# Fluid Flow Modelling Through Fractured Soils

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**Abstract** The main goal of this study is to simulate fluid flow across a fractured medium and to visualize its motion as a function of several parameters such as the tortuosity, the inlet pressure and the geometry of the fracture. Using the concept of double porosity, we have developed a hybrid model based on the Finite Element Method. The hydraulic characterization of the medium is realized with a 3D geometry, while the transport process of radionuclides (RNs) through the interface fracture—porous matrix is done in 2D. The results show that the sorption increases when the flow rate decreases. Moreover, an increment in the inlet pressure reduces the residence time of the RNs in the fracture.

## 1 Introduction

Deep geological repository (DGR) systems are needed to isolate residual long-lived radionuclides (RNs) produced by human activity. DGRs are based on the multiple barriers concept, in which the barriers work together to provide containment. The

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natural (or geological) barrier is the host rock formation itself. Since the retention of RNs within the natural barrier delays or prevents RN migration, it can be considered one of the most important safety functions of the deep geological repository (Astudillo 2001).

In crystalline rocks, fractures play an important role in the transport because the permeability of the fractured network is greater than the permeability of the rock and therefore fractures represent a highly effective pathway for transport. However, in most cases the flow occurs through a few preferential pathways which form channel clusters of flow or macro—channels that intercepts between them (Tsang and Neretnieks 1998). Therefore, the permeability of the fractured soil depends on the degree of interconnectedness of the fractures, and should also take into account a dependence on scale, which has been observed from laboratory to field scales (Brace 1980; Clauser 1992). The study of flow in a fractured medium is more difficult than the study of flow in a porous media because of the complex geometry characterizing fractured systems.

Early research work has mainly focused on the plane of the fracture and assumed that the flow through a fracture is similar to that between two parallel plates for which the Navier-Stokes equations can be applied. One of the most relevant results is the expression for calculating the velocity of the fluid in the fracture, which is known as the cubic law. Actually, several conceptual models have been developed for describing fluid flow in fractured porous media. Four concepts have dominated the research: (a) The explicit discrete fracture, (b) dual continuum, (c) discrete fracture network and (d) single equivalent continuum. Each method can be distinguished on the basis of the storage and flow capabilities of the porous medium and the fracture.

The aim of this study is to simulate the fluid flow across a fracture and visualize how its motion is affected when several parameters such as the tortuosity, the inlet pressure and the aperture are modified. The mathematical model is based on the substitution of the mass balance equation by the Darcy's law and the derivation of expressions for the pressure and the velocity in discrete blocks within the system. The pressure-velocity equations are related to the rock and fluid properties. In this model, one of the most important parameters is the "aperture" of the fracture. In order to understand fluid flow in a fractured medium, we need to analyze its motion and take into account the variation of its velocity at the fracture scale depending on the variable aperture and the inlet pressure.

The influence of the tortuosity of the fracture on the flow is studied by using a derivation of the Reynolds equation, also known as the "cubic law" equation. In this relation, it is very important the "aperture" term, which is assumed to be a non-constant function of the *x*- and *y*-coordinates (COMSOL 2008). We use the aperture data by defining an interpolation function, which is used as the aperture in the cubic-law equation.

Another related problem is to understand how the flow of the fluid affects the radionuclides' concentration in provisional or permanent sites for nuclear waste disposal. Since the retention of RNs within the natural barrier delays or prevents RN migration, the geophysical and chemical parameters can be considered as some of the most important safety functions in deep geological repositories.

In this work the conditions that govern the flow through the fracture and the relations governing the matrix-fracture exchanges along the fracture surface are studied. In our simulations, we use a pre-calculated result for the pressure-velocity fields, which is then used in the estimation of the RN transport rate along the fracture and across the porous matrix.

In addition, the water flow rate affects the residence time of the RNs in the fracture (Zimmerman et al. 2002) and their interactions with the rock, as well as the extent of the matrix diffusion. With respect to this problem, several simulations have been performed to establish a comparison between the different inlet pressure values of the fracture and the RNs' transfer rate at the fracture—porous matrix interface.

Using the concept of double porosity, we have developed a hybrid model based on the Finite Element Method and simulated it using the COMSOL software. The hydraulic characterization of the medium is realized with a 3D geometry, while the transport process of radionuclides (RNs) through the interface fracture—porous matrix is done in 2D.

The proposed model contains an interchange layer, where the contaminant travels by diffusion through the porous matrix, and this reduces the RNs' concentration in the aqueous stream of the fracture. The model has been validated through comparisons with analytical solutions (see Souley and Thoraval 2011).

The results obtained indicate that the sorption increases when the flow rate decreases. Moreover, an increment in the inlet pressure reduces the RNs residence time in the fracture.

# 2 The Problem

## *2.1 The Mathematical Model*

Barenblatt et al. (1960) introduced the dual continuum approach which is based on an idealized flow medium which is constituted by a primary porosity or the "solid" matrix and a secondary porosity created by fracturing, jointing or dissolution. The porous medium and the fractures are envisioned as two separated but overlapping continua. This concept implies the definition of two coupled equations, the first one for the flow through the solid matrix and the second one for the flow in the fracture. Darcy's law governs the velocity in the matrix blocks, while the flow in the boundary of a fracture is established by taking into account the fracture's thickness. In this work we consider saturated conditions.

The time-dependent fluid flow in the matrix block is governed by Darcy's law

$$\left[\chi_f \theta_{s+} \chi_s \left(1-\theta_s\right)\right] \frac{\partial p}{\partial t} - \nabla \cdot \left(\frac{k_m}{\eta} \nabla p\right) = 0; \quad \Omega \text{ matrix block}, \tag{1}$$

where the dependent variable $p$ is the fluid pressure in the pore space [Pa] $\theta_s$, is the void fraction, or porosity of the matrix blocks [$m^3/m^3$], $\chi_f$ and $\chi_s$ are the compressibilities [1/Pa] of the fluid and solid, respectively, $k_m$ gives the permeability of the matrix blocks [$m^2$], and $\eta$ is the fluid dynamic viscosity [$Pa \cdot s$]. Here we use a predefined velocity variable that gives the Darcy velocity variable: $\boldsymbol{u}_{esdl} = -\left(\frac{k_m}{\eta}\nabla p\right)$, which is a volume flow per unit area.

In the fracture, we modify the coefficients of the Darcy's law to account for a relatively small flow resistance on the fracture and the fracture thickness:

$$\left(S_{frac}\, d_{frac}\right)\frac{\partial p}{\partial t} - \nabla \cdot \left(\frac{k_{frac}}{\eta} d_{frac} \nabla p\right) = 0; \qquad \Omega\, fracture \tag{2}$$

where $S_{frac}$ is the fracture-storage coefficient [1/Pa], $k_{frac}$ describes the fracture permeability [$m^2$], and $d_{frac}$ is the fracture thickness or aperture [m]. Because the thickness appears in the fracture flow equation, the predefined variable $\mathbf{u}_{\mathrm{esdl}}$ gives the volume flow rate per unit fracture length on the fracture:

$$\mathbf{u}_{\mathrm{esdl}} = -\frac{k_{frac}}{\eta} d_{frac} \nabla_{\mathrm{T}} \mathrm{T} p; \qquad \Omega\, fracture \tag{3}$$

where $\nabla_{\mathrm{T}} \mathrm{T} p$ denotes the gradient operator restricted to the fracture's tangential plane.

Fluids in fractured porous media move quickly through the fractures but also migrate, albeit relatively slowly, through the tiny pores within the surrounding matrix blocks. In this work, the fluid mass transfer between the porous matrix and the fractures occurs at the interface layer. The transfer occurs according to the interface conditions described by dimensionless partition coefficients.

Based mainly on the formulation given in Sudicky and Frind (1982) and Gonzalez-Galan et al. (2013), we consider convection–diffusion within the fracture, here modified to take into account a finite length of fracture, diffusion in the interface layer and convection-diffusion within the porous matrix:

$$\nabla \cdot (-D\nabla C_1 + C_1 \boldsymbol{u}) = 0; \qquad in\ \Omega_{fracture} \tag{4}$$

$$\nabla \cdot (-D_m \nabla C_2) = 0; \qquad in\ \Omega_{interface} \tag{5}$$

$$\nabla \cdot (-D\nabla C_3 + C_3 \boldsymbol{u}) = 0; \qquad in\ \Omega_{matrix} \tag{6}$$

where $C_i$ denotes the concentration of the contaminant (mol/$m^3$) in the respective phases, $D$ denotes the diffusion coefficient ($m^2$/s) in the liquid phases, and $D_m$ is the diffusion coefficient in the membrane, while $\boldsymbol{u}$ denotes the velocity (m/s) in the respective liquid phase.

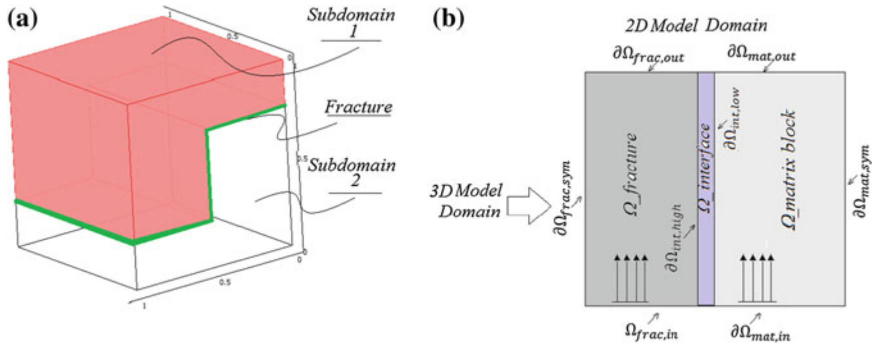


**Fig. 1** Implementation of the suggested mix model in 3D (**a**) and in 2D (**b**)

## 2.2 Set up of the Model

A synthetic example is developed in order to show the resolution methodology for the RNs' transport problem in fractured porous media as is illustrated in Fig. 1.

In the 3D model we study the flow across the fracture and the matrix block. This model consists of a solid block that represents the porous matrix. The fracture in this model is represented by a sequence of interior boundaries. Because the thickness appears in the fracture flow equation, the predefined variable $\mathbf{u}_{\mathrm{esdl}}$ gives the volume flow rate per unit fracture length on the fracture. In this model, we first calculate the pressure and velocity fields of both the fracture and the porous matrix. We then analyze the interchange mechanics between the porous matrix and the fracture with a 2D model.

## 2.3 Boundary Conditions

Along all faces of the block a zero flow boundary condition is applied. The boundaries of the fracture are edges that intersect the porous media block. Conditions in these edges are: at the inlet edge the pressure is constant, $p = p_0$, and at the outlet it decreases linearly with time: $p = p_0 - t \cdot 10\left[\frac{Pa}{s}\right]$. There is no flow through the other edges so that $-\frac{k_{frac}}{\eta} d_{frac} \nabla p = 0$.

For the RNs' transport, the contaminant must be dissolved in the interface layer in order to be transported through it. At the inlet of the model domain, we define the following concentration conditions: $c_1 = c_0$ at the boundary $\Omega_{frac,in}$ and $c_3 = 0$ at the boundary $\partial\Omega_{mat,in}$. At the outlet, we assume that the convective contribution to the mass transport is much larger than the diffusive contribution: $\nabla \cdot (-D\nabla C_i + C_i \boldsymbol{u}) \cdot \boldsymbol{n} = C_i \boldsymbol{u} \cdot \boldsymbol{n}$ at $\partial\Omega_{frac,out}$ and $\partial\Omega_{mat,out}$. Here $\mathbf{n}$ is the normal unit vector to the respective boundary. Furthermore, we assume that there is no transport on the symmetry boundaries: $\nabla \cdot (-D\nabla C_i + C_i \boldsymbol{u}) \cdot \boldsymbol{n} = 0$ at $\partial\Omega_{frac,sym}$ and $\partial\Omega_{mat,sym}$. We

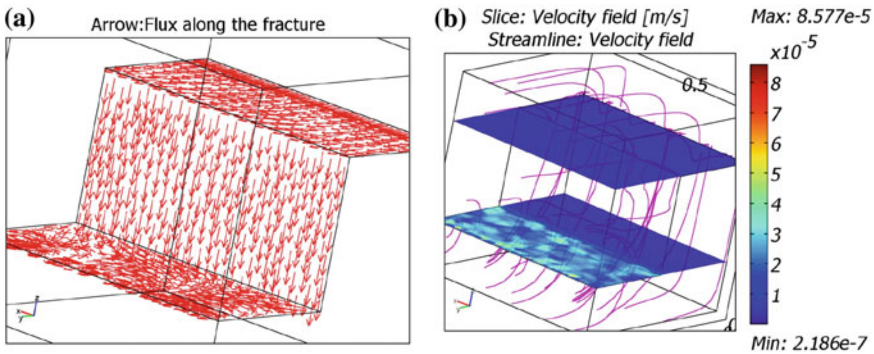


**Fig. 2** **a** Flux along the fracture and **b** flow velocity field through the porous matrix

also assume symmetry at the horizontal boundaries of the interface: $(-\mathrm{D_m}\nabla \mathrm{C_2})\cdot \boldsymbol{n} = 0$ at $\partial\Omega_{int,high}$ and $\partial\Omega_{int,low}$.

The interface conditions between the liquid and membrane phases for the concentration are described by the dimensionless partition coefficient:

$$K = \frac{c_2^{int}}{c_1^{frac}} - \frac{c_2^{int}}{c_3^{mat}}.$$

## 3 Results

In the simulation, the fluid moves from the left to the right through the block, entering at the upper fracture edge and exiting at the lower edge. Initially, the fluid does not move within the volume. The walls of the block are impermeable to the flow except at the fracture edges. The fracture is divided in three sections, the first is the upper one and is located in the z = 0.75 plane, the second section is a vertical plane at y = 0.5, and finally the third section is located in the z = 0.25 plane (see the geometry in Fig. 1). The fracture has a thickness of 0.1 mm in the first two sections. The third section of the fracture has a variable thickness defined by the aperture, which is an interpolation function over a sample data that corresponds to an aperture with a fractal dimension of 2.6.

In panel (a) of Fig. 2 the arrows show the flux along the fracture. In panel (b) we show the velocity streamlines of fluid through the porous matrix. The slice shows the velocity field between the planes z = 0.25 and z = 0.75.

The fracture is far more permeable to the fluid than the matrix block, and the influence of its variable thickness over the velocity field is clearly seen in Figs. 2 and 3. The first two sections have a constant aperture; its velocity field has a regular distribution, while in the third section the velocity field is not uniform because in this section the fracture has a variable thickness.

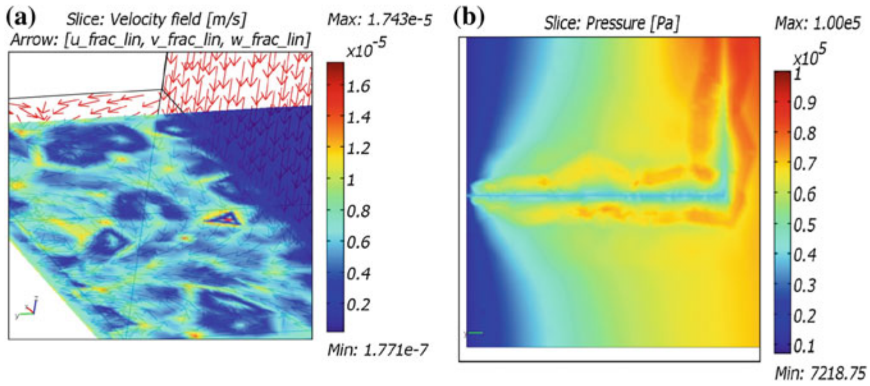


**Fig. 3** Detail of the fracture. The velocity and pressure fields are strongly influenced by the variable thickness of the fracture in this region **a** Enlarged view of the velocity field in the near area to the exit of the fracture. In this zone, is shown the strong influence of the fracture's thickness. **b** Lateral view of the pressure field in the same area next to the out of the fracture

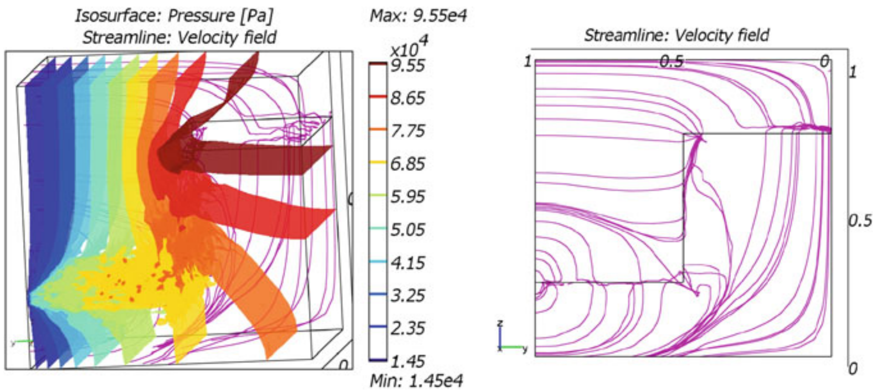


**Fig. 4** In right panel, the isosurfaces show the pressure field in the porous matrix. In left panel, the streamlines are the velocity field through the domain of simulation, in a lateral view

In Fig. 3 we show in detail the region mentioned above. The left panel shows the velocity field while in the right panel we show the pressure from a lateral view of this section. We can see that the pressure field is highly affected by the variable thickness, and accordingly the velocity field (represented by arrows) follow an irregular trajectory according to the variable pressure field estimated.

The isosurfaces depicted in Fig. 4 are pressure contours throughout the block. The pressure is continuous across the fracture from block to block. Even so, the bends in the isosurfaces indicate different flow regimes in the fracture and the matrix blocks. This is observed in the region for a non-constant fracture aperture. The pressure and the velocity fields do not have a uniform structure as is indeed observed in other regions of the system.

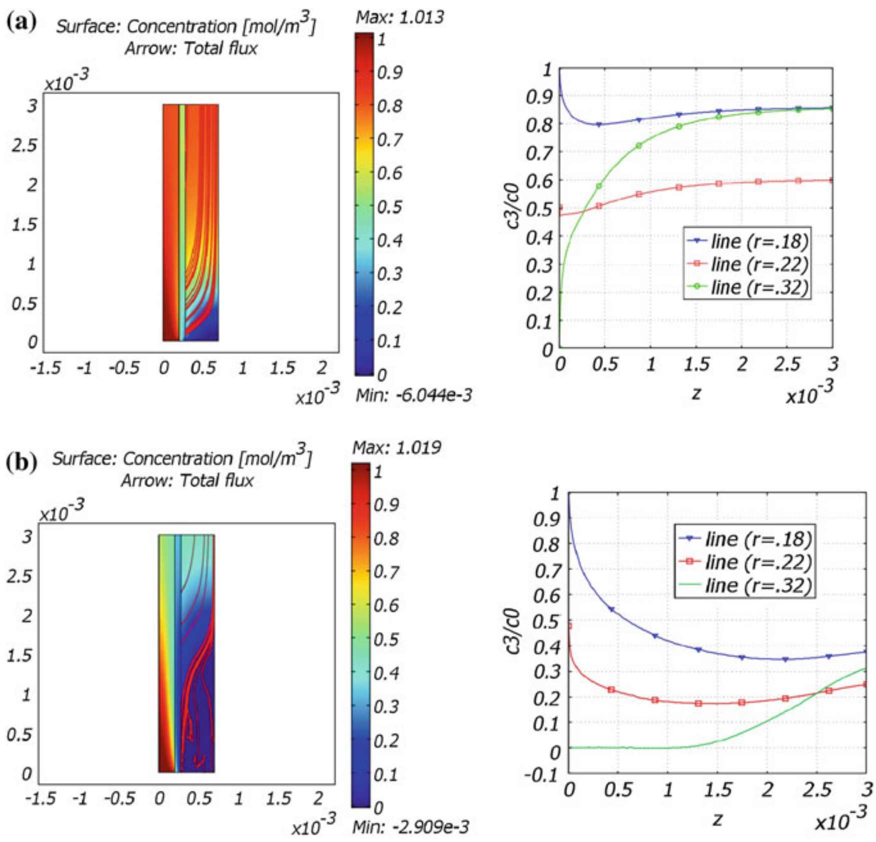


**Fig. 5** Concentration profiles in the fracture interface (*left panels*) and the porous matrix (*right panels*) for **a** $p_0 = 10$ [Pa] and **b** $p_0 = 10^4$ [Pa]

The streamlines in the right panel indicate velocities in the porous matrix. The fluid moves from the inlet to the outlet along the fracture with a velocity field that is uniform across the block. The figure indicates that the linear velocity in the matrix is significantly smaller than the average linear velocity along the fracture.

On the other hand, the surface plot and the plot in Fig. 5 visualize the concentration distribution throughout the three model subdomains: the fracture region inside the model on the left side, the interface fracture-porous matrix in the middle, and the porous matrix to the right. In the left panels of Fig. 5, the concentration profiles in each region are shown (three lines in $r = 0.18$ [mm], $r = 0.22$ [mm] and $r = 0.32$ [mm] which correspond to the three regions mentioned above). The plots in the right panels show the distribution of concentration along each sub-domain. The figure also shows the concentration jump that arises at the boundary between the fracture and the interface membrane. Finally, we can also see that the concentration absorbed by the porous matrix is influenced by the filtration process and the velocity flux.

## 4 Conclusions

In this work, we described a mathematical model for simulating flow in a fractured porous media, using the dual media approach. According to the proposed model and the results obtained, one of the most important parameters is just the fracture aperture.

We have showed how to model the flow in a discrete fracture by considering the interactions between the fluid and the porous matrix. The fracture is defined as a 2D domain within the other 3D domains; this prevents us of using a large number of grid elements along the fracture that reduces the computational time significantly, thus making our method computationally very efficient.

The velocities in the porous matrix are of the order of $5 \times 10^{-7}$ [m/s], while they are $2 \times 10^{-6}$ [m/s] along the fracture. As expected, this result indicate that the linear velocity in the matrix is significantly smaller than the average linear velocity along the fracture. The variation of the aperture parameter in the fracture produced a large variation in the pressure, and therefore also in the velocity. There is enough evidence to conclude that on the fracture surface there is a large variation in the flow resistance due to the variability in the opening and contact areas, which creates tortuous flow lines over the plane of the fracture. As was shown, the concentration inside the fracture decreases markedly over the first sections of the interface from the inlet.

Moreover, the simulations were performed for a wide range of pressures at the inlet of the fracture and the obtained results clearly indicate that for high inlet pressures the radionuclides flow through the fracture with high velocities, which shortens the residence time and reduces the interaction with the porous matrix. Thus, the radionuclides' concentration in the porous matrix gets smaller as the inlet pressure increases.

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