Seismic Performance of Tuned Mass Dampers

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Abstract In a fundamental parametric study the seismic performance of Tuned Mass Dampers (TMDs) is investigated. Earthquake excited vibration-prone structures are modeled as elastic single-degree-of-freedom oscillators and they are equipped with a single TMD. The TMD performance is assessed by means of response reduction coefficients, which are generated from the ratio of the structural response with and without TMD attached. It is found that TMDs are effective in reducing the dynamic response of seismic excited structures with light structural damping. The results of the presented study are based on a set of 40 recorded ordinary ground motions.

1 Introduction

New technologies and refined methods of analysis permit the design and construction of more slender, and hence, in many cases more vibration-prone structures with rather light damping. One effective measure to protect buildings against excessive large vibration amplitudes is the installation of Tuned Mass Dampers (TMDs). A TMD is a control device with a single-degree-of-freedom (SDOF) of either mass-spring-dashpot type, or a pendulum-dashpot system. The Tuned Liquid Column Damper (TLCD) is a variety of the TMD, which is based on the same mode of operation [1].

The natural frequency of the TMD is tuned closely to the dominant mode of the vibration-prone structure. Thus, the kinetic energy is transferred from

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the vibrating main structure to the TMD, where it is subsequently dissipated by its viscous element. Optimal design of TMDs is discussed in various papers, e.g. [2 - 6].

TMDs have been proven to be effective in reducing the dynamic response of structures induced by narrow-band periodic excitation such as wind and traffic loads. However, the effectiveness of TMDs to mitigate earthquake induced vibrations is still a topic of controversial discussion. For example, [7] reports about weak seismic performance of TMDs with very small mass ratios. On the other hand, in [6] it is shown that for a large mass ratio TMDs become very effective in minimizing the structural response.

In this paper the seismic performance of TMDs, i.e. their effectiveness and robustness, is assessed. The presented parametric study of SDOF structures covers a wide range of structural periods between $0.05s$ and $5.0s$, and mass ratios between 2% and 8%. The results are based on a set of recorded ordinary ground motions.

2 Applied Procedure

2.1 Mechanical Model

A SDOF oscillator with mass M, stiffness K and viscous damping coefficient R (or expressed alternatively by the non-dimensional damping coefficient ζ_S) is utilized to represent a vibration-prone structure. The base acceleration $\ddot{x}_g$ induces structural vibrations, which are characterized by the displacement X of mass M with respect to the base. To this SDOF system a TMD is attached, which is itself a SDOF oscillator with mass m, stiffness k, and damping r (or ζ_T, alternatively). The relative displacement x of mass m is related to the base. Together, structure and TMD form a non-classically damped system with two-degrees-of-freedom (displacements X and x). An example of a structure-TMD system is shown in Figure 1.

2.2 Seismic Excitation

For this study a set of "real" earthquake records is employed to excite the structural model. This set of ordinary ground motion records, denoted as LMSR-N, contains 40 ground motions recorded in Californian earthquakes of moment magnitude between 6.5 and 7 and closest distance to the fault rupture between 13 km and 40 km on NEHRP site class D according to FEMA 368, 2000, [8]. This set of ordinary records has strong motion duration characteristics, which are not sensitive to magnitude and distance. A statistical
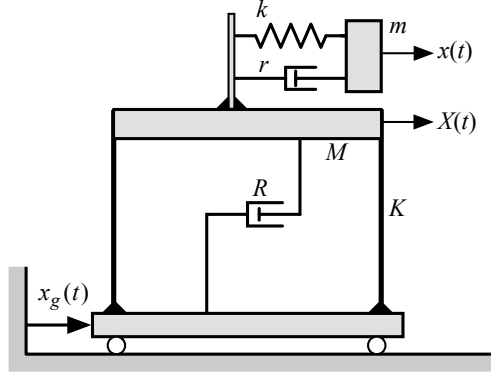
Fig. 1 Mechanical model of structure-TMD system subjected to seismic excitation

evaluation of this bin of records and its detailed description are provided in [8].

2.3 Applied Tuning Procedures

The effectivity of TMDs to mitigate the dynamic structural response depends on appropriate, or better, "optimal" tuning of its parameters, i.e. the natural frequency ω of the decoupled TMD expressed by the frequency ratio δ

$$\delta = \frac{\omega}{\Omega} \quad , \quad \omega = \sqrt{\frac{k}{m}} \quad , \quad \Omega = \sqrt{\frac{K}{M}} \tag{1}$$

and the damping ratio ζ_T [2]. In Eq. (1) Ω denotes the natural frequency of the structure without TMD. Assuming that ordinary earthquake excitation can be approximated with sufficient accuracy by a stationary white noise random process the appropriate structural response quantity to be minimized is the variance σ_x^2 of the structural displacement X. The variance σ_x^2 is related to the constant white noise spectral density S_0 by [3]

$$\sigma_x^2 = E\left[x^2(t)\right] = S_0 \int\limits_{-\infty}^{\infty} |H(\nu)|^2 \, d\nu \tag{2}$$

$H(\nu)$ is the complex frequency response function [9],

$$H(\nu) = \frac{\mu + Z(\nu)}{\mu \left(1 - \alpha^2 + 2i\zeta_S\alpha\right)} \alpha^2 \nu^2 \quad , \quad Z(\nu) = \frac{\delta^2 + 2i\zeta_T\alpha\delta}{\delta^2 - \alpha^2 + 2i\zeta_T\alpha\delta} \tag{3}$$

where μ is the mass ratio and α an excitation frequency ratio,

$$\mu = \frac{m}{M} \quad , \quad \alpha = \frac{\nu}{\Omega} \tag{4}$$

Mathematically, the optimization of the TMD parameters requires a performance index J_0, which complies with σ_x^2 [4],

$$J_0 = S_0 \int_{-\infty}^{\infty} |H(\nu)|^2 \, d\nu \tag{5}$$

Subsequently, J_0 is minimized with respect to δ and ζ_T. For an undamped main structure ($\zeta_S = 0$) the optimization procedure leads to analytical expressions for the TMD parameters [3], which depend on the mass ratio μ only,

$$\delta_{opt} = \frac{\sqrt{1 - \mu/2}}{1 + \mu} \quad , \quad \zeta_{Topt} = \sqrt{\frac{\mu\,(1 - \mu/4)}{4\,(1 + \mu)\,(1 - \mu/2)}} \tag{6}$$

In this study the parameters of the TMDs are optimized also utilizing recorded earthquake motion records. Since ordinary ground motions are random processes with in general wide-banded frequency content the performance index according to white noise excitation is utilized for the optimization procedure. However, the actual spectral density $S_i(\nu)$ of the considered record, which is a function of frequency ν, must be employed. Thus, the performance index reads [6]

$$J_i = \int_{-\infty}^{\infty} |H(\nu)|^2 \, S_i(\nu) d\nu \tag{7}$$

For a given structure, and a given ground motion record this performance index is used to determine the optimal tuning frequency and optimal viscous damping coefficient of the TMD. The procedure is repeated for all 40 records. Subsequently, the median TMD parameters of the 40 individual optimized TMD parameters are employed to derive the structural response.

2.4 Representation of Outcomes

The effectiveness of the optimized TMD is presented by means of so-called response reduction coefficients. Two types of response reduction coefficients are defined: The response reduction coefficient $R_{m,i}$ is the ratio of the structural peak displacement with attached TMD to the structural peak displacement without TMD induced by the ith earthquake record, while $R_{\sigma,i}$ is generated from the ratio of the displacement standard deviation with TMD to the

displacement standard deviation without TMD,

$$R_{m,i} = \frac{\max |X_i|_{TMD}}{\max |X_i|_{NOTMD}} \quad , \quad R_{\sigma,i} = \frac{\sqrt{\int X_i^2 dt}\Big|_{TMD}}{\sqrt{\int X_i^2 dt}\Big|_{NOTMD}} \tag{8}$$

The response reduction coefficients for all 40 records are evaluated statistically. In particular, their medians R_m and R_σ are utilized to assess the TMD performance.

3 Assessment of the Seismic TMD Performance

In the following the results of parametric studies involving a series of structure-TMD systems are discussed. Thereby, each system is characterized by the natural period T_S of the stand-alone main structure, $T_S = 2\pi/\Omega$, and the mass ratio μ. After finishing all simulations for a particular structure another system with different T_S and μ is examined. The period T_S is changed stepwise with increments of $0.05s$, starting from $0.05s$ up to $5.0s$: $0.05s \le T_S \le 5.0s$. I.e. very stiff to soft structures are covered by the considered periods. The range of mass ratios, $0.02 \le \mu \le 0.08$, correlates to the mass ratios of practically applied TMDs.

In Figure 2 the response reduction coefficients R_m and R_σ are depicted as a function of structural period T_S and mass ratio μ. Viscous damping of the main structure is selected to be 1% ($\zeta_S = 0.01$). TMDs of this parametric study are tuned according to the assumption of white-noise ground acceleration, compare with Eq. (5). The median response reduction coefficients R_m shown in Figure 2(a) reveal that the median peak displacements are reduced for all combinations of T_S and μ, since R_m is always smaller than 1. A reduction from 10% to 40% can be observed. As expected the response diminishes with increasing mass ratio. Furthermore it can be seen that the effectiveness of TMDs is better for short period structures than for long period systems.

The response reduction coefficients R_σ of displacement standard deviations are plotted in Figure 2(b). They exhibit values between 0.35 and 0.65. These results demonstrate that for this set of earthquake records TMDs are capable to reduce the vibration amplitudes of seismic excited structures.

Figure 3 shows the distribution of response reduction coefficients R_m and R_σ for the same set of main structures. However, tuning of the attached TMDs is based on the optimization procedure including the actual earthquake records, see Eq. (7). Comparison of these outcomes with the results of Figure 2 reveals that the influence of the applied tuning procedure on the performance of TMDs is of negligible magnitude, since the median TMD parameters instead of the individual optimized TMD parameters are employed. Thus, for this study simplified tuning of the TMD parameters for station-

ary white noise base acceleration is justified, although real recorded ordinary ground motions induce structural vibrations. Subsequently, the control effec-
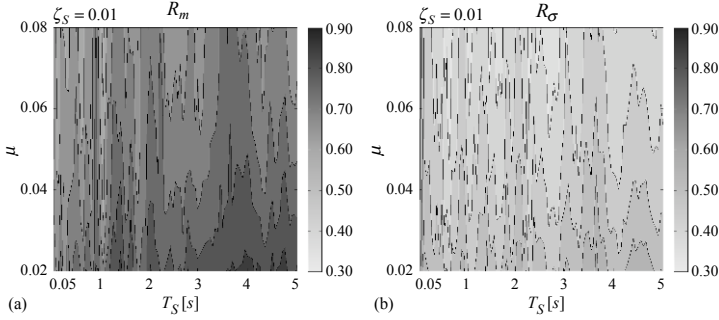

Fig. 2 Response reduction coefficients, optimal TMD tuning assuming white noise base excitation: (a) peak displacement, (b) standard deviation of displacement
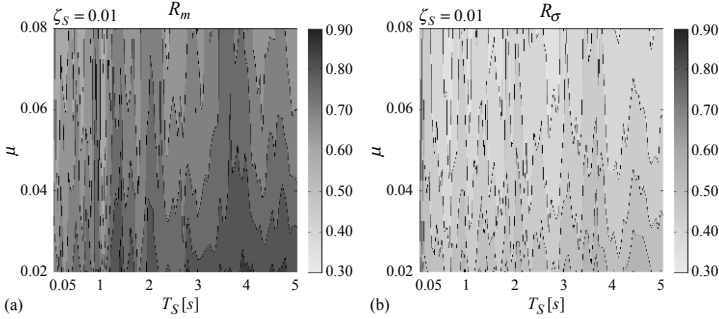

Fig. 3 Response reduction coefficients, optimal TMD using actual seismic ground motions records: (a) peak displacement, (b) standard deviation of displacement

tiveness of TMDs for structures with heavier damping is discussed. Viscous structural damping of the main structure is increased to 3% ($\zeta_S = 0.03$). TMD parameters are optimally tuned for white noise ground acceleration. Figure 4 verifies that the effectiveness of TMDs declines for main structures with heavier structural damping. For the considered mass ratios and structural periods the peak displacements can be reduced at most up to 30% (i.e. a R_m of 0.70), but in average not more than 15% to 20%, compare with Figure 4(a). For the standard deviation of displacements a reduction of up to 45% can be achieved (i.e. a R_σ of 0.55). Eventually, the robustness of the seismic TMD performance to uncertainty in its parameters is studied. Exemplarily, a structure-TMD system with the following properties is considered: Mass
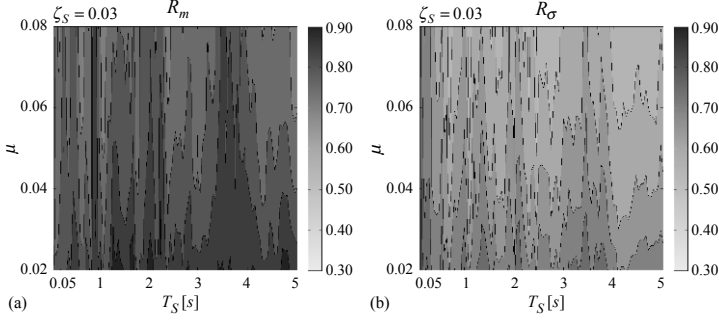
Fig. 4 Response reduction coefficients, optimal TMD tuning assuming white noise base excitation: (a) peak displacement, (b) standard deviation of displacement
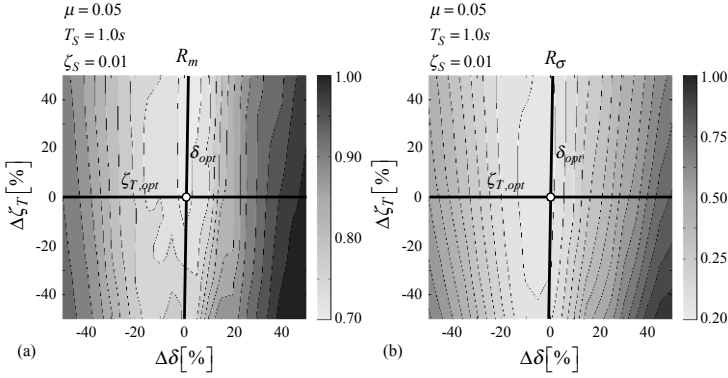

Fig. 5 Response reduction coefficients, effect of mistuned TMD parameters: (a) peak displacement, (b) standard deviation of displacement

ratio $\mu = 0.05$, period of the decoupled main structure $T = 1.0s$, damping of the main structure $\zeta_S = 0.01$. Tuning of the TMD utilizing the performance index J_0, Eq. (5), leads to the following optimal TMD parameters: $\delta_{opt} = 0.935$, $\zeta_{T,opt} = 0.110$. The response reduction coefficients R_m and R_σ are determined for this optimal TMD configuration. Subsequently, the frequency ratio and the damping coefficient are stepwise mistuned from -50% to 50% compared to the corresponding optimal value. For each mistuned system the response reduction coefficients are plotted as a function of the deviation from optimal conditions. The results are visualized in Figure 5, where a horizontal black line refers to results based on the optimal damping coefficient $\zeta_{T,opt}$, and the vertical black line highlights response reduction coefficients derived utilizing the optimal frequency ratio δ_{opt}. The intersection point of both lines identifies results for optimal tuning parameters. Both, R_m (Figure 5(a)) and R_σ (Figure 5(b)) reveal that the seismic TMD performance

is robust with respect to mistuning of the TMD damping coefficient ζ_T as long as δ is optimal. However, mistuning of δ has a grave effect on the TMD effectiveness. In particular, if δ is much larger than δ_{opt} the TMD is not able to function properly. However, mistuning of δ less than 3% can be accepted for this particular system and set of ground motions.

4 Conclusion

The results presented in this study suggest that the application of Tuned Mass Dampers (TMDs) with mass ratios between 2% and 8% is an appropriate measure to mitigate the dynamic response of structures subjected to ordinary seismic ground motions. This statement applies both for stiff and soft structures. The seismic effectiveness of an optimally tuned TMD decreases with increasing initial structural damping of the vibrating structure. Reviewing the results obtained in this study reveals that optimal tuning of the TMD parameters under the assumption of white noise base acceleration is sufficiently accurate. The seismic performance of the TMD is robust against mistuning of the viscous element in the TMD. However, accurate tuning of the TMD natural frequency is essential for its effectiveness.

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