Research of Adaptive RQ System M/M/1 with Unreliable Server

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Abstract. The paper considers a single-line retrial queueing (RQ) system with an unreliable server controlled by a adaptive random multiple access protocol. The study is carried out using the method of asymptotic analysis under conditions of heavy system load. In this paper, the main characteristics of the system were found.

Keywords: Retrial queue · Adaptive random multiple access protocol · Unreliable server

1 Introduction

Unreliable servers can be used in telecommunications, call centers and data networks. For example, faulty hardware or software can lead to network failures, loss of communications, interruptions in data transmission, or incorrect processing of information. Research into unreliable devices in such areas helps to identify the causes of failures and develop methods to prevent them.

RQ queuing system with unreliable server and adaptive random multiple access protocol is a system that combines elements of queuing, unreliable server and adaptive random multiple access protocols in a data network.

In such systems, a large number of customers or clients are served using unreliable devices that may be subject to failures or malfunctions. To ensure efficiency and reliability, such a system can be equipped with an adaptive random multiple access protocol, which provides mechanisms for optimizing the use of available resources and managing data transmission in the face of varying network load, interference, or frequent server failures.

Adaptive Random Multiple Access Protocol is a method of controlling access to a common data link that allows multiple devices to share available resources. It is a form of multiple access protocol that allows devices to compete for access to data communications.

Unlike static protocols, an adaptive random multiple access protocol can change its parameters depending on current network conditions such as load, collisions and delays. This allows you to optimize the use of the available data channel and significantly reduce the likelihood of collisions (a situation in which

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two or more devices try to transmit data at the same time, resulting in signal loss).

Many scientific works are devoted to the study of various models of data transmission networks and random multiple access protocols. There are various modifications of access protocols [1–8].

In papers [9–12], authors investigate models with adaptive access protocols. The papers [13–20] consider the study of queuing systems with a dynamic access protocol.

In this paper, we study a single-channel RQ system with an unreliable device controlled by an adaptive access protocol. The server is considered unreliable if it periodically fails and requires time to be restored. Which, accordingly, can lead to a decrease in the efficiency of the system and an increase in waiting time for service.

2 Description of the Mathematical Model

Let's consider an RQ system with an adaptive access protocol, the input of which receives a simple flow of requests with parameter λ. The time for servicing a customer by the server is distributed exponentially with the parameter μ_1. We assume that the server is unreliable. An unreliable device can be in one of the following states: idle, busy or under repair. When a new customer arrives and the server is idle, then the servicing immediately begins.

If at this moment another customer arrives, and the device is busy, then the received customer goes into orbit and waits for the opportunity to occupy the device during the next attempt. After a random delay, a request with intensity $\sigma = 1/T(t)$ again tries to occupy the server for service, where $T(t)$ is the state of the adapter at time t (see Fig. 1).

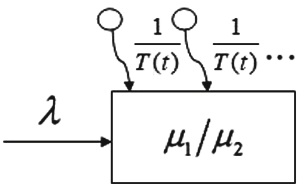

Fig. 1. Model of adaptive retrial queueing system M/M/1 with unreliable server

The working time is distributed exponentially with parameter γ_1, if server is idle and with parameter γ_2, if the server is busy. As soon as a breakdown occurs, the server is sent to repair and the servicing customer goes into the orbit. During repairing, all incoming customers go into the orbit. The recovery time is distributed exponentially with parameter μ_2.

The goal of the research is to study such a system, as well as to determine its main characteristics and to find the throughput of the system and the stationary probability distribution of server states.

Let $i(t)$ be the number of customers in the orbit at time t and $k(t)$ determine the state of the server as follows:

$$k(t) = \begin{cases} 0, \text{if the server is idle,} \\ 1, \text{if the server is busy,} \\ 2, \text{if the server is under repair.} \end{cases}$$

The process of changing adapter states $T(t)$ is defined as follows:

$$T(t+\Delta t) = \begin{cases} T(t) - \alpha\Delta t, \text{if } k(t) = 0, \\ T(t+\Delta t) = T(t), \text{if } k(t) = 1, \\ T(t) + \beta\Delta t, \text{if } k(t) = 2, \end{cases}$$

where $\alpha > 0,\ \beta > 0$ are adapter parameters, the values of which are indicated.

If the device is idle, then $T(t)$ decreases linearly with intensity α; if the device is busy, then $T(t)$ does not change; if the device is under repair, then $T(t)$ increases linearly with intensity β.

3 The Method of Asymptotic Analysis

Let us denote

$$P(k,\ i,\ T,\ t) = \frac{\partial P\{k(t) = k,\ i(t) = i,\ T(t) < T\}}{\partial T}$$

- the probability that at time t the server is in state k and i customers in the orbit.

The probability distribution $P(k,\ i,\ T,\ t)$ satisfies the following system of equations:

$$\begin{cases} P(0, i, T-\alpha\Delta t,\ t+\Delta t) = (1-\lambda\Delta t)\left(1-\frac{i}{T}\Delta t\right)(1-\gamma_1\Delta t)\,P(0, i,\ T, t) \\ +\mu_1\Delta t P(1, i, T, t) + \mu_2\Delta t P(2, i, T, t) + o(\Delta t), \\ P(1, i, T,\ t+\Delta t) = (1-\lambda\Delta t)(1-\mu_1\Delta t)(1-\gamma_2\Delta t)\,P_1(1, i, T,\ t) \\ +\lambda\Delta t P(0, i, T,\ t) + \frac{i+1}{T}\Delta t P(0, i+1, T, t) + \lambda P(1, i-1, T, t) + o(\Delta t), \\ P(2, i,\ T+\beta\Delta t, t+\Delta t) = (1-\lambda\Delta t)(1-\mu_2\Delta t)\,P(2, i, T, t) \\ +\gamma_1\Delta t P(0, i, T, t) + \gamma_2\Delta t P(1, i-1, T, t) + \lambda\Delta t P(2, i-1, T, t) + o(\Delta t). \end{cases}$$

Let us compose a system of Kolmogorov differential equations for the stationary probability distribution $P(k,\ i,\ T)$:

$$\begin{cases} -\alpha\dfrac{\partial P(0,i,\ T)}{\partial T} = -\left(\lambda+\dfrac{i}{T}+\gamma_1\right)P(0,i,T)+\mu_1P(1,i,T) \\ +\mu_2P(2,i,T), \\ \dfrac{\partial P(1,i,\ T)}{\partial T} = -(\lambda+\mu_1+\gamma_2)P(1,i,\ T)+\lambda P(0,i,T) \\ +\dfrac{i+1}{T}P(0,i+1,T)+\lambda P(1,i-1,T), \\ \beta\dfrac{\partial P(2,i,\ T)}{\partial T} = -(\lambda+\mu_2)P(2,i,T)+\gamma_1P(0,i,T) \\ +\gamma_2P(1,i-1,T)+\lambda P(2,i-1,T). \end{cases} \tag{1}$$

Let us denote the partial characteristic functions

$$\begin{aligned} H_k(u_1,\ u_2) &= \sum_i e^{-u_1 i}\int_0^\infty e^{-u_2T}P(k,i,T)dT \\ &= P\{k(t)=k\}M\{e^{-u_1i(t)-u_2T(t)}\,|k(t)=k\}. \end{aligned} \tag{2}$$

Functions $H_k(u_1,\ u_2)$ have the following properties:

$$\sum_i e^{-u_1 i}\int_0^\infty e^{-u_2T}iP(k,i,T)dT = -\frac{\partial H_k(u_1,u_2)}{\partial u_1},$$

$$\sum_i e^{-u_1 i}\int_0^\infty e^{-u_2T}\frac{\partial P(k,i,T)}{\partial T}dT = u_2H_k(u_1,u_2),$$

$$\sum_i e^{-u_1 i}\int_0^\infty e^{-u_2T}\frac{1}{T}P(k,i,T)dT = \int_{u_2}^\infty H_k(u_1,x)dx.$$

Using the Eq. (2) and the properties of characteristic functions from System (1) we obtain:

$$\begin{cases} (\alpha u_2-\lambda-\gamma_1)H_0(u_1,u_2)+\displaystyle\int_{u_2}^\infty\frac{\partial H_0(u_1,x)}{\partial u_1}dx+\mu_1H_1(u_1,u_2) \\ +\mu_2H_2(u_1,u_2)=0, \\ -(\lambda(1-e^{-u_1})+\mu_1+\gamma_2+u_2)H_1(u_1,u_2)+\lambda H_0(u_1,u_2) \\ -e^{u_1}\displaystyle\int_{u_2}^\infty\frac{\partial H_0(u_1,x)}{\partial u_1}dx=0, \\ -(\lambda(1-e^{-u_1})+\mu_2+\beta u_2)H_2(u_1,u_2)+\gamma_1H_0(u_1,u_2)+ \\ \gamma_2e^{-u_1}H_1(u_1,u_2)=0. \end{cases}$$

We introduce a parameter

$$\rho = \frac{\lambda}{\mu_1},$$

that characterizes the system load, then we get

$$\begin{cases} \left(\frac{\alpha u_2 - \gamma_1}{\mu_1} - \rho\right) H_0\left(u_1, u_2\right) + \frac{1}{\mu_1}\int\limits_{u_2}^{\infty} \frac{\partial H_0(u_1, x)}{\partial u_1} dx + H_1\left(u_1, u_2\right) \\ + \frac{\mu_2}{\mu_1} H_2\left(u_1, u_2\right) = 0, \\ -\left(\rho(1 - e^{-u_1}) + 1 + \frac{\gamma_2 + u_2}{\mu_1}\right) H_1\left(u_1, u_2\right) + \rho H_0\left(u_1, u_2\right) \\ - \frac{e^{u_1}}{\mu_1}\int\limits_{u_2}^{\infty} \frac{\partial H_0(u_1, x)}{\partial u_1} dx = 0, \\ -\left(\rho(1 - e^{-u_1}) + \frac{\mu_2 + \beta u_2}{\mu_1}\right) H_2(u_1, u_2) + \frac{\gamma_1}{\mu_1} H_0\left(u_1, u_2\right) \\ + \frac{\gamma_2}{\mu_1} e^{-u_1} H_1\left(u_1, u_2\right) = 0. \end{cases} \tag{3}$$

There are no exact analytical methods for solving the System (3), so we will find the main characteristics of the adaptive system.

Let's study the System (3) under the condition of heavy load. Let us define the throughput S of an adaptive RQ system as the exact upper bound of system load values ρ for which there is a steady-state regime with $\rho \uparrow S$.

Let us denote

$$\varepsilon = S - \rho.$$

Assuming that $\varepsilon \to 0$ in the System (2), we will perform the following substitutions:

$$\rho = S - \varepsilon,\ u_1 = \varepsilon w_1,\ u_2 = \varepsilon w_2,\ H_k(u_1, u_2) = F_k(w_1, w_2, \varepsilon).$$

Then, we obtain:

$$\begin{cases} F_0\left(w_1, w_2, \varepsilon\right)\left(\frac{\alpha \varepsilon w_2 - \gamma_1}{\mu_1} - (S - \varepsilon)\right) + \frac{1}{\mu_1}\int\limits_{u_2}^{\infty} \frac{\partial F_0(w_1, x, \varepsilon)}{\partial w_1} dx \\ + F_1\left(w_1, w_2, \varepsilon\right) + \frac{\mu_2}{\mu_1} F_2\left(w_1, w_2, \varepsilon\right) = 0, \\ F_0\left(w_1, w_2, \varepsilon\right)(S - \varepsilon) - \frac{e^{\varepsilon w_1}}{\mu_1}\int\limits_{w_2}^{\infty} \frac{\partial F_0(w_1, x, \varepsilon)}{\partial w_1} dx \\ + F_1\left(w_1, w_2, \varepsilon\right)\left((S - \varepsilon)(e^{-\varepsilon w_1} - 1) - 1 - \frac{\gamma_2 + \varepsilon w_2}{\mu_1}\right) = 0, \\ F_0\left(w_1, w_2, \varepsilon\right)\frac{\gamma_1}{\mu_1} + F_1\left(w_1, w_2, \varepsilon\right)\frac{\gamma_2}{\mu_1} e^{-\varepsilon w_1} \\ + F_2(w_1, w_2, \varepsilon)\left((S - \varepsilon)(e^{-\varepsilon w_1} - 1) - \left(\frac{\mu_2 + \beta \varepsilon w_2}{\mu_1}\right)\right) = 0. \end{cases} \tag{4}$$

Theorem 1. *The values of the parameters* S *and* y *in the adaptive RQ-system are determined by the equalities*

$$S = \frac{\alpha\mu_2 - \beta\gamma_1}{(1-\beta)\gamma_1 + (\alpha+1)\mu_2 + \gamma_2(\alpha+\beta)},$$

$$y = \frac{(\alpha\mu_2 - \beta\gamma_1)((\alpha+\beta)\gamma_2^2 + ((\alpha+1)\mu_2 + \alpha\mu_1 - \beta\gamma_1 + \gamma_1)\gamma_2)}{(\gamma_2(\alpha+\beta) + (\alpha+1)\mu_2 + \gamma_1(1-\beta))(\beta\gamma_2 + \mu_2)} + \frac{\mu_1(\alpha\mu_2 + \gamma_1(1-\beta))}{(\gamma_2(\alpha+\beta) + (\alpha+1)\mu_2 + \gamma_1(1-\beta))(\beta\gamma_2 + \mu_2)}.$$

where $\alpha > 0,\ \beta > 0$ *are adapter parameters, the values of which are indicated.*

Proof. Let us denote $\lim\limits_{\varepsilon\to 0} F_k(w_1, w_2, \varepsilon) = F_k(w_1, w_2)$ and for $\varepsilon \to 0$, we get

$$\begin{cases} -F_0(w_1,w_2)\left(\dfrac{\gamma_1}{\mu_1} + S\right) + \dfrac{1}{\mu_1}\displaystyle\int_{w_2}^{\infty}\dfrac{\partial F_0(w_1,x)}{\partial w_1}dx + F_1(w_1,w_2) \\ + \dfrac{\mu_2}{\mu_1}F_2(w_1,w_2) = 0, \\ F_0(w_1,w_2)S - \dfrac{1}{\mu_1}\displaystyle\int_{w_2}^{\infty}\dfrac{\partial F_0(w_1,x)}{\partial w_1}dx - F_1(w_1,w_2)(1 + \dfrac{\gamma_2}{\mu_1}) = 0, \\ F_0(w_1,w_2)\dfrac{\gamma_1}{\mu_1} + F_1(w_1,w_2)\dfrac{\gamma_2}{\mu_1} - F_2(w_1,w_2)\dfrac{\mu_2}{\mu_1} = 0. \end{cases} \quad (5)$$

We will look for the solution $F_k(w_1, w_2)$ of the System (5) in the form:

$$F_k(w_1, w_2) = F_k\Phi(w_1, w_2) = R_k(S, y)\varphi(w_2 + w_1 y). \quad (6)$$

Assuming that the function $\varphi(w)$ is equal to zero at infinity, we obtain

$$\int_{w_2}^{\infty}\frac{\partial F_0(w_1,x)}{\partial w_1}dx = -yR_0(S,y)\varphi(w_2 + yw_1).$$

Then we rewrite the System (5):

$$\begin{cases} -R_0(S,y)\left(\dfrac{\gamma_1}{\mu_1} + S\right) - \dfrac{y}{\mu_1}R_0(S,y) + R_1(S,y) + \dfrac{\mu_2}{\mu_1}R_2(S,y) = 0, \\ R_0(S,y)S + \dfrac{y}{\mu_1}R_0(S,y) - R_1(S,y)(1 + \dfrac{\gamma_2}{\mu_1}) = 0, \\ R_0(S,y)\dfrac{\gamma_1}{\mu_1} + R_1(S,y)\dfrac{\gamma_2}{\mu_1} - R_2(S,y)\dfrac{\mu_2}{\mu_1} = 0. \end{cases} \quad (7)$$

Let us add a normalization condition to the System (7):

$$R_0 + R_1 + R_2 = 1.$$

We obtain:

$$\begin{cases} -R_0\left(S,y\right)\left(\dfrac{\gamma_1}{\mu_1}+S\right)-\dfrac{y}{\mu_1}R_0\left(S,y\right)+R_1\left(S,y\right)+\dfrac{\mu_2}{\mu_1}R_2\left(S,y\right)=0, \\ R_0\left(S,y\right)S+\dfrac{y}{\mu_1}R_0\left(S,y\right)-R_1\left(S,y\right)(1+\dfrac{\gamma_2}{\mu_1})=0, \\ R_0\left(S,y\right)\dfrac{\gamma_1}{\mu_1}+R_1\left(S,y\right)\dfrac{\gamma_2}{\mu_1}-R_2(S,y)\dfrac{\mu_2}{\mu_1}=0. \\ R_0+R_1+R_2=0. \end{cases} \tag{8}$$

Then from the System (8) we find expressions for the stationary distribution of server states:

$$R_0=\frac{\mu_2(\gamma_2+\mu_1)}{(S\mu_1+\gamma_1+\mu_2+y)\gamma_2+(S\mu_2+\mu_2+\gamma_1)\mu_1+\mu_2 y},$$

$$R_1=\frac{\mu_2(S\mu_1+y)}{(S\mu_1+\gamma_1+\mu_2+y)\gamma_2+(S\mu_2+\mu_2+\gamma_1)\mu_1+\mu_2 y},$$

$$R_2=\frac{(S\mu_1+\gamma_1+y)\gamma_2+\gamma_1\mu_1}{(S\mu_1+\gamma_1+\mu_2+y)\gamma_2+(S\mu_2+\mu_2+\gamma_1)\mu_1+\mu_2 y}.$$

To find the values S and y, we sum up all the equations of the System (4) and for $\varepsilon\to 0$, we obtain

$$\begin{aligned} &F_0\left(w_1,w_2,\varepsilon\right)\left(\frac{\alpha\varepsilon w_2}{\mu_1}\right)-\frac{e^{\varepsilon w_1}-1}{\mu_1}\int\limits_{u_2}^{\infty}\frac{\partial F_0(w_1,x,\varepsilon)}{\partial w_1}dx \\ &+F_1\left(w_1,w_2,\varepsilon\right)((S-\varepsilon)(e^{-\varepsilon w_1}-1)+\frac{\gamma_2}{\mu_1}(e^{-\varepsilon w_1}-1)-\frac{\varepsilon w_2}{\mu_1}) \\ &+F_2\left(w_1,w_2,\varepsilon\right)((S-\varepsilon)(e^{-\varepsilon w_1}-1)-\frac{\beta\varepsilon w_2}{\mu_1})=0. \end{aligned}$$

Dividing resulting equation by ε, we get:

$$\begin{aligned} &F_0\left(w_1,w_2,\varepsilon\right)\left(\frac{\alpha w_2}{\mu_1}\right)-\frac{e^{\varepsilon w_1}-1}{\mu_1\varepsilon}\int\limits_{u_2}^{\infty}\frac{\partial F_0(w_1,x,\varepsilon)}{\partial w_1}dx \\ &+F_1\left(w_1,w_2,\varepsilon\right)((S-\varepsilon)\frac{(e^{-\varepsilon w_1}-1)}{\varepsilon}+\frac{\gamma_2}{\mu_1}\frac{(e^{-\varepsilon w_1}-1)}{\varepsilon}-\frac{w_2}{\mu_1}) \\ &+F_2\left(w_1,w_2,\varepsilon\right)((S-\varepsilon)\frac{(e^{-\varepsilon w_1}-1)}{\varepsilon}-\frac{\beta w_2}{\mu_1})=0. \end{aligned}$$

Then using the Taylor expansion, we obtain:

$$F_0\left(w_1, w_2, \varepsilon\right) \frac{\alpha w_2}{\mu_1} - \frac{w_1}{\mu_1} \int\limits_{u_2}^{\infty} \frac{\partial F_0(w_1, x, \varepsilon)}{\partial w_1} dx$$

$$- F_1\left(w_1, w_2, \varepsilon\right) \left(w_1 (S + \frac{\gamma_2}{\mu_1} + \frac{w_2}{\mu_1}\right) - F_2\left(w_1, w_2, \varepsilon\right) \left(S w_1 + \frac{\beta w_2}{\mu_1}\right) = 0.$$

Applying (6) to the equation, we get:

$$\begin{aligned} &\alpha w_2 \varphi(w_2 + w_1 y) R_0(S, y) + y w_1 \varphi(w_2 + w_1 y) R_0(S, y) \\ &- \varphi(w_2 + w_1 y) R_1(S, y)(\mu_1 w_1 S + w_1 \gamma_2 + w_2) \\ &- \varphi(w_2 + w_1 y) R_2(S, y)(\mu_1 w_1 S + \beta w_2) = 0. \end{aligned}$$

Let us write the equation in the form:

$$\begin{aligned} &\alpha w_2 R_0(S, y) + y w_1 R_0(S, y) - R_1(S, y)(\mu_1 w_1 S + w_1 \gamma_2 + w_2) \\ &- R_2(S, y)(\mu_1 w_1 S + \beta w_2) = 0. \end{aligned} \tag{9}$$

Then we rewrite the Eq. (9):

$$\begin{aligned} &w_1(y R_0(S, y) - \mu_1 S R_1(S, y) - \gamma_2 R_1(S, y) - \mu_1 S R_2(S, y)) \\ &+ w_2(\alpha R_0(S, y) - R_1(S, y) - \beta R_2(S, y)) = 0. \end{aligned}$$

In order to turn the equation into an identity in w_1 and w_2, it is enough to require the following equalities:

$$\begin{cases} y R_0(S, y) - \mu_1 S R_1(S, y) - \gamma_2 R_1(S, y) - \mu_1 S R_2(S, y) = 0, \\ \alpha R_0(S, y) - R_1(S, y) - \beta R_2(S, y) = 0. \end{cases} \tag{10}$$

By substituting $R_0(S, y)$, $R_1(S, y)$, $R_2(S, y)$ into the System (10), we get:

$$S = \frac{\alpha \mu_2 - \beta \gamma_1}{(1 - \beta)\gamma_1 + (\alpha + 1)\mu_2 + \gamma_2(\alpha + \beta)},$$

$$\begin{aligned} y = &\frac{(\alpha \mu_2 - \beta \gamma_1)((\alpha + \beta)\gamma_2^2 + ((\alpha + 1)\mu_2 + \alpha \mu_1 - \beta \gamma_1 + \gamma_1)\gamma_2)}{(\gamma_2(\alpha + \beta) + (\alpha + 1)\mu_2 + \gamma_1(1 - \beta))(\beta \gamma_2 + \mu_2)} \\ &+ \frac{\mu_1(\alpha \mu_2 + \gamma_1(1 - \beta))}{(\gamma_2(\alpha + \beta) + (\alpha + 1)\mu_2 + \gamma_1(1 - \beta))(\beta \gamma_2 + \mu_2)}. \end{aligned}$$

Definition. Throughput S is the upper limit of those load values $\rho = \frac{\lambda}{\mu_1}$, for which there is the steady-state regime.

The inequality

$$\frac{\lambda}{\mu_1} \leq S$$

determines the condition for the existence of a steady-state regime for the considered adaptive system.

So Theorem 1 is proved.

4 Numerical Example

We consider a system with parameters:

$$\mu_1 = 5, \quad \mu_2 = 2, \quad \gamma_1 = 0.03, \quad \gamma_2 = 0.03, \quad \lambda = 1, \quad \beta = 1.$$

Table 1 shows the values of S and y for a given system for different α.

Table 1. Values of S and y for different α

α	0,2	0,4	0,8	1	5	10	100
S	0,036	0,155	0,314	0,370	0,703	0,779	0,860
y	0,049	0,376	1,426	2,070	18,838	41,502	455,880

According to the data in Table 1, we can conclude that as α increases, the value of throughput S increases, and the value of y also increases significantly.

For adaptive RQ systems, under the limiting condition of heavy system load, random processes $i(t)$ and $T(t)$ are linearly dependent with some parameter y equal to the ratio of linearly dependent random processes $i(t)/T(t)$.

When $\alpha = 0,9$, the throughput of the adaptive RQ system $S = 0,334$ and $y = 2$, which corresponds to the throughput of the dynamic RQ system M/M/1 $S = 0,334$ at $y = 2$, which confirms the asymptotic equivalence of the adaptive and dynamic RQ systems with the simplest incoming flow of customers.

Consequently, adaptive RQ systems are asymptotically, under heavy load conditions, equivalent to dynamic RQ systems with the specified value of the parameter y, calculated in the work [21].

5 Conclusion

In this paper, we study the adaptive RQ-system M/M/1 with an unreliable server. The study was carried out using the method of asymptotic analysis under conditions of heavy system load. As a result, the main characteristics of the system, the stationary distribution of server states, and the throughput of the system under consideration are found.

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