Actuarial Gains in Life Annuities Due to Declining Health: LTC

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Abstract. Life annuities are actuarial products based on technical assumptions, such as mortality. The degeneration of the human body leads, in addition to generating long-term care, to a higher mortality of the dependent than that of the general and insured population. Therefore, the period for receiving this benefit would be shorter. The aim of this paper is to determine the economic impact of the change in the beneficiary's status when receiving this life annuity. It should be stressed that, in the life annuity, the biometric risk is borne by the insurer and that a lower payment expectancy due to the pension beneficiary's change to dependent status entails a benefit, as this gain is not distributed to the beneficiary. A surplus is created by paying out the same benefit. Thus, the use of an appropriate mortality assumption results in a reduction of the mathematical payout provision, which frees up capital and results in a lower solvency capital requirement.

Keywords: Actuarial fairness · life annuities · Long Term Care

1 Introduction

The aim of this paper is to determine the impact of a life annuity when the health status changes to severely dependent/highly dependent with no return. The annuity contract does not include mortality other than that of the general population, however, the change in mortality due to a non-returning health condition may lead to a transformation of the annuity's purpose into a long-term care (LTC) benefit. If the same benefit is paid, the use of an appropriate mortality assumption will reduce the value of the annuity, leading to the release of capital and a lower solvency capital requirement.

This paper makes a breakthrough: it determines the procedure for calculating the surplus for not differentiating mortality in the life annuity. The main contribution lies in quantifying this surplus. On the one hand, as we do not know when the beneficiary becomes severely dependent, this information should be provided by the beneficiary himself/herself. On the other hand, the total effect on the insurer will depend on the demographic composition of its life annuity portfolio.

The next section deals with measurement. The third section includes the methodology for valuation. The next section provides a representative application of the Spanish

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market to illustrate this. The discussion and implications of the model are included, and the paper ends with the relevant conclusions.

2 Measuring Economic Impact

The economic effect can be captured with an Actuarial Gain/Loss (AGL) analysis. *AGL* calculates the economic value of the differences between actuarial assumptions and reality, generating a deficit or surplus. It is a common actuarial technique that can be used to analyse both financial assets and biometric liabilities separately or jointly [7] - Fig. 1.

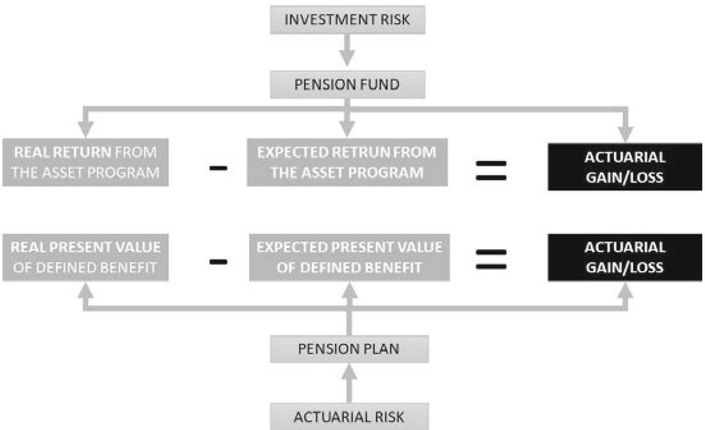

Fig. 1. Main components of Actuarial Gain/Loss. Source: Own elaboration.

The literature analyses actual investment performance on the interest rate assumption of the valuation [1, 3, 4, 14, 21]. It also considers the effect of discretionary choice of actuarial assumptions and their gradual appropriateness. Its simplicity and analytical power have brought it to the forefront of pension information systems [10], and in the amortisation of pension deficits [13, 17, 19].

3 Methodology: Actuarial Gain/Loss

We take as a starting point the actuarial model proposed by [5, 6, 11, 12] contemplating high degrees of dependency without return [4]. The beneficiary's health status changes at age x as he/she becomes severely or highly dependent. Then, under a new survival function and with the initial benefit, the actuarial value of the benefit actually received ($VajR_x$) is given by the expression (1),

$$VajR_x = \int_x^w b_t \cdot e^{-\int_x^w {}^d\mu_t^m dt} \cdot e^{-\int_x^w \delta(t)dt} \cdot dt \qquad (1)$$

Being,

b_t: Benefit function.
${}^{d}\mu_t^m$: Instantaneous mortality rate of a severely or highly dependent person at the t-th instant.
$e^{-\int_x^w {}^{d}\mu_t^m dt}$: Probability of survival of an individual of age x as a function of the instantaneous mortality rate of a dependent person.
$\delta(t)$: instantaneous interest rate.
$e^{-\int_x^w \delta(t)dt}$: Financial discount function up to age x, through the instantaneous interest rate.

The *AGL* is therefore defined as the difference between the actuarial value of the benefit actually received and the benefit expected to receive under the initial assumptions:

$$AGL_x = VajR_{x+1} - E(Vaj)_{x+1} \tag{2}$$

where,

AGL_x: Actuarial Gain/Loss generated at age x by the change in the beneficiary's health status.
$E(Vaj)_{x+1}$: Present value of the benefit expected to be received under the initial health status at age $x + 1$.

It can be determined for each beneficiary in each of the future years in case he/she reaches future ages and becomes a dependent person.

$$VaAGL_x = \sum_{h=x}^{w} AGL_h \cdot {}^{r}q_h^{(d)} \cdot {}^{r}_{h-x}p_x^{(r)} \cdot v^{h-x} \tag{3}$$

$VaAGL_x$: Actuarial value of the *AGL* due to the change in the beneficiary's health, calculated at age x.
${}^{r}q_h^{(d)}$: Probability that a retirement beneficiary at age h will be severely or severely dependent at that age, being exposed to another cause of exit (mortality).
${}^{r}_{h-x}p_x^{(r)}$: Probability that a retirement beneficiary of age x will reach age h without death or change of health status.
v^{h-x}: Financial discount factor from age h to age x.

In the case of the annuity, it is the pension beneficiary who is aware of his or her health status, and if he or she changes to a dependent, only he or she can inform the insurer. Therefore, the incidence of the dependent's mortality is instantaneous from the moment of notification.

4 Dependent Mortality Versus Overall Mortality: Discussion

[12] establishes the life expectancy of an individual in the most severe stages of dependency. [8] start from a general mortality and propose additive displacement on the instantaneous mortality rate. However, [16] indicate that dependents will have an over-mortality that can be expressed by a multiplicative correction - θ - on the mortality probability of the general population:

$${}^{d}q_x^m = \theta \cdot q_x^m \tag{4}$$

This correction can be variable at each age, although [16] indicated that a fixed correction adjusts the mortality of older dependents better than other types of approximations. However, it overestimates mortality at lower ages and underestimates at higher ages. Therefore, it is better to perform an additive adjustment (ε) considering age as an independent variable in a functional form [18].

$$^{d}q_x^m = q_x^m + \varepsilon \quad \text{where} \quad \varepsilon = f(x) \tag{5}$$

As a result, mortality rates are lower at younger ages and increase with the degree of dependency [15]. [20] determined the probability of death for severe dependency; they used general mortality tables and adjusted them to the HID 98–01 statistics for France:

$$^{d}q_x^m = \begin{cases} q_x^m + \frac{\delta}{1+\gamma^{x_i-x}} & \forall x_i < 95 \\ q_x^m \cdot (1+\beta) + \frac{\delta}{1+\gamma^{x_i-x}} & \forall x_i \geq 95 \end{cases} \tag{6}$$

δ: Maximum value to be incorporated as a function of age at which it converges asymptotically.
γ: Slope factor.
x_i: Age of inflection at which the curve changes shape from convex to concave.
β: Multiplicative factor on overall mortality (Table 1).

Table 1. Overmortality factors for severe dependency in Spain. Source: [20].

Factors	Men	Women
δ	0.245	0.165
γ	1.135	1.09
x_i	62.50	58.61
β	0.1142	0.0962

Mortality rates for disabled persons ($^{d}q_x^m$) and invalids ($^{i}q_x^m$) converge to the general population (q_x^m) as age increases (Fig. 2).

As a final result, for both men and women, there is an actuarial gain, i.e. a smaller capital sum needed to guarantee the benefit in the event of a change in health status. The surplus is greater the younger the beneficiary is. The main implication is that, to pay out the same benefit, there is money left over, so a surplus will be generated as it is not distributed to the life annuity beneficiary.

The surplus decreases as the beneficiary becomes severely dependent at older ages, as can be seen in Fig. 3, taking minimum values from the age of 100 onwards in the case of retirement beneficiaries. As for the disability pension, it takes minimum values for men, with a deficit per euro of benefit for women who change their health status to severe dependency from the age of 90.

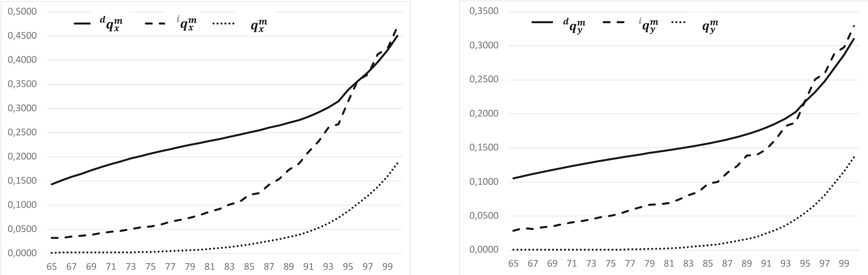

Fig. 2. Mortality differential after retirement age at 65. (**x**) Men; (**y**) Women. Source: Own elaboration. Databases: [2, 9]; Mortality Tables with factors from [20].

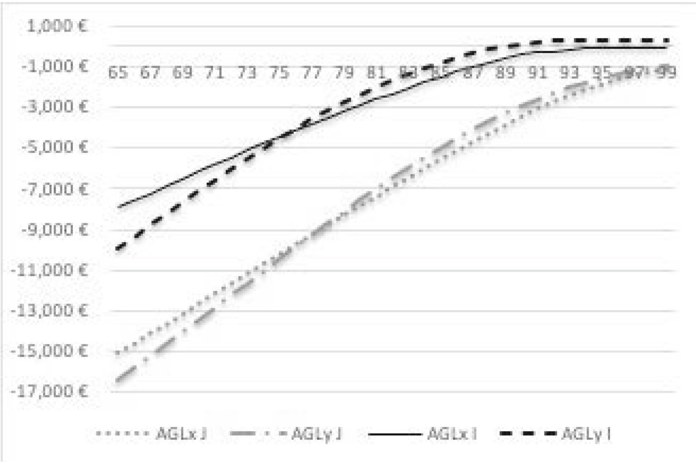

Fig. 3. Evolution of the surplus by change of mortality tables and by euro benefit, according to age and origin (Retirement -J- or Invalidity -I-). Source: Own work.

5 Conclusion

Logically, it is in the insurer's interest that the beneficiary informs the insurer of his or her change in health. This new is compulsory, as the risk status of the insured person changes. However, the beneficiary himself/herself does not perceive a change of benefit after this information, but it is the insurer itself that takes advantage. There is no incentive to update the death risk status information from the beneficiary point of view.

It is necessary to establish a mutually beneficial incentive. So, part of the gain can pass on to the beneficiary, with the mandatory benefit increase to help with LTC. There is an incentive for the beneficiary and partially the insurer also reduces the capital at risk.

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