Chapter 45
Verrocchio's Tombslab for Cosimo de' Medici: Designing with a Mathematical Vocabulary

Kim Williams

Introduction

The tombslab by Verrocchio commemorating Cosimo de' Medici, patriarch of the wealthiest of Florentine families, is a relatively small memorial marker laid in the floor of the crossing in the basilica of San Lorenzo in Florence (Fig. 45.1). In spite of its size, it contains interesting lessons on the rich relationships between mathematics and design. Forentine sculptor Andrea del Verrocchio (ca. 1435–1488) was one of the best known artists in Florence at the time, and his workshop was a breeding ground for master artists such as Leonardo da Vinci and Pietro Perugino (Adomo 1991; Bule et al. 1992). As a basis for his composition, Verrocchio used a vocabulary of geometrical figures. The centrepoint in the composition plays a dual role, organizing the figures of the composition and relating the tombslab as a whole to the particular architectural setting in which it was placed. Further, the figures relate to each other through a system of proportions derived from the Pythagorean musical scale. The symbolism of the tombslab is derived from all of these elements: the sacred significance attached to the figures is reinforced by the colours used in the composition, the proportional relationships, and the position of the tombslab in space.

Cosimo de' Medici died in 1464. The tombslab which Verrocchio designed to commemorate him was laid in San Lorenzo in 1467. The slab appears in the pavement in the centre of the crossing; Cosimo's remains are buried in the ground in the crypt below (Baldini and Nardini 1984; Burns 1979). The actual

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Fig. 45.1 Verrocchio's tombslab for Cosimo de' Medici in San Lorenzo. Drawing: author

tomb and the tombslab are connected by a massive pier in the crypt. The central element in the composition of the tombslab is a rectangle of red porphyry. Elongated half-circles are placed on each side of the rectangle. In the top and bottom half-circles are inscription panels[1]; in those on either side are green porphyry *mandorle*. This inner composition is circumscribed by a circle, which is itself circumscribed by a square. In the interstices between the half-circles and the outer circle are small green porphyry roundels. All the geometrical figures are outlined in white marble. An outer square of black marble frames the whole composition. Bronze shields with the Medici symbol, *palle* or balls, in red porphyry, appear in the interstices between the circle and the outer square.[2] Centred on three of the four sides of the outer square are small, square, bronze grilles which provide light for the crypt below. The fourth grille was apparently obliterated when the altar was redesigned in the 1600s[3] (Fig. 45.2).

The tombslab has most intrigued those interested in Renaissance tomb design because it is so different from the kind of tomb markers found at the time (Clearfield 1981). It is devoid of any figural representation of the deceased, for example, such

[1] The inscriptions read, "Here lies Cosimo de' Medici. Publicly declared Father of his Country" and "Lived 75 years, 3 months, and 20 days."

[2] The shields may be a later addition.

[3] In addition to obliterating the fourth grille, the present altar step shaves about 1 cm off the tombslab's upper edge.

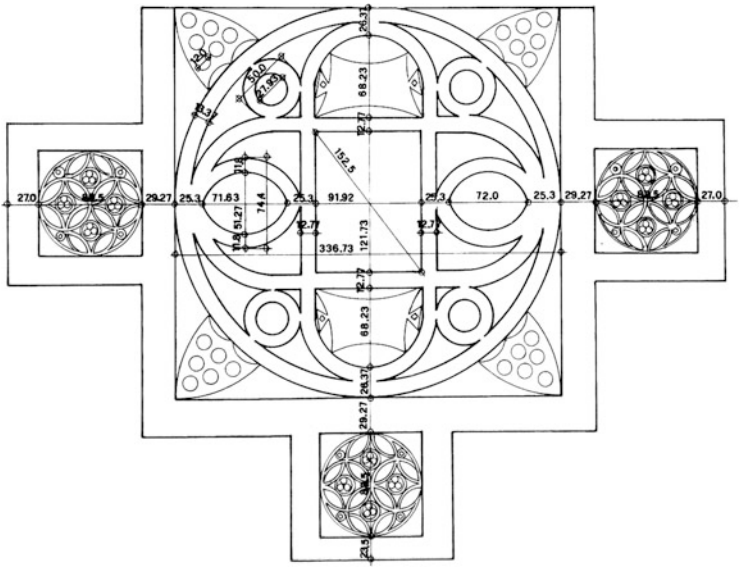

Fig. 45.2 Survey of the tombslab with the key dimensions (all dimensions are in centimetres). Drawing: author

as a bust or portrait. In addition, there are no readily recognized Christian symbols used in the marker. Although Cosimo was the wealthiest and most influential Florentine of his time, his marker is simple to the point of austerity. The simplicity of his tomb marker is unabated in spite of the rich materials used. In part, the restraint demonstrated by the design may be due to the personality of Cosimo: he was known to have avoided overt displays of his wealth and position. To understand fully the significance of the tombslab, however, it is necessary to study the geometrical figures which appear in the composition. My conclusion is that the slab, far from being a terse pagan marker, professes a belief in a divinely-created cosmos. We will first examine the use of the figures as carriers of sacred ideas, and then examine the intriguing relationships suggested by the proportions of the figures.

The Forms of the Tombslab

Verrocchio used five distinct geometrical figures in his composition. These were references to philosophical ideas and readily understood as such by those who were familiar with the neo-Platonic philosophy popular in fifteenth century Florence. In the centre of the panel is a rectangle of red porphyry. The rectangle measures some 92 cm in width and 122 cm in length, with a diagonal of 152.5 cm (see Fig. 45.2 for

exact dimensions).[4] The relationship between these dimensions becomes somewhat clearer when they are converted into fifteenth century Florentine units of measure: the rectangle measures 31 soldi in width by 41 soldi in length, with a diagonal equal to some 51 soldi.[5] In other words, the rectangle is formed of two right triangles whose sides relate to each other in the ratio of 3:4:5. I believe that the significance of the 3:4 rectangle in Verrocchio's design is linked to that of the triangles of which it is formed. Triangles in general, as the first plane figures, have always been given special significance. A triangle is formed by three points; Pythagoreans considered three the first "real" number and, therefore, divine. Plato gave a cosmic symbolism to triangles, writing in the *Timaeus* that the world is composed of triangles (Plato 1961: 53c–e). The 3:4:5 triangle, sometimes called an "Egyptian triangle" because it was studied and used by the Egyptians, notably in the Pyramid of Cheops, and sometimes called the "Pythagorean triangle" because it provides a ready proof for the Pythagorean formula $a^2 + b^2 = c^2$, has a special place among triangles. It is the only triangle whose sides form an arithmetic series. In addition, the sum of the lengths of the sides, 3 + 4 + 5, is 12, a particularly significant number. A circle divided into 12 equal segments symbolized the division of the heavens into 12 zodiacal regions. If one imagines the circumference of this circle as a line, then the line can be opened and refolded to form the 3:4:5 triangle (Fig. 45.3). Further, the proportions of the 3:4:5 triangle are related mathematically to the value for the musical whole tone (the mathematics of this proportion will be explained in detail in the next section.) For Neo-Platonists, the musical whole tone was a divine value, symbolic of the Creator, in that it is a number which cannot be divided evenly into two, and is therefore eternal and unchanging.

Thus, the symbolism of the 3:4 centre rectangle, composed of two 3:4:5 triangles, is informed by their symbolizing the cosmos, or creation, as well as their symbolizing the Creator.

The overlapping ovals created by the addition of half-circles to all sides of the rectangle form what is known as a "Solomon's knot." The "knot" has neither beginning nor end, and is therefore symbolic of immortality and eternity. "Solomon's knot" is an ancient pavement motif, and was commonly used in the fifteenth century as a decorative motif. Leonardo da Vinci used it as the basis of a design for a centrally planned church.[6] It was probably used by Verrocchio with much the same intention as it was by Leonardo: to underline the sacred nature of the design by imparting to it the symbolism of immortality and eternity carried by the motif. What differs between Verrocchio's "knot" and that of Leonardo is the

[4] My analysis of the tombslab is based on my own survey. Because accuracy of the values used is critical to the accuracy of the analysis, I follow the recommendations of Howard Saalman (1979), taking each measurement three times and using the average value of the three as the working value. These are the values which appear in Fig. 45.2.

[5] A Florentine braccio was the unit of measure used at the time the tombslab was constructed. 1 braccio = 58.4 cm. It was subdivided into 12 crazie (1 crazia = 4.95 cm) or 20 soldi (1 soldo = 2.92 mm); each soldo was further subdivided into 20 denari; see Zervas (1979).

[6] For an illustration of Leonardo's design, see Pevsner (1972: 202, Fig. 143).

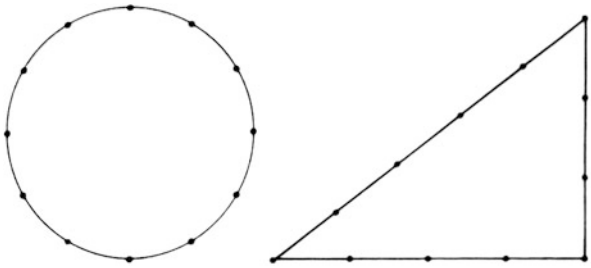

Fig. 45.3 A *circle* divided into 12 segments may be opened and recomposed into a 3:4:5 *triangle*. Drawing: author

proportions of the centre, Verrocchio having used the 3:4 rectangle while Leonardo used a square.

On either side of the central rectangle, in the interstices of the "Solomon's knot," appear two *mandorle*. The *mandorla* ("almond" in Italian) is also known as a *vesica*, or a *vesica pisces* ("fish bladder" in Greek.) The *vesica* is the fish-shaped symbol for Christ, and is linked to the celebration of the Eucharist. This sacrament had a great importance for Cosimo de' Medici: he had expressly requested that his tomb be located at the foot of the altar, so as to symbolize his being present at the celebration of the Eucharist. The *vesica* also represents a zone of intersection between two intersecting circles, one representing the heavens and the other representing earth (in neo-Platonic symbolism the two circles would represent the world of "being" and the world of "becoming"). The *vesica* itself, the intersection, is symbolic of the mediator between the two. For Christians, the mediator is Christ, who was both God and man.

The circle-in-a-square circumscribes the "Solomon's knot." We find references to these two shapes in Plato's *Timaeus* (1961: 54d–55c). The cube and the sphere represented for Plato the earth and the universe respectively, and the two-dimensional forms of square and circle have analogous meanings. As did the geometrical problem of squaring the circle, the circle-in-a-square graphically symbolized the perfecting of the imperfect. It was also a symbol for the cosmos.

Finally, it is important to consider the role of the centre point in the tombslab. Geometrically, a point lacks dimension, possessing only the characteristic of position. In Verrocchio's composition, the centre point serves a fundamental role in organizing the design, and so is an integral part of Verrocchio's language of form. As a reference for both the crossing of the basilica and of the tombslab, it also contributes significantly to the symbolism of the tombslab. The fact that the tombslab appears in a pavement (rather than on a wall, for instance) introduces the element of living man into the composition, by involving not only the deceased commemorated by the slab, but the spectator as well, who, in order to view the memorial, is drawn into the centre of the crossing. The space of the crossing had become increasingly significant in fifteenth century Italian architecture, supplanting the presbytery as the most sacred location within the church. Considering the symbolic content of the design, it is my conclusion that, if the symbolism of the design of Cosimo's marker makes reference to the order of the cosmos, its placement in the centre of the crossing is symbolic of man's

central position in that cosmos. This attitude towards man is predominant in fifteenth century humanism.

The colours of the materials used in the tombslab reiterate the symbolism attached to the figures. The red porphyry used for the central rectangle is symbolic of divinity, and also carried connotations of royalty. The colour used for the central rectangle, then, underlines the reference made to the Creator through the 3:4:5 proportions, and may refer to the sovereignty of Cosimo de' Medici as well. In addition, purple was, and is today, a liturgical colour representing sorrow and penitence, used as the colour for Advent and Lent by the Roman Catholic church, and as such is an appropriate colour for a tomb marker. The green porphyry used for the *vesica* shapes refers to resurrection and immortality. As a liturgical colour green symbolizes the predominance of life over death, as the green of new leaves symbolizes winter vanquished by spring. Thus green is apt as a colour for the *vesica*, a shape representing Christ. White is a conventional colour for purity and therefore holiness. The white used to outline the forms of the tombslab alludes to the sacred nature of the message of the memorial.

In terms of the geometrical figures which appear in the design, and the colours in which they are presented, the symbolism of the tombslab is far from pagan. Instead, it is evocative of an indivisible, eternal sovereign Creator of the universe; of the Christ, mediator between heaven and earth, present in the Eucharist; of the desire that the terrestrial achieve the perfection of the celestial.

The Proportions of the Tombslab

Analysis of the panel in terms of abstract geometry presents some difficulties. Geometrical figures are bounded by lines; lines have length but no width. The lines in the tombslab have both length and width. Thus it is not always clear which of the dimensions are to be considered. However, it is clear that the proportions of the forms are interrelated, and this is particularly important with regards to a study of pattern, because the resulting order in the composition integrates the forms. I believe that mathematical proportions related to the musical scale predominate in the composition (as opposed to a system of proportions based on an irrational such as $\sqrt{2}$, ϕ, or π, though irrational proportions can be found in the composition, as we shall see presently).[7] The use of musical proportions implies a "harmonic" treatment of the elements of the composition, and they are found quite frequently in the proportions of elements in Renaissance architecture.

Let us note some of the proportional relationships (please refer to Fig. 45.2). The diameter of the outer circle is equal to twice the long side of the 3:4 rectangle plus

[7] The musical proportions are very closely related to systems of proportion based upon irrational values, so that the use of one does not necessarily preclude the other.

the short side (121.73 + 91.92 + 121.73 = 335.38, a deviation from 336.78 of 0.4 %). The width of the tombslab *vesica* is one-third the diagonal of the 3:4 rectangle (152.5 ÷ 3 = 50.83, a 0.9 % deviation from the actual 51.27). The outer diameter of the small roundels tangent to the "Solomon's knot" is equal to the diagonal of the rectangle. The width of the strips which outline the figures is equal to one quarter of the width of the *vesica* (51.27 ÷ 4 = 12.82, a 0.4 % deviation from 12.77). It is also equal to one-twelfth of the diagonal of the 3:4 rectangle (152.5 ÷ 12 = 12.71, a 0.5 % deviation from 12.77). The outer diameter of the roundels is equal to one-third of the diagonal of the 3:4 rectangle (152.5 ÷ 3 = 50.83, a 1.6 % deviation from 50.0). The outer diameter of the roundels is also equal to two-thirds of the outer width of the *vesica* (74.87 × 2/3 = 49.91, a 0.2 % deviation from 50.0). The doubled border is equal to one-half of the width of the *vesica* (51.27 ÷ 2 = 25.63, a 1.3 % deviation from 25.3, and a 2.8 % deviation from 26.37). We are seeing, then, proportions of 1:2, 1:3, 1:4, 2:3 and 3:4. The diagonal of the rectangle appears to be a key dimension for generating the others.

A search for a geometrical method by which to describe the tombslab has led me to recognize that the composition is based upon three squares *ad quadratum*, the innermost of which is subdivided into modules with musical proportions by extensions of the 3:4 rectangle (Fig. 45.4). The corners of the outermost square come very close to falling exactly on the centrepoints of the grilles (deviating by only 1.5 %).

In an *ad quadratum* composition the √2 figures prominently, and it is worthwhile to point out that it is found in other elements of the tombslab. For instance, we noted that while Verrocchio and Leonardo both used the "Solomon's knot," Verrocchio departed from the norm by placing a 3:4 rectangle in the centre instead of a square. A similar variation is found in his use of the *vesica*. The classic *vesica* is formed by two overlapping circles; the circumference of each circle passes through the centre of the other. The width of the *vesica* created is equal to the radius, or one-half of the diameter, of the circles which overlap to create it. It can be circumscribed by a rectangle of proportions 1:√3. Another way to think of it is as being based upon two equilateral triangles. In contrast, the tombslab *vesica* may be circumscribed by a rectangle of proportions 1:√2, and the width of the vesica is three-quarters of the diameter of the circles which overlap to create it. It may be thought of as based upon two triangles with sides in the ratios of 1:√2:3 (Fig. 45.5).

While no documentation exists of the tombslab which would shed light on Verrocchio's actual process for its composition, it is likely that some kind of comprehensive system was used to generate the dimensions of the geometrical figures within the composition. I have been unable in my own work so far to solve the mystery, but two separate systems developed by others are particularly suggestive, and each involves the 3:4:5 triangle. Let us consider first an examination by Jay Kappraff of the geometry of that triangle, which reveals the relationship between that triangle and the Pythagorean musical scale:

> Let the triangle ABC (Fig. 45.6) have lengths in the ratio: L_1:L_2:L_3 = 3:4:5. It follows from trigonometry that the angle bisector of angle A cuts the opposite side L_1 in a length a = L_2/3, while the angle bisector of angle B cuts its opposite side L_2 in b = L_1/2. In other words,

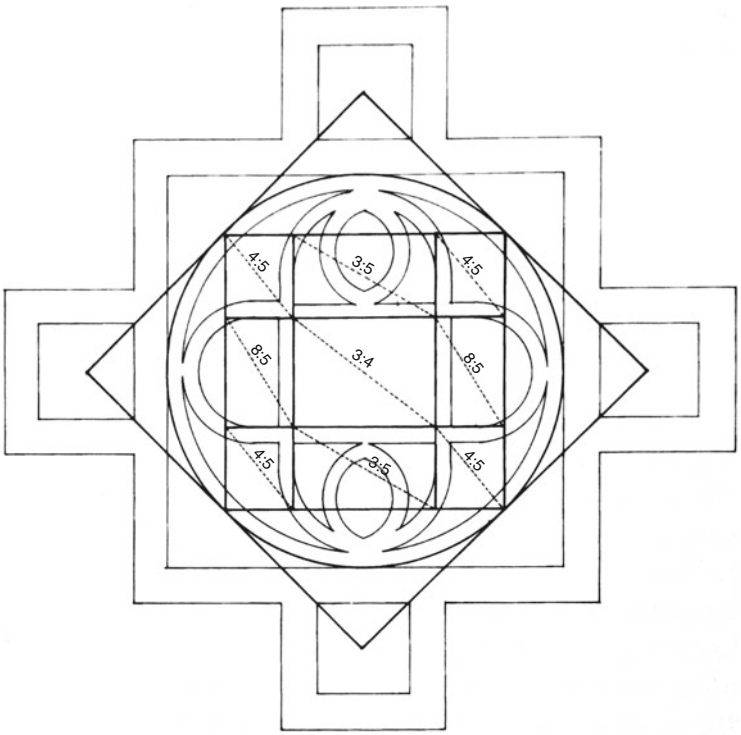

Fig. 45.4 A geometric analysis of the proportions of the tombslab. Drawing: author

tan A/2 = 1/3 and tan B/2 = 1/2. These two fractions have special significance in terms of the Pythagorean musical scale. They form the basis of the Pythagorean tetrachord: 1:1, 1:2, 1:3, and 2:3, respectively the unison, octave, fifth above an octave, and the fifth. Now it follows that 2a:3b:c = 3:4:5. From this it follows that a/b = 8/9, the ratio of a whole tone in the Pythagorean scale (Kapraff 1993: 10).

Dr. Kappraff's study provides one important way to begin to understand just how integrated the dimensions of the tombslab are. This system would provide a justification for the "Solomon's knot" being based upon a 3:4 rectangle rather than a square, and has the further advantage of providing a means of generating both the dimensions of the figures in the tombslab and the resulting musical proportions between them.

Another very interesting geometrical system with regards to the tombslab is the "New Jerusalem" geometry, proposed by John Michell (1988; Kappraff 1991: 4–6) (Fig. 45.7). The point of coincidence between this system and the Verrocchio tombslab lies in the use of the relationships between the 3, 4, 5 and the 11. In the tombslab, each of the three sides of the triangle relate to the diameter of the circle as parts of 11, that is to say, that the three sides of the triangles are related to the diameter of the circle in the following ratios:

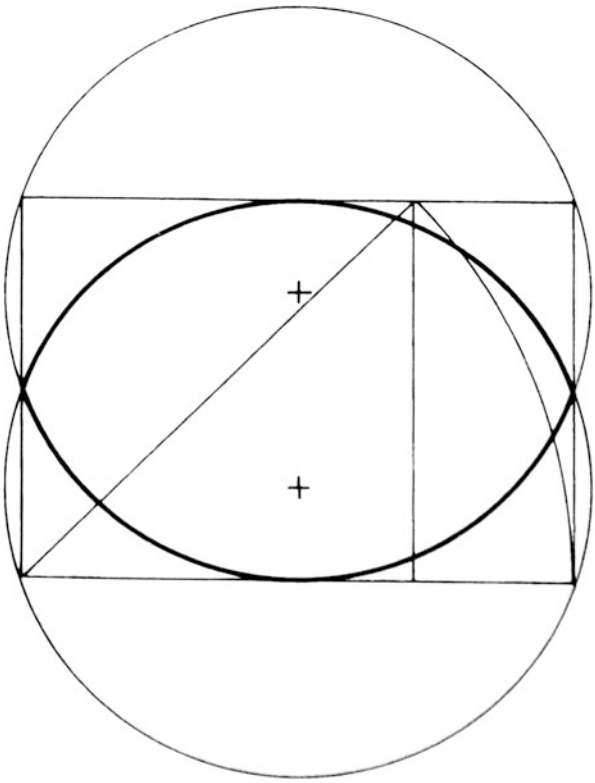

Fig. 45.5 The proportions of the tombslab *vesica*. Drawing: author

$$\frac{91.92}{336.73} :: \frac{3}{11}; \frac{121.73}{336.73} :: \frac{4}{11}; \frac{152.50}{336.73} :: \frac{5}{11}.$$

I have been particularly interested in these proportions because they are found in two other contexts within the basilica of San Lorenzo. First, Filippo Brunelleschi, architect of the basilica, based his plan for the basilica on a module which measures 11 *braccia* to a side. This module is sometimes described in literature on Brunelleschi as a square with a circle inscribed in it. Second, Vasari's pavement design for Michelangelo's New Sacristy in the basilica features a checkerboard design in which the floor plane is subdivided into fields of proportions 3:11, 4:11, 5:11, and 11:11. Turning now to Michell's geometric construction, we see it is based upon a square, the side of which measures 11 units and is created by the addition of two 3:4:5 triangles to a 3 × 3 square. One of Michell's claims for the geometric construction is that a circle which passes through the centre points of the four lateral squares has a circumference equal to the perimeter of the 11 × 11 square, effectively squaring the circle. Some of the similarities between the New Jerusalem construction and the tombslab design are striking, in that we have a circle inscribed in a square, with four smaller squares on each side (analogous to the grilles in the tombslab design) and the recurrent theme of the 3, 4, 5 and

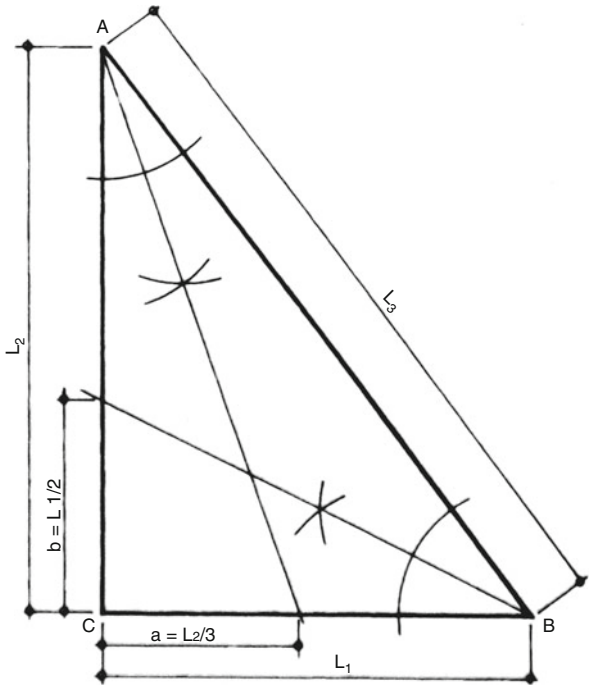

Fig. 45.6 The 3:4:5 triangle is related to the musical 8:9 whole tone. Drawing: author

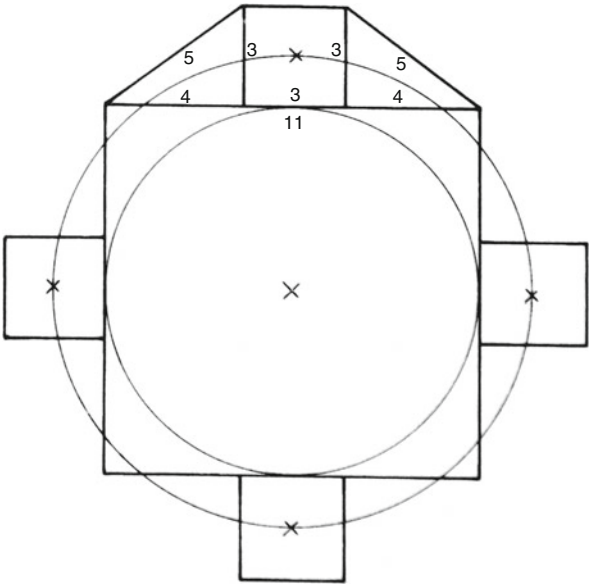

Fig. 45.7 John Michell's "New Jerusalem" geometry, an approximate method for squaring the *circle*. Drawing: author

11 proportions. But extension of the 3 × 3 squares centred on the sides of the New Jerusalem construction won't account for the placement of the 3 × 4 rectangle in the tombslab design, nor am I able to find the generation of the musical proportions within the system.

It may not be possible to develop a system which accounts for all of the properties of Cosimo's funerary marker. At any rate it is clear that mathematics played a major part in Verrocchio's creative process, resulting in a beauty that mathematicians are most apt to appreciate.

Acknowledgments I am indebted to Jay Kappraff for sharing his unpublished paper with me. All images in this chapter are by the author.

Biography Kim Williams was a practicing architect before moving to Italy and dedicating her attention to studies in architecture and mathematics. She is the founder of the conference series "Nexus: Relationships between Architecture and Mathematics" and the founder and editor-in-chief of the *Nexus Network Journal* . She has written extensively on architecture and mathematics for the past 20 years. Her latest publication, with Stephen Wassell and Lionel March, is *The Mathematical Works of Leon Battista Alberti* (Basel: Birkhäuser, 2011).

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