Chapter 3
Mathematics *in*, *of* and *for* Architecture: A Framework of Types

Michael J. Ostwald and Kim Williams

aetiology | ˌaētēˈäləjēl *noun*
The investigation or attribution of the cause or reason for something, often expressed in terms of historical or mythical explanation.

teleology | ˌtelēˈäləjē | *noun*
The explanation of phenomena by the purpose they serve rather than by postulated causes.

Introduction

The frontispiece of the thirteenth century *Bible Moralisee* conserved in Vienna portrays a Christ-like figure leaning over a primordial world and using a pair of compasses to measure and inscribe its limits (Fig. 3.1). Titled 'God as architect of the world', it depicts the use of a mathematical instrument to determine the functional, symbolic and aesthetic properties of the universe. The pair of compasses is a symbol of all of the possible ways in which mathematics is used to support design. Such symbols are useful for reinforcing the simple message that the creative impulse relies on mathematics to translate a concept into reality. At the same time, however, this symbolism masks the fact that the relationships between architecture and mathematics are both richer and more diverse than the sign implies. The purpose of the present chapter is to look behind the symbol of the

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Fig. 3.1 'God as architect of the world'. *Bible Moralisée*, Paris (ca. 1220–1230) (Image: Osterreichische Nationalbibliothek, Vienna, Codex Vindobonensis 2554, fol. Iv. Reproduced by permission)

pair of compasses and to begin to identify the different ways in which mathematics is used in architecture.

The *Bible Moralisee* was an illuminated manuscript in the medieval tradition that used images to communicate important biblical themes. The illuminations were evocative visual counterparts to the myths, beliefs, parables and morality tales, originally transmitted orally, that sought to educate people about the world. The conflation of God as both architect and geometer in the frontispiece is especially noteworthy because it communicates mathematics' fundamental contribution as intermediary between the creative impulse and the product of that divine vision (Kline 2001). What is often forgotten in this reading of the frontispiece is that the analogy not only communicates something about God's power and wisdom, but also about the accepted role and skills of the architect. The allegorical effectiveness of this image relies on the viewer being aware that architects use mathematics to create structure. This message is reinforced by the representation of God stepping through a timber portal, with one foot resting in the quotidian world of the designer or artisan as user of geometry, and the other transcending this as maker of the universe (Husband 2009). The frontispiece of the *Bible Moralisee* is a culturally-coded representation of the vital bond that exists between architecture and mathematics. Yet, while it presents this relationship as both natural and necessary, it says nothing about the connection itself.

A common question in architectural scholarship asks why architects use mathematics (Kappraff 1990; Rossi 2004; Goldberger 2009). Despite multiple answers being offered (Scruton 1983; Evans 1995), the majority of such responses have served a rhetorical purpose, providing the impetus for a personal manifesto or theory (Salingaros 2006). For example, Mario Salvadori (2014) asks, '[c]an there be any relationship between architecture and mathematics?', and after considering several responses, concludes that architecture simply cannot exist without mathematics. Salvadori's answer, like many of the others that have been offered, is eminently reasonable but it does not provide a holistic insight into the different ways architects use mathematics.

Here we will identify some of the types of applications of mathematics that conventionally occur in architecture, drawing on historic and contemporary myths and models to propose a framework for classifying the ways architects use numbers and geometry. We commence by examining connections between architecture and mathematics first from a *causal* or mythopoeic perspective, and second from an *effects*-based viewpoint. Here, the causes and effects are disconnected, each informing and shaping the framework, but unable to be directly correlated through that mechanism. This discontinuity is unavoidable because relationships between architecture and mathematics are not predicated on a singular need, desire or process; they serve a multiplicity of different and sometimes conflicting agendas. Cause and effect cannot be perfectly aligned under such conditions, but there are ways of investigating the two that are informative and useful for this purpose.

The study of the cause or genesis of an occurrence is called *aetiology*. This approach to understanding the origin of an idea or relationship is often undertaken through an investigation of the founding myths of a discipline. The present chapter commences by examining the classic Western myths of the first building—the primitive hut of the ancients—and the first architect, Daedalus. The purpose of this strategy is to reveal the presence of mathematics within the earliest accounts of architecture. Such myths distil a series of ideas in such a way that their essential message is retained while other peripheral issues are excised (Kirk 1975). A study of myths reveals the values, superstitions and beliefs that are the historic cornerstone of a discipline (Bettelheim 1978). The myths of the primitive hut and Daedalus are crucial indicators of architectural attitudes towards geometry, pattern and metrology and they resonate with other canonical value structures including the Vitruvian triad of *firmitas*, *utilitas*, and *venustas*; terms that were aptly translated by Sir Henry Wotton as *firmness*, *commodity* and *delight* (Kostof 1977; Johnson 1994).[1]

Whereas aetiology supports the consideration of causes without effects, the examination of effects without causes is called *teleology*. A teleological investigation of a relationship seeks to comprehend it in terms of its outcome and without reference to its source. In the second major section of this chapter a more modern myth—the collectively accepted model of the design process—is reviewed to reveal the breadth and depth of uses of mathematics in more recent times. This model has endured for many hundreds of years, embedded as it is in the practices of the architectural discipline through pedagogical, fiduciary and curatorial mechanisms such that, despite countless practical changes, the primary creative systems continue to be conceptualised in this way (Miller 1995; Ostwald 2012).

Combining both the aetiological and the teleological readings of the relationships between architecture and mathematics allows us to propose a framework of types. Three purposive agendas are at the core of this framework: the use of knowledge *for* supporting the design process, the desire to embed knowledge *in* an aesthetic construct, and the application *of* knowledge through design analysis. Within this framework 13 different types of mathematical applications in architecture are identified. These are: logic; measurement; surveying; modularity; performance and prediction; generation; aesthetics; symbolism and semiotics; phenomenality and rationalism; inspiration; surface articulation; analysis and informatics.

The framework proposed in this chapter is not intended to provide a definitive epistemology; rather, its purpose is more akin to a genealogist's study of kinship and consanguinity. It investigates the natural mathematical relations or bloodlines that have historically sustained architecture. Furthermore, the goal of this chapter is not to explain why these different applications of mathematics occur in architecture, but to provide a mechanism for recording the different types of applications and for understanding them holistically, as either occurring in a particular stage of the design process, or in support of a specific architectural quality. Through this dual aetiological and teleological process the breadth of

[1] "Well building hath three Conditions. *Commoditie*, *Firmenes* and *Delight*" (Wotton 1624: 1).

approaches, applications and techniques—all symbolically represented by the pair of compasses in the frontispiece of the *Bible Moralisee*—is revealed.

Myths of Architecture: An Aetiology

A common practice in the historiography of many disciplines is to link the origins of ideas to specific incidents, either real or imagined. For example, in 1665, while convalescing at the family home in Lincolnshire, Isaac Newton observed an apple falling from a tree. In his later life he would recount this event, describing it as the catalyst for his formulation of a universal theory of gravitation (Hall 1999). The story of Newton and the apple has since become one of the enduring myths of modern science. However, despite being allegedly based on real events, the term 'myth' is appropriate here because there are multiple conflicting versions of Newton's account (Brewster 1835). Indeed, five decades passed between the windfall occurring and Newton describing its significance. Newton actually invested several decades of his life in detailed research into the topic of gravity but when called upon in his later life to explain the genesis of his work, he repeated variations of this account of the falling apple. An aetiological perspective of this event is not concerned with its historical veracity but with the reason Newton chose to present his work in this way, emphasising the manner in which it uses an everyday occurrence to evoke the presence of a universal system of physical laws (Berkun 2010).

Every discipline has an equivalent origin myth, a tale that serves to elucidate and authorise a set of actions or values. In Western mythology the two great origin myths of architecture are both, as is typical of the genre, largely apocryphal. This is why they should only be read as a post-rationalised or figurative explanation of why certain acts should continue or particular relationships are important. The two origin myths of Western architecture describe the construction of the first building and the skills of the first architect. Whether one can be said to precede the other is a point of minor contention, but the myth of the first building, the archetypal primitive hut, is deliberately composed without the presence of an architect and so it is the first that is considered here.

Joseph Rykwert (1981) argues that throughout history architects have returned to the idea of the first house, the primitive hut of the ancients, whenever they have sought to make sense of the purpose of architecture. According to Rykwert, an interest in the primitive hut has been a constant throughout history: '[it] seems to have been displayed by practically all peoples at all times, and the meaning given to this elaborate figure does not appear to have shifted much from place to place, from time to time' (Rykwert 1981: 183). The myth of the primitive hut provides a philosophical foundation for understanding, questioning or reinvigorating architecture. Alberti, Laugier, Perrault, Viollet-le-Duc, Ruskin, Le Corbusier and Wright have each studied the primitive hut in its various incarnations (Harries 1993; Vogt 1998). Whether they have attempted to find its site, reconstruct its form, or study its construction, they have been drawn to seek inspiration from its imagined properties (Mitias 1999). Rykwert maintains that the primitive hut provides a 'point of reference for all speculation on the essentials of building' (Rykwert 1981: 183) including the relationship between architecture and systems of knowledge (like mathematics). The earliest extant version of this myth, from which most others can be traced, is found in Vitruvius's *De Architectura*.

Marcus Vitruvius Pollio, writing around the time of the Emperor Augustus in the first century B.C., provides an imagined account of a primitive race of men who 'were born like the wild beasts, [and lived in] woods, caves, and groves' (Vitruvius 1914: 38). During a storm, the branches of some trees near the tribe's cave 'caught fire, and so the inhabitants of the place were put to flight, being terrified by the furious flame' (38). After the storm had subsided, they gathered around the flames and learnt to sustain them, and the fire in turn kept the tribe safe from predators. To maintain both the fire and the community that had formed around it, a shelter had to be constructed. This compulsion to create a structure in a specific location, rather than to inhabit an existing cave or hollow, was to be the impetus for the first building:

> At first they set up forked stakes connected by twigs and covered these walls with mud. Others made walls of lumps of dried mud, covering them with reeds and leaves to keep out the rain and the heat. Finding that such roofs could not stand the rain during the storms of winter, they built them with peaks daubed with mud, the roofs sloping and projecting so as to carry off the rain water (Vitruvius 1914: 39).

A woodcut illustration in the 1521 edition of Vitruvius by Cesare Cesariano depicts a large fire surrounded by a primitive tribe. In the foreground people are gathering branches to feed the flames, while in the background, glimpsed through the smoke-haze, the branches of the living trees can be seen entwined together, suggesting a pitched or woven-roofed form. A second woodcut by Cesariano—much like subsequent ones from later editions of *De Architectura* and those in *Vitruvius Teutsch*—is less allegorical in its intent, displaying a more literal representation of the first hut. In that woodcut, rows of evenly spaced, vertically-arrayed tree trunks each end in a forked bough, which creates a natural cradle for a horizontal timber spar to connect the columns and create an edge to the roof. Between these columnar trunks with their forked pinnacles, smaller branches have been woven (Fig. 3.2). The regularly spaced, if roughly hewn, rafters and beams are also plaited together, creating an alternating surface of branches and grass, woven as if 'in imitation of the nests of swallows' (Vitruvius 1914: 38).

The architecture of the Vitruvian primitive hut is founded, initially at least, on the crafts of weaving or plaiting; the regular interleaving of elements forms a reinforced surface which is also a recurring geometric pattern. Starting with living branches and leaves, in groves or bowers, and then including loose grass and partially dressed timber, woven structures formed the basis for tents, screens and simple roofs. The first structures were created using felled trees as columns, arrayed in such a way that their forked joints created natural supports, and sized and spaced to achieve a consistent wall. These timber frames were the basis for

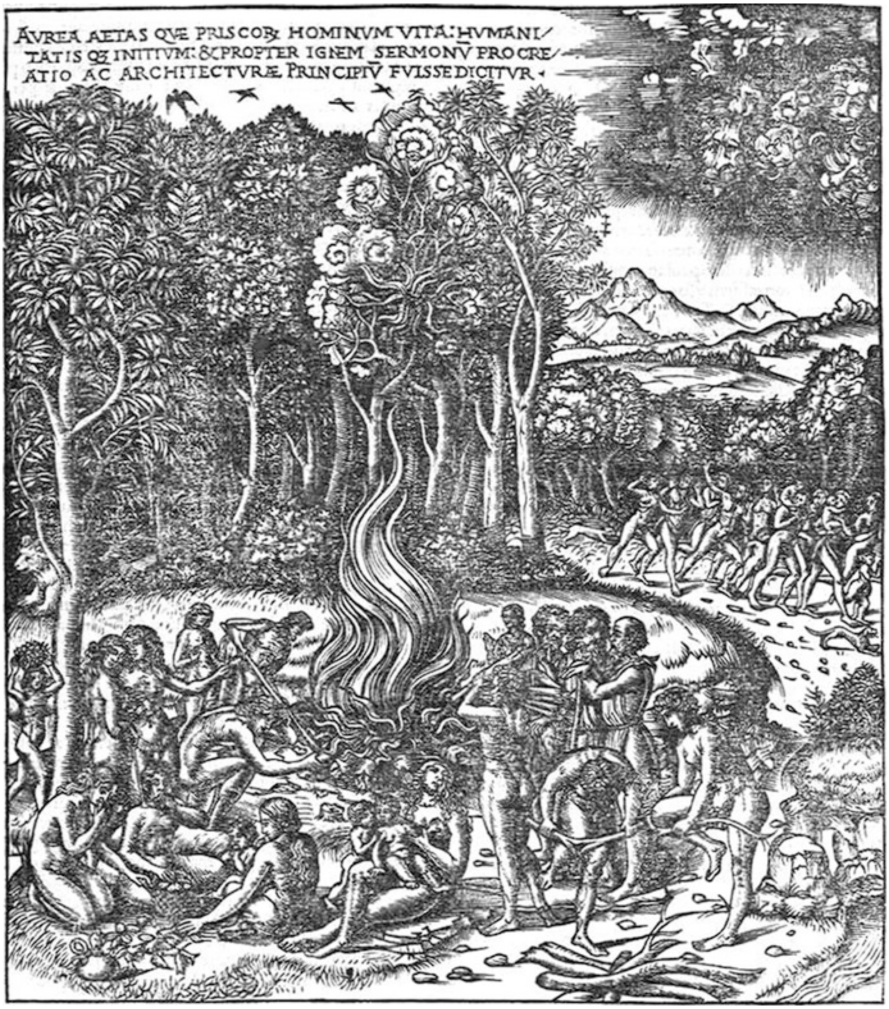

Fig. 3.2 The primitive hut according to Cesariano's edition of Vitruvius. Image: Cesariano (1521: Bk. II, ch. 1, p. XXXI v

subsequent woven and layered enclosures. Vitruvius states that variations of these techniques can be seen in the primitive dwellings of many cultures, including the Colchians of Pontus (near present day Georgia on the Black Sea). The Colchians would commence by laying.

> . . . down entire trees flat on the ground to the right and the left, leaving between them a space to suit the length of the trees, and then place above these another pair of trees, resting on the ends of the former and at right angles with them. These four trees enclose the space for the dwelling. Then upon these they place sticks of timber, one after the other on the four sides, crossing each other at the angles, and so, proceeding with their walls of trees laid perpendicularly above the lowest, they build up high towers. The interstices, which are left on account of the thickness of the building material, are stopped up with chips and mud.

> As for the roofs, by cutting away the ends of the crossbeams and making them converge gradually as they lay them across, they bring them up to the top from the four sides in the shape of a pyramid (Vitruvius 1914: 39).

In the Colchian hut, stacked logs, carefully sized, spaced and cut to measure, create both structure and enclosure. The form of this dwelling is square in plan with a pyramid-shaped roof. In the various examples of the primitive hut the importance of measurement (typically relative to other elements in a building), structural stability (intuitively or empirically determined), geometry (for the creation of symmetrical and stable forms in three dimensions) and pattern (in the construction and expression of woven forms) are all reinforced.

A second architectural aetiology is found in Greek mythology where Daedalus, the father of Icarus, is characterized as the first architect. Daedalus was an Athenian craftsman who is credited with the design of Ariadne's dancing floor and the Labyrinth at Knossos. Whether he was a real person or an amalgam of several different designers is unknown. Homer, Euripides and Ovid describe his actions in poetic terms, dwelling on his invention of animated statues, the golden thread of Ariadne and the waxed and feathered wings of Icarus that famously melted, sending Daedalus's son plummeting to his death. In contrast, Pliny the Elder treats Daedalus as a historic figure, with a known parentage and birthplace.

In one of the earliest references to Daedalus, Homer's epic poem the *Illiad* (written in the seventh or eighth century B.C.) describes a 'cunningly wrought dancing-floor like unto that which in wide Cnosus Daedalus fashioned of old for fair-tressed Ariadne' (Homer 1924: 590). Produced in 415 B.C., Euripides' play *Hecuba* refers to Daedalus's almost godlike power to give life to inanimate objects. Aristotle, in Book I of *Politics* (ca. 330 B.C.), presents Daedalus as a legendary sculptor and Plato in Book III of *Laws*, refers to the great inventions of Daedalus. In Ovid's *Metamorphoses* (ca. 8 A.D.) Daedalus is described as 'an architect of wonderful ability' who 'built with intricate design' (Ovid 1922: 152). In 78 A.D., Pliny the Elder's *Naturalis Historia* commends Daedalus on being the 'first person who worked in wood' (Pliny 1893: 226). Pliny states, 'it was [Daedalus] who invented the saw, the axe, the plummet, the gimlet, glue, and isinglass' (1893: 226). Notwithstanding the obvious fallacy of Pliny's statement (axes existed long before Daedalus is thought to have been born), Horace (Horatius Flaccus), Virgil (Virgili Maronis), William Shakespeare and John Ruskin, amongst many others from antiquity to modern times, have portrayed Daedalus as a master sculptor, inventor and architect.

In mythology Daedalus's most famous work is the Labyrinth at Knossos. Ovid's account of the origins of the maze commences with the unnatural birth of the bull-headed man, the Minotaur. King Minos, seeking to imprison the Minotaur, commissioned Daedalus to design and construct a maze:

> This he planned of mazey wanderings that deceived the eyes, and labyrinthic passages involved. So sports the clear Maeander, in the fields of Phrygia winding doubtful; back and forth it meets itself, until the wandering stream fatigued, impedes its wearied waters' flow; from source to sea, from sea to source involved. So Daedalus contrived innumerous paths, and windings vague, so intricate that he, the architect, hardly could retrace his steps (Ovid 1922: 152).

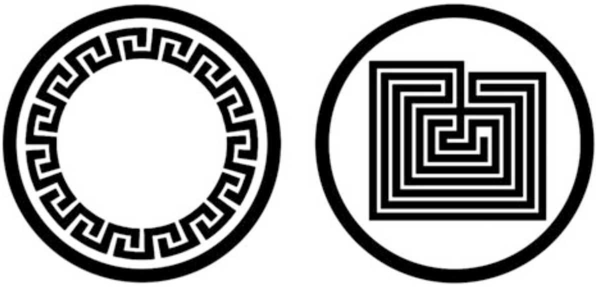

Fig. 3.3 Representation of the Labyrinth at Knossos reconstructed from silver coins (ca. 400 B.C.). Image: Michael Dawes

Thus imprisoned, the Minotaur had to be appeased with the lives of Athenian youths and maidens. This sacrificial rite continued for many years until Theseus, guided through the maze by Ariadne's golden thread, slew the Minotaur and escaped the Labyrinth (Castleden 1990).

Opinion is divided over whether Daedalus built Ariadne's *choros*—an intricate dancing floor—before or after Theseus's escape from the Labyrinth (Ovid 1922; Nichols 1995). Part of the confusion relates to the language used to describe the two designs. Indra Kagis McEwen (1993) has demonstrated that several of Daedalus's inventions share a common etymology. That is, the Greek words used to describe the act of dancing, a patterned dancing floor and a maze are all related to the concepts of weaving or animation. Noting this connection, Kerenyi (1976) gave Ariadne the title 'Mistress of the Labyrinth', a reference not only to the Minotaur's maze, but also to the elaborate formal structure of Ariadne's dance and to its divine or transcendent aspiration. Kern (2000) suggests that the geometry of the dancing floor, itself a symbol of the ritual and possibly erotic conjoining of two bodies, was repeated at larger scale in the Labyrinth, which explains why the two share the same geometric pattern and language. For these reasons, in Greek mythology Daedalus's claim to the title architect is not a result of his ability to oversee the construction of a building, but rather of his capacity to weave geometry into space and form which has both symbolic and phenomenal significance.

In the examples of the *choros* and the Labyrinth, geometry is placed in the service of design in three broad ways, each of which is aligned to one of the Vitruvian triad of architectural qualities. First, it delineates and structures space (firmness): both the dancing floor and the maze are geometrically defined and controlled. Second, it fulfils a program function (commodity): the maze is a geometric structure with a distinct spatial function—to disorientate, restrain, or beguile visitors. The function of the *choros* was to enable the 'crane dance', a tightly constrained marriage ritual. Finally, geometry provides a decorative motif (delight): the geometric weave of the *choros* and the maze has since appeared on coins, reliefs, pottery and in wood carvings (Fig. 3.3).

Homer notes that the Daedalic geometric weave is found in the decoration on Achilles' shield, and Ruskin traces the aesthetic and moral importance of 'Daedalic Right Line' in Gothic architecture (Moore and Ostwald 1997). Like the application of measurement, structure and pattern in the primitive hut, the presence of geometric function, foundation and fascination in the work of Daedalus

Table 3.1 Mathematical applications in the foundation myths of architecture

Application		Myth		
Type	General definition	Primitive hut	Daedalus	Instance
Measurement	The use of mathematics to record and communicate dimensional information	■	□	Sourcing or modifying materials to achieve consistent, relative dimensions
Surveying	The use of mathematics to derive and translate locational or site-related measures	■	□	Information relating to the position and relative spacing of columns and the efficient sourcing or transportation of materials
Performance and prediction	The use of mathematics to inform decisions about structural, acoustic, environmental, visual and related physical properties	■	□	Empirically or intuitively derived estimates of the size of structural members for stability and endurance
Surface articulation	The use of mathematics to achieve an efficient or controlled coverage of a defined plane	■	■	Empirically or intuitively derived methods for achieving a waterproof, or wind-proof woven or thatched surface. The use of geometry to achieve an intricate, patterned surface covering
Generation	The use of algorithms or rules to evolve or parameterise aspects of a design	⦸	■	The Labyrinth is a mathematical construct with a distinct set of geometric and spatial parameters
Inspiration	The use of mathematics as influence, motivation or animation	⦸	■	Both the form of the dancing floor and the Labyrinth are geometric mazes
Aesthetics	The use of mathematics to achieve a particular appearance or visual effect	□	■	The woven path of Ariadne conforms to a pre-determined symmetrical field, within which separate circular and orthogonal patterns reinforce the overall structure
Symbolism and semiotics	The use of mathematics to represent or communicate something about a building	□	■	The geometric decoration of Achilles shield (likened to Daedalus's dance floor) is intended to communicate both a connection to Ariadne and to the heavens
Phenomenality and rationalism	The use of mathematics to evoke a connection by way of the senses or the mind	⦸	■	The geometric path on Ariadne's dance floor evokes and enables a particular physical and sensual ritual—the 'crane dance'

(continued)

Table 3.1 (continued)

Application		Myth		
Type	General definition	Primitive hut	Daedalus	Instance
Logic	The reasoned or disciplined application of knowledge	□	□	Underpinning the majority of the applications of mathematics found in the two myths is the presence of a reasoned and consistent use of information

Key: ■ = application explicit, □ = application inferred, ⃠ = application absent

reinforces the early, mythopoeically delineated set of relationships between architecture and mathematics.

When the two foundation myths are viewed together, they present complementary visions of the role of architecture and of the architect (Table 3.1).

The primitive hut stresses the importance of construction, structure and utility, while the work of Daedalus emphasises aesthetic, inspirational and phenomenal applications. Furthermore, despite their differing emphases, both myths contain references to a larger set of pragmatic and poetic applications. For example, a crucial function of the primitive hut is to shelter a community, both physically and spiritually. Social and cultural concerns are present in this myth, even if its brevity curtails them. The primitive huts described by Vitruvius also possess symmetrical cross sections and plans, something that is especially significant when viewed in the context of the larger body of his theory which uses geometry to evoke divine relations. Similarly, in the Daedalus myth, technical skills are praised along with the ability to work with particular materials. His capacity to measure and survey is also assumed as a basic prerequisite skill of his craft. In addition, while not explicitly stated in either myth, there is an implication that underlying all of the basic actions and decisions is a capacity to think logically and consistently. Thus, the correct size for a rafter in the primitive hut was not calculated, it was determined either empirically (by loading different size beams until structural failure occurred) or intuitively (by using a knowledge of the size of rafters that had worked in the past). For this reason, and despite identifying nine different rudimentary applications of mathematics in these myths, the central role of logic, the tenth type, cannot be ignored.

Finally, it is possible to conceptualize each of these ten types of applications as serving at least one of the core qualities of architecture. For example, if we accept the Vitruvian triad then performance-related applications of mathematics may be associated with firmness and aesthetic applications are related to delight. However, some other types, like measurement, can be mapped to two categories—firmness and commodity—while surface articulation and logic can potentially be used to fulfil parts of all three Vitruvian qualities (Fig. 3.4).

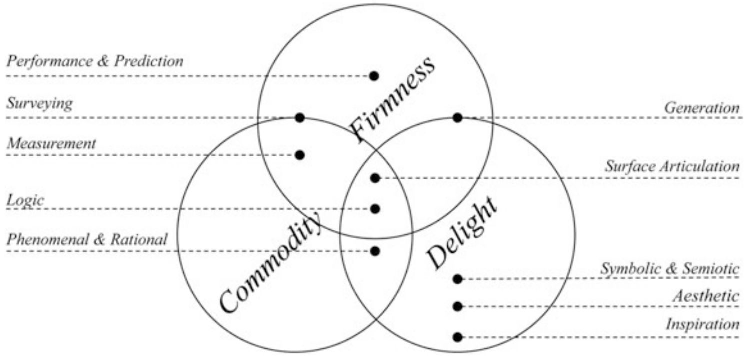

Fig. 3.4 Conceptual mapping of application types against the Vitruvian triad (definitions in Table 3.1)

Models of Architecture: A Teleology

The focus of this section shifts from historic myths to contemporary models. Just as aetiology and teleology have a close, but inverse relationship, so too do myths and models. Ian Barbour defines a myth as an archetypal event that reinforces a pattern of behaviour in society. Myths 'integrate the community around common memories and common goals'; they 'are neither true nor false; they are useful fictions which fulfil these important social functions' (Barbour 1974: 3). The modern counterpart of the myth is the model or paradigm. For Barbour, the model is 'a symbolic representation of selected aspects of the behaviour of a complex system' (3). Whereas a myth describes the world, a model is an 'imaginative tool for ordering experience' (3). The critical difference between myths and models is that the myth derives a universal message from a specific event (thereby relating the particular to the general), while the model starts with a universal system from which a specific response is derived (progressing from the general to the particular) (Coupe 2009). Thus, despite the way models are positioned in contemporary discourse as encapsulating a global truth, they have innate fictional, imagined or conceptual properties that are similar to those of myth. It is also often assumed that the model is more cogently founded in reason, observation or data, but the myth too, represents a body of received wisdom.

The primary role of the architect has historically been, and remains to the present day, the visualization of a design and the communication of this intent, in such a way as to support the construction of a building. The same is also true if the architect's purpose is to refurbish an existing structure, to design a landscape for a park or create a new urban space. While the tools and technologies available to architects have changed over many centuries, the conceptual process of designing and executing a building has remained a surprisingly durable one. There are many subtle variations of this model of the design process, although the majority are conceptualised as an iterative or staged sequence with occasional recursive loops. This model of the design process as a system is found in educational settings

(Pressman 1993; Anderson 2011) and there is evidence that it is used by professionals (Rowe 1987; Lawson 2005; Pressman 2012).[2] Two of the more common variations of the model are framed around cognitive and contractual processes. The cognitive variation commences with problem definition, analysis and synthesis stages, prior to conceptual and schematic diagramming, and finally solution proposition, testing (the recursive loop) and realisation (Pressman 2012). The more contractual or practical variation commences with client briefing, conceptual design, schematic design, developed and detail design, and construction. Several of these steps allow for a limited return to the previous stage to revise or correct any errors which have occurred in the process or to take account of any revisions—to the brief, budget or site conditions—which require a more substantial redesign. More nuanced variations of this contractual model note that there are parallel approval and review processes and that design often continues throughout the construction period and through to post-occupancy evaluation and optimisation. The cognitive variation of the model continues to cycle through the same stages, but with each subsequent series the focus is on a smaller sub-problem within the larger design. Although there are differences between these variations, they both describe a simplified and universal vision of the role of the architect in society. This model, and especially the contractual or practical variation, is useful for identifying the various ways in which architecture uses mathematics.

A necessary precursor to the design process is the production of a design brief, a document which defines the practical and functional limits of a project. The brief typically comprises a list of functional zones, along with information about the scale, critical dimensions and performance criteria. For example, a brief might state that a particular house requires a living room which is at least 15 m^2 in floor area, with a minimum ceiling height of 3.5 m, and with a south-facing wall which is mostly (between 5 and 8 m^2) glass, at least 30 % of which is operable. These measures or conditions are a numeric reflection of the need to accommodate a certain size of social gathering in a space that doesn't feel vertically constrained, is illuminated with natural light, and allows for some natural ventilation.[3]

[2] A common and reasonable concern that has been raised with the standard design process model is that design is not necessarily a linear or systematic process. Design is often characterised as an 'ill-defined' or 'wicked' problem (Brown et al. 2010). Design problems, unlike many mathematical ones, rarely have a single ideal solution. Instead, design involves handling a range of challenges that are described by scientists and engineers as either 'non-trivial' or 'sub-optimal'. Design involves balanced compromise between issues, some of which may be described with great rigour (like structural stability and material strength) while others cannot (like the symbolic power of a building, or the message its iconography communicates to society). This is why the design process model, which may be appropriate for simple or formulaic buildings, is much less useful for more complex building types.

[3] For some complex building types, a much higher level of performance is specified in the architectural brief including lighting levels, acoustic reverberation times and structural bearing capacities. In the last few decades it has also become common for technically advanced buildings, like hospitals, to rely on a relative performance brief. For example, a client might state that a new oncology centre for Rome must function at least as well as the recently completed oncology centre

Once the brief is defined, then the architect engages in a process of parametrically-informed idea generation, wherein he or she seeks to derive a solution to the constraints and opportunities of a brief and a site. This so-called 'conceptual design' stage draws on the architect's ability to manage multiple, sometimes conflicting requirements, simultaneously juggling both relative spatial issues (like the relationship between a living room, a dining room and a kitchen) and absolute ones (like the orientation of the site and the address or access to the building). These interconnecting performance parameters may often be solved in a larger number of alternative spatial configurations and thus the architect must be guided by a vision or set of values, often embodied in a *parti* or organising principle, which assists in determining which conceptual design variations to present to a client. The vision or inspiration for a design remains ever-present throughout the remainder of the project, but its core aesthetic, poetic or representational agenda is typically delineated at this stage, along with possible strategies for achieving this vision. Furthermore, the architect's core values become evident at this point, including the factors driving their design aspiration, from ecological to social, technical and poetic values. Many of these factors involve geometry in an aesthetic, symbolic, semiotic or inspirational role.

Whereas in the concept design stage spatial and contextual relations are described in a topological manner (that is, through connections and relations rather than absolute dimensions), in the schematic design stage, the concept and *parti* of a design are given scale and dimensionality, in accordance with the original brief, along with relative proportions. The first sense of structure and three-dimensional massing (width, depth, height, bulk) is typically tested at this stage, along with an early sense of fenestration and materiality. A preliminary estimate of the cost of the design, typically based on 'square metre' or 'floor rates', is also calculated to determine if the client's brief and budget are viable. For particular building types, the schematic design stage can also include simple modelling and simulation of performance requirements, like the volume of indirect natural light in an art gallery, clear sight lines in a theatre or overshadowing caused by a tall building.

Once the schematic design has been approved, the next stage requires the refinement of its principles. Depending of the building type, the developed design stage can commence with extensive testing and modelling of design variations to optimise important factors (light, security, efficiency, environmental impact) and with each refinement the spatial program evolves while seeking to maintain the topological and geographic relations agreed with the client in the previous stages, but which are now forced to change in response to more detailed design considerations. As the design is finalised, its overarching dimensions and properties are delineated and cost estimates made prior to seeking approval to commence construction.

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in Sydney, but accommodate a 25 % growth in treatment capacity. Such a brief involves both the measuring of the properties of the reference structure and then the interpretation and interpolation of these performance criteria into the new design with increased capacity.

Prior to construction commencing, construction documentation must be produced to translate the design into a system which allows for multiple contractors to undertake the work. At this time, structural engineers complete and certify their designs of columns, beams and bracing, mechanical engineers design services and equipment, and other specialist consultants use a range of mathematical and computational approaches and techniques to determine and specify systems which can be installed during the construction process (Ambrose and Tripeny 2012). Some of the sub-contractors involved in this stage can include pre-fabrication and curtain wall consultants, professionals who extract information from architectural drawings and models to quantify the time it will take to manufacture components, the implications for tolerances and batching (storing pre-fabricated elements prior to construction) and site handling. Meanwhile, the architect often coordinates all of these activities, defining the dimensions and limits for each part of the building.

Several variations of the design process model end with a ‘post occupancy’ stage, in which the completed building is analysed to optimise or assess its performance. Increasingly, theories and techniques have been developed which can be applied to support improved social interaction, wayfinding and security in buildings, amongst other factors. Such mathematical techniques are useful for refurbishment and improvement and also for scholarly analysis. With the rise of global information and positioning systems, data developed from a building may also be applied to much larger models of suburbs or cities (Hilton 2007). Whereas architectural analysis is typically focussed on extracting information from a building so as to better understand its properties, the field of spatial or urban informatics combines information from multiple buildings, transport networks and infrastructure systems to analyse larger regions (Foth 2009).

If the complete set of applications listed in this section are categorised, a set of thirteen types is identified, each of which can be cross-referenced to the stages in the design process model in which they are likely to occur (Table 3.2).

Some application types are concentrated (but not exclusively present) in certain stages. For example, applications of mathematics associated with modularity are typically less important early in the design process, but become more significant in the detail design and construction stages. Modularity may well be a consideration in earlier stages in the design of particular building types, or for architects whose theories rely on systematised construction, but it is more likely to be used in the detail design stage (Kroll 1986). Similarly, the use of mathematics to generate the form of a design is something that is most likely to occur in the concept and schematic stages, and is often indirectly evolved from the brief itself. In certain projects such generative or parametric techniques might continue to be important in the detail design stage as well, but this is less common. Aesthetic and phenomenal considerations are likely to be more prominent in these same early stages (concept and schematic design) and play a lesser, supporting role, in later parts of the process. Viewed in this way, the different types of mathematical applications in

Table 3.2 Mathematical applications in the traditional model of the design process

Application		Stage in the design process model						
Type	General definition	Brief	Concept	Schematic	Developed	Detail	Const.	Post Occ.
Logic	The reasoned or disciplined application of knowledge	■	■	■	■	■	■	■
Measurement	The use of mathematics to record and communicate dimensional information	■	⦸	□	■	■	■	⦸
Surveying	The use of mathematics to derive and translate locational or site-related measures	■	□	□	■	□	■	⦸
Modularity	The use of mathematics for achieving coordination and consistency within a larger system	⦸	□	□	■	■	■	⦸
Performance and prediction	The use of mathematics to inform decisions about structural, acoustic, visual, environmental and related physical properties	⦸	⦸	□	■	■	□	⦸
Surface articulation	The use of mathematics to achieve an efficient or controlled coverage of a defined plane	⦸	⦸	□	■	■	⦸	⦸
Analysis	The use of mathematics to better understand the properties of a design	⦸	⦸	□	■	□	⦸	■

(continued)

Table 3.2 (continued)

Application		Stage in the design process model						
Type	General definition	Brief	Concept	Schematic	Developed	Detail	Const.	Post Occ.
Informatics	The use of mathematics to visualise or characterise architectural, urban and regional spatial and formal properties.	⃠	■	□	⃠	⃠	⃠	■
Generation	The use of algorithms or rules to evolve or parameterise aspects of a design	□	■	■	⃠	⃠	⃠	⃠
Inspiration	The use of mathematics as influence, motivation or animation	⃠	■	■	□	⃠	⃠	⃠
Aesthetics	The use mathematics to achieve a particular appearance or visual effect	⃠	■	■	□	⃠	⃠	⃠
Symbolism and semiotics	The use of mathematics to represent or communicate something about a building	⃠	■	■	⃠	⃠	⃠	⃠
Phenomenality and rationalism	The use of mathematics to evoke a connection by way of the senses or the mind	⃠	■	■	□	⃠	⃠	⃠

Key: ■ = common application, □ = less common application, ⃠ = rare application

the design process can be understood in terms of their shifting potential at various times in the project, rather than as a set of absolute values. This means, for example, that while deriving inspiration from mathematics is something that usually happens early in a process (Hahn 2012), it can also, in certain circumstances, be useful for final detail design decisions (Jencks 1985).

The complete set of application types found in the contemporary design process also include variations of those identified in the review of foundation myths. However, whereas the first group could be readily conceptualised as serving core architectural values, as exemplified in the Vitruvian triad, the second, larger group have taken on a more directed quality. That is, the categories define types by their use or application for different purposes. Thus, while it is possible to map the more extensive set of contemporary types to the categories of firmness, commodity and delight, this is less useful for the applications found in modern architectural practice. For this reason, the following section considers an alternative, triadic framework.

A Framework of Types

There are three overarching categories in the proposed framework for classifying the types of applications of mathematics found in architecture. These categories distinguish between mathematics that is used *for* the support of the design and construction process, that which is visible *in* the design product, and finally mathematics which is a property *of* the design itself. The first of the three categories could be thought of as encompassing all the factors conventionally considered under the Vitruvian rubric 'firmness', as well as some of those associated with the more functional dimensions of 'commodity'. This first part of the framework, mathematics *for* architecture, is related to, amongst other things, stability, function and environmental performance. The second category, mathematics *in* architecture closely correlates to the classic Vitruvian quality, 'delight', and generally comprises aesthetic, sensual or intellectual properties. The final category in the framework has no clear parallel in Vitruvius, although some of the derived properties of 'commodity' in the sense of usefulness or function may resonate with its purpose. The mathematics *of* architecture is concerned with reasoning and analysis about spatial and formal relations present in a design. Analysis, as a stand-alone activity, was uncommon in ancient times but it has since become increasingly important. Collectively the three purposive categories—*for*, *in* and *of*—provide an indicative way of classifying application types (Fig. 3.5).

Importantly though, while the majority of the 13 applications identified in this chapter are aligned to one of the three categories, a few potentially cross between them depending on their purpose or application (Table 3.3).

Furthermore, logic, as a foundation or core value for any reasoned practice, is a member of all three sets, although it has been listed here as part of the analytical

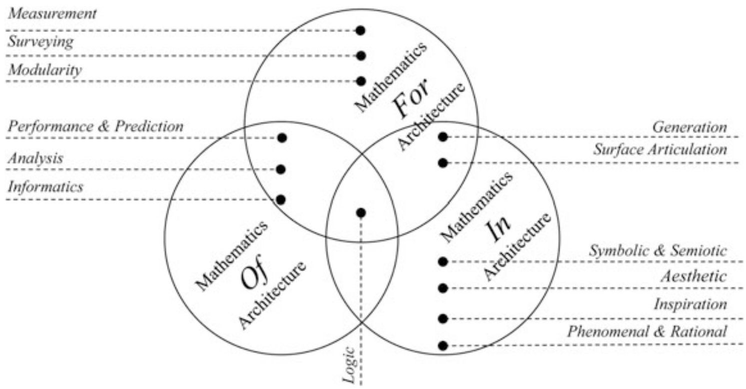

Fig. 3.5 Conceptual mapping of application types against the proposed framework (definitions in Table 3.2)

group of applications. Each of the three categories is described in more detail in what follows.

The first category in the framework—mathematics *for* architecture—includes practical techniques and tools that support architectural design, construction and conservation. These applications occur as a natural part of the design process informing decision-making about functional issues. They enable the development of intrinsic properties of a building that are critical for its stability, environmental performance and programmatic function, but are not necessarily expressed or visually apparent in its final form. This means that the mathematical application itself—for example, the calculation of the depth of a beam or the level of insulation required in a wall—is completed as a precursor to construction commencing. The outcomes of these calculations—the stability of the beam or the wall's capacity to mitigate heat—shape the ongoing function of the building, but the application itself is over and its result is implicit in the finished building rather than explicit. For example, surveying is a critical mathematical technique at certain stages in the design process, but once these stages are completed the active role of mathematics in the process is over. This first category is the most extensive in its list of types, but also the most temporal, because most of these only occur briefly and as part of the design process.

The specific application types that are exclusively allied with this first category include measurement, surveying and modularity. Measurement is associated with the use of mathematics to record and communicate dimensional information, whereas surveying applies and develops such measures in the context of a specific site. Applications of measurement, like those of logic, are so ubiquitous that they are rarely acknowledged in architecture, whereas site-specific surveying techniques have become more specialised over time. Modularity describes those practices that support coordination and consistency in the construction processes.

Table 3.3 A framework of types: mathematics, *in*, *of* and *for* architecture

Primary category	Definition	Type	Examples
Mathematics *for* Architecture	Practical or functional tools or techniques for the support of architectural design, construction and conservation	Measurement—the use of mathematics to record and communicate dimensional information	Theories/Techniques: relative measures, (cubits, rods and rulers), decimal and non-decimal systems, absolute measures (metric)
		Surveying—the use of mathematics to derive and translate locational or site-related measures	Theories/Techniques: levelling, traversing, triangulation and topography, laser scanning, GPS, GIF
		Modularity—the use of mathematics for achieving coordination and consistency within a larger design system	People: Moshe Safdie, Lucien Kroll, Richard Meier Buildings: Habitat (Montreal), Medical Faculty, Louvain University (Brussels), Sainsbury Centre (Norwich)
		Performance and Prediction—the use of mathematics to inform decisions about structural, acoustic, visual, environmental and related physical properties	People: Ove Arup, Renzo Piano, Norman Foster, Future Systems Buildings: Menil Collection (Houston), London City Hall (London)
		Generation—the use of algorithms or rules to evolve or parameterise aspects of a design	People: George Stiny, William Mitchell, Patrik Schumacher Buildings: Water Cube (Beijing), British Museum Atrium (London) Theories: Parametric design, generative design, shape grammar
Mathematics *in* Architecture	Geometric or numeric properties which are demonstrated, visible or sensible in architecture	Aesthetic—the use of mathematics to achieve a particular appearance or visual effect	People: Brunelleschi, Andrea Palladio, Le Corbusier, Hans Van Der Laan. Buildings: Parthenon (Athens), Chartres Cathedral (Chartres), Notre Dame Cathedral, (Paris), Unité d'Habitation (Marseilles) Styles: Medieval, Gothic, Palladian, Renaissance, Mannerism Theories: Golden Section, The Modulor, The Plastic Number

(continued)

Table 3.3 (continued)

Primary category	Definition	Type	Examples
		Symbolic and Semiotic—the use of mathematics to represent or communicate something about a building	People: Leon Battista Alberti, John Ruskin, Robert Venturi, Charles Correa Buildings: Hagia Sophia (Istanbul), Vana Venturi House (Philadelphia), Jawahar Kala Kendra (Jaipur) Styles: Medieval, Gothic, Postmodern
		Phenomenal and Rational—the use of mathematics to evoke a connection by way of the senses or the mind	People: Walter Burley Griffon, Richard Neutra, Louis Kahn, Stephen Holl Buildings: Salk Institute (La Jolla), Casa Del Fascio (Como), Simmons Hall (Cambridge, Mass.) Styles: Rationalism, Organic Modernism, Regionalism
		Inspirational—a use of mathematics as influence, motivation or animation	People: Oscar Niemeyer, Zaha Hadid, Peter Eisenman Buildings: National Congress of Brazil (Brasília), Sydney Opera House (Sydney) Styles: Modernism, Deconstructivism, Generative
		Surface articulation—the use of mathematics to achieve an efficient or controlled coverage of a defined plane	People: Antonio Gaudí, Lab, ARM Buildings: Park Güell (Barcelona), Federation Square (Melbourne), Storey Hall (Melbourne) Theories: Tessellations and Tilings, Aperiodic and Quasi-periodic Tiling
Mathematics *of* Architecture	Logical and analytical methods for quantifying or determining various properties of architecture	Analysis—a use of mathematics to better understand the properties of a design	People: Christopher Alexander, Bill Hillier, Lionel March Theories: Space Syntax, Fractal Analysis, Graph Theory, Fuzzy Theor

(continued)

Table 3.3 (continued)

Primary category	Definition	Type	Examples
		Informatics—the use of mathematics to visualise or characterise architectural, urban and regional spatial and formal properties	People: John James, Michael Benedikt, Michael Batty Theories: Tochymetry, Isovists, GIS mapping
		Logic—the reasoned or disciplined application of knowledge	Theories: Inductive, abductive and deductive reasoning; computational and heuristic reasoning

In a sense, these are specialised routines for relative or rule-based measurement. The remaining application types associated with this primary category can also be found in other secondary categories as well. They include the use of mathematics to predict and optimise the performance of a design, the use of rules or parameters to generate and evaluate design alternatives, and applications of plane-filling geometry (tiling, weaving and patterns). Specific examples of mathematical applications in this category include static and dynamic load calculations for structural stability, computational thermo-fluid dynamics for determining wind load, and thermal conductivity formulas for estimating human comfort levels.

The second of the three categories—mathematics *in* architecture—encompasses geometric or numeric properties that are intentionally designed into and are demonstrated in the form and materiality of a building. This category includes those applications of mathematics that are visible in the architecture, but are not necessarily products of its structure, construction or other functional or performance-related factors. Therefore, this category encompasses applications which augment or supplant those associated with the basic needs for stability and shelter. They could be described as being extrinsic factors because they are integral to the expression of a building, whereas those in the previous category were intrinsic to the function of the building. The type of mathematics that is found *in* architecture is expressed in ways that can be seen, sensed or read in the completed building. It includes the use of mathematics as inspiration for a design, the use of numbers and geometry to perform symbolic or semiotic functions, and properties that can be sensed either intellectually (aesthetic properties) or sensually (phenomenological properties). It is possible, and indeed likely, that the structure of a building will play a role in the expression of mathematics in architecture, but this is not necessarily a factor of the practical performance of that structure (its load-bearing or bracing capacity), but of the meaning or message it conveys visually or perceptually. Thus, while acknowledging that the meaning of symbols changes over time and that inspiration and phenomena cannot be consistently transmitted from a building to a viewer, this category could be understood as pertaining to those mathematical properties of architecture which are enduring or continue to operate after the building is complete. Some specific examples of applications in this category include proportional systems (like the golden mean and the Modulor), symbols which use geometry or number to communicate religious or cultural ideas (the Star of David), rationalist applications of Phileban solids in architecture, and phenomenological uses of geometry to evoke connections to nature.

The final category—mathematics *of* architecture—comprises analytical methods and approaches that are used for quantifying or determining various properties of a completed building or its context. These are mathematically-derived properties, rather than innate ones. They are the by-products of other design decisions which can be understood or modelled mathematically. The types of applications found in this category are focussed on the analysis of information about buildings and cities, for the purpose of understanding or optimising some aspect of a design. This category could also be understood as relating to those mathematical properties of a building that are only apparent when the building is subjected to a methodical investigation using approaches which are not otherwise intrinsic in the design. Examples of this category include space syntax and fractal analysis techniques, isovist analysis and spatial cognition and urban spatial informatics.

The set of types which make up this framework represents a compromise between accuracy and usefulness. At one extreme, it is possible to group almost all of the applications into just two or three categories that broadly correspond to the three overarching groups that are present in the final framework presented here. But, as the examples, tables and diagrams demonstrate, there are multiple overlaps between the three which undermine their utility. At a much finer-grained level, a notably larger list of types was originally identified which separated out multiple specific applications of mathematics, almost a third of which were used for structural and environmental calculations. However, these mathematical and computational approaches have changed over time and with increasing processing power, techniques which were impractical a decade ago, are now in common use. What has not changed is the core intent of all of these applications of mathematics—to ensure that the performance of a part of a building meets a given standard. Thus, a more extensive list of applications was merged into a single type: performance and prediction.

A different challenge was present in the topics of measurement, surveying and modularity. It could be argued that the first two are the same and that the third is simply a specialised application of rule-based measurement. However, measurement, like logic, is part of the base language of architectural design, whereas surveying is, to extend the metaphor, a separate dialect with a specific purpose and application. Modularity is a more contingent type because it is also potentially related to the aesthetic consideration of proportions. Nevertheless, in the pre-fabrication process and as part of a design approach, a separate tradition has developed around the topic of modularity to such an extent that it is worthy of separation from the other types.

The decision to merge the symbolic and semiotic uses into a single type was made to overcome the lack of distinction between them in many applications. For the first of these, some historic uses of symbolic geometry are clear in their application, while in much postmodern architecture, numbers are used as signs and with an understanding of their semiotic and linguistic properties. Nevertheless, for the majority of cases such a segregation is irrelevant because the design is simply called upon to communicate an idea using one technique or another. A similar logic was behind the decision to merge the phenomenal and the rational into a single type. Proponents of phenomenological design tend to deny or understate the role of the mind in responding to architecture and conversely, supporters of rationalist design tend to consider the senses a debased extension of the mind which distracts it from higher thoughts. Despite these differences, both phenomenal and rational approaches rely on geometry and form to elicit either a physical reaction or a mental one. It is their common desire for provocation that binds them together, much as it is impulse to communicate that led to symbolism and semiotics being similarly grouped.

Conclusion

In the thirteenth century, the pair of compasses in the hands of 'God the architect' symbolised the complete set of tools and devices used by designers to translate a vision into reality. The central message—that mathematics serves to translate the imagined into the physical—was reinforced by God the architect's stance, framed by a constructed portal and poised midway between the heavens and the earth. The pair of compasses encapsulates the many different types of applications of mathematics in architecture, with the majority present, in some rudimentary way at least, in even the earliest myths of this discipline. The more extensive set of application types in use today shares a clear lineage to these ancestral cases. The specific formulas used by architects and engineers may have changed, and, amongst other things, their capacity to work with non-orthogonal geometries has also improved, but the fundamental purpose of the application of mathematics in architecture has endured throughout history.

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Kim Williams was a practicing architect before moving to Italy and dedicating her attention to studies in architecture and mathematics. She is the founder of the conference series "Nexus: Relationships between Architecture and Mathematics" and the founder and Co-Editor-in-Chief of the *Nexus Network Journal*. She has written extensively on architecture and mathematics for the past 20 years. Her latest publication, with Stephen Wassell and Lionel March, is *The Mathematical Works of Leon Battista Alberti* (Basel: Birkhäuser, 2011).

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