Privations, Negations and the Square: Basic Elements of a Logic of Privations

Stamatios Gerogiorgakis

Abstract I try to explain the difference between three kinds of negation: external negation, negation of the predicate and privation. Further I use polygons of opposition as heuristic devices to show that a logic which contains all three mentioned kinds of negation must be a fragment of a Łukasiewicz-four-valued predicate logic. I show, further, that, this analysis can be elaborated so as to comprise additional kinds of privation. This would increase the truth-values in question and bring fragments of (more generally speaking) Łukasiewicz-n-valued predicate logics into the scene.

Keywords Privation · Negation · Łukasiewicz-n-valued predicate logic

Mathematics Subject Classification Primary 03B65 · Secondary 03B50

1 Motivating Arguments and Some History

Negation is what makes the square a square "of opposition" and a very intensively investigated logical constant.

Privation, a kind of truth-value inversion expressed by nouns like 'immorality', 'irresponsibility', 'unreadiness', 'anarchy', 'senselessness', 'blindness', 'deafness', 'ignorance', etc., is usually associated with negation in a very general sense of the word. Unlike "standard" negation however, in the recent decades privation has been neglected or suppressed in the discussion on the square and in logic altogether. This is one of these very usual sins, which are understandable but inexcusable.

It is inexcusable, since the very first hints to a logic of privations date already in the Middle Ages to be repeated now and then until the 19th century.

Greek and Arab Aristotelians dealt with three sorts of negation (cf. [16, pp. 173–184]):

1. the simple negation, which they understood in a way which roughly resembles our external negation—"external" because in our semi-formal English the particle "not" occurs, in this kind of negation, "externally" as part of the introductory expression: "It is not the case that . . . ";

For discussions concerning privation and amendments in the first drafts of this paper, I would like to thank Prof. Dr. Hans Burkhardt, Georgios Karageorgoudis (both Munich) and, last but not least, two anonymous referees.

J.-Y. Béziau, D. Jacquette (eds.), *Around and Beyond the Square of Opposition,* 229–239
Studies in Universal Logic, DOI 10.1007/978-3-0348-0379-3_16, © Springer Basel 2012

2. the negation of the predicate; and
3. the privation

Rough examples in English would be respectively: 'It is not the case that Ebenezer Scrooge is a generous person'; 'Ebenezer Scrooge is a non-generous person'; 'Ebenezer Scrooge is a stingy person'. NB being stingy is a privation of a virtue: generosity.

The negation of the predicate was known to the ancients and the medievals by two alternative names: infinite term (cf. Aristotle [3, 16a32]; Boethius [6, editio prima, I, chapter 2]) and negation by transposition (cf. Ammonius [2, pp. 161–162]; Alexander [1, p. 397]; John Italos [11, pp. 61–62, i.e. question 49]). The reason for giving it the name "negation by transposition" is the structure of the classical languages: in Greek and in Latin the negation of the predicate is formulated by taking a negative sentence with the usual syntax of these languages (i.e. with the negation particle just in front of the verb) and moving ("transposing") the negation particle ('ouk' or 'ou' in Greek, 'non' in Latin) to come immediately before the predicate. A standard Latin negation would be: 'Bucephalus non est asinus' (= It is not the case that (there is something like) Bucephalus (and that it) is a donkey) and the negation by transposition would be: 'Bucephalus est non asinus' (Bucephalus is no donkey). 'Non' was transposed from the first to the second place, to formulate the negation by transposition. The English syntax does not allow negation by transposition to be formulated by means of a transposition literally.

Some medieval Arab Aristotelians tended to confuse privations and negations of the predicate because of the linguistic production rules of Arabic: There is nothing like a monolectic privation with affixes in Arabic and the privation of the Indogermanic languages can only be expressed in this language by means of a periphrastic formulation analogous to the Indogermanic negation of the predicate—cf. [16, p. 178]. Regardless of the reasons which gave rise to it, there is something true in the idea of the Arab logic to regard the negation of the predicate as some kind of privation. In what follows we are going to see that the negation of the predicate is, like privation, a *strong* kind of negation.

John Italos [11, p. 62, i.e. question 49], an 11th-century Norman from Sicily who wrote in Greek, observed that the sentence 'Being just is predicated of humans' is true in just one case: obviously when the humans in question are just. Italos continues with the statement that the sentence: 'Being non-just is predicated of no humans' is true in three cases. He does not explicitly say so, however good candidates would be the following three cases: case (1) the humans in question are just (note that this case made also the sentence 'Being just is predicated of humans' true) or, case (2), there are no humans or, case (3), there is nothing which is non-just. Further, Italos observed that the sentence: 'Being unjust is predicated of no humans' is true in five cases. By this he rather meant the following five cases: the inclusive disjunction of the cases 1 till 3, or, case (4), injustice is a predicate which does not apply to the humans of the domain (like, for example, it does not apply to insects) or case (5), there is nothing unjust. Italos remarked that the humans' in question being unjust implies their being non-just, which implies their not being just. However, according to Italos these implications do not hold *vice versa*. And this is exactly what my interpretation of Italos does justice to.

Another example, not found in Italos but representative of what he had in mind, might be more illuminating: If Homer is blind, then Homer sees nothing. But it is not correct to say that if Homer sees nothing, then Homer is blind, since Homer may be unable to see only because the room is dark. Further, if Homer sees nothing, then seeing is not

predicated of the reference of this name. But if seeing is not predicated of the reference of the name 'Homer', then from this does not follow that Homer is an existing individual and that other predicates of real people can be attached to him: like, say, being in a dark room, being blind and so on. In other words, if the only thing which we know about Homer is that it is not the case that he is there and watches around, then from this does not follow that he is there and watches nothing. He might not be there at all. Cf. also Wolfson [16, pp. 179–181].

After the 15th century, the distinction between simple (= external) negation and privation was criticized as unintelligible by authors who were anti-Aristotelians or nominalists or both; among many others by Valla [14, pp. 114–115], and Gassendi [7, lib. I, ex. 6, cap. 1]. Their arguments against privation reflect basic nominalistic beliefs which primarily concern *ontology* and affect the logic of privation only insofar one decides to *indulge in nominalist ontology*. It is interesting, however, to review these arguments, in order to understand how privation was dismissed from natural science, initially, and from logic subsequently. Since Aristotle, privation was defined with reference to natural properties. Privation-talk presupposes that an individual has properties, which are natural for a universal, to which it belongs, and others which are not natural for it. We say, for example, that Stevie Wonder is deprived of sight, because it is natural for humans to see, but we do *not* say that he is deprived of hearing ultrasounds. The nominalistic cluster of arguments against privation-talk denies the ontology which underlies properties natural for members of the one universal to have and unnatural for members of another universal to have. Based on this, nominalists stress that the distinction between simple negation and privation has to be dismissed. This view has had some very influential supporters not only in the early modern period but also in very recent times, above all Quine. Horn [10, p. 112], reviews briefly the contemporary discussions on this point.

These historical remarks concerning the topic: nominalist ontology and privation may suffice. The logical issue concerning privation is not directly linked to nominalist ontological concerns. The logic of privation formalizes the privation-talk of natural language. The ontological issue, which the traditional nominalists and their recent epigones raise, does not affect the way in which we express privation in natural language. While nominalist ontology rejects the idea of real universals with natural properties, the logical issue consists in taking privation-talk for granted and in attempting to model it in logic. In what follows, I will leave ontology aside and concentrate on the logical issue. In other words, I will take privation-talk seriously and by doing this I will engage in an ontology, which is not nominalistic.

This is not to say that I will put all issues raised by nominalism aside. My analysis does deal with some logical issues, which nominalism raises, though not with the ontological ones. A logical issue of nominalistic spirit, which my analysis will account for, was anticipated by Aristotle [4, 1022b32-33], who observed that no privation is meant like the next privation: an individual mole is not said to be *deprived* of sight in reference to creatures which have eyes (moles do have eyes). Therefore, it is *one kind* of blindness which is expressed in a mole's being blind and *another* kind of blindness which is expressed in a human's being blind as a result of an accident—in the sense that these individuals are deprived of one-and-the-same thing in different grades. The one is deprived of something she never had, the other is deprived of something she once had. Another, perhaps more appropriate or revealing example, would be the difference between blindness and half-blindness. Let me coin a new "-ism" for this feature of natural language: the pluralism of

privations. At the end of this paper, I will attempt to show that the pluralism of privations does not dismiss privation.

Despite the criticism, to which the notion of privation was exposed in the 16th and 17th centuries, Wolff [15, §§109–110] and Kant [12, A70 = B95; A72-73 = B97-98] in the 18th century expressed the view that the sentences are according to their quality affirmative, negative or infinite. Kant's example was the difference between 'The soul is not mortal' (a negative sentence) and 'The soul is non-mortal' (an infinite sentence). We know what the case must be for the negative sentence 'The soul is not mortal' to be true: it has to fail to fulfill the sentence: 'The soul is mortal'. This is the case when the soul does not exist or is inorganic or is divine. But we do not know as exactly as this, what the case must be for the infinite sentence 'The soul is non-mortal' to be true. The things which are non-mortal form the complementary set of all mortal entities—and this set has infinite members. Since this set is infinite, we can never be certain whether there exist entities in it, perhaps the soul itself, which would surprise us in the sense that neither mortality nor non-mortality, or even both would be attributed to them. However, when we state that the soul is *not* mortal (instead of *non*-mortal), we claim that the soul fails to be an element of a finite set: the set of all mortal entities. And since we have defined this finite set, in principle we can be certain that mortality either is or is not attributed to the soul—for this line of argument cf. Kant [12, A72-73 = B97-98].

Subsequently there were Kantian scholars who expressed the view that it is one thing to negate the verb ('The soul is-not mortal') and another thing to negate the predicate ('The soul is non-mortal'); for a discussion cf. Hamilton [9, pp. 177–179]. But the issues, which Kantianism raised in this respect, got forgotten at the latest when this philosophical tradition declined in the second half of the 20th century.

Albeit inexcusable, the negligence of privation in logic is, as I said, understandable. It is understandable, since the syntactical means to express privation, unlike those which express external negation, do not seem to be propositional operators. I will argue that this is a false impression, that privations *are* propositional functions after all. I will show this by using an hexagon and a square of opposition.

2 Polygons of Opposition Edged by Privations, Negations of the Predicate, and External Negations

Take the following examples of a privation, a negation of the predicate, and an external negation. These are exactly the kinds of negation, which are known to Aristotelian logic.

Privation The philosopher's stone is immortal
Negation of the predicate The philosopher's stone is non-mortal
External negation[1] It is not the case that the philosopher's stone is mortal

[1]Some contemporary authors like Blau [5, p. 49] and Horn [10, pp. 122–132], call the negation of the predicate: 'presupposing' negation, because, unlike the external negation it presupposes the existence of the entity expressed in the subject. Especially Blau (loc. cit.) calls it 'strong negation' because it entails the external negation but is not entailed by it. Since in Blau's three-valued logic there are exactly two negations: (a) a formal construction, which formalizes the negation of the predicate/by transposition, and (b) a weak negation which formalizes the external negation of the natural language, the former is the only strong negation in Blau's three-valued logic. But, if I am allowed to anticipate one of my results, the negation of the predicate is *not* the strongest kind of negation in the logic of privation.

My example of an external negation is a true sentence. A stone cannot be said to be mortal. This seems to make my example of a negation of the predicate true (this conclusion is premature and inaccurate, but for now, let me continue). The philosopher's stone belongs to the entities which are neither mortal nor immortal. To say that a stone would belong to the entities which are immortal, is to commit a category mistake. The main difference between my example of a negation of the predicate and my example of a privation is that the privation invites a category mistake, the negation of the predicate does not. In fact, a negation of a predicate never involves category mistakes, privations sometimes do (cf. [10, pp. 110–122, i.e. chapter 2.3]). This is one of the reasons why the privation above is also not true: it is a category mistake to assign a stone immortality. To avoid the category mistake one must avoid to speak of immortality in this context, and say instead that the philosopher's stone belongs to the entities which are non-mortal. However effective for avoiding category mistakes, this trick, however, does not save from unjustified existential import.

Unjustified existential import is the other reason for which my example of a privation is not true (my example of a privation fails to be true because of two reasons). My example of a privation presupposes that the philosopher's stone exists. If the philosopher's stone existed, then it would be true to say that it is non-mortal, since it would be neither mortal nor immortal. But, alas, the philosopher's stone does not exist. To refer to the philosopher's stone is analogous to referring to entities like the present king of France, James Bond and Pegasus. Note that referring to the predicates which are not predicated of the present king of France is totally different than referring to the predicates which are not predicated of the present queen of the Netherlands. It is not true to say of Beatrix that she is bald, because she has plenty of hair on her head. And it is not true to say of the present king of France that he is bald (or that he is not bald alike), because he does not exist. To make a long argument short, my example of an external negation is true, my example of a negation of the predicate, however, is not true because it presents an unjustified existential import.

The hexagon in Fig. 1 may serve as a starting point to portray the subalternations between my examples.

The bottom sentences of Fig. 1 are true. Each of them contradicts to one of the top sentences, which, *eo ipso*, fail to be true. The sentences on the top right and at the bottom left, contradict each other in every respect. In the top right sentence an unjustified existential import and a category mistake are committed. The bottom left sentence discards the unjustified existential import and the category mistake. Analogous is the case with the bottom right sentence: it denies the unjustified existential import and the category mistake committed in the top left sentence.

I left the remaining contradiction between the middle right and the middle left sentence last, because it presents a disharmony compared to the other two contradictions of the hexagon. To begin with the problems involved, the middle right and the middle left sentences are a contradictory pair. Therefore they cannot be both false. But they both fail to be true for the following reasons: The middle right sentence fails to be true because of unjustified existential import. The middle left sentence appears syntactically to be an external negation of the middle right sentence. If classical logic is adequate to model the semantic nuances of this hexagon (this is a hypothesis, which we will have to drop in due course), then the middle left sentence is true, since it is an external negation of the middle right sentence, which is not true. But it cannot be true! The middle right sentence

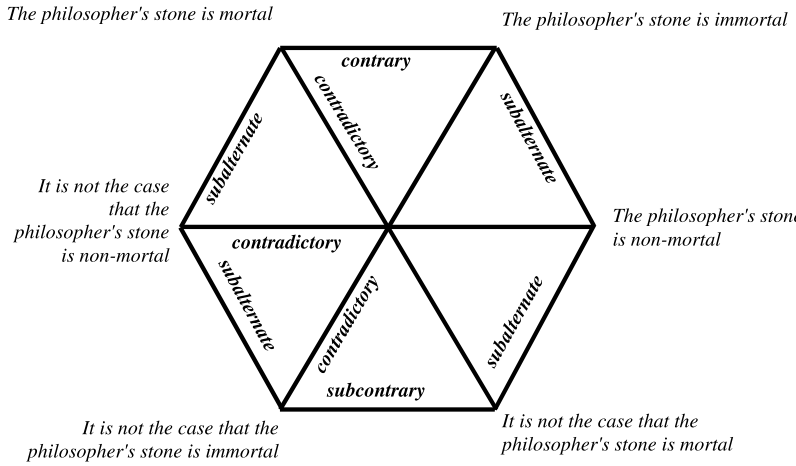

Fig. 1 An hexagon with three different kinds of negation

commits an unjustified existential import and no category mistake. It is not clear whether the middle left sentence says that the philosopher's stone is a non-existing non-mortal (in which case it is true) or a non-existing (maybe even an existing) mortal (in which cases it is not true). The relation between the middle right and the middle left sentence is not of the same kind as the contradictions between the two other contradictory pairs of the hexagon, where it was clear that the one sentence committed a category mistake, the other did not. The fact that the middle left sentence is susceptible to interpretations which fail to be true, although syntactically it contradicts a sentence which also fails to be true, is my main motivation to consider classical logic as inadequate for this context. This is also my reason to abandon the hexagon. An hexagon insinuates that all sentences connected by a diagonal, including the middle right and the middle left corners, contradict in the same way. But as we have just seen, the sentences of the middle right and the middle left corner are opposed to each other in a way different, than the way in which the other two contradictory pairs of the hexagon are opposed to each other. I would like to call the opposition between the middle right and the middle left corner: "relativization" (instead of contradiction). The reasons for this name are going to be clear once these sentences are assigned adequate values in a multivalued setting. Since the hexagon in Fig. 1 insinuates a misleading account of these "nuances", I suggest the good-old-square in Fig. 2 as a more fitting representation of the oppositions between my examples.

The top left and top right sentences of Fig. 2 are false for two reasons: they presuppose the existence of the philosopher's stone and they commit a category mistake. The middle right sentence is false for just one reason: unjustified existential import (but no category mistake). The middle left sentence is opposed to the middle right sentence in a way different than the way, in which the bottom right and the bottom left sentences are opposed to the top left and the top right sentences respectively.

I take it that a four-valued system is adequate to cope with the semantic "nuances" of my examples which edge the polygons of Fig. 1 and Fig. 2. In what follows, I will try to substantiate this claim.

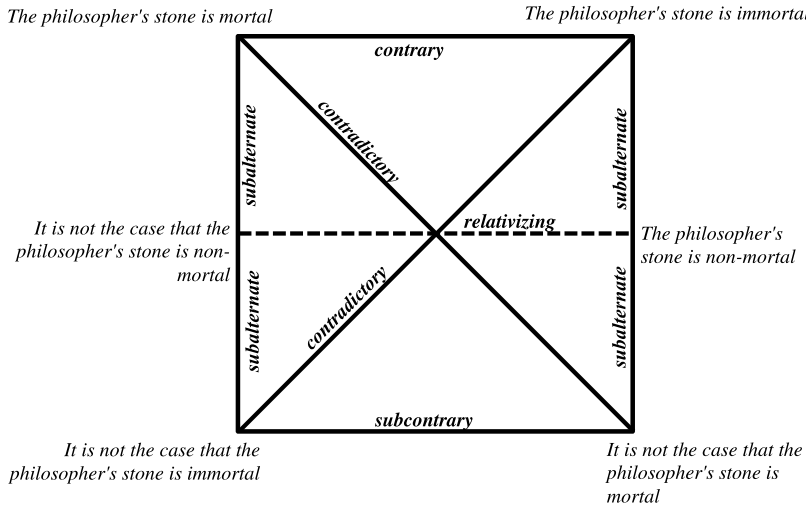

Fig. 2 A square with three different kinds of negation

3 What Is a Negation?

Privation is a kind of negation which is used in natural languages, but has never been used in a formal context. Drawing a semantics of privation is, for this reason, a task akin to redefining or reinterpreting negation in formal contexts. The following definition of negation is useful for my purposes.

Definition of Negation A negation (comprising external negation, negation of the predicate and privation among others) is any monadic propositional function, such that it renders at least the maximum and the minimum truth values of a sentence which serves as the argument of the function, into other values.

Remark on the Definition of Negation This definition implies that identity and the constant functions, which assign every sentence one single truth value, are not negations.

This captures, I think, the common element, which all sorts of negation, which are in use in logic, share. For, first, you need some truth-value change in a negation. This has to be at least the maximum and the minimum value. When a meteorologist predicts: 'It will rain tomorrow', then she might mean that this is true, or at least, that it is more true than false. To deny the accuracy of the meteorologist's prediction, is to say that, what she says is false or, at least, more false than true. If I take her prediction to be true granted that she means it to be true, but to be more false than true, granted that she means it to be more true than false, then, if anything, I cannot be said to deny the accuracy of her prediction. What I would do in such a case is maybe doubt the meteorologist's ability to maintain probabilities rather than necessities. But I would not *deny* the accuracy of the meteorologist's prediction *in every respect*. As one sees, a monadic propositional function, whose values deviates only from any intermediate value of the sentence which serves as its argument, but does not deviate from the maximum and the minimum values of this sentence, is *not* a negation.

In other words, for a monadic propositional function $f(p)$, if

- $v_i[p]$ is the intermediate value of the sentence p which serves as the argument of $f(p)$; and
- $v_x[f(p)]$ is the value of $f(p)$ if $v_i[p]$ is the value of p; and
- $v_y[f(p)]$ and $v_z[f(p)]$ are the values of $f(p)$ if p is assigned the values $v_{max}[p]$ and $v_{min}[p]$ respectively; and if finally
- $v_y[f(p)] = max$ and $v_z[f(p)] = min$

then $f(p)$ is *not* a negation, whatever the value $v_x[f(p)]$.

In order to have a negation, however, it is not necessary to have $v_y[f(p)] = min$ and $v_z[f(p)] = max$. Post's cyclical negation does not function in this way. Moreover, it is not necessary to change the intermediate values of the initial sentence. The strong negation in Blau's three valued logic does not. But, of course, one can change also some intermediate values along with the maximal and minimal values of the initial sentence to obtain a negation thereof. The symmetrical negation in Łukasiewicz's four-valued logic does exactly this.

Theorem *For any n-valued logic there are* $n^{n-2} \times (n-1)^2 - (n-2)$ *negations in the sense of the aforementioned Definition of Negation.*

Proof This number follows from the following considerations. The number of the rows of any truth-table which delivers the truth-functional definition of a monadic function in an n-valued logic is n. The number of all monadic functions of an n-valued logic is n^n. But since negations are only those monadic operations, in which the maximum and the minimum values of the initial sentence must be rendered into other values, in every possible truth table which delivers the truth-functional definition of a *negation in the sense of the aforementioned Definition of Negation*, there are two rows which cannot contain all possible values of the initial sentence. That is, there are n^{n-2} possible combinations of values in $n-2$ rows which are combined with, i.e. multiplied by $(n-1)^2$ possible combinations of values in 2 rows (the highest and the lowest). We have to subtract from this product the number of those monadic functions which are constant and give a value other than the maximum or the minimum. These are $n-2$. □

Remark on the Proof The two constant functions which give the maximum and the minimum values, are, of course, also not welcome as negations, but we have already excluded them by demanding the maximum and the minimum values $v_{max}[p]$ and $v_{min}[p]$ to result anything but $v_y[f(p)] = max$ and $v_z[f(p)] = min$ respectively. For the same reason we have already excluded identity from negations.

Corollary 1 *Since I am attempting to square the semantic "nuances" of my examples which edge the square in Fig. 2 by operating in a four-valued system, let me remark that in every four-valued logic there are* 142 *negations in the above sense.*

Among these kinds of negation, there are some awkward ones. One might doubt, for example, whether the function which turns true and false sentences to more-true-than-false and intermediate values to true, is a negation. I would say that it is. It is a considerably weak denial of maximal and minimal truth values.

4 The Semantics of the Logic of Privations

I attempt to model privation-talk in the logic of privations (LP), which I take to be a fragment of the Łukasiewicz-four-valued predicate logic; a fragment of an extension of the Łukasiewicz-four-valued propositional logic, that is, which is large enough to contain quantifiers, predicate symbols, individual variables and constants and indexical operators.[2]

LP is only a fragment of the Łukasiewicz-four-valued predicate logic because quantifiers and individual variables occur in LP only as parts of singular terms: definite descriptions or indexical/demonstrative expressions.

The reason for this proviso is the following: one basic idea of LP is that external negations are subalternate to privations. For example 'It is not the case that Stevie Wonder can see' is subalternate to 'Stevie Wonder is blind' (cf. the top left and the bottom left sentences of Fig. 2). Subalternations of this kind, however, *do not apply* in expressions like 'Some singers…', etc. Note, for example, that 'Some singers are blind' does *not* imply: 'It is not the case that some singers can see', since it is true that there are blind singers, it is not true however that all singers cannot see. In order to ensure, now, that external negations are subalternate to privations, by turning English contexts into LP-contexts, one has to restrict context to English sentences, whose grammatical subject is a proper name or, if it is any other (common) noun, it is governed by an indexical/demonstrative ('here', 'there', 'now', 'this', 'that', etc.) or by the definite article ('the').

LP contains the usual symbols for individual constants, individual variables and logical constants plus some symbols for alternative kinds of negation. Where we have to go into some detail, are the logical constants: $\rightarrow$, $\neg$, $-$, $\sim$, $\vdash$.

The truth values are: 1 (true), 2 (more-true-than-false), 3 (more-false-than-true), 4 (false).

The arrow ($\rightarrow$) symbolizes implication with the meaning which this operation has in Łukasiewicz's four-valued propositional logic. The following table defines truth-functionally the propositional function $p \rightarrow q$:

$p \backslash q$	1	2	3	4
1	1	2	3	4
2	1	1	3	3
3	1	2	1	2
4	1	1	1	1

The other logical constants in question have to be understood as follows

- '$\neg$': weak negation;
- '$-$': presupposing negation (the formal counterpart of the negation of the predicate in English—note that it has been often held to be a privation in the history of philosophy);
- '$\sim$: symmetric negation (Łukasiewicz's four-valued negation);
- '$\vdash$': privation.

[2]Gottwald [8, pp. 55–57] gives a brief exposition of the construction of the Łukasiewicz-four-valued *predicate* logic; he [8, pp. 45–47] also gives the axioms and the deduction rules of the Łukasiewicz-four-valued propositional logic. For a brief but full account of the semantics of the Łukasiewicz-four-valued propositional logic cf. [13, pp. 29–32].

The truth-functional definition of these kinds of negation is given by the following table:

p	$\neg p$	$-p$	$\sim p$	$\vdash p$
1	4	4	4	4
2	1	3	3	4
3	1	4	2	4
4	1	1	1	1

Now, let me assign the sentences of the square in Fig. 2 values between 1 and 4.

- Top right: v['The philosopher's stone is immortal'] = 4 (NB, a privation which is false for two reasons: unjustified existential import and category mistake);
- Top left: v['The philosopher's stone is mortal'] = 4 (an affirmative sentence which is false for the same reasons as its contrary—contrary sentences can be simultaneously false);
- Middle right: v['The philosopher's stone is non-mortal'] = 3 (NB, a presupposing negation, which is false for one reason: unjustified existential import);
- Middle left: v['It is not the case that the philosopher's stone is non-mortal'] = 2 (a *symmetric negation* (!) of the presupposing middle right negation—readers can very easily persuade themselves that, when a sentence has the value 3, as is here the case, then the symmetrical negation of this sentence gives the value 2);
- Bottom right: v['It is not the case that the philosopher's stone is mortal'] = 1 (a symmetric negation, forming a contradictory pair with the false top left affirmative sentence);
- Bottom left: v['It is not the case that the philosopher's stone is immortal'] = 1 (a *symmetric negation* of a privation, forming a contradictory pair with the false top right privation).

As one sees the external negation of English in the sentences which edge the square of Fig. 2 has to be formalized with a *symmetric*, *not* with a weak negation.

The peculiar name which I have chosen to express the opposition between the middle right and the middle left sentences ('relativizing' in Fig. 2 instead of 'contradictory' in Fig. 1) has to do with the truth-values 3 and 2 which I just assigned these sentences: the one says that the philosopher's stone is non-mortal—because it does not belong to the things which are either mortal or immortal. The other adds for consideration or, rather, relativizes, that it is not existing either, so that it is natural for it to fail being non-mortal.

As one sees, four values, some negations more and an understanding of subalternation as Łukasiewicz's four-valued implication are sufficient to solve the puzzle which arises out of a natural understanding of the middle-right and the middle-left sentences.

One last remark: In my outline of LP, I used only five negations (privation, presupposing negation, symmetric negation, symmetric negation of a presupposing negation, symmetric negation of a privation), although in the Corollary 1 spoke of 142 negations in a four-valued logic. Are the rest kinds irrelevant? The answer is no! The square could be edged by alternative kinds of negation, instead of those which I presented. And, what is much more, many additional kinds of negations and privations among them might be introduced. For example lacking magnificence, undoubtedly a privation, allows many intermediate grades of privation: lacking magnificence but not excellence; then lacking magnificence and excellence, but not goodness; moreover lacking magnificence, excellence,

goodness, but still being mediocre; finally being bad. To be able to make a full account of the different sorts of truth, falsity and contradiction in a corresponding square, we might need to speak not of a four-valued logic, but in a more general manner of an n-valued logic. The number of the values would be dependent on the number of the negations and privations involved.

References

1. Alexander: In Priora Analytica. Wallies, M. (ed.). Reimer, Berlin (1883)
2. Ammonius: In Aristotelis De interpretatione commentarius. Busse, A. (ed.). Reimer, Berlin (1895)
3. Aristotle: De interpretatione. Cooke, H.P. (ed.). Harvard University Press, Cambridge (1938)
4. Aristotle: Metaphysica. Jaeger, W. (ed.). Clarendon, Oxford (1957)
5. Blau, U.: Die dreiwertige Logik der Sprache. de Gruyter, Berlin (1977)
6. Boethius: Commentarius in librum Aristotelis Peri hermeneias. Meiser, C. (ed.). Teubner, Leipzig (1877)
7. Gassendi, P.: Exercitationes paradoxicae adversus Aristoteleos. Amsterdam (1649)
8. Gottwald, S., et al.: Nichtklassische Logik. Eine Einführung. Akademie Verlag, Berlin (1988)
9. Hamilton, W.: Lectures on Metaphysics and Logic, vol. 2. Gould and Lincoln, Boston (1860)
10. Horn, L.R.: A Natural History of Negation. CSLI, Palo Alto (2001)
11. Italos, J.: Quaestiones quodlibetales. Joannou, P. (ed.). Studia Patristica et Byzantina, vol. 4. Buch-Kunstverlag, Ettal (1956)
12. Kant, I.: Kritik der reinen Vernunft. Suhrkamp, Frankfurt a. M. (1974)
13. Sinowjew, A.A.: Über mehrwertige Logik. Ein Abriss. VEB, Berlin (1968)
14. Valla, L.: Retractatio totius dialectice cum fundamentis universe philosophie (=Repastinatio dialectice et philosophie, vol. 1). Zippel, G. (ed.). Antenore, Padua (1982)
15. Wolff, Ch.: Philosophia rationalis sive logica, 2nd edn. Frankfurt a. M. and Leipzig (1732)
16. Wolfson, H.A.: Infinite and privative judgments in Aristotle, Averroes and Kant. Philos. Phenomenol. Res. **8**, 173–187 (1947)

S. Gerogiorgakis (✉)
University of Erfurt, Hauspostfach 64, PSF 900221, 99105 Erfurt, Germany
e-mail: stamatios.gerogiorgakis@uni-erfurt.de