bragdon (1866–1946), an American architect who worked in architectural practice until the 1920s before becoming a stage designer. While Bragdon's early architectural designs were undertaken in a revivalist Italianate tradition, he is best known today as a proponent of a particular variant of the Arts and Crafts movement. Like his contemporaries Louis Sullivan and Frank Lloyd Wright, Bragdon showed a proclivity for ornamental and handcrafted methods, but was also informed by new industrial materials and production techniques. Indeed, the architecture of these three Americans varied considerably from that of the English Arts and Crafts movement, characterised by the early designs of Edwin Lutyens or the country houses of Charles Voysey. Instead, Bragdon is more correctly associated with the Prairie Style, not only because of the way this approach mediated between progressive and revivalist traditions, but also because of its rigorous approach to geometry. In 'Geomantic (Re)Creation: Magic Squares and Claude Bragdon's Theosophic Architecture' (Chap. 66) Eugenia Victoria Ellis examines Bragdon's fascination with theosophy, number symbolism and divine proportions and projections. Ellis traces the presence of the 'magic square' construction—a mathematical procedure with both geometric and geomantic significance—in Bragdon's First Universalist Church in Rochester, New York.

The next two chapters, both by eminent Wright scholar Leonard K. Eaton, are concerned with the Prairie Style and Usonian houses of Frank Lloyd Wright (1867–1959). Wright was born less than a year after Claude Bragdon, and he is typically regarded as having defined the basic principles of the Prairie Style. This style is characterized by horizontally exaggerated buildings, with low, sloping roofs, wide eaves, extensive terraces and clearly defined exterior spaces (Hess 2006). Organised internally around a central fireplace, Wright's prairie houses were less cellular than most housing of their era, characterised by a free flow of space that prefigured developments in European modern planning by over a decade (Lind

1994). Wright's prairie architecture has been celebrated for its rigorous geometric planning and geometric detailing and ornamentation (Koning and Eizenberg 1981). Both of Eaton's chapters are concerned with these geometric properties, which evoke connections to music and nature. In 'Mathematics and Music in the Art Glass Windows of Frank Lloyd Wright' (Chap. 67), Eaton examines Wright's predilection for embedding the geometric language of a house into the design of its windows. Within the context of patterns, both visual and musical, Eaton analyses five windows from Wright's Meyer May house in Grand Rapids, Michigan. Eaton especially notes in these designs the way in which Wright seemingly anticipates developments in non-figurative art, using a series of geometric ratios, which were largely intuitively defined, to create a system of recurring patterns. Eaton observes that, despite Wright's desire to emulate the structure or timbre of Beethoven, the geometric language of the windows more clearly prefigures the Modernist compositions of Bartok.

Recurring geometric patterns are also at the centre of Eaton's second chapter, 'Fractal Geometry in the Late Work of Frank Lloyd Wright' (Chap. 68), a reading of the plan geometry of the Palmer House. Like Louis Sullivan, Wright had a deep interest in geometry and patterns found in nature (Kubala 1990). This has led to the observation that the geometric patterns developed by these architects are potentially precursors to fractal geometry, a branch of geometry that was only formalised in the late 1970s. In this chapter Eaton notes the presence of triangular geometry, repeated across multiple scales, from siting elements through to decisions regarding design details. Eaton's argument is that Wright possessed an intuitive grasp of the way geometric systems, when repeated across multiple scales, have an affinity to natural systems. Thus, Eaton's argument could be defined as one tying Wright's architecture to fractal geometry, by way of analogy.

In the following chapter, 'Characteristic Visual Complexity: Fractal dimensions in the architecture of Frank Lloyd Wright and Le Corbusier' (Chap. 69), Michael Ostwald, Josephine Vaughan and Chris Tucker develop a very different connection between Wright's architecture and fractal theory. Ostwald (a co-editor of the present work) and his colleagues calculate the fractal dimensions of five of Wright's houses and then compare these results with the fractal dimensions of five of Le Corbusier's houses. Ostwald's and Eaton's chapters clarify the difference between 'fractal geometry' and 'fractal dimensions'. The former is an algorithmically defined process which generates a complex, deep and ordered figure from a recursive rule (Mandelbrot 1977; Ostwald 2001, 2003). In contrast, any object can have a fractal dimension, a measure of its spread of information across all scales of observation. There are several mathematical methods for determining the fractal dimension of an object and Ostwald, Vaughan and Tucker demonstrate one of the first computational applications of this approach. Thus, where previous chapters have sought to uncover the mathematical principles inherent in a design, this one instead describes the application of mathematics to architectural analysis.

The subject of the next chapter is the work of Walter Burley Griffin (1876–1937) and Marion Mahony Griffin (1871–1961), architects who, in their early careers, worked in Wright's Oak Park Studio in Chicago. In 1911 the Griffins submitted a master-plan concept for an international competition to design a new Australian capital city, Canberra (Reid 2002). The Griffins, like many of their Modernist contemporaries, were fascinated by the relationship between geometry and nature. In their case, this passion was furthered through their reading of Rudolph Steiner's spiritual and psychological theory, anthroposophy, as well as through various detailed investigations of the symbolic potential of geometry (Navaretti and Turnbull 1998). These proclivities are the subject of Graham Pont's and Peter Proudfoot's chapter, 'From Cosmic City to Esoteric Cinema' (Chap. 70), on the Griffins's attempts to evoke cosmic or spiritual ideals in architecture and urban design.

A different type of geometric application, for the purpose of constructing complex surfaces, is the subject of John Poros's chapter, 'The Ruled Geometries of Marcel Breuer' (Chap. 71). In the years following the Second World War, engineers and architects including Pier Luigi Nervi (1891–1979) and Felix Candela (1910–1997) had designed a series of structurally efficient shell surfaces using a technique that relied on ruled lines (Ching 2007). Marcel Breuer (1902–1981), a Hungarian-born Modernist architect who later designed the famous Whitney Museum of American Art in New York, not only adopted such ruled geometries to achieve structural solutions, but he also used them to shape space and form to achieve unexpected and sometimes contradictory results (Hyman 2001).

Another famous example of the curved surface in twentieth-century architecture is the subject of Alessandra Capanna's chapter, 'Conoids and Hyperbolic Paraboloids in Le Corbusier's Philips Pavilion' (Chap. 72). Designed for the Brussels World Fair and produced in collaboration with Iannis Xenakis (1922–2001), the Philips Pavilion uses conoids and hyperbolic paraboloids to create an enclosure for experiencing recorded music. Le Corbusier (1887–1965) designed the pavilion at a time when he was interested in double-ruled quadric surfaces, a geometric approach that is related to the one described in Poros's chapter about Breuer. A related, Modernist, curvilinear or parabolic approach to form is seen in the work of Brazilian architect Oscar Niemeyer (1907–2012). More than any other architect of the mid-twentieth century Niemeyer rejected simple orthogonal geometry, embracing sensuous, serpentine curves, shallow domes and partial conic sections. Niemeyer's language of curved form is the subject of Benamy Turkienicz and Rosirene Mayer's chapter, 'Oscar Niemeyer Curved Lines: Few Words, Many Sentences' (Chap. 73). Through a graphic comparison of the forms present in Niemeyer's canonical works, Turkienicz and Mayer demonstrate a particular set of recurring geometric tropes.

Dom Hans van der Laan (1994–1991) was born in the Netherlands. Prior to becoming a monk, he was trained as an architect, and his few completed buildings, like the Monastery church in Tomelilla, Sweden, were designed for the Benedictine order. Richard Padovan's chapter, 'Dom Hans Van Der Laan and the Plastic Number' (Chap. 74) is about one of van der Laan's major contributions to architecture and mathematics: a proportional system based on the 'plastic number'. Like the golden ratio and similar irrationals, plastic numbers are mathematical constants. However, unlike the so-called 'metallic ratios', plastic numbers are not derived from quadratic equations. Instead, they form a proportional system which is derived from cubic equations. The term ‘plastic’ refers to the extent to which a form is perceptible, in and of itself, while a plastic number is the ratio that describes the lower and upper limits of the human ability to perceive differences of size amongst three-dimensional objects. Padovan describes the basic ratios of the plastic number, 3:4 and 1:7, and how they have been used in architecture.

The final two chapters in Part IX are about the architecture of Louis Kahn (1901–1974). Kahn famously cultivated a metaphysical approach to materiality and form which inspired many attempts to interpret his architecture as being emphatically geometric in its proportions. Despite this, his buildings rarely reveal simple ratios or systems. Instead, Kahn described his geometric intentions as being mediated by the practicalities of siting, construction technology and budget. Like the French Rationalists, Kahn’s use of Phileban forms was largely for their timeless or Platonic appeal, and not necessarily for their specific geometric properties. Nevertheless, spurred on by Kahn’s association with Anne Tyng (who had completed research on complex geometries), various scholars have sought to find deeper mathematical patterns in Kahn’s architecture. In ‘Louis Kahn’s Platonic Approach to Number and Geometry’ (Chap. 75), Steven Fleming analyses several claims regarding such hidden geometric systems in Kahn’s architecture. Fleming uses measured drawings and a detailed knowledge of the construction techniques applied in these buildings to show that Kahn’s work rarely, if ever, displays the precise type of geometric order which he has since become famous for. Indeed, on many occasions it is apparent that Kahn avoided creating forms which would have been perfect squares, or provided ideal golden sections. In ‘The Salk: A Geometrical Analysis Supported by Historical Evidence’ (Chap. 76), Steven Fleming undertakes an analysis of the historical evidence surrounding the planning and geometry of Kahn’s Salk Institute in La Jolla, California, while in parallel with this, co-author Mark Reynolds pursues a graphical analysis of geometry in the same building. Through a combination of Fleming’s scholarly and Reynolds’s intuitive geometric readings of the building, they uncover several large-scale patterns, reminiscent of those identified by previous scholars but not as clearly supported by a closer analysis of the working drawings and dimensions chosen by Kahn.

Part X: Contemporary Approaches to Design and Analysis

There are eight chapters in Part X, the first three of which concern surfaces, including minimal surfaces (soup bubbles) and two forms of topographic tiling (aperiodic and quasi-periodic). The architectural potential of each of these three systems is demonstrated using designs from historic and modern eras. The next three chapters describe different methods for using geometry to generate architecture. These include classical proportional systems, linear algebra and perspective hypercube constructions. The final pair of chapters contains accounts of the use of computational techniques for the analysis of geometry in historic structures.

Soap bubbles and soap films are the minimal surfaces required to define a three-dimensional space. Michele Emmer, in 'Architecture and Mathematics: Soap Bubbles and Soap Films' (Chap. 77), describes the history of the development of theories of soap bubble geometry in the late nineteenth century. Emmer then provides an overview of examples of historic applications of such isoperimetric forms, including the spherical designs of Ledoux and Boullée and the more literal application of minimal surfaces in Frei Otto's tensile structures.

Late twentieth-century applications of topographic tiling are the subject of the next pair of chapters. In a mathematical sense, tiles are a system of geometric patternation that fills a surface using a finite set of shapes without gaps or overlaps. Tiles are one of the oldest forms of decoration in architecture and square, rectangular, triangular and hexagonal sets have been used throughout history. Such periodic tiling systems (so-called because they repeat a core pattern at fixed intervals) are well known in architecture and, since the seventeenth century, have been considered of only limited interest to mathematicians. However, in the mid-twentieth century Hao Wang set out to discover if an aperiodic tile set exists, that is, one that would perfectly fill a plane, but would never repeat the same pattern. In 'Aperiodic Tiling, Penrose Tiling and the Generation of Architectural Forms' (Chap. 78), Michael Ostwald provides a background to the history of periodic tiling and then describes the search for an aperiodic tile set. Thereafter he analyses the application of one of the most efficient aperiodic sets—the Penrose tiling created by mathematician Roger Penrose (b. 1931)—in the refurbishment of Storey Hall in Melbourne, Australia. Ostwald's chapter finishes with a discussion of various tiling properties that are yet to be fully examined by architects. This interest in tiling surfaces is continued in 'Paving the Alexanderplatz Efficiently with a Quasi-Periodic Tiling' (Chap. 79), by Ulrich Kortenkamp, which describes a method for tiling a large, non-rectangular space. Using a refined version of the Penrose tiling system, Kortenkamp creates a polygonal set of four tiles to produce a quasi-periodic surface for a public plaza in Berlin.

The next three chapters examine mathematical systems for generating architectural form, but the approaches they take are from very different traditions and follow divergent historic trajectories. In 'Generation of Architectural Forms Through Linear Algebra' (Chap. 80), Franca Caliò and Elena Marchetti provide a mathematical taxonomy reliant on linear algebra which they use to classify architectural forms from different eras. They acknowledge that architects largely derive such forms intuitively, but demonstrate that these forms also have underlying mathematical rules. Through this process they establish a core set of forms which they use to demonstrate how variations in the underlying mathematics can produce alternative geometric constructions. In 'The Praxis of Roman Geometrical Ordering in the Design of a New American Prairie House' (Chap. 81), Donald J. Watts describes an equally rigorous, but innately revivalist approach to design using proportional and geometric systems. Watts demonstrates the application of classic Roman geometrical ordering systems in the design of a 1980s prairie style house in Kansas. Possibly the only Postmodern work in this volume, Watts's design is a homage to, and analysis of, two historic styles and their associated geometric themes. 'Exploring Architectural

Form in Perspective: A Fractal Hypercube-Building' (Chap. 82) by Tomás García-Salgado commences with a discussion of relatively recent applications of perspective, including references to twentieth-century architecture and cinema, to bring into focus the way perspective can be used to challenge representation. From this foundation García-Salgado demonstrates a perspective construction of a hypercube which he then "fractalises" to embed a smaller hypercube variant in one corner. Through this operation—geometry mediated through perspectival representation—García-Salgado generates a conceptual method for building design.

The final chapters in Part X examine computationally-based systems of measurement and analysis. In 'The Compass, the Ruler and the Computer: An Analysis of the Design of the Amphitheatre of Pompeii' (Chap. 83), Sylvie Duvernoy and Paul Rosin describe the application of two methods of measurement for historic structures. The first of these uses geometry to replicate or reconstruct the method used in ancient times for generating the form of a building; the example presented is a Roman amphitheatre. Duvernoy and Rosin next use modern digital tools to undertake an arithmetical analysis of the same structure. Finally, the two sets of results are compared, and in the case of the amphitheatre in Pompeii, the subject of their analysis, they independently arrive at the same conclusion. The final chapter in Part X, 'Correlation of Laser-Scan Surveys of Irish Classical Architecture with Historic Documentation from Architectural Pattern Books' (Chap. 84) by Maurice Murphy, Sara Pavia and Eugene McGovern, describes the application of three-dimensional surface modelling, derived from a laser scan, of historic architecture. This is another technically based and computationally intensive method for first recording, and then supporting the analysis of, the form of a historic building.

Part XI: Theories and Applications of Computing in Architecture

Four of the chapters in Part XI describe applications of digital technology in architectural design. These chapters typically celebrate the creative potential of the computer, identifying ways in which software allows architects to apply new mathematical knowledge in design. In contrast, three chapters consider computational design issues in a different light. One of these, the opening chapter in this section, offers an overview of the rise of computers in architecture, while the other two, are less sanguine in their assessment of the way architects have adopted and used computers for design, identifying ethical and philosophical dilemmas faced by architects in the digital design process. Taken together these seven chapters provide a balanced view of the challenges and opportunities of computational design.

Lionel March was one of the earliest innovators in the use of computational and contemporary mathematical approaches (like graph theory) in architectural design and analysis (March 1976). Starting in the 1960s, and working for over four decades at the forefront of this field, March completed several books on both historic and contemporary connections between architecture and mathematics (March and Steadman 1974; March 1998). In 'Architecture and Mathematics Since 1960' (Chap. 85), he looks back on this period of great change and provides an account of his personal experience in the field. That account commences with a consideration of the way computers (and computationally informed ways of thinking) have altered the relationship between architecture and mathematics, potentially returning architecture to its former, revolutionary, trajectory. March borrows Karl Friedrich Froebel's three categories of mathematical thought—the quantitative, qualitative and relational—to structure his review of the developments since the 1960s.

The next three chapters address closely related topics concerning the way in which digital modelling and animation software has provided architects with a means of generating form using parametric or algorithmic rules. This approach has been widely if somewhat controversially praised in architecture, triggering a plethora of publications and a growing body of research which tends to describe these advances as scientific or evolutionary, thereby seemingly providing authority for architecture through appeals to nature (Szalapaj 2005; Steadman 2008). In 'Bio-Organic Design. A New Method for Architecture and the City' (Chap. 86), Alessandra Capanna provides a synopsis of these emerging design paradigms which use software to generate complex, naturalistic forms. Citing advances in complexity theory and non-linear dynamics, since the 1980s architects have used the so-called "new sciences", coupled with advances in hardware and software, to visualize striking new forms. Andrejz Zarzycki develops this idea in 'Formal Mutations: Variation, Constraint, Selection' (Chap. 87), to consider the role of formal permutations in design decision-making. This type of outcome occurs because some of the parametric modelling and animation software used by architects does not generate a single design solution, rather it produces a myriad of alternative forms, each of which fulfil the starting parameters to a greater or lesser extent. Such variations often evocatively called 'mutations' by architects, are presented as striking and original alternatives for an architect to consider. In 'The Role of Mathematics in the Design Process under the Influence of Computational and Information Technologies' (Chap. 88), Arzu Gönenç Sorguç takes a larger scale view of these developments when she notes that the new focus of design is shifting towards process, rather than product, although digital manufacturing and printing techniques are also automating production at a similarly rapid rate. Sorguç also asks whether the emphasis of architecture has now shifted away from design and more towards presentation; that is, from ideation to representation. Focusing on the role of mathematics in the design process, Sorguç analyses this question in the context of contemporary software-driven design methods.

One of the first computational theories of generative design was developed by George Stiny in the 1970s and is called a "shape grammar" (Stiny 1980, 2008). Shape grammars could be regarded as early versions of more recent parametric approaches to design. Both of these methods use rules to generate or refine a design. In 'A Grammar for Dynamic and Autonomous Design in 3D Virtual Environments' (Chap. 89), Ning Gu demonstrates an auto-generative variation of the shape grammar for use in virtual environments. Gu's work shows that such a design grammar can use agent-based reasoning to begin to optimize the process of generating design variations. Whereas the earlier chapters by Capanna and Zarzycki highlighted the evocative and creative potential of computationally evolved forms, Gu's chapter is more in the spirit of March's research, which calls for the rigorous application of methods to make such processes useful.

Two of the final three chapters in Part XI are more critical of the use of computer technology in design. In 'Geometric Transformations and the Ethics of the Curved Surface in Architecture' (Chap. 90) Michael Ostwald reminds the reader that throughout history systems of geometry have typically been used with a knowledge and transparent demonstration of the basic mathematical principles underpinning that geometry. These properties are often associated with beliefs in correct, right or ideal applications of knowledge in architecture (Watkin 1977; Evans 1997). Grouped under the general heading of ethical considerations, these values have shaped some of the greatest architectural works of each generation (Harries 1997). However, in the late twentieth century, and despite the growing number of examples of applications of mathematics in architecture, the average architect's knowledge of mathematics has probably never been lower. This occurs because the computer allows a designer to create forms without necessitating even the most basic awareness of where these forms are coming from or what rules (geometric or numeric) are shaping them. Architecture's often shallow and opportunistic appropriation of geometry is something that this chapter warns against (Evans 1995; Ostwald 2010).

The future of architecture is sure to continue to include mathematics as a core source of inspiration and validation. Even today numbers surprise and fascinate. The field of number theory, aimed at discovering special qualities of numbers and their combinations, might still prove to be fertile ground for the architect's imagination. In 'Equiangular Numbers' (Chap. 91) mathematicians Henry Crapo and Claude Le Conte de Poly Barbut describe a class of numbers with peculiar properties that have yet to find an application in architectural design, but which future architects might find intriguing enough to pursue.

Finally, in his argument against the potential for digital architecture Alberto Pérez-Gómez follows a historical trajectory coupled with an ethical or humanist foundation. While Ostwald's ethical dispute is against the complacency and obfuscation which is common in digital design processes, Pérez-Gómez has repeatedly argued against such architecture, on the grounds that it lacks both phenomenal and spiritual depth. Pérez-Gómez (1992) was famously critical of both the loss of sense of purpose and of a more transcendent aspiration in much contemporary architecture and he sees these problems exacerbated in computational design practices (Pelletier and Perez-Gomez 1994). In particular, in 'Architecture as Verb and the Ethics of Making' (Chap. 92), Pérez-Gómez reiterates the trend identified earlier by Arzu Gönenç Sorguç that digital designers valorise the process rather than its product, but argues that this is a fundamentally flawed approach. Furthermore, this practice relies too much on the appearance of being scientifically based, rather than actually understanding the science on which its premise rests.

Through a review of the theories of Luca Pacioli and Le Corbusier, Pérez-Gómez suggests that the relationship between architecture and mathematics is at its most productive (both in ethical terms and in terms of appropriately aspirational or transcendent architecture) when designers demonstrate a deeper awareness of the challenges and opportunities of the geometric systems they are using. In a sense, Pérez-Gómez's work reinforces the observation expressed at the start of the present chapter, that the most productive and meaningful exchanges between architecture and mathematics have tended to occur when each are on a converging, but forward-looking trajectory.

Conclusion

In our opening chapter to Volume I, 'Relationships Between Architecture and Mathematics' (Vol. I, Chap. 1) we position the content of that volume against the backdrop of changing relationships between a field of knowledge—mathematics—and a discipline of practice—architecture. While the chapters in Volume I examined the works of particular architects and mathematicians, or provided descriptions of famous buildings or theories of geometry and design, taken collectively they also constructed a narrative thread through an era where architecture and mathematics were both respected, and often closely related, pursuits. However, impelled by increasingly specialised knowledge, supported by differing educational approaches, and triggered by the growing separation of the arts from the sciences, the years after 1500 begin to tell a different story. Nevertheless, while all of these forces have worked to divide the architectural profession from the discipline of mathematics, the two remain connected, as the chapters in Volume II demonstrate.

In order to begin to explain how, with so many forces separating the two, architecture and mathematics have continued to work productively together, here we have suggested a different framework. Drawn from critical theory and the history and philosophy of science, it has adopted three ways of viewing history as a trajectory though time and space. These three ways of conceptualising trends in aesthetics and knowledge—the revolutionary, the reactionary and the revivalist—allow us to see that the majority of the positive, productive or fruitful connections which have been proposed between architecture and mathematics appear to have occurred when both have followed a more rigorous forward trajectory, driven by revolutionary or reactionary agendas. At such moments, when the two disciplines have an equally progressive outlook, the gap between them has been at least partially bridged. Across 45 chapters and the work of 50 architectural historians, and designers, mathematicians, engineers, philosophers and computer scientists, this volume not only traces over 500 years of the history of the relationship between architecture and mathematics, but it is also drawn to the future, offering technical and philosophical views which will remain of relevance for many years, and will continue to shape new, creative opportunities for architecture and mathematics.

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