An Identification Method of the Time Dependence of the Impact Force by Using Acoustic Response and FEM Analysis

Tomoaki Tsuji, Takafumi Kurimoto and Toshikazu Shibuya

Abstract The radiated sound from the impacted body must have the information with respect to the impact force. We have proposed the method in order to identify the impact force by analyzing the radiated sound from the impacted body. Normally the impact position is unknown and important to measure the impact position. Therefore, in this study, we propose the method to identify the impact position and force by using the radiated sound from the impact body. In the present method, the relationship between the impacted force and sound pressure is obtained by FEM simulation. In order to identify the impact position, the sound pressure, which is measured at the other position, is used. The efficiency of the present method is confirmed by using the many experiments of the plate as the impacted body.

1 Introduction

A lot of studies that identify the impact force have been reported. For instance, Inoue et al. [1-3] identified the impact force by using the strain response caused in the elastic body. In this method, the relation between the known impact force and the strain response is requested, in order to determine the transfer function between the impact force and the strain response. On the other hand, the radiated sound from the impacted body must have

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the information with respect to the impact force. If the impact force is identified by measuring the radiated sound, measurement will be made leaving from the impact body, that is non-contact measurement. Author et al. [4, 5] proposed the identify method of the impact force by measuring the radiated sound. In this method, it is necessary to measure the impact force directory before the identification experiment, in order to obtain the transfer function between the impact force and the radiated sound pressure. However, it is difficult to measure the impact force directory. Authors proposed the method to identify the impact force without the preliminary experiment [6]. In this method, the relation between the known impact force and the radiated sound is given by using FEM simulation.

Normally the impact position is unknown, especially in the case of the impact from the flying object. But, it is difficult to investigate the impact position. Therefore, in this study, we propose the method to identify the impact position and force by using the radiated sound from the impact body. In the present method, the relation between the impacted force and sound pressure is obtained by FEM simulation. By this relationship, the time dependence of the radiated sound pressure and impact force can be shown by series form. The unknown coefficient is determined by the least square method using the measured sound pressure. In order to identify the impact position, the sound pressure, which is measured at the other position, is used. The efficiency of the present method is confirmed by using the many experiments of the plate as the impacted body.

2 Experimental set up and measurement

In this study, we propose the method to identify the placement of the impact and the impact force by using the sound pressure measured at different place. Figure 1 shows the outline of the measurement system. The radiated sound from the impacted body is measured by the two microphones, and recorded by the digital oscilloscope. Moreover, the strain data by the strain gauge is recorded. The sound and strain values are transferred into the computer and analyzed. Figure 2 shows the configuration of the impact body

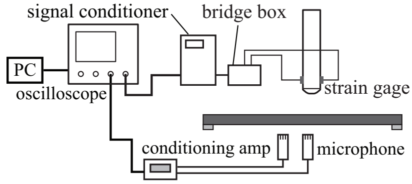

Fig. 1 The measuring system of the sound and the strain.

as the aluminum bar (ϕ30mm$\times$300mm) and the impacted body as the aluminum plate (201mm×501mm×10mm). The impact bar is freely dropped from height $H$and impacts to the plate at the point x as shown in the figure. Two microphones are in set with the distance 40mm and under the plate with height h. The contact area between the impact bar and the impacted plate approximately the circular rejoin with 10mm diameter. The impacted plate is supported by the silicon rubber seats (20mm×20mm×10mm) under the four corners of the plate. The impact bar falls to the plate through the plastic cylinder of the inside diameter 31mm. The strain gauges are attached on the side of the impact bar at 20mm from the under side of the bar as shown in Fig.2. The impact force can be measured directly by these strain gauges. The

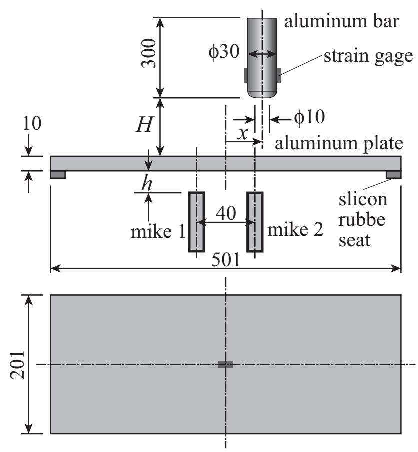

Fig. 2 The configuration of the impact bar and the aluminum plate.

radiated sound pressure is measured by the microphone. The Type and the configuration and the performance of this microphone are listed in Table 1. The microphone is calibrated to output the voltage, which is linear to the sound pressure. The actual measurement element of the sound pressure is in the interior from the surface cap of the microphone. By the preliminary experiment, we decided this actual measurement element is in 5mm from the front surface of the microphone.

If the stress distribution in the section of the impact bar is assumed as

Type	4190
configuration	ϕ12.7mm×17.6mm
bandwidth of the frequency	3 ∼ 20kHz
company	*Bruel & Kjaer*

Table 1 The type and the configuration and the performance of the microphone.

uniform, the time dependence of the impact force $P_{bar}(t)$ at the impact bar can be given by the strain $\varepsilon_{bar}(t)$ at the side of the bar as follow.

$$P_{bar}(t) = E_{bar}\varepsilon_{bar}(t)\frac{\pi}{4}d_{bar}^2, \tag{1}$$

with E_{bar} : the modulus of the longitudinal elasticity and d_{bar}: diameter of the impact bar.

This impact force $P_{bar}(t)$ should equal to the time dependence of the impact force on the plate surface. Thus, we will use these data as the actual measured values of the impact force in order to confirm the identified impact force by the present method.

3 The identification method of the impact force by using the radiated sound and FEM analysis

It is common using the stepwise function as input data, in order to get any response. But, it is difficult to get good accuracy in FEM calculation because of discontinuity. Thus, the following impact force $P_{sim}(t)$ respect with time t is introduced as the input impact force.

$$P_{sim}(t) = P_0\left[\frac{1}{2}\left\{1-\cos(\pi\frac{t}{T})\right\}H(t)H(T-t)+H(t-T)\right], \tag{2}$$

with T : interval time, P_0: magnitude of the impact force and $H(t)$: Heaviside's step function.

The time dependence of the radiated sound pressure $p_{sim}(t)$ for the given impact force$P_{sim}(t)$ can be calculated by FEM analysis. Then, the time dependence of the arbitrary impact force $P(t)$ and sound pressure $p(t)$ can be given by using these functions $p_{sim}(t)$ and $P_{sim}(t)$ as follows.

$$P(t) = \sum_{n=1}^{N} A_n P_{sim}(t+t_n) \quad , \quad p(t) = \sum_{n=1}^{N} A_n p_{sim}(t+t_n), \tag{3}$$

with $t_n = \Delta t \times (n-1)$, Δt: time interval and A_n: unknown coefficient.

The measured sound pressure by the mike 1 is used to identify the impact force $P(t)$, and the one by the mike 2 is used to identify the impact position x. The time dependence of the measured sound pressure is denoted as $p_{exp1}(t)$ and $p_{exp2}(t)$. The square error E_{error1} between the measured values $p_{exp1}(t)$ and the identified values $p(t)$ in Eq.(3) can be shown as follow.

$$E_{error1} = \sum_{i=1}^{N\max}\left\{\sum_{n-1}^{N} A_n p_{sim1}(t_i+t_n) - p_{\exp 1}(t_i)\right\}^2, \tag{4}$$

with t_i(i=1,2. . . N_{max}): time when the sound pressure is measured and N_{max}: number of values

The unknown coefficient A_n can be given by the least square method as the solution of the following simultaneous linear equations.

$$\sum_{n=1}^{N}\sum_{i=1}^{N\max} A_n p_{sim1}(\tau_i + t_n) p_{sim}(\tau_i + t_m) = \sum_{n=1}^{N} p_{\exp}(\tau_i) p_{sim}(\tau_i + t_m) \qquad (5)$$

If the values of A_n are obtained by Eq.(5), the time dependence of the impact force $P(t)$ for the sound pressure $p_{exp1}(t)$ can be identified by Eq.(3). However, the sound pressure $p\ (t)$ and force $P\ (t)$ are respect with the impact position x, that is the value of x must be known before calculation. Once x is expected as $x = x^{*}$, the values of A_n can be given by solving Eq.(5). Moreover, the impact force can be given by Eq.(3). For this impact force, we can calculate the sound pressure at the position of the mike 2 by using Eq.(3). The square error of this expected sound pressure $p_{sim2}(t)$ and measured one $p_{exp2}(t)$ can be given as follow.

$$E_{error2} = \sum_{i=1}^{N\max} \left\{ \sum_{n-1}^{N} A_n p_{sim2}(t_i + t_n) - p_{\exp 2}(t_i) \right\}^2 \qquad (6)$$

The impact position x can be decided by minimizing Eq.(6) with respect to the value of x^{*}. Figure 3 shows the FEM model for the FEM simulation. The

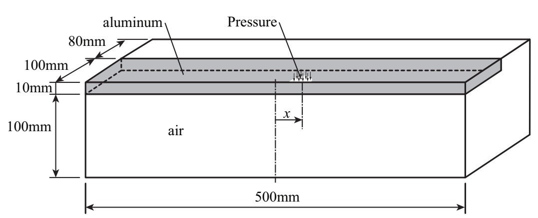

Fig. 3 The simulation model of the plate with the air.

mechanical properties of the plate and the air layer are shown in Table 2 and 3. Since the symmetry of the model, the half region should be considered. The boundary conditions for the support are very complicated, since the silicone lubber should be deformed and contacted in any area to the plate. However, within such a short duration, the supporting condition makes no influence [6]. Thus, we use free condition that is no support. The region of air is considered under and beside the plate as shown in Fig.3. The sound wave is reflected at these open boundaries. However, the reflected sound at the open boundary does not reach the position where the sound pressure is measured until time 0.5ms, because the moving distance of the sound wave is 173mm. Thus, the air layer boundary makes no influence to the calculated values at the position where the sound pressure is measured. The mechanical properties and the other conditions for the plate and the air are listed in Table 2 and 3, respectively. The impact force is applied to the circular region with

material	Aluminum
configuration	100×500×10 mm
Young's modulus	70 GPa
Poisson's ratio	0.34
density	2700 kg/m^3
N. of element	3705

Table 2 The mechanical properties of the aluminum plate for the FEM analysis.

material	air
configuration	180×500×100 mm
acoustic velocity	346 m/s
density	1.3 kg/m^3
N. of element	24020

Table 3 The mechanical properties of air for the FEM analysis.

diameter $d = 10$mm as the uniformly distributed pressure. The input impact force $P_{sim}(t)$ in Eq.(2) with $T = 0.1$ms and $P_0 = 1$MPa is shown in Fig.4. Time dependent FEM analysis is performed by the commercial FEM code by Ansys inc. The radiated sound pressure at the mike 1, for the impact position $x = 0$ is shown in Fig.5. By substituting this data into Eq.(3), the arbitrary sound pressure can be given.

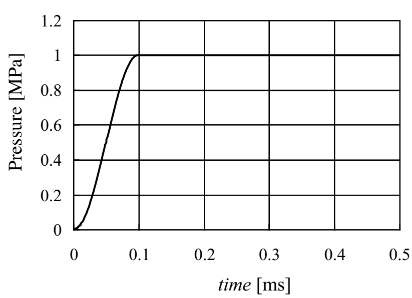

Fig. 4 The time dependence of the pressure as the input for the FEM simulation ($T = 0.1$ms).

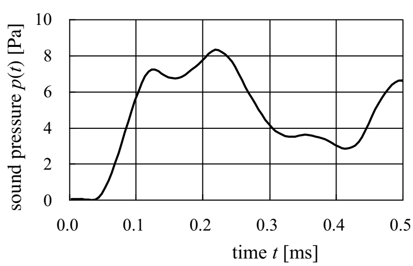

Fig. 5 The time dependence of the sound pressure by FEM simulation at the mike 1 (h= 10mm, x = 0mm).

4 The identification of the impact position and impact force

The arbitrary sound pressure and impact force can be given by unknown coefficients A_n. In order to identify the impact force from the radiated sound pressure, the unknown coefficients A_n is determined by minimizing Eq.(4), that is solving Eq.(5). The radiated sound pressure is measured by dropping from height H = 30mm to the impact position $x = 0$. The experimental data $p_{exp1}(t_i)$ is obtained through t = 0ms to 1ms by dividing N_{max} = 200 points. The time interval is Δ = 0.005ms. The calculated sound pressure are satisfactory converged with N = 40 to the measured sound pressure. Thus, in the following calculation, the value of N = 40 is used.

Figure 6 shows the time dependence of the identified sound pressure at the mike 2 with the values of x^* = -20, 0, 20mm. In the figure the measured sound pressure is shown as gray line. The identified sound pressure at the mike 2 with $x^* = 0$ is in good agreement with the measured one. Thus, the impact position should be $x = 0$. In order to obtain the impact position x, which minimize E_{error2} in Eq.(6), the relationship between x^* and E_{error2} is shown in Fig.7. By this figure, $x^* = 0$ minimizes the E_{error2} that is the impact position is $x = 0$. This position is exactly same as the impact position. Figure

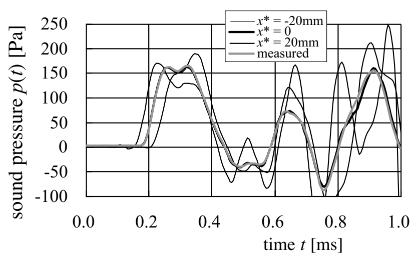

Fig. 6 The comparison between the measured and the simulated sound pressure at the mike 2 with the values of x^* = -20, 0, 20mm (N = 40).

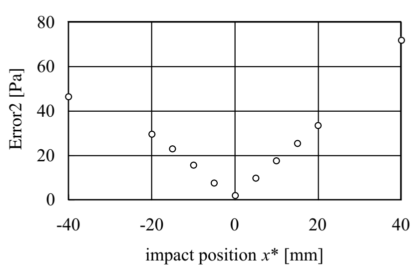

Fig. 7 The relationship between the expected impact position x^* and the least square error of the sound presser by the mike 2 during t = 0 to 0.5ms, when the impact position is $x = 0$.

8 shows the time dependence of the identified impact force by using Eq.(3) with the expected impact position x^* = -20, 0, 20mm. The measured impact

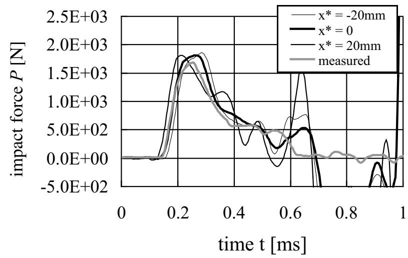

Fig. 8 The time dependence of the impact force with x^* = -20mm, 0, 20mm, when impacted at $x = 0$.

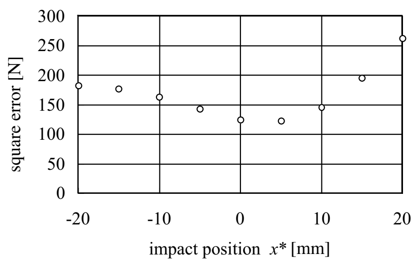

Fig. 9 The relationship between the expected impact position x^* and the square error of the impact force during $t = 0$ to 0.5ms, when the impact position is $x = 0$.

force by the strain gage is shown as gray line. The values with $x^* = 0$ is good agreement with the measured values until time t = 0.5ms. After t= 0.5ms, the identified impact force is not agree with the measured one. It may be caused by the influence of the supporting condition. It is confirmed, that the impact position is identified by the sound pressure of the mike 2 before t = 0.5ms. Figure 9 shows the relationship between the respected impact position x^* and the square error in between the measured and the identified impact force. By this figure, the error takes the minimum value in between $x^* = 0$ to 5mm. Therefore, it is confirmed that the impact position and the impact force can be identified by the sound pressure, which is recorded by the two microphones. In order to confirm the accuracy of the present method, the im-

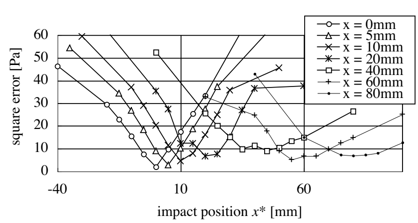

Fig. 10 The relationship between the expected impact position x^* and the least square error of the sound presser by the mike 2 during $t = 0$ to 0.5ms, with various values of the impact position $x = 0$ to 80mm.

pact experiments are made with various values of the impact position. Figure 10 shows the relationship between the respect impact position x^* and E_{error2} in Eq.(6) of the mike 2. Increasing the value of x, the graph becomes flatter. Then the determination of the impact position becomes difficult. However, we can obtain the impact position when the impact position is far from the microphone such as $x = 80$mm.

The time dependence of the identified impact force with the impact position $x = 10$, 20 and 40mm are shown in Fig.11. The starting time of impact

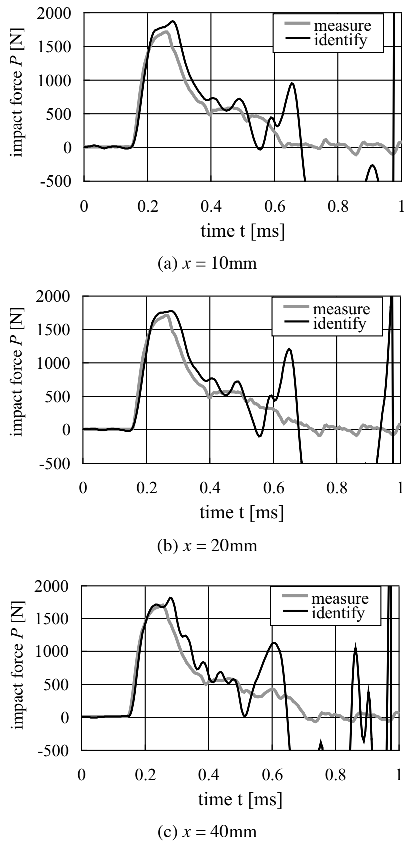

Fig. 11 The comparison between the measured and identified impact force with the impact position $x = 10$, 20, 40mm.

and the maximum values of impact force are in good agreement with the directly measured ones by the strain gages. In Fig.11(c), the identified impact force is wavering after time $t = 0.3$ms, because the impact position x is leaving from the center of the microphones.

Figure 12 shows the relationship between the impact position x and the square error of the impact force to the measured data during $t = 0$ to 0.5ms. The error is almost same, when the impact position x is smaller than 20mm, that is the impact position is in between the two microphones. The error is increasing with the increment of the impact position, because the impact position is leaving from the center of the microphones.

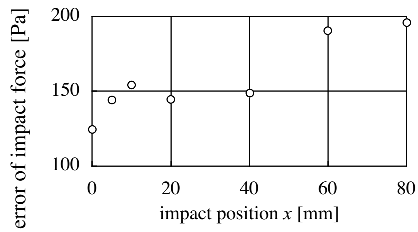

Fig. 12 The relationship between the impact position x and the square error of the identified impact force respected with the measured one during $t = 0$ to 0.5ms.

5 Conclusion

In this study, we proposed the method to identify the impact position and the impact force by using the radiated sound pressure with two microphones. By using FEM analysis, the time dependence of the impact force and the impact position can be given without any special experiments before the impact experiment. The present method is confirmed by the many experiments. The results are listed as follows.

1. The impact position and the time dependence of the impact force can be identified by the present method.
2. The impact position can be determined when the impact position far from two microphones.
3. The impact force is satisfactory identified, when the impact position is in between the two microphones.

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