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Bogdan Mielnik: Contributions to Quantum Control

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To Professor Bogdan Mielnik on his 75th Birthday

Abstract. In this article two main aspects of quantum control, which require basically different mathematical techniques will be addressed. In the first one the systems are characterized by stationary Hamiltonians, while in the second they are ruled by time-dependent ones. Both trends were initiated in Mexico by Bogdan Mielnik, who has played a central role in the development of a research group on quantum control at Cinvestav.

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1. Introduction

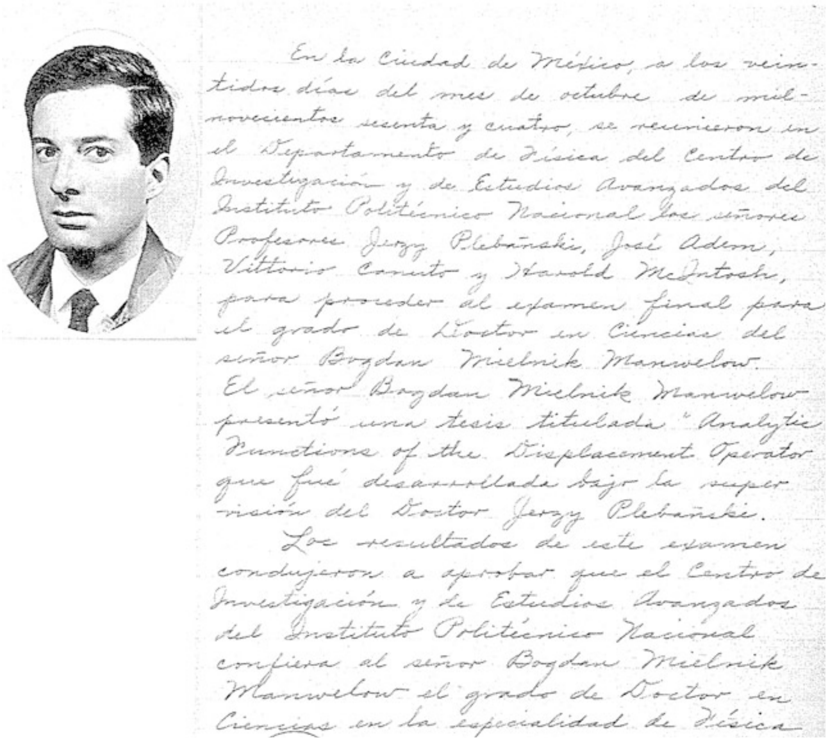
I would like to describe here the genesis and development of the quantum control group created by Bogdan Mielnik (BM) at the Center for Advanced Studies (Cinvestav) in Mexico City. Indeed, the beginning of this story is strongly tied to the birth of our Physics Department at Cinvestav, which deserves some words.

In 1962, while working at the Institute of Theoretical Physics of Warsaw University, Jerzy Plebański was invited by the outstanding Mexican physiologist Arturo Rosenblueth to develop a Physics Department at the recently created Cinvestav, at the north of Mexico City. In that invitation, it was suggested that Jerzy should also invite a younger assistant from Poland, to help him do the job. Plebański accepted Rosenblueth's invitation, and he arrived to Mexico in the late summer of 1962. His younger fellow, who turned out to be Bogdan Mielnik, arrived to Mexico on November 13th, 1962 as Jerzy's assistant and his Ph.D. student. From this period (1963) is the photograph in which Jerzy Plebański, Bogdan Mielnik and Anna Plebańska stay in front of the Pyramid of Quetzalcóatl, at the Teotihuacán ceremonial center (see Figure 1).

Figure 1. Jerzy Plebański, Bogdan Mielnik and Anna Plebańska in front of the Pyramid of Quetzalcóatl, at the Teotihuacán ceremonial center (1963).

During his first stay at Mexico, Mielnik taught courses on the mathematical foundations of quantum mechanics. As for research, he was working on the finite difference calculus and pseudo-hermitian operators. On October 22nd, 1964, he submitted his PhD Thesis entitled *Analytic functions of the displacement operator* [1] (see also [2]). Incidentally, it is worth mentioning that Bogdan Mielnik was the first Ph.D. graduate of our Physics Department at Cinvestav. A copy of the official document is shown in Figure 2.

In April 1965, after finishing his PhD, Mielnik returned to Poland. In the following years, he maintained interest in the operator calculus, leading to the explicit algebraic solution of the continuous Baker-Campbell-Hausdorff (BCH) problem [3,4], which remained open for about 60 years since the original BCH papers. In the period 1966–1976, Mielnik wrote and published his seminal papers on the geometric structure of quantum theories [5–8]. Due to the wide impact of these works, he was invited, in the period 1975–1980, to several prestigious institutions, both in Europe and in the United States, such as the Institute of Theoretical Physics in Gothenburg and the Royal Institute of Technology in Stockholm (Sweden), King's College and Imperial College (United Kingdom), Rockefeller University (USA), among others (see, e.g., [9,10]). In particular, in 1976 and 1978 he got back to Mexico to deliver talks at the *International Symposium on Mathematical Physics* in the old *Hotel del Prado*, destroyed by the earthquake in 1985, and the *Latin American Symposium on General Relativity* (Silarg) [11,12]. From that time



En la Ciudad de México, a los veintidós días del mes de octubre de mil-novecientos sesenta y cuatro, se reunieron en el Departamento de Física del Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional los señores Profesores Jerzy Plebański, José Adem, Vittorio Canuto y Harold McIntosh, para proceder al examen final para el grado de Doctor en Ciencias del señor Bogdan Mielnik Manwelow.
El señor Bogdan Mielnik Manwelow presentó una tesis titulada "Analytic Functions of the Displacement Operator que fué desarrollada bajo la supervisión del Doctor Jerzy Plebański.
Los resultados de este examen condujeron a aprobar que el Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional confiera al señor Bogdan Mielnik Manwelow el grado de Doctor en Ciencias en la especialidad de Física

FIGURE 2. The copy of the Mielnik's Ph.D. certificate (October 22nd, 1964) from the official Cinvestav roster (from S. Quintanilla, *Recordar hacia el mañana. Creación y primeros años del Cinvestav* 1960–1970, Cinvestav, Mexico, 2002).

(1976) is the nice photograph in which Bogdan Mielnik, Anna Plebańska, Virginia T. Rosenblueth and Pleblański's daughter Magdalena appear in front of some already non-existent buildings in the Reforma Avenue in Mexico City (see Figure 3).

In November 1981, Mielnik visited Cinvestav, in what was supposed to be a short-term visit. This seemingly current event became crucial for our Department and for Mielnik's life. In December 13th, 1981, while he was still in Mexico, the martial law was declared in Poland. The situation seemed to be hard in Warsaw and thus Augusto García, at the time Head of the Department, proposed Mielnik to stay longer at Cinvestav. He decided to accept this invitation which, as the years passed by, turned into a permanent stay. During that time Mielnik pursued his studies on dynamical manipulation [13], and he also wrote his short seminal article, about the generation of new Hamiltonians isospectral to the harmonic oscillator through a variant of the factorization method [14]. In the early 1983 I met Bogdan Mielnik as a student of his course in quantum mechanics. I was subsequently

Figure 3. Bogdan Mielnik, Anna Plebańska, Virginia T. Rosenblueth and Magdalena Plebański in *Paseo de la Reforma*, Mexico City (January 1976).

involved, already as Mielnik's MSc student, in applying the recently developed modified factorization to the Coulomb potential. The photograph of Mielnik in his office in *Física II* (see Figure 4) is from that time (1986). In the following years 1986–1987, he spent a sabbatical leave at the Institute of Theoretical Physics of Warsaw University.

In the period 1987–1990, Mielnik got a double appointment at the Physics Department of Cinvestav and the Institute of Theoretical Physics of Warsaw University. In 1989 he was nominated the Full Professor at the Institute of Theoretical Physics of Warsaw University. In parallel he has been a Permanent Professor at Cinvestav.

During all the time that he spent in Mexico, Mielnik has produced outstanding works, becoming the founder of the quantum control school currently existing at Cinvestav. The motivation of this subject is to control typically quantum phenomena such as diffraction, interference, wave-packet spreading, decoherence, etc. Our dream is to build a handbook of unitary operations that can be dynamically achieved.

On the other hand, for stationary systems the equivalent goal would be to construct Hamiltonians with an *a priori* prescribed spectrum. The first steps in that direction have been given by employing the well-known *factorization method*, which is worth describing shortly.

FIGURE 4. Bogdan Mielnik at his office in *Física II*, Cinvestav (1986).

2. Control of systems with time-independent Hamiltonians

When dealing with stationary systems, an obvious target to manipulate is the Hamiltonian spectrum. The simplest available technique for spectral manipulation is the factorization method, which is equivalent to the intertwining technique, Darboux transformation and supersymmetric quantum mechanics. The way in which the factorization method works can be simply illustrated through the harmonic oscillator potential.

The harmonic oscillator Hamiltonian in natural units, with $\hbar = m = \omega = 1$, reads

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{x^2}{2}. \tag{1}$$

The standard factorizations in terms of the annihilation a and creation a^+ operators are given by:

$$H = aa^+ - \frac{1}{2}, \tag{2}$$

$$H = a^+a + \frac{1}{2}, \tag{3}$$

where

$$a = \frac{1}{\sqrt{2}}\left(\frac{d}{dx} + x\right), \tag{4}$$

$$a^+ = \frac{1}{\sqrt{2}}\left(-\frac{d}{dx} + x\right). \tag{5}$$

From these expressions the following intertwining relationships can be derived:

$$Ha^+ = a^+(H+1), \tag{6}$$

$$Ha = a(H-1), \tag{7}$$

which imply that, by acting the operator a (a^+) onto an eigenfunction of H with eigenvalue E, a new eigenfunction of H is obtained with eigenvalue $E-1$ ($E+1$). By using all these ingredients, it is straightforward to derive the complete set of eigenfunctions $\psi_n(x)$ and eigenvalues $E_n = n + 1/2$ of H, for $n = 0, 1, \dots$

In 1983 Mielnik asked a simple question [14]: Is the factorization of the harmonic oscillator Hamiltonian given in equation (2) unique? In order to answer, he looked for more general first-order differential operators

$$b = \frac{1}{\sqrt{2}}\left[\frac{d}{dx} + \beta(x)\right], \tag{8}$$

$$b^+ = \frac{1}{\sqrt{2}}\left[-\frac{d}{dx} + \beta(x)\right], \tag{9}$$

such that

$$H = bb^+ - \frac{1}{2}. \tag{10}$$

It turns out that the unknown function $\beta(x)$ must satisfy the Riccati equation

$$\beta' + \beta^2 = x^2 + 1, \tag{11}$$

whose general solution is given by

$$\beta = x + \frac{e^{-x^2}}{\lambda + \int_0^x e^{-y^2}dy}. \tag{12}$$

The key point now is that the product b^+b, in general, is no longer reduced to the harmonic oscillator Hamiltonian, but it leads to a different operator:

$$\widetilde{H} = b^+b + \frac{1}{2} = -\frac{1}{2}\frac{d^2}{dx^2} + \widetilde{V}(x), \tag{13}$$

where

$$\widetilde{V}(x) = \frac{x^2}{2} - \frac{d}{dx}\left[\frac{e^{-x^2}}{\lambda + \int_0^x e^{-y^2}dy}\right]. \tag{14}$$

However, there are still intertwining relationships that look similar to those of equations (6) and (7),

$$\widetilde{H}b^+ = b^+(H+1), \tag{15}$$

$$Hb = b(\widetilde{H}-1). \tag{16}$$

Thus, the eigenfunctions $\widetilde{\psi}_n$ of $\widetilde{H}$ can be easily constructed from those of H:

$$\widetilde{\psi}_{n+1} = \frac{b^+\psi_n}{\sqrt{n+1}}, \quad n = 0, 1, \dots \tag{17}$$

Moreover, there is an additional eigenstate of $\widetilde{H}$ associated to the eigenvalue $E_0 = 1/2$ and simultaneously annihilated by b, which is given by:

$$\widetilde{\psi}_0 \propto \exp\left[-\int_0^x \beta(y)dy\right]. \tag{18}$$

In order to avoid singularities in $\widetilde{V}(x)$ and in $\widetilde{\psi}_n(x),\ n = 0, 1, \ldots$, the inequality $|\lambda| > \sqrt{\pi}/2$ must hold. Thus, in this λ-domain it turns out that $\widetilde{H}$ is a new Hamiltonian isospectral to the harmonic oscillator.

FIGURE 5. Bogdan Mielnik, David Fernández and Oscar Rosas, during a break at the Conference *Symmetries in quantum mechanics and quantum optics*, Burgos (Spain), September of 1998.

It is worth noting that the modified factorization described here represented a breakthrough in the generation of exactly solvable quantum mechanical potentials. Indeed, the intertwining relation (15) admits several generalizations that were proposed shortly after. An obvious one consists in departing from a given generic Schrödinger Hamiltonian H of the form (13) and look for a new one $\widetilde{H}$ such that

$$\widetilde{H}B^+ = B^+H, \tag{19}$$

where the initial potential $V(x)$ and the intertwining operator B^+ are not necessarily the harmonic oscillator and a first-order operator respectively. In particular,

Figure 6. Boris Samsonov, David Fernández, Bogdan Mielnik and Oscar Rosas at Mielnik's office (March of 2001).

the generalization for B^+ being of first-order and general $V(x)$ was proposed by Sukumar in 1985, who proved that a solution of the stationary Schrödinger equation associated to H and a given factorization energy ϵ such that $\epsilon \leq E_0$ is required to generate the new potential $\widetilde{V}(x)$ through non-singular transformations. On the other hand, Andrianov, Ioffe and Spiridonov (1993) suggested that B^+ should be of order greater than one with general $V(x)$, and this suggestion was later studied by Andrianov, Ioffe, Cannata, Dedonder (1995), Bagrov and Samsonov (1995) and a member of our research group (Fernández 1997). It is important to notice that in the higher-order case several seed solutions of the stationary Schrödinger equation associated to diverse factorization energies are required in order to implement the transformation (for a review containing further discussion, the reader can consult [15]).

The case where $V(x)$ is the harmonic oscillator potential and B^+ is of second-order was explored in detail in 1998 by members of our group [16]. A photograph taken during a break at the Conference *Symmetries in quantum mechanics and quantum optics* which was held at Burgos, Spain, can be seen on Figure 5. Subsequently, the so-called confluent algorithm, for which the involved factorization energies tend to a common value, was explored in 2000 by Mielnik, Nieto and Rosas-Ortiz [17], and later by Fernández and Salinas-Hernández. The situation when $V(x)$ is periodic has been also analyzed in the interval 2000–2010 (see, e.g., [18, 19]). Some members of our team elaborating the last problem appear on the photo of Figure 6.

Before finishing this section, I would like to remark that in 2003 the Conference *Progress in supersymmetric quantum mechanics* took place at Valladolid, Spain. An overview article opening the special issue of *J. Phys. A: Math. Gen.* dedicated to the topic of the Conference, that has quickly became a hit of the factorization subject, is strongly recommended (see [15]).

3. Control of systems with time-dependent Hamiltonians

For systems ruled by time-dependent Hamiltonians the quantum control has to be implemented in a different way. First of all, it is well known that the evolution operator induced by a self-adjoint Hamiltonian is unitary. Thus, it is natural to consider the inverse problem: Can any unitary operator be achieved as the result of a dynamical evolution? In other words, can a set of prescribed external conditions be designed for the system to evolve in such a way that its evolution operator becomes, at a certain time, the required unitary operator? The answer to this question was suggested by Mielnik in 1977 [20]: provided there are no superselection rules, any unitary operator can be dynamically approximated. Moreover, there is a generic prescription, proposed in 1986, in order to induce an arbitrary unitary evolution [21, 22]: (i) first of all, let us choose the system that performs a circular dynamical process such that $U_0(\tau) = I$, that is called an evolution loop (EL); (ii) then, by perturbing the EL, the small deviations of this process will eventually induce any given unitary operation (see an illustration of this process in Figure 7).

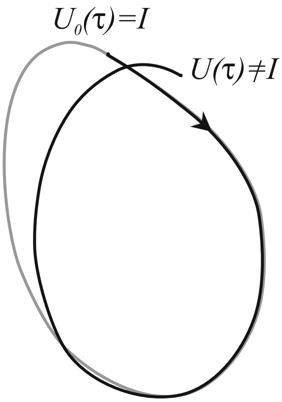


Figure 7. The deviation of the evolution loop induced by a perturbation.

3.1. One-dimensional systems

Let us note that the one-dimensional harmonic oscillator is the simplest system having an EL. Thus, it is natural first to look for EL in one-dimensional systems ruled by Hamiltonians of the form

$$H(t) = \frac{p^2}{2} + g(t)\frac{q^2}{2}, \tag{20}$$

where q, p are the quantum mechanical coordinate and momentum operators such that

$$[q, p] = i, \tag{21}$$

and the evolution operator $U(t)$ of the system satisfies

$$\frac{dU(t)}{dt} = -iH(t)U(t), \quad U(0) = I. \tag{22}$$

A curious and interesting result that was found in 1977 deserves some discussion [20]. For the periodic sequence of pulses such that

$$g(t) = \frac{1}{\lambda}\delta(t-\lambda) \quad \text{for} \quad 0 < t \leq \lambda, \tag{23}$$

periodically extended for $t > \lambda$, the following holds

$$\underbrace{e^{-i\frac{1}{\lambda}\frac{q^2}{2}}e^{-i\lambda\frac{p^2}{2}}\cdots e^{-i\frac{1}{\lambda}\frac{q^2}{2}}e^{-i\lambda\frac{p^2}{2}}}_{\text{12 factors}} \equiv I, \tag{24}$$

where the equivalence symbol $\equiv$ interrelates any two unitary operators which differ only by a c-number phase factor. This means that the system has an evolution loop of period $\tau = 6\lambda$. A schematic representation of this dynamical process is given in Figure 8. Notice that, as a bonus, it is possible now to invert the natural free evolution:

$$e^{i\lambda\frac{p^2}{2}} \equiv \underbrace{e^{-i\frac{1}{\lambda}\frac{q^2}{2}}e^{-i\lambda\frac{p^2}{2}}\cdots e^{-i\lambda\frac{p^2}{2}}e^{-i\frac{1}{\lambda}\frac{q^2}{2}}}_{\text{11 factors}}. \tag{25}$$

Let us stress that the evolution loop of equation (24) is not the only one that can be produced through Hamiltonians of the form (20) [22]. In particular, it turns out that

$$\left(e^{-i3\tau\frac{p^2}{2}}e^{-i\frac{1}{\tau}\frac{q^2}{2}}\right)^3 \equiv I, \tag{26}$$

which implies that it is possible once again to invert the natural free evolution:

$$e^{i3\tau\frac{p^2}{2}} \equiv e^{-i\frac{1}{\tau}\frac{q^2}{2}}e^{-i3\tau\frac{p^2}{2}}e^{-i\frac{1}{\tau}\frac{q^2}{2}}e^{-i3\tau\frac{p^2}{2}}e^{-i\frac{1}{\tau}\frac{q^2}{2}}. \tag{27}$$

A representation of the evolution loop of equation (26) is also given in Figure 8 [23].

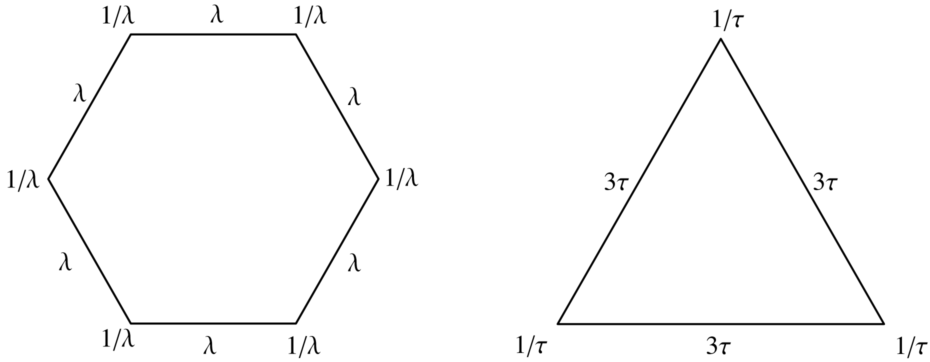


FIGURE 8. Representation of the evolution loops of equations (24) (left) and (26) (right).

3.2. Three-dimensional systems

As our three-dimensional system let us consider a charged particle interacting with homogeneous time-dependent magnetic fields. A possible experimental setup is illustrated in Figure 9. In a neighborhood of the origin, the magnetic field can be considered approximately homogeneous, and the corresponding Hamiltonian takes

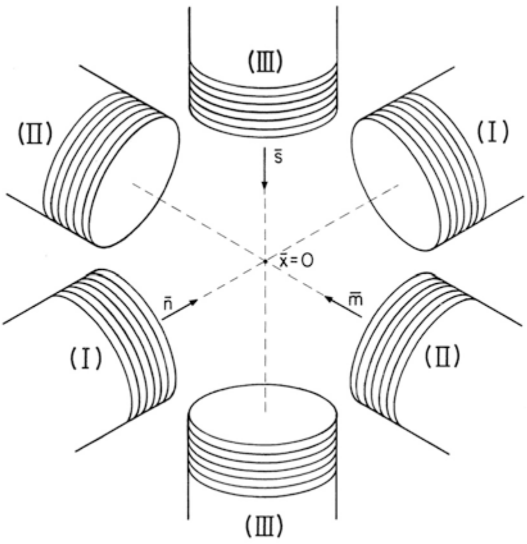


FIGURE 9. An experimental setup to manipulate charged particles.

the form:

$$H(t)=\frac{1}{2m}\left(\mathbf{p}+\frac{e}{2c}\mathbf{r}\times\mathbf{B}(t)\right)^2=\frac{1}{2m}\left[\mathbf{p}^2+\left(\frac{e\mathbf{B}(t)}{2c}\right)^2\mathbf{r}_\perp^2\right]-\frac{e\mathbf{B}(t)\cdot\mathbf{L}}{2mc}, \quad (28)$$

a non-relativistic Hamiltonian with time dependent $\mathbf{B}(t)$ representing the first step of the Einstein-Infeld-Hoffman (EIH) method in classical electrodynamics (see the discussion in [24]). Our first choice was the following rotating magnetic field [25]:

$$\mathbf{B}(t)=B\cos(\omega t)\mathbf{m}+B\sin(\omega t)\mathbf{n}, \quad (29)$$

for which we wanted to find the evolution loops. Unfortunately, we were unable to find them for this system. Despite that, the corresponding quantum mechanical problem was explicitly solved. We found a regime where the charged particle is confined to a neighborhood of the origin (the trapping region). However, there exists also the domain of parametric resonance, where the charged particle is quickly ejected off the trapping zone (see also [26–28]). These results constitute the core of my PhD Thesis [29], supervised by Mielnik. The dissertation was delivered on September 19th, 1988 (a photograph of José Luis Lucio, Bogdan Mielnik and David Fernández, after the event, can be seen in Figure 10).

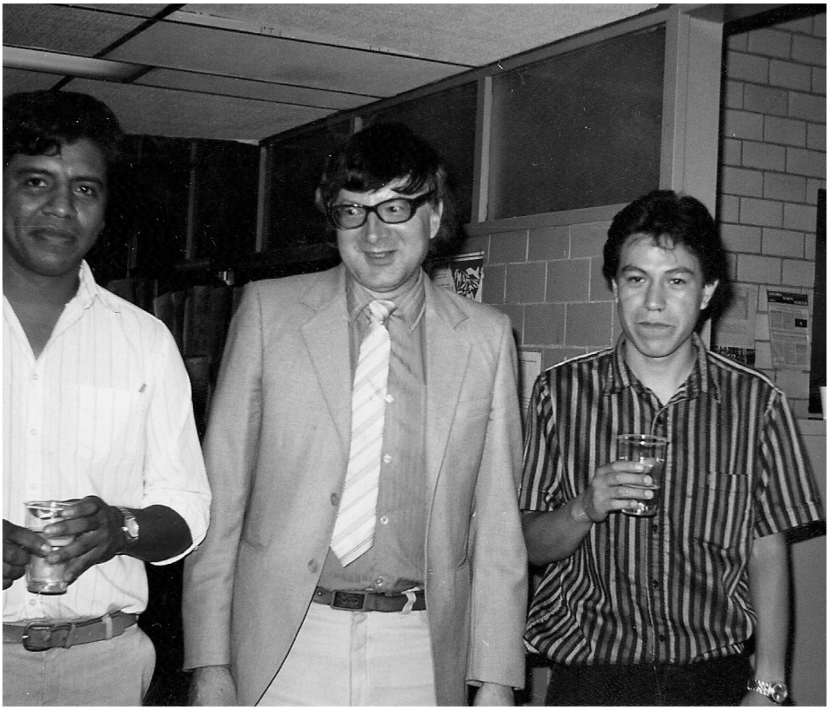


FIGURE 10. José Luis Lucio, Bogdan Mielnik and David Fernández at *Física I*, September 19th, 1988.

An alternative magnetic field was explored afterwards [30] (see also [31–33]):

$$\mathbf{B}(t) = \begin{cases} B(t)\mathbf{m} & \text{for } t \in [0, 2T) \\ B(t-2T)\mathbf{n} & \text{for } t \in [2T, 4T) \\ B(t-4T)\mathbf{s} & \text{for } t \in [4T, 6T) \end{cases}, \tag{30}$$

where

$$B(t) = \begin{cases} B_1 & \text{for } t \in [0, t_1),\ 0 < t_1 < T, \\ B_2 & \text{for } t \in [t_1, T), \\ -B_2 & \text{for } t \in [T, T+t_2),\ t_2 = T - t_1, \\ -B_1 & \text{for } t \in [T+t_2, 2T). \end{cases} \tag{31}$$

For the three-dimensional system we were particularly interested, as in the one-dimensional case, in inverting the natural free evolution. In order to do that, we first of all switched to the following dimensionless quantities:

$$\gamma_1 = \alpha_1 t_1', \quad \gamma_2 = \alpha_2 t_2', \quad \alpha_1 = \frac{eB_1 T}{2mc}, \tag{32}$$

$$\alpha_2 = \frac{eB_2 T}{2mc}, \quad t_1' = \frac{t_1}{T}, \quad t_2' = \frac{t_2}{T}. \tag{33}$$

It turns out that the free evolution is induced when the previous parameters satisfy the following relationships:

$$\alpha_1 = \frac{\gamma_1 \tan(\gamma_1) - \gamma_2 \tan(\gamma_2)}{\tan(\gamma_1)}, \qquad \alpha_2 = \frac{\gamma_1 \tan(\gamma_1) - \gamma_2 \tan(\gamma_2)}{-\tan(\gamma_2)}, \tag{34}$$

$$t_1' = \frac{\gamma_1 \tan(\gamma_1)}{\gamma_1 \tan(\gamma_1) - \gamma_2 \tan(\gamma_2)}, \qquad t_2' = \frac{-\gamma_2 \tan(\gamma_2)}{\gamma_1 \tan(\gamma_1) - \gamma_2 \tan(\gamma_2)}. \tag{35}$$

The evolution operator, at the time $\tau = 6T$ where the application of the magnetic field ends, thus becomes:

$$U(\tau = 6T) = \exp\left(-\frac{i}{\hbar}\frac{p^2}{2m}T'\right) = \exp\left(-\frac{i}{\hbar}\frac{p^2}{2m}\tau\chi\right), \tag{36}$$

where the effective time $T' = \tau\chi = 6T\chi$ depends on the distortion parameter χ, which in turn depends on the angles γ_1 and γ_2 in the following way:

$$\chi = \frac{1}{3} + \frac{2}{3}\cos^2(\gamma_2)\frac{\tan^2(\gamma_1) - \tan^2(\gamma_2)}{\gamma_1 \tan(\gamma_1) - \gamma_2 \tan(\gamma_2)}. \tag{37}$$

Notice that the required restrictions $t_1' > 0$ and $t_2' > 0$ are satisfied for $n\pi < \gamma_1 < (n+1/2)\pi$ and $(m-1/2)\pi < \gamma_2 < m\pi$, or for $(m-1/2)\pi < \gamma_1 < m\pi$ and $n\pi < \gamma_2 < (n+1/2)\pi$, $m, n \in \mathbf{Z}^+$. Moreover, depending on the values taken by χ in the admissible domain of (γ_1, γ_2), three physically different situations arise:

$$\begin{cases} \chi > 1 & \text{accelerating the free evolution} \\ 0 \leq \chi \leq 1 & \text{slowing the free evolution} \\ \chi < 0 & \text{inverting the free evolution} \end{cases} \tag{38}$$

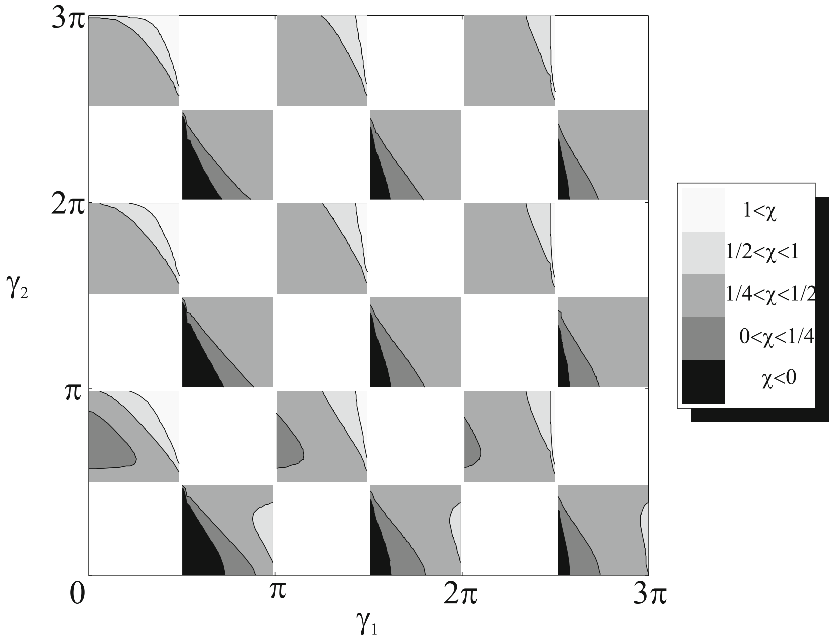


FIGURE 11. The *chessboard of distorted time*, where the manipulated free evolution is induced for a charged particle in the magnetic field of equations (30)–(31).

A plot summarizing these results is shown in Figure 11 [32]. In particular, it is worth noticing the existence of regions where the inversion of the natural free evolution is produced (see the domain in which $\chi < 0$).

A photograph of Bogdan Mielnik, at the time of elaborating [30], is shown in Figure 12.

I have illustrated here, with the example of the free evolution, the way in which we learned to implement the dynamical manipulation. Of course, there are other unitary operations which have been of interest for our group. In particular, it is worth mentioning the squeezing operations, which were explored in detail by Francisco Delgado during the elaboration of his MSc and PhD Thesis [34], under the supervision of Mielnik as well (see also [35–37]). A photograph of both, taken after the MSc dissertation of Francisco Delgado, February 2nd, 1992, is shown in Figure 13.

On the other hand, with Sara Cruz the possible physical meaning of the Floquet operator and its usefulness to achieve several interesting unitary operators was explored in [38]. As a result of its originality and the large amount of new facts contained in Sara's Thesis [39–41] (also under BM supervision), in April 2006 she was awarded the 2005 *Arturo Rosenblueth prize* to the best PhD thesis written at Cinvestav (see photograph in Figure 14).

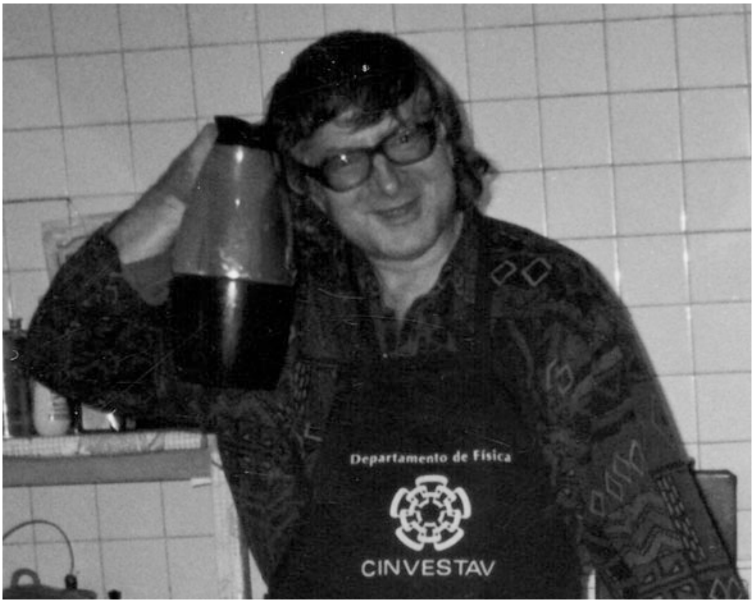


FIGURE 12. The tea time at Paranagua 42-4 in 1993, during some discussions on magnetic control.

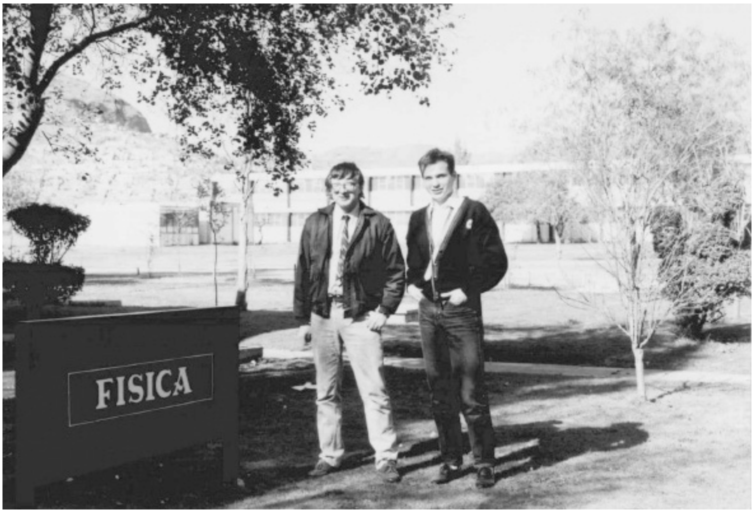


FIGURE 13. Bogdan Mielnik and Francisco Delgado, in front of Physics Department, Cinvestav (February 2nd, 1992).

Figure 14. Sara Cruz was awarded (April, 2006) the 2005 *Arturo Rosenblueth Prize* to the best PhD Thesis defended at Cinvestav during the year 2005 (here with her Thesis adviser, Bogdan Mielnik).

In addition, after studying physical problems leading to indefinite Hilbert spaces and non-hermitian Hamiltonians [42], Bogdan Mielnik and Alejandra Ramírez have explored some non-commutative coordinate operators naturally arising when dealing with a charged particle interacting with several magnetic field configurations [43]. Their most recent papers [24, 44, 45] contain several results reported in Alejandra's Thesis. A photograph of Alejandra Ramírez and Bogdan Mielnik, during their participation at the *XXII Workshop on Geometrical Methods in Physics* which was held at Białowieża in 2003 is shown in Figure 15.

It is important to notice that the school of quantum control at Cinvestav also involves colleagues who only did their MSc Thesis under Mielnik's advice. Although they obtained PhDs later on in other areas, we guess that they still conserve some interdisciplinary spirit. I would like to mention specifically:

Gerardo Herrera (September 1987): his MSc Thesis has to do with the dynamical manipulation of a one-dimensional Schrödinger particle in a quadratic potential with a time-dependent frequency [46], the corresponding Hamiltonian is given in equation (20). It was shown that the parity, scale and Fourier transformations can be dynamically induced. A photograph taken after the presentation of the documentary about Plebański's life, January 26th, 2005, containing Gerardo Herrera (at the time Head of the Physics Department), Rosalinda Contreras (then Director of Cinvestav) and Bogdan Mielnik, is shown in Figure 16.

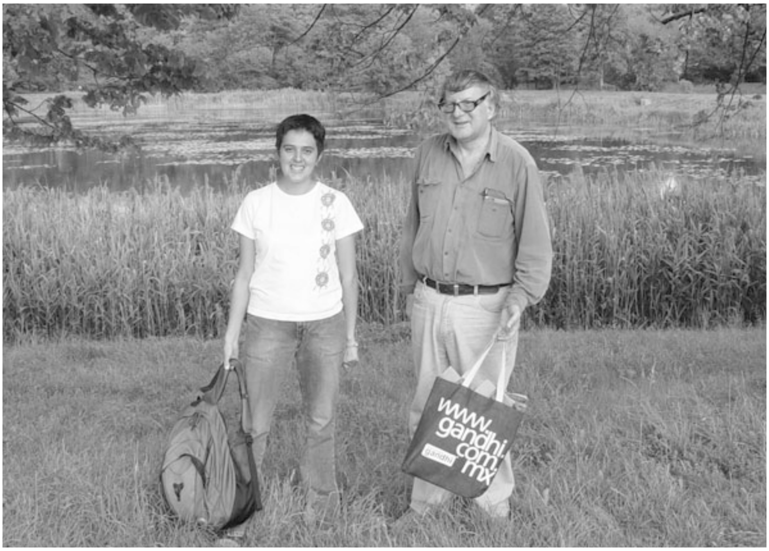


Figure 15. Alejandra Ramírez and Bogdan Mielnik at Białowieża, June 2003.

Figure 16. Gerardo Herrera (Head of the Physics Department), Rosalinda Contreras (Director of Cinvestav) and Bogdan Mielnik in front of the Arturo Rosenblueth Auditorium, Cinvestav, January 26th, 2005.

Diego Sanjinés (May 23th, 1990): his MSc Thesis addresses the connection between the stationary one-dimensional Schrödinger equation and the classical dynamical problem of an oscillator with time-dependent frequency [47]. It was shown that the stability of the classical problem is closely related to the quantum mechanical problem of eigenvalues.

Francisco Solis (August 1990): in this MSc Thesis, the stability of the motion of charged particles in the vicinity of the nodal points of a monochromatic standing plane wave was analyzed [48]. These results were compared with those obtained from the ponderomotrix potential of Kapitza and Landau.

Marco Antonio Reyes (February 1992): his MSc Thesis has to do with a non-perturbative numerical approach designed for calculating the energy levels of one-dimensional or spherically symmetric potentials [49]. The method was implemented with the angular form of the Riccati equation as a starting point [50].

Since one of the aims of this Conference was to celebrate the 70th and 75th Birthdays of Lech Woronowicz and Bogdan Mielnik respectively, I find interesting to show a photograph (see Figure 17) containing both, along with Jerzy Plebański, at the *Conference to celebrate the Jerzy Plebański's* 75*th Birthday* which was held in Mexico City in September of 2002 [51].

Figure 17. Lech Woronowicz, Bogdan Mielnik and Jerzy Plebański (September of 2002).

FIGURE 18. The group of quantum control at the Physics Department of Cinvestav (December 8th, 2010). From left to right and top to bottom: Bogdan Mielnik, Rodrigo Muñoz (upper row); David Bermúdez, Nicolás Fernández, Oscar Rosas, Alonso Contreras, David Fernández, Encarnación Salinas (second row); Sara Cruz, Iván Cabrera, Gerardo Herrera (front row).

In order to provide a global view of Mielnik's scientific work, I would like to close this section by mentioning that he has also contributed substantially to a better understanding of conceptual and polemic problems in quantum mechanics [37, 52–56].

4. Conclusions

By means of these specific examples I have tried to illustrate the way in which we approach the problem of quantum control at the Physics Department of Cinvestav. It is always difficult to evaluate which scientific results will turn important for the future theories or applications, but undoubtedly Professor Bogdan Mielnik has been quite essential in growing up a quantum control group which we hope can compete on the international arena (see Figure 18). On behalf of the group, I would like to express our best wishes to him:

For teaching us the way of doing science.
For teaching us that work has to be done patiently and carefully.
For an atmosphere of permanent creation and discussion.
Long live Professor Bogdan Mielnik!

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