Elastodynamic Doppler Effects by a Moving Interface

Kazumi Watanabe and Naobumi Sumi

Abstract A unified mathematical technique for analyzing a one-dimensional Doppler effects by a moving interface is presented. Exact and closed form expressions for stress waves are obtained. The solution for the stress has no restriction not only for the motion of the interface, but also for the wave nature, impulsive or time-harmonic. As an application example, the Doppler frequency shifts by the uniform and back and forth motions of the interface are discussed for the time-harmonic wave.

1 Introduction

There are two types of the Doppler frequency shift. The one is induced by a moving source and the other by a moving reflector. The latter is named as "scattering" Doppler effects and is called "moving mirror problem" for light and electromagnetic waves. The Doppler frequency shift in electromagnetic and acoustic waves is widely used as the sensing principle, such as laser and ultrasonic velocity meters. The outline of the existing theoretical work for the Doppler effects by the uniformly moving reflector/interface can be found in a relatively updated work by Huang [1]. The Doppler effect by a non-uniformly moving edge of a string has been discussed by Censor[2] and he applied his technique to the Doppler effect for the electromagnetic waves. An approximation technique for the back and forth motion of a mirror has been developed by Van Bladel and De Zutter[3]. In addition to [1], the Doppler

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effects by a uniformly moving electromagnetic interface have also been discussed by Yeh[4], and by Daly and Gruenberg[5] for 2D oblique incident wave. However, the work on the scattering Doppler effects for elastic wave is very scarcely because of its less applicability for solid media. At the present time, the authors can not find any application of the scattering Doppler effects in elastic solid media. But, as the natural extension of academic interest and a hopeful application in the future, the Doppler effect by the moving interface in elastic media is an attractive subject. If the moving interface in the elastic media is considered as a model of the dynamic deformation or phase transformation, the scattering Doppler effects in the solid will play some roles for developing sensing instruments which detect the dynamic deformation.

The present paper develops a unified mathematical technique for analyzing the 1D scattering Doppler effects by the moving interface which separates two dissimilar elastic media. The mathematics developed here is a revised and generalized version of Censors[2]. Our solution is valid for all types of wave form and the interface motion. Applying this general solution to the case of the back and forth motion of the interface, the Doppler effects are discussed for a time-harmonic incident wave.

2 Elastodynamic Scattering Doppler Effect

Let us consider two dissimilar elastic half spaces and take x-axis as shown in Fig. 1. Their interface is moving along the x-axis and is on $x = 0$ at time $t = 0$ and its traveling distance is an arbitrary time function, $l(t)$. We employ the numerical subscripts, 1 and 2, to distinguish two materials and discuss the Doppler effects for the dilatational wave. The 1D dilatational wave field is governed by the equations,

$$\frac{\partial^2 u_x}{\partial x^2} = \frac{1}{c_d^2}\frac{\partial^2 u_x}{\partial t^2}\ , \quad \sigma_{xx} = (\lambda + 2\mu)\frac{\partial u_x}{\partial x}\ , \quad c_d = \sqrt{(\lambda + 2\mu)/\rho}, \qquad (1)$$

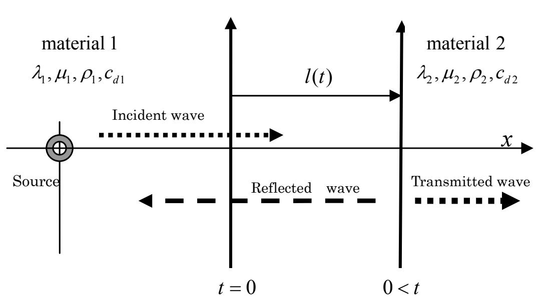


Fig. 1 A moving interface between two dissimilar elastic half spaces.

where u_x and σ_{xx} are the displacement and stress, λ, ν, ρ are Lame's constants and density, and c_d is the velocity of the dilatational wave. Here, we have assumed that *the motion of the interface does not cause any additional disturbances in the wave field* and thus the governing equation (1) holds for all time and the whole region.

We consider an incident wave in the material 1 and it has the arbitrary wave form as

$$u_{x1}^{(i)} = u_0 f\left(t - x/c_{d1}\right) \ , \quad \sigma_{xx1}^{(i)} = -\frac{u_0}{c_{d1}}(\lambda_1 + 2\mu_1) f'\left(t - x/c_{d1}\right), \tag{2}$$

where the arbitrary wave form function $f(z)$ has a single variable z and its derivative is denoted by

$$f'(z) = df(z)/dz \tag{3}$$

In general, as we do not know the frequencies of reflected and transmitted waves in advance, these waves are assumed in the form of Fourier integral with respect to the frequency (This idea is the same as Censors[2]). They are

$$u_{x1}^{(r)} = u_0 \int_{-\infty}^{+\infty} \frac{1}{\varpi} R(\varpi) e^{+i\varpi(t + x/c_{d1})} d\varpi, \tag{4}$$

$$\sigma_{xx1}^{(r)} = \frac{iu_0}{c_{d1}}(\lambda_1 + 2\mu_1) \int_{-\infty}^{+\infty} R(\varpi) e^{+i\varpi(t + x/c_{d1})} d\varpi, \tag{5}$$

for the reflected wave, and

$$u_{x2}^{(t)} = u_0 \int_{-\infty}^{+\infty} \frac{1}{\varpi} T(\varpi) e^{+i\varpi(t - x/c_{d2})} d\varpi, \tag{6}$$

$$\sigma_{xx2}^{(t)} = -\frac{iu_0}{c_{d2}}(\lambda_2 + 2\mu_2) \int_{-\infty}^{+\infty} T(\varpi) e^{+i\varpi(t - x/c_{d2})} d\varpi, \tag{7}$$

for the transmitted wave, where two unknown functions, $R(\varpi)$ and $T(\varpi)$, would be called as the spectrum amplitude.

In order to determine the unknown spectrums, we employ boundary conditions at the moving interface, $x = l(t)$. They are the continuities of the displacement and stress,

$$u_{x1}^{(i)} + u_{x1}^{(r)} = u_{x2}^{(t)} \ , \quad \sigma_{xx1}^{(i)} + \sigma_{xx1}^{(r)} = \sigma_{xx2}^{(t)} \ , \quad x = l(t). \tag{8}$$

Substituting Eqs. (2)-(7) into Eq.(8), we have the coupled integral equations for the spectrum amplitudes,

$$\int_{-\infty}^{+\infty} \frac{1}{\varpi} R(\varpi) e^{+i\varpi\{t+l(t)/c_{d1}\}} d\varpi - \int_{-\infty}^{+\infty} \frac{1}{\varpi} T(\varpi) e^{+i\varpi\{t-l(t)/c_{d2}\}} d\varpi = = -f\left(t - l(t)/c_{d1}\right), \tag{9}$$

$$\int_{-\infty}^{+\infty} R(\varpi) e^{+i\varpi\{t+l(t)/c_{d1}\}} d\varpi + \frac{1}{Z} \int_{-\infty}^{+\infty} T(\varpi) e^{+i\varpi\{t-l(t)/c_{d2}\}} d\varpi = = -if'\left(t - l(t)/c_{d1}\right), \tag{10}$$

where Z is the impedance ratio defined by

$$Z = \frac{\lambda_1 + 2\mu_1}{\lambda_2 + 2\mu_2} \frac{c_{d2}}{c_{d1}} = \sqrt{\frac{(\lambda_1 + 2\mu_1)\rho_1}{(\lambda_2 + 2\mu_2)\rho_2}}. \tag{11}$$

In order to make the same integration form for each spectrum amplitude, we differentiate Eq. (9) with respect to time t,

$$\{1 + M_1(t)\} \int_{-\infty}^{+\infty} R(\varpi) e^{+i\varpi\{t+l(t)/c_{d1}\}} d\varpi - \{1 - M_2(t)\} \times \times \int_{-\infty}^{+\infty} T(\varpi) e^{+i\varpi\{t-l(t)/c_{d2}\}} d\varpi = i\{1 - M_1(t)\} f'\left(t - l(t)/c_{d1}\right), \tag{12}$$

where Mach numbers which are varying with time are defined by

$$M_j(t) = \frac{1}{c_{dj}} \frac{dl(t)}{dt} \; ; \quad j = 1, 2. \tag{13}$$

Then, Eqs. (10) and (12) constitute the algebraic simultaneous equations for the integral of the spectrum amplitude and are solved as

$$\int_{-\infty}^{+\infty} R(\varpi) e^{+i\varpi\{t+l(t)/c_{d1}\}} d\varpi = +i \frac{1 - M_1(t) - Z\{1 - M_2(t)\}}{1 + M_1(t) + Z\{1 - M_2(t)\}} f'\left(t - \frac{l(t)}{c_{d1}}\right), \tag{14}$$

$$\int_{-\infty}^{+\infty} T(\varpi) e^{+i\varpi\{t-l(t)/c_{d2}\}} d\varpi = -i \frac{2Z}{1 + M_1(t) + Z\{1 - M_2(t)\}} f'\left(t - \frac{l(t)}{c_{d1}}\right). \tag{15}$$

The form of the integration in the above equations is very close to that of the Fourier transform, but the exponents of exponential function is slightly different from that of the standard Fourier transform. However, we can find a transform couple for the Fourier transform with non-uniform parameter. That is

$$F(x) = \int_a^b f(\xi) \exp\{+i\xi h(x)\} d\xi \ ; \quad -\infty < x < +\infty$$

$$f(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(x) \exp\{-i\xi h(x)\} h'(x) dx \ ; \quad a < \xi < b \tag{16}$$

where a and b are constants, and $h(x)$ is a monotonically increasing function.

If we assume that the velocity of the moving interface is subsonic for both materials, the two time functions in the argument of the exponential function in Eqs. (14) and (15),

$$T_1(t) = t + l(t)/c_{d1} \ , \quad T_2(t) = t - l(t)/c_{d2}, \tag{17}$$

are monotonically increasing. Then, these functions should be understood as the non-uniform parameter $h(x)$ and apply the transform formula (16) to Eqs. (14) and (15), we have

$$R(\varpi) = \frac{i}{2\pi} \int_{-\infty}^{+\infty} C^{(r)}(t) f'\left(t - l(t)/c_{d1}\right) e^{-i\varpi T_1(t)} T_1'(t) dt \tag{18}$$

$$T(\varpi) = \frac{-i}{2\pi} \int_{-\infty}^{+\infty} C^{(t)}(t) f'\left(t - l(t)/c_{d1}\right) e^{-i\varpi T_2(t)} T_2'(t) dt \tag{19}$$

where $T_j'(t) = dT(t)/dt$, and reflection and transmission coefficients which are not constants are

$$(C^{(r)}(t), C^{(t)}(t)) = \frac{(1 - M_1(t) - Z\{1 - M_2(t)\}, 2Z)}{1 + M_1(t) + Z\{1 - M_2(t)\}}. \tag{20}$$

We have just obtained the spectrum amplitude in the form of Fourier inversion integral. It is no need to evaluate the integral, since this form is preferable for the subsequent treatise. Substituting Eqs. (18) and (19) into the stress wave of Eq. (5) and (7) respectively, and changing the order of integration,

$$-\frac{\sigma_{xx1}^{(r)}}{\lambda_1 + 2\mu_1} = \frac{u_0}{2\pi c_{d1}} \int_{-\infty}^{+\infty} C^{(r)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_1'(\tau) d\tau \times$$
$$\times \int_{-\infty}^{+\infty} e^{-i\varpi\{T_1(\tau) - (t + x/c_{d1})\}} d\varpi, \tag{21}$$

$$-\frac{\sigma^{(t)}_{xx2}}{\lambda_1+2\mu_1} = \frac{u_0}{2\pi c_{d1}} \int\limits_{-\infty}^{+\infty} C^{(t)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_2'(\tau) d\tau \times$$
$$\times \int\limits_{-\infty}^{+\infty} e^{-i\varpi\{T_2(\tau)-(t-x/c_{d2})\}} d\varpi. \tag{22}$$

These inner integrals are easily evaluated by applying the integration formula for the Dirac's delta function,

$$\int\limits_{-\infty}^{+\infty} e^{-i\varpi x} d\varpi = 2\pi\delta(x). \tag{23}$$

Eqs. (21) and (22) yield to the form of the single integral,

$$-\frac{\sigma^{(r)}_{xx1}}{\lambda_1+2\mu_1} =$$
$$= \frac{u_0}{c_{d1}} \int\limits_{-\infty}^{+\infty} C^{(r)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_1'(\tau) \delta\left(T_1(\tau) - (t + x/c_{d1})\right) d\tau, \tag{24}$$

$$-\frac{\sigma^{(t)}_{xx2}}{\lambda_1+2\mu_1} =$$
$$= \frac{u_0}{c_{d1}} \int\limits_{-\infty}^{+\infty} C^{(t)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_2'(\tau) \delta\left(T_2(\tau) - (t - x/c_{d2})\right) d\tau. \tag{25}$$

Further, we apply the integration formula,

$$\int\limits_{-\infty}^{+\infty} \delta(T(\tau) - x) g(\tau) T'(\tau) d\tau = g(T^{-1}(x)). \tag{26}$$

where $T^{-1}(x)$ is the inverse function of $x = T(\tau)$.

Finally, we have the exact closed form solution for the stress wave,

$$-\frac{\sigma^{(r)}_{xx1}}{\lambda_1+2\mu_1} = \frac{u_0}{c_{d1}} C^{(r)}(t_1) f'\left(t_1 - l(t_1)/c_{d1}\right), \tag{27}$$
$$-\frac{\sigma^{(t)}_{xx2}}{\lambda_1+2\mu_1} = \frac{u_0}{c_{d1}} C^{(t)}(t_2) f'\left(t_2 - l(t_2)/c_{d1}\right), \tag{28}$$

where t_1 and t_2 are inverse functions defined by

$$
\begin{aligned}
t_1 &= T_1^{-1}(t + x/c_{d1}) \quad \Leftrightarrow \quad t + x/c_{d1} = T_1(t_1), \\
t_2 &= T_2^{-1}(t - x/c_{d2}) \quad \Leftrightarrow \quad t - x/c_{d2} = T_2(t_2).
\end{aligned} \tag{29}
$$

As for the displacement, Eqs. (18) and (19) are substituted into Eqs. (4) and (6) respectively, and the order of integration is also exchanged. We have

$$
\begin{aligned}
\frac{u_{x1}^{(r)}}{u_0} = +\frac{i}{2\pi} \int_{-\infty}^{+\infty} C^{(r)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_1'(\tau) d\tau \times \\
\times \int_{-\infty}^{+\infty} \frac{1}{\varpi} e^{-i\varpi\{T_1(\tau) - (t + x/c_{d1})\}} d\varpi,
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
\frac{u_{x2}^{(t)}}{u_0} = -\frac{i}{2\pi} \int_{-\infty}^{+\infty} C^{(t)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_2'(\tau) d\tau \times \\
\times \int_{-\infty}^{+\infty} \frac{1}{\varpi} e^{-i\varpi\{T_2(\tau) - (t - x/c_{d2})\}} d\varpi.
\end{aligned} \tag{31}
$$

The integration formula,

$$
\int_{-\infty}^{+\infty} \frac{1}{\varpi} e^{-i\varpi x} d\varpi = \begin{cases} -\pi i\,; & x > 0, \\ +\pi i\,; & x < 0, \end{cases} \tag{32}
$$

is applied to Eqs. (30) and (31). Then, the displacement wave is given in the form of integral,

$$
\begin{aligned}
\frac{u_{x1}^{(r)}}{u_0} = +\frac{1}{2} \int_{\tau = T_1^{-1}(t + x/c_{d1})}^{+\infty} C^{(r)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_1'(\tau) d\tau - \\
- \frac{1}{2} \int_{-\infty}^{\tau = T_1^{-1}(t + x/c_{d1})} C^{(r)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_1'(\tau) d\tau,
\end{aligned} \tag{33}
$$

$$\frac{u_{x2}^{(t)}}{u_0} = -\frac{1}{2} \int\limits_{\tau=T_2^{-1}(t-x/c_{d2})}^{+\infty} C^{(t)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_2'(\tau) d\tau + \\ + \frac{1}{2} \int\limits_{-\infty}^{\tau=T_2^{-1}(t-x/c_{d2})} C^{(t)}(\tau) f'\left(\tau - l(\tau)/c_{d1}\right) T_2'(\tau) d\tau. \tag{34}$$

Consequently, the reflected and transmitted stress waves are obtained exactly. However, it is little bit regrettable that the displacement wave is in the form of integral, not in the closed form. Some applications of this exact solution are shown and the Doppler effects are also discussed in the subsequent sections.

3 Time-Harmonic Wave

When the incident wave is sinusoidal with frequency ω,

$$f(z) = \cos(\omega z)\ , \quad f'(z) = -\omega \sin(\omega z) \tag{35}$$

the stress wave for any motion of the interface is given by

$$\frac{\sigma_{xx1}^{(i)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} \sin\left\{\omega(t - x/c_{d1})\right\}, \tag{36}$$

$$\frac{\sigma_{xx1}^{(r)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} C^{(r)}(t_1) \sin\left[\omega\left\{t_1 - l(t_1)/c_{d1}\right\}\right]\ ,\ t_1 = T_1^{-1}(t + x/c_{d1}), \tag{37}$$

$$\frac{\sigma_{xx2}^{(t)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} C^{(t)}(t_2) \sin\left[\omega\left\{t_2 - l(t_2)/c_{d1}\right\}\right]\ ,\ t_2 = T_2^{-1}(t - x/c_{d2}). \tag{38}$$

Here, the displacement wave is left to the integral form of Eqs. (33) and (34), since the interface motion $l(t)$ is not specified.

Fortunately, the exact expression for the stress gives us a good chance to discuss the Doppler effects. The arguments, so called phase, in the reflection and transmission waves are time-dependent,

$$\Theta_r(t,x) = \omega\left\{t_1 - l(t_1)/c_{d1}\right\}\ , \quad \Theta_t(t,x) = \omega\left\{t_2 - l(t_2)/c_{d1}\right\}. \tag{39}$$

We define the instantaneous frequency for each wave. Differentiating Eq. (39) with respect to time and with aids of the nature of the inverse function defined by Eq. (29), the instantaneous frequency is derived as

$$\omega_r(t,x) = \frac{\partial \Theta_r(t,x)}{\partial t} = \frac{1 - M_1(t_1)}{1 + M_1(t_1)}\omega, \tag{40}$$

for the reflected wave, and

$$\omega_t(t,x) = \frac{\partial \Theta_t(t,x)}{\partial t} = \frac{1 - M_1(t_2)}{1 - M_2(t_2)}\omega, \tag{41}$$

for the transmitted wave, where the non-uniform Mach numbers are defined by Eq. (13). The Doppler frequency shifts, which are thus time-dependent, are

$$\frac{\Delta\omega_r}{\omega} = \frac{\omega_r(t,x) - \omega}{\omega} = -\frac{2M_1(t_1)}{1 + M_1(t_1)}, \tag{42}$$

$$\frac{\Delta\omega_t}{\omega} = \frac{\omega_t(t,x) - \omega}{\omega} = -\frac{M_1(t_2) - M_2(t_2)}{1 - M_2(t_2)}. \tag{43}$$

Then, we readily learn that the Doppler frequency shifts depend only on the Mach numbers and their equation forms are unchanged for any motion of the interface. This guarantees the approximation method [3].

3.1 Uniform motion

When the interface moves uniformly with velocity V,

$$l(t) = Vt, \tag{44}$$

$$M_j = V/c_{dj}\ , \quad j = 1, 2, \tag{45}$$

the coefficients of reflection and transmission are constant,

$$C_r \equiv C^{(r)}(t) = \frac{1 - M_1 - Z(1 - M_2)}{1 + M_1 + Z(1 - M_2)}\ ,\ C_t \equiv C^{(t)}(t) = \frac{2Z}{1 + M_1 + Z(1 - M_2)}, \tag{46}$$

and two inverse functions are expressed exactly,

$$t_1 = T_1^{-1}(\tau) = \frac{\tau}{1 + M_1}\ , \quad t_2 = T_2^{-1}(\tau) = \frac{\tau}{1 - M_2}. \tag{47}$$

Then, the stress wave yields

$$\frac{\sigma_{xx1}^{(i)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} \sin\{\omega(t - x/c_{d1})\}, \tag{48}$$

$$\frac{\sigma_{xx1}^{(r)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} \frac{1 - M_1 - Z(1 - M_2)}{1 + M_1 + Z(1 - M_2)} \sin\left\{\frac{1 - M_1}{1 + M_1}\omega(t + x/c_{d1})\right\}, \tag{49}$$

$$\frac{\sigma_{xx2}^{(t)}}{\lambda_1 + 2\mu_1} = \frac{\omega u_0}{c_{d1}} \frac{2}{1 + M_1 + Z(1 - M_2)} \sin\left\{\frac{1 - M_1}{1 - M_2}\omega(t - x/c_{d2})\right\}, \tag{50}$$

and the Doppler frequency shifts are

$$\omega^{(r)} = \frac{1-M_1}{1+M_1}\omega \ , \quad \frac{\Delta\omega^{(r)}}{\omega} = \frac{\omega^{(r)}-\omega}{\omega} = -\frac{2M_1}{1+M_1}, \tag{51}$$

for the reflected wave, and

$$\omega^{(t)} = \frac{1-M_1}{1-M_2}\omega \ , \quad \frac{\Delta\omega^{(t)}}{\omega} = \frac{\omega^{(t)}-\omega}{\omega} = -\frac{M_1-M_2}{1-M_2}, \tag{52}$$

for the transmitted wave.

3.2 Back and forth motion

When the motion of the interface is back and forth, and its maximum velocity is subsonic for both materials,

$$l(t) = l_0 \sin(\lambda t) \ , \quad M_j^* = \lambda l_0 / c_{dj} < 1 \ , \quad j = 1, 2. \tag{53}$$

Then Mach numbers are periodic functions of time,

$$M_j(t) = M_j^* \cos(\lambda t) \ , \quad j = 1, 2. \tag{54}$$

and the reflection and transmission coefficients, defined by Eq. (20), are also periodic. In this case, two inverse functions $t_j = T_j^{-1}(\cdot)$ have no explicit expressions and we have to obtain t_j numerically, based on their definitions,

$$\begin{aligned} t + x/c_{d1} &= T_1(t_1) = t_1 + (l_0/c_{d1})\sin(\lambda t_1), \\ t - x/c_{d2} &= T_2(t_2) = t_2 - (l_0/c_{d2})\sin(\lambda t_2). \end{aligned} \tag{55}$$

After getting t_j numerically, the periodic Doppler frequency shift and amplitude modulation are given by

$$\frac{\Delta\omega_r}{\omega} = -\frac{2M_1^*\cos(\lambda t_1)}{1+M_1^*\cos(\lambda t_1)}, \tag{56}$$

$$C^{(r)}(t_1) = \frac{1-M_1(t_1)-Z\{1-M_2(t_1)\}}{1+M_1(t_1)+Z\{1-M_2(t_1)\}}, \tag{57}$$

for the reflected wave, and

$$\frac{\Delta\omega^{(t)}}{\omega} = -\frac{M_1^*-M_2^*}{1-M_2^*\cos(\lambda t_2)}\cos(\lambda t_2), \tag{58}$$

$$C^{(t)}(t_2) = \frac{2Z}{1+M_1(t_2)+Z\{1-M_2(t_2)\}}. \tag{59}$$

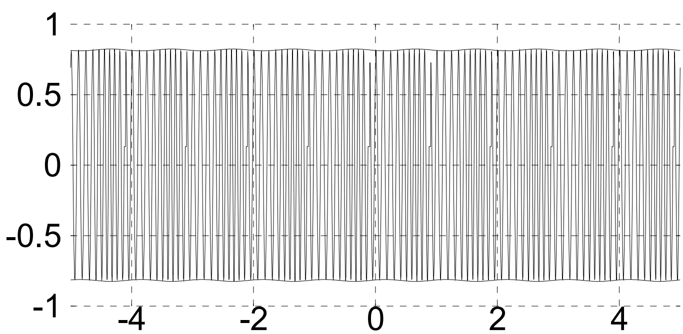
for the transmitted wave.

In the practical use, it is not convenient to obtain the inverse, $t_j = T_j^{-1}(\cdot)$, numerically. But, fortunately, if the Mach numbers are sufficient small, $|M_j| \ll 1$, the inverse functions can be approximated as

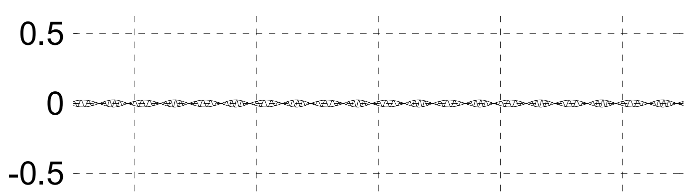
$$\begin{aligned} t_1 &= T_1^{-1}(t + x/c_{d1}) \approx t + x/c_{d1}, \\ t_2 &= T_2^{-1}(t - x/c_{d2}) \approx t - x/c_{d2}. \end{aligned} \tag{60}$$

Then, we will have exact expressions for frequency shifts and the stress response.

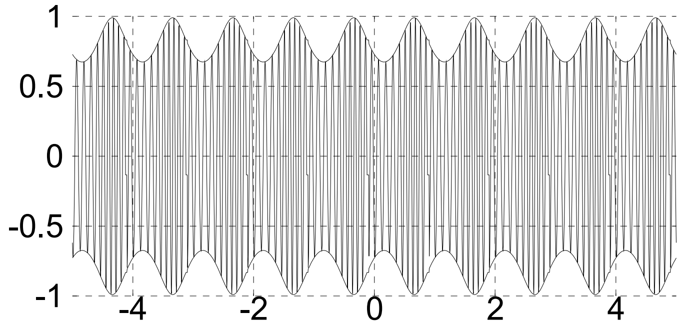
Figure 2 shows a typical time-response of the reflected stress wave for three impedance ratios. The computations are carried out based on the numerical inversion for the inverse function. The amplitude of the high frequency carrier wave is modulated and its amplitude modulation becomes clearer especially in Fig. 2(c). However, the frequency modulation which is defined by Eq. (56) is less visible, since the Mach number $M_1^* = 0.1$ is so small. The amplitude modulation is caused by the periodic change of the amplitude equation (20) with its frequency λ of the back and forth motion. Thus, this amplitude modulation may be a key signal for detecting the interface motion.



$(a)\ Z = 10$



$(b)\ Z = 1$



$(c)\ Z = 0.1$

Fig. 2 Typical wave forms for reflected stress wave $(\lambda_1+2\nu_1)^{-1}(c_{d1}/\omega u_0)\sigma^{(r)}_{xx1}$. $(l_0\omega/c_{d1} = 1\ ,\ l_0\lambda/c_{d1} = 0.1\ ,\ c_{d1}/c_{d2} = 0.5\ ,\ x/l_0 = -10)$

4 Conclusion

A unified mathematical technique for the 1D elastodynamic Doppler effect by the moving interface has been developed. The exact closed form solution obtained is valid not only for the arbitrary interface motion, but also for every wave form. The solution is applied to the case of the standard uniform motion, and of the back and forth motion. It is shown that the amplitude modulation takes place when the interface motion is periodic, and reflected and transmitted waves include not only the Doppler frequency shit, but also amplitude modulation. The frequency of the amplitude modulation is the same as that of interface motion. These amplitude and frequency modulations may be useful information for developing a motion sensor for detecting the dynamic deformation of solids.

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