Infinite-Server Queueing System with Two States of Service and Abandonments

Radmir Salimzyanov(✉) and Alexander Moiseev

Tomsk State University, Tomsk, Russian Federation
rsalimzyanov@yahoo.com, moiseev.tsu@gmail.com

Abstract. Infinite-server queueing system with Poisson arrival process, two states of service and abandonments is considered in the paper. Such system can be used as a simple mathematical model of a subscriber communication network based on IAB (Integrated Access and Backhaul) technology with two mobile nodes. Joint probability distribution of the number of customers in the states of service is obtained under asymptotic condition of high intensity of the arrival process. Numerical experiments are performed to estimate precision and applicability area of the approximation built on the results of the asymptotic analysis.

Keywords: IAB · asymptotic analysis · infinite-server queue · service abandonments

1 Introduction

Integrated Access and Backhaul (IAB) is a technology that provides fast and cost-effective deployment on millimeter waves (mmWave) due to self-connection in the same spectrum [1]. Wireless autonomous reverse transmission uses the same wireless channel to cover and connect to other base stations (BS), which leads to increased productivity, more efficient use of spectrum resources, and lower equipment costs, as well as to reducing dependence on the availability of wired reverse transmission at each location of the access node [2].

Mathematical modeling of an IAB-based network using queueing theory is a promising research direction. In addition to the mentioned standard [3], there have been studies conducted by various authors on the coverage of BS [4], signal transmission speeds under different conditions, and the utilization of fifth-generation networks with IAB on the Internet of Things [5]. However, the question of modeling of such systems still remains open.

In this paper, we propose a mathematical model of IAB-based network with two mobile nodes in the form of infinite-server queueing system with two states of service and abandonments. This model takes into account the roaming of a user from one node to another during the entire service time and the possibility of early leaving the system.

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In paper [6], the authors considered a similar model in which there are two phases of service, but due to the specifics of the real problem, the phases of service are considered as sequential and each of them has its own service parameter (there is no a separate total time of successful service). Some results and literature reviews on models with abandonments of service (customers impatient in service) can be found in [7,8].

The rest part of the paper is organized as follows. In Sect. 2, the problem is formulated and a mathematical model in the form of a queueing system is proposed. In Sect. 3, the system of Kolmogorov equations is formulated and its exact solution obtained under the condition of equivalence of the local and global balance equations is provided. In Sect. 4, the asymptotic analysis method is applied for solution of the problem for a wider class of systems than the exact solution may be used for. As a result, an approximation of the joint probability distribution of the number of customers in the states of service is obtained. For estimating precision of the approximation and its applicability area, series of numerical experiments have been conducted. Their results are presented in Sect. 5. Conclusions are formulated in Sect. 6.

2 Problem and Mathematical Model

Making necessary assumptions and generalizations, we can depict the behavior of the entire system as follows. Let us consider an IAB system consisting of one donor and two mobile network customer service nodes (Fig. 1). Users move between two communication nodes and the following options are possible in the system:

- abandonment of service – user goes beyond the range of his or her communication node and does not connect to any other node (Fig. 1: a, b);
- internal migration – user goes beyond the range of his or her communication node, but immediately after that, he (or she) enters into the range of another communication node and can continue servicing (Fig. 1: a, b, and c);
- successful service completion – user completes his or her work and logs out of the system.

For this model, we are interested in how much the system is loaded, e.g. how many users are connected to each node, taking into account their possible migrations.

For modeling the system described above, we propose a mathematical model in the form of an infinte-server queue with two states of servicing (Fig. 2). The input flow is a Poisson arrival process with intensity λ. An incoming customer occupies any available server and starts its service in state 1 or 2 with probabilities v_1 or v_2, respectively. Duration of the service is an exponentially distributed random variable with parameter μ. While the customer is servicing, during time period of length Δt, it can move from state i to state k ($i, k \in \{1, 2\}$) with probability $\alpha_{ik}\Delta t$ (internal migration) or leave the system without completing

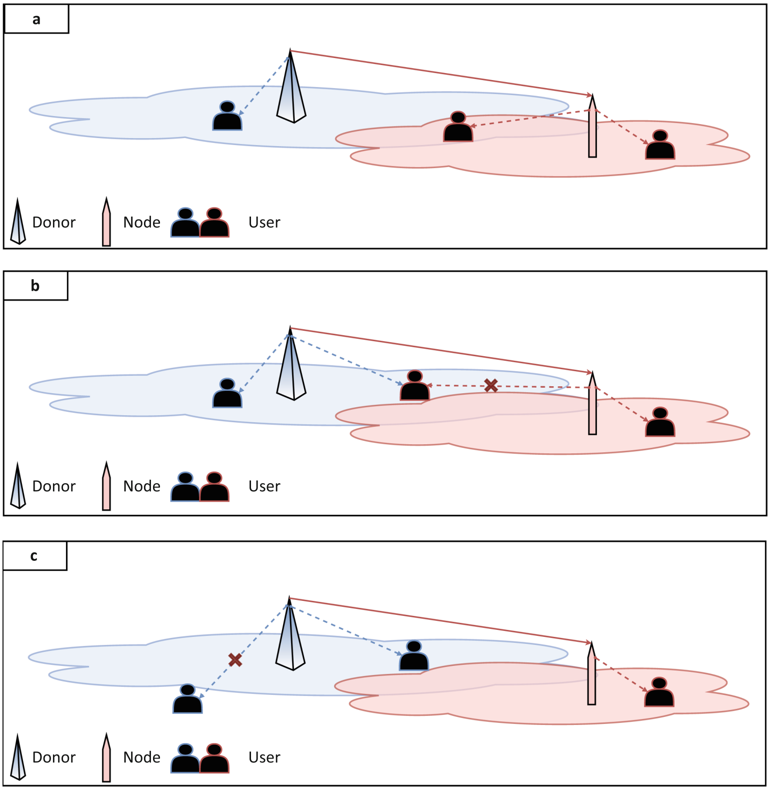


Fig. 1. System model

its service with probability $\alpha_{i0}\Delta t$ (abandonment of service). At the end of the service (successful service completion), the customer also leaves the system.

Let us denote the number of customers serviced in state i at instant t by $n_i(t)$ $(i = 1, 2)$. The problem is to find joint probability distribution of the number of customers in the states

$$P(n_1, n_2) = \Pr\{n_1(t) = n_1, n_2(t) = n_2\}$$

which we consider in a steady-state regime.

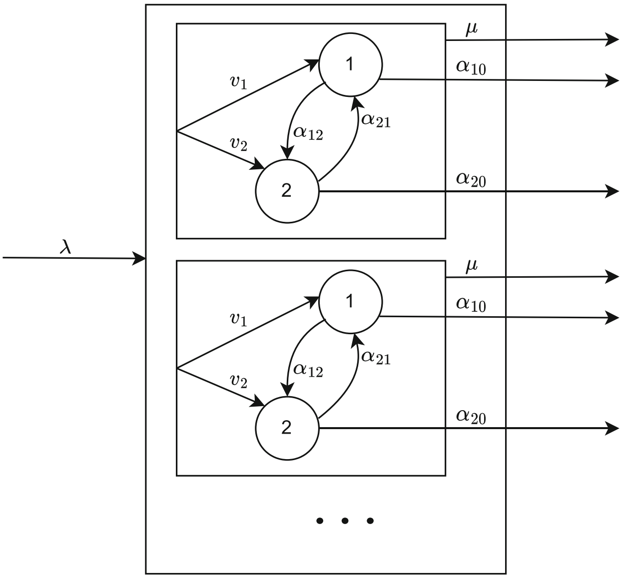


Fig. 2. Mathematical model

3 Kolmogorov Equations and Exact Solution

Described problem was considered in our recent paper [9]. The system of Kolmogorov equations for distribution $P(n_1, n_2)$ may be written as follows:

$$\begin{gathered} P(n_1,n_2)[\lambda + n_1\mu + n_2\mu + n_1\alpha_{10} + n_2\alpha_{20} + n_1\alpha_{12} + n_2\alpha_{21}] = \\ P(n_1+1,n_2)(n_1+1)[\mu+\alpha_{10}] + P(n_1,n_2+1)(n_2+1)[\mu+\alpha_{20}] \\ +P(n_1-1,n_2)v_1\lambda + P(n_1,n_2-1)v_2\lambda \\ +P(n_1-1,n_2+1)(n_2+1)\alpha_{21} + P(n_1+1,n_2-1)(n_1+1)\alpha_{12}. \end{gathered} \tag{1}$$

In [9], the following exact solution of the system was obtained:

$$\begin{gathered} P(n_1,n_2) = \frac{1}{n_1!n_2!}\left[\frac{v_1\lambda}{\mu+\alpha_{10}}\right]^{n_1}\left[\frac{v_2\lambda}{\mu+\alpha_{20}}\right]^{n_2} P(0,0), \\ P(0,0) = \left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\frac{1}{n_1!n_2!}\left[\frac{v_1\lambda}{\mu+\alpha_{10}}\right]^{n_1}\left[\frac{v_2\lambda}{\mu+\alpha_{20}}\right]^{n_2}\right)^{-1} \end{gathered} \tag{2}$$

under condition of the equivalence of the local and global balance equations which has the following form for the considered model:

$$\alpha_{21}[\mu+\alpha_{10}]v_2 = \alpha_{12}[\mu+\alpha_{20}]v_1. \tag{3}$$

Solution (2) can be applied only when condition (3) is satisfied and can not be used in other cases [10].

4 Asymptotic Analysis

Condition (3) imposes severe constraints, and solution (2) is almost inapplicable in practice. So, it is necessary to find a solution of system (1) for a wider range of model parameters. Because direct solution of the problem seems unreachable, we propose to use the asymptotic analysis method [11,12] for obtaining the solution.

Let us introduce the characteristic function

$$H(u_1, u_2) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} e^{ju_1 n_1} e^{ju_2 n_2} P(n_1, n_2) \tag{4}$$

(here $j = \sqrt{-1}$) and make corresponding transformations in (1). We obtain

$$\begin{gathered} H(u_1, u_2)(v_1 \lambda (e^{ju_1} - 1) + v_2 \lambda (e^{ju_2} - 1)) + \\ + j \frac{\partial H(u_1, u_2)}{\partial u_1} (\mu + \alpha_{10} + \alpha_{12} - e^{-ju_1} (\mu + \alpha_{10} - e^{ju_2} \alpha_{12})) + \\ + j \frac{\partial H(u_1, u_2)}{\partial u_2} (\mu + \alpha_{20} + \alpha_{21} - e^{-ju_2} (\mu + \alpha_{20} - e^{ju_1} \alpha_{21})) = 0. \end{gathered} \tag{5}$$

We look for the solution of (5) under the condition of increasing intensity of the incoming flow: $\lambda \to \infty$.

4.1 First-Order Asymptotic

As the solution will be sought under the condition of increasing intensity of the incoming flow, we introduce the following notation:

$$\varepsilon = \frac{1}{\lambda},$$

where $\varepsilon \to 0$ while $\lambda \to \infty$. Also, we introduce the following notations:

$$u_1 = \varepsilon w_1, \qquad u_2 = \varepsilon w_2, \qquad H(u_1, u_2) = F_1(w_1, w_2, \varepsilon).$$

Let us make these substitutions in Eq. (5):

$$\begin{gathered} F_1(w_1, w_2, \varepsilon) \frac{1}{\varepsilon} \{ v_1 (e^{j\varepsilon w_1} - 1) + v_2 (e^{j\varepsilon w_2} - 1) \} + \\ + j \frac{\partial F_1(w_1, w_2, \varepsilon)}{\partial w_1} \frac{1}{\varepsilon} \{ \mu + \alpha_{10} + \alpha_{12} - e^{-jw_1 \varepsilon} (\mu + \alpha_{10} + e^{j\varepsilon w_2} \alpha_{12}) \} + \\ + j \frac{\partial F_1(w_1, w_2, \varepsilon)}{\partial w_2} \frac{1}{\varepsilon} \{ \mu + \alpha_{20} + \alpha_{21} - e^{-jw_2 \varepsilon} (\mu + \alpha_{20} + e^{j\varepsilon w_1} \alpha_{21}) \} = 0. \end{gathered}$$

Using the expansion

$$e^{j\varepsilon w_k} = 1 + j\varepsilon w_k + +O(\varepsilon^2),$$

after completing limit transition $\varepsilon \to 0$, we obtain

$$\begin{gathered}F_1(w_1,w_2)(jw_1v_1+jw_2v_2)+\\ +j^2\frac{\partial F_1(w_1,w_2)}{\partial w_1}(w_1\mu+w_1\alpha_{10}-w_2\alpha_{12}+w_1\alpha_{12})+\\ j^2\frac{\partial F_1(w_1,w_2)}{\partial w_2}(w_2\mu+w_2\alpha_{20}-w_1\alpha_{21}+w_2\alpha_{21})=0.\end{gathered} \tag{6}$$

Let us rewrite Eq. (6) in the following form:

$$\begin{gathered}F_1(w_1,w_2)jw_1v_1+j^2\frac{\partial F_1(w_1,w_2)}{\partial w_1}(w_1\mu+w_1\alpha_{10}+w_1\alpha_{12})+\\ j^2\frac{\partial F_1(w_1,w_2)}{\partial w_2}(-w_1\alpha_{21})=0,\\ F_1(w_1,w_2)jw_2v_2+j^2\frac{\partial F_1(w_1,w_2)}{\partial w_1}(-w_2\alpha_{12})+\\ j^2\frac{\partial F_1(w_1,w_2)}{\partial w_2}(w_2\mu+w_2\alpha_{20}+w_2\alpha_{21})=0.\end{gathered}$$

Dividing the first equation by $F(w_1,w_2)w_1$ and the second one by $F(w_1,w_2)w_2$, we obtain

$$\begin{gathered}jv_1+j^2\frac{\partial F_1(w_1,w_2)}{\partial w_1}\frac{1}{F_1(w_1,w_2)}(\mu+\alpha_{10}+\alpha_{12})+\\ j^2\frac{\partial F_1(w_1,w_2)}{\partial w_2}\frac{1}{F_1(w_1,w_2)}(-\alpha_{21})=0,\\ jv_2+j^2\frac{\partial F_1(w_1,w_2)}{\partial w_1}\frac{1}{F_1(w_1,w_2)}(-\alpha_{12})+\\ j^2\frac{\partial F_1(w_1,w_2)}{\partial w_2}\frac{1}{F_1(w_1,w_2)}(\mu+\alpha_{20}+\alpha_{21})=0.\end{gathered} \tag{7}$$

We will look a solution of this system in the following form:

$$F_1(w_1,w_2)=\exp\{jw_1a_1+jw_2a_2\},$$

where a_1 and a_2 are some constants. Making corresponding substitutions in (7), we derive

$$\begin{gathered}jw_1(v_1-a_1\mu-a_1\alpha_{10}-\alpha_{12}a_1+a_2\alpha_{21})+\\ jw_2(v_2-a_2\mu-a_1\alpha_{10}-\alpha_{12}a_1+a_2\alpha_{21})=0,\end{gathered}$$

which we write in the form

$$\begin{cases}v_1-a_1(\mu+\alpha_{10}+\alpha_{12})+a_2\alpha_{21}=0,\\ v_2-a_2(\mu+\alpha_{20}+\alpha_{21})+a_1\alpha_{12}=0.\end{cases}$$

Solving this system, we obtain

$$a_1 = \frac{v_1 + v_2\alpha_{21}}{(\mu+\alpha_{20}+\alpha_{21})(\mu+\alpha_{10}+\alpha_{12})+\alpha_{12}\alpha_{21}},$$
$$a_2 = \frac{\alpha_{12}v_1 + v_2(\mu+\alpha_{10}+\alpha_{12})}{(\mu+\alpha_{10}+\alpha_{12})(\mu+\alpha_{20}+\alpha_{21})+\alpha_{12}\alpha_{21}}.$$

4.2 Second-Order Asymptotic

Let us perform the following substitution in Eq. (5):

$$H(u_1,u_2) = H^{(2)}(u_1,u_2)\exp\{ju_1\lambda a_1 + ju_2\lambda a_2\}, \tag{8}$$

where $H^{(2)}(u_1,u_2)$ is the characteristic function of two-dimensional centered random process $\{n_1(t)-a_1\lambda, n_2(t)-a_2\lambda\}$. We obtain

$$\begin{gathered} H(u_1,u_2)\{-\lambda + j^2(\mu+\alpha_{10}+\alpha_{12})\lambda a_1 + j^2(\mu+\alpha_{20}+\alpha_{21}) \\ -j^2e^{-ju_1}(\mu+\alpha_{10})\lambda a_1 - j^2e^{-ju_1}(\mu+\alpha_{20})\lambda a_2 \\ +v_1\lambda e^{ju_1} + v_2\lambda e^{ju_2} - j^2e^{-ju_1}\alpha_{12}e^{ju_2}\lambda a_1 - j^2e^{-ju_2}\alpha_{21}e^{ju_1}\lambda a_2\} \\ +\frac{\partial H^{(2)}(u_1,u_2)}{\partial u_1}\left\{j(\mu+\alpha_{10}+\alpha_{12}) - je^{-ju_1}(\mu+\alpha_{10}) - je^{-ju_1}\alpha_{12}e^{ju_2}\right\} \\ +\frac{\partial H^{(2)}(u_1,u_2)}{\partial u_2}\left\{j(\mu+\alpha_{20}+\alpha_{21}) - je^{-ju_2}(\mu+\alpha_{20}) - je^{-ju_2}\alpha_{21}e^{ju_1}\right\} = 0. \end{gathered}$$

By making the following substitutions:

$$\lambda = \frac{1}{\varepsilon^2}, \quad u_1 = \varepsilon w_1, \quad u_2 = \varepsilon w_2,$$
$$H^{(2)}(u_1,u_2) = F_2(w_1,w_2,\varepsilon),$$

we derive

$$\begin{gathered} F_2(w_1,w_2,\varepsilon)\frac{1}{\varepsilon^2}\{-1 + j^2(\mu+\alpha_{10}+\alpha_{12})a_1 + j^2(\mu+\alpha_{20}+\alpha_{21}) \\ -j^2e^{-j\varepsilon w_1}(\mu+\alpha_{10})a_1 - j^2e^{-j\varepsilon w_1}(\mu+\alpha_{20})a_2 \\ +v_1e^{j\varepsilon w_1} + v_2e^{j\varepsilon w_2} - j^2e^{-j\varepsilon w_1}\alpha_{12}e^{j\varepsilon w_2}a_1 - j^2e^{-j\varepsilon w_2}\alpha_{21}e^{j\varepsilon w_1}a_2\} \\ +\frac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1}\frac{1}{\varepsilon}\left\{j(\mu+\alpha_{10}+\alpha_{12}) - je^{-j\varepsilon w_1}(\mu+\alpha_{10}) - je^{-j\varepsilon w_1}\alpha_{12}e^{j\varepsilon w_2}\right\} \\ +\frac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_2}\frac{1}{\varepsilon}\left\{j(\mu+\alpha_{20}+\alpha_{21}) - je^{-j\varepsilon w_2}(\mu+\alpha_{20}) - je^{-j\varepsilon w_2}\alpha_{21}e^{j\varepsilon w_1}\right\} = 0. \end{gathered}$$

Using expansions

$$e^{j\varepsilon w_k} = 1 + j\varepsilon w_k + \frac{(j\varepsilon w_k)^2}{2} + O(\varepsilon^2)$$

and

$$e^{-j\varepsilon w_k} = 1 - j\varepsilon w_k + \frac{(j\varepsilon w_k)^2}{2} + O(\varepsilon^2),$$

we obtain

$$\begin{aligned} -\frac{1}{2}F_2(w_1,w_2)\Big\{w_1^2(v_1 + a_1(\mu + \alpha_{10} + \alpha_{12}) + \alpha_{21}a_2) \\ +w_2^2(v_1 + a_2(\mu + \alpha_{20} + \alpha_{21}) + \alpha_{12}a_1) - 2w_1w_2(\alpha_{12}a_1 + \alpha_{21}a_2)\Big\} \\ -\frac{\partial F_2(w_1,w_2)}{\partial w_1}jw_1\Big\{\alpha_{12} - \alpha_{10} - \mu\Big\} + \frac{\partial F_2(w_1,w_2)}{\partial w_1}jw_2\alpha_{12} \\ -\frac{\partial F_2(w_1,w_2)}{\partial w_2}jw_1\Big\{\alpha_{21} - \alpha_{20} - \mu\Big\} + \frac{\partial F_2(w_1,w_2)}{\partial w_2}jw_2\alpha_{21} = 0. \end{aligned} \tag{9}$$

We will look for a solution of this equation in the form

$$F(w_1,w_2) = \exp\{-\frac{1}{2}w_1^2K_{11} - \frac{1}{2}w_2^2K_{22} - w_1w_2K_{12}\}, \tag{10}$$

where K_{11}, K_{22}, and K_{12} are some constants.

Substituting (10) into (9), we obtain:

$$\begin{aligned} w_1^2\Big\{-\frac{1}{2}(v_1 + a_1(\mu + \alpha_{10} + \alpha_{12}) + \alpha_{21}a_2) - 2K_{11}(\alpha_{12} - \alpha_{10} - \mu) + K_{12}\alpha_{21}\Big\}+ \\ +w_2^2\Big\{-\frac{1}{2}(v_1 + a_2(\mu + \alpha_{20} + \alpha_{21}) + \alpha_{12}a_1) + K_{12}\alpha_{12} + 2K_{22}\alpha_{21}\Big\}+ \\ +w_1w_2\Big\{\alpha_{12}a_1 + \alpha_{21}a_2 - K_{12}(\alpha_{12} - \alpha_{10} - \mu) + 2K_{11}\alpha_{12}- \\ -2K_{22}(\alpha_{21} - \alpha_{20} - \mu) + K_{12}\alpha_{21}\Big\} = 0. \end{aligned}$$

After some derivations, we obtain the following expressions for evaluation of constants K_{11}, K_{22}, K_{12}:

$$K_{11} = 2\frac{v_1 + a_2\alpha_{21} + K_{12}\alpha_{21}}{\alpha_{12} + \alpha_{10} + \mu},$$

$$K_{22} = 2\frac{v_2 + a_1\alpha_{12} + K_{12}\alpha_{12}}{\alpha_{21} + \alpha_{20} + \mu},$$

$$K_{12} = \frac{\alpha_{12}\dfrac{v_1 + \alpha_{21}a_2}{\alpha_{12} + \alpha_{10} + \mu} + \alpha_{21}\dfrac{v_2 + \alpha_{12}a_1}{\alpha_{21} + \alpha_{20} + \mu} - (a_1\alpha_{12} + a_2\alpha_{21})}{\alpha_{12} + \alpha_{10} + \mu + \alpha_{21} + \alpha_{21} + \mu - \dfrac{\alpha_{12}\alpha_{21}}{\alpha_{12} + \alpha_{10} + \mu} - \dfrac{\alpha_{12}\alpha_{21}}{\alpha_{21} + \alpha_{20} + \mu}}.$$

4.3 Approximation of Joint Probability Distribution of the Number of Customers in States of Service

Taking into account derived expressions for constants $a_1, a_2, K_{11}, K_{22}, K_{12}$ and using expression (8), we obtain the following approximation for characteristic function of the number of customers in states of service in the steady-state regime:

$$H(u_1,u_2) \approx \exp\left\{ju_1\lambda a_1 + ju_2\lambda a_2 - u_1u_2\lambda K_{12} - \frac{u_1^2\lambda K_{11}}{2} - \frac{u_2^2\lambda K_{22}}{2}\right\}, \quad (11)$$

which can be applied for enough big values of the arrival process intensity λ. So, the probability distribution of the number of customers in the states of service in the steady-state regime $P(n_1, n_2)$ is a two-dimensional Gaussian distribution with vector of mathematical expectations

$$\mathbf{m} = \lambda\begin{bmatrix}a_1 & a_2\end{bmatrix} \quad (12)$$

and covariance matrix

$$\mathbf{K} = \lambda\begin{bmatrix}K_{11} & K_{12}\\ K_{12} & K_{22}\end{bmatrix}. \quad (13)$$

Because (11) represents characteristic function of continuous random variable with possible negative values, we need in constructing of probability distribution for integer non-negative values which can be applied as an approximation for the probability distribution of the number of customers. To do this, we propose to use the following cumulative distribution function (c.d.f.):

$$F(i,k) = \frac{G(i+0.5, k+0.5) - G(i-0.5, k-0.5)}{1 - G(-0.5, -0.5)}, \quad (14)$$

where $i, k \in \{0, 1, \dots\}$ mean the number of customers in service states 1 and 2 respectively, $G(i,k)$ is a c.d.f. of two-dimensional Gaussian distribution with vector of mathematical expectations (12) and covariance matrix (13).

5 Numerical Example

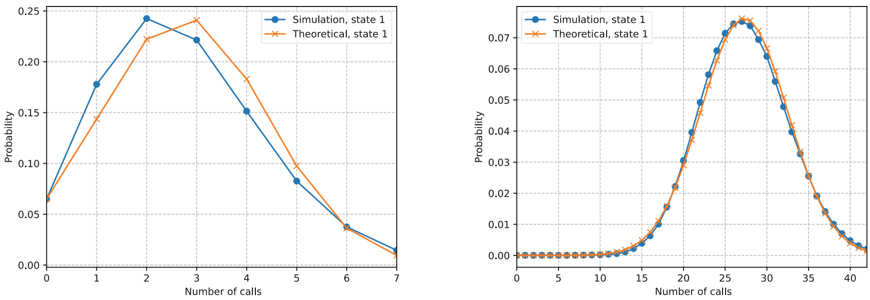
To evaluate the accuracy of approximation (14), we conduct the following experiment: for different values of parameter λ, using simulations, we obtain an empirical probability distribution function and compare it with approximation (14). For the comparison, we will take into account only marginal distributions for the corresponding states of service. For accuracy estimation, we use the Kolmogorov distance

$$\Delta = \max_i |F(i) - F_{\text{sim}}(i)|,$$

where $F_{\text{sim}}(i)$ is an empirical c.d.f. built on the base of results of simulations, and $F(i)$ is a marginal one-dimensional Gaussian c.d.f. built on the base of expression (14). Choosen values of system parameters are given in Table 1. Also, we preformed similar numerical comparison of the approximation with the exact solution obtained under the condition of equivalence of the local and global balance equations (3). The experiments were conducted with the same values of infinite (Table 1), for which condition (3) is satisfied.

Table 1. Values of parameters for numerical experiments

Parameter	Value
μ	0.1
v_1	0.3
v_2	0.7
α_{12}	12.83
α_{21}	1
α_{10}	1
α_{20}	0.1



a) $\lambda = 10$ b) $\lambda = 100$

Fig. 3. Probability distribution of the number of customers in state 1 for $\lambda = 10$ and $\lambda = 100$

Figure 3 shows a comparison of the stationary probability distributions of the number of customers serviced in state 1 for different intensities of the arrival process. Table 2 shows corresponding values of the Kolmogorov distance. We can see that accuracy of the theoretical approximation increases with increasing of λ. The same results for state 2 can be found in Table 3. We consider the results can be an acceptable if Kolmogorov distance $\Delta \leq 0.05$ (highlighted in boldface in the tables). So, as we see from the tables, we reach the acceptable results for the obtained approximation for values $\lambda \geq 10$.

Table 2. Kolmogorov distance between probability distributions of the number of customers in state 1 for various values of λ: Δ_{sim} – approximation against simulation; Δ_{ex} – approximation against exact solution

λ	1	5	10	15	20
Δ_{sim}	0,1388	0.0712	**0,0354**	**0.0227**	**0.0165**
Δ_{ex}	0,1387	0.0711	**0,0348**	**0.0229**	**0.0170**

Table 3. Kolmogorov distance between probability distributions of the number of customers in state 2 for various values of λ: Δ_{sim} – approximation against simulation; Δ_{ex} – approximation against exact solution

λ	1	5	10	15	20
Δ_{sim}	**0,0288**	**0.0058**	**0,0031**	**0.0016**	**0.0016**
Δ_{ex}	**0,0284**	**0.0056**	**0,0027**	**0,0018**	**0.0013**

6 Conclusion

Mathematical model for subscriber communication network using IAB technology with two mobile nodes is proposed in the paper. The model is formulated in the form of an infinite-server queueing system with two states of service and abandonments. The method of asymptotic analysis is applied to find the joint two-dimensional probability distribution of the number of customers in the first and second states of service. Obtained result in the form of an approximation can be applied in the case when the condition of equivalence of the local and global balance equations is not met but it is limited by enough big intensity of the arrival process. Conducted numerical experiments approve applicability of the obtained approximation. We think that the approach may be applied for models with an arbitrary number of service states and for models with non-Poisson arrivals and non-exponential service times.

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