The Cost of Longevity Risk Transfer by Capital Solution De-risking Strategy

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Abstract. In this paper, we develop a longevity swap de-risking strategy to mitigate the impact of the longevity risk related to payments that depend on how long individuals are going to live. In order to ensure the development of an effi-cient capital market for longevity risk transfers, the longevity hedge would allow longevity risk to be shared efficiently and fairly between the parties. Our results show that the fixed proportional risk premium that the counter-party requires to take on the longevity risk varies by changing the mortality model adopted to represent the evolution of the longevity of the population underlying the swap and that, as the risk premium changes, the total transfer of longevity risk may become inefficient.

Keywords: longevity risk · de-risking strategy · longevity swap

1 Introduction

Every institutions and governments making payments that depend on how long individuals are going to live face with longevity risk, the risk that individuals live longer than expected. In particular, defined benefit pension plan sponsors, annuity providers are transferring these obligations, to life (re)insurers via insurance and capital solutions such as for instance buy-outs, buy-ins, longevity swaps and so on. Nevertheless, as the demand of longevity risk protection increases, the key question consists in capability of (re)insurance sector to cope with future increasing potential liabilities of the longevity risk exposures [2,4]. The innovative capital market solutions for transferring longevity risk consist in several forms of transactions, each differing in the types of risk transferred and the categories of risk created, including longevity (or survivor) bonds, longevity (or survivor) swaps, mortality (or q-)forward contracts, and reinsurance sidecars (also called strategic reinsurance vehicles). According to Reinsurance Group of

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America (RGA) that has completed a new US$1.7 billion longevity risk transfer, the longevity swap arrangement covers roughly 11,000 single premium immediate annuity contracts, transferring the longevity related risk away [7].

A longevity swap transaction is based on periodic fixed payments that are paid to the swap counterparty in exchange for periodic payments according to the difference between the actual and expected pension or annuity mortality experience. When the longevity swap is index-based, the mortality experience is represented by the standardised population cohorts ("index swaps"). De-risking strategies may broaden, above all by considering that risk-mitigation instruments should not involve material basis risk that is intrinsically inherent the longevity hedges. Furthermore, regulatory restrictions could affect the technical forms and options. In particular, the feasibility of the de-risking transactions depends on the appropriate cost of the longevity risk transfer. It is well-known that the degree of the cost-efficient longevity de-risking solutions may stimulate or on the contrary deflate the market's potential for further risk transfers.

In this paper we investigate how the fixed proportional risk premium that the counterparty requires to take on the longevity risk varies with the underlying mortality model adopted. Our results show that, as the risk premium changes, the transfer of longevity risk may become more or less effective.

The remainder of the paper is organized as follows: in Sect. 2 we introduce de-risking strategy based on longevity swap and define the optimization problem that determines the optimal proportion of risk that should be transferred. Section 3 shows the main findings of the numerical application. Section 4 concludes.

2 De-risking Strategies

Let us consider a portfolio of annuitants all aged x_0 at time 0, we define ${}_sp_{x,t}$ the probability that an individual alive at time t, with age x, survives to age $x+s$ at year $t+s$, and ${}_s\hat{p}_{x,t}$ its conditional expected value. We denote with $v = \frac{1}{1+r}$ the discount factor with the discount rate r (assumed to be deterministic), we define the conditional expected value of a life annuity $a(x(t))$ as $\mathbb{E}[a_{\overline{k(x)}}|p_{x,t}, {}_2p_{x,t}, \dots] = \sum_{s=1}^{\omega-x} v^s\, {}_s\hat{p}_{x,t}$.

Let A_0 and V_0 the asset and portfolio expected liabilities at time 0. The initial unfunded liabilities, UL_0, are given by $V_0 - A_0$. The insurer benefit liability at time t, is the discounted expected value of future benefits, with B_t the total annual benefit in t. B_t is given by the product of the individual benefit b, assumed to be equal for all the insureds, and the number of survived annuitants, n_t: $B_t = b \cdot n_t$. Let J_t be the return on assets from $t-1$ to t at $j(t-1,t)$ rate, this implies that $J_t = A_{t-1} \cdot j(t-1,t)$. Denoting K_t the capital flow in the year t, the portfolio asset is given by:

$$A_t = A_{t-1} + J_t + K_t - B_t \tag{1}$$

while UL_t, without de-risking strategy, is obtained as follows:

$$UL_t = V_t - A_{t-1} - J_t + B_t \tag{2}$$

If a time t the unfolded liabilities are greater than zero, $UL_t > 0$, the insurer experiences a portfolio loss and vice versa. Furthermore, we assume that, the insurer amortizes the unfunded liability year by year, that is $K_t = UL_t \ \ \forall t$.

We consider a de-risking strategy for longevity risk based on a longevity swap (LS), written on living policyholders, with hedge cost HC^{LS}. We denote with $in_t^{LS} = max(K_t^{LS}, 0)$ and $out_t^{LS} = max(-K_t^{LS}, 0)$ the present value of capital inflows and capital outflows, subject to constant penalty factors ψ_1 and ψ_2, respectively. ψ_1 represents the opportunity cost due to the need to increase the capital and ψ_2 the opportunity cost due to lock capital that could have been invested otherwise. The total portfolio cost TPC^{LS} of the strategy is obtained as:

$$TPC^{LS} = HC^{LS} + \sum_{t=1}^{\omega-x} \frac{in_t^{LS}(1+\psi_1) - out_t^{LS}(1+\psi_2)}{(1+r)^t} \tag{3}$$

Denoting with HCF_t^{LS} the hedging cash flows from de-risking strategy, the unfunded liabilities with the de-risking strategy is given by:

$$UL_t^{LS} = V_t - A_{t-1} - J_t + B_t - HCF_t^{LS} \tag{4}$$

We denote with TUL^{LS} the total unfunded liabilities over the entire time horizon of the de-risking strategy based on LS:

$$TUL^{LS} = \sum_{t=1}^{\omega-x} \frac{UL^{LS}}{(1+r)^t} \tag{5}$$

We consider a plain vanilla longevity swap written on n_0 survivors. We define the fixed leg of longevity swap at time t as $b \cdot n_0 \cdot_t \hat{p}_x(1+\pi)$ (where π is the fixed proportional risk premium that the counterpart requires to take on longevity risk) and the floating leg as $b \cdot n_0 \cdot {}_tp_x$. At each t, $t = 1, 2, \ldots$ the LS payoff is given by the difference between the floating and the fixed leg: $b \cdot n_0 [{}_tp_x - {}_t\hat{p}_x(1+\pi)], \quad t = 1, 2, \ldots$

Setting π so that the swap value is zero at the inception date, the swap price is null, $HP^{LS} = 0$. With a hedging proportion of h^{LS}, the hedging cost is equal to:

$$HC^{LS} = -h^{LS} \cdot b \cdot n_0 \cdot \mathbb{E}\left[\sum_{t=1}^{\omega-x} d(0,t)[{}_tp_x -_t \hat{p}_x(1+\pi)]\right] \tag{6}$$

Following [6], the optimal hedge level for the de-risking strategy can be obtained solving an optimisation problem where the insurer aims to minimizing the Conditional Value-at-Risk of the total unfunded liabilities at a fixed confidence level α, $CVaR_\alpha(TUL^{LS})$, with respect to h^{LS}, subjected to the constraint that the total cost does not exceed a fixed amount c. This is formalised in the following non-linear optimization problem:

$$\min_{h^{LS}} CVaR_\alpha[TUL^{LS}]$$

$$\begin{aligned} & sub \\ & \mathbb{E}[TPC^{LS}] \leq c \\ & \mathbb{E}[TUL^{LS}] \leq 0 \\ & 0 \leq h^{LS} \leq 1 \end{aligned} \tag{7}$$

As already noted in [6], the de-risking strategy is strongly influenced by its cost (which depends on π). On the other hand, the cost of the LS is also related to the assumptions on the evolution of mortality in terms of both trend and volatility. Therefore, the choice of the mortality model is a determining factor in defining the optimal de-risking strategy. In the following numerical application, we verify how the optimal risk transfer rate h^{LS} is influenced by the choice of the underlying mortality model, and how the existence of an information asymmetry on mortality trends between protection seller (short position on LS) and protection buyer (long position on LS) is crucial.

3 Numerical Application

We consider a portfolio of immediate temporary life annuities (with term $T = 20$), written on a cohort of males all aged 65 at issue ($t = 0$) with $n_0 = 10,000$. For sake of simplicity we assume $b = 1$, so $B_t = n_t$. Expenses and taxes are not considered in the valuation. The single premium, Π_X, is determined according to the Standard Deviation Principle ($\Pi_X = \mathbb{E}[X] + \lambda \cdot SD[X]$). We fix λ at 20%. We consider two different mortality models, the traditional Lee-Carter model [5] (LCA in the following) and the Lee-Carter model including a frailty factor proposed by [3] (denoted with ATFLCA). We estimate LCA and ATFLCA models for English 50–90 aged male population. For LCA model, we used data about death rates and exposures to risk only and we refer to the Human Mortality Database. For ATFLCA model, we also used data relating to the co-morbidity trend in the population, and we referred to the English Longitudinal Study on Ageing (ELSA) [1]. Performing 10,000 simulation, we obtain the evolution of the UL_t, the total unexpected losses, TUL, and the total portfolio costs TPC, without hedging. We then introduce the optimization problems setting a constraints for $\mathbb{E}[TPC]$: the maximum level c for the expected total cost related to strategy j is set in relation to its initial value (without hedging): $c = 0.5 \cdot \mathbb{E}[TPC]$. The following assumption are adopted in the evaluations:

- the initial asset, A_0, are equal to the total portfolio single premium;
- we assume a flat rate of return on asset ($j(t-1,t) = r = 0.02 \quad \forall t$);
- the penalty factors in the TPC are: $\psi_1 = \psi_2 = 0.2$.

Adopting a demographic technical basis determined through the LCA model (see Table 1 columns 2 and 3), the initial portfolio liabilities are: $V_0 = 168,102$ while the total portfolio single premium, P, are: 168,810. The risk premium of the longevity swap, π, is set at 0.421%. This value was determined consistently with the standard deviation principle adopted for the determination of the single premium. Without hedging the $\mathbb{E}[TUL]$ is negative, denoting an expected profit, but the portfolio is characterized by a positive $CVaR_{99.5\%}[TUL]$ (with an average of the losses beyond the VaR at 99.5% of 8,239.21. The penalties ψ_1 and ψ_2 strongly reduce the profit, but $\mathbb{E}[TPC]$ is still negative. Results show that the optimal strategy minimizing $CVaR_{99.5\%}[TUL^LS]$ is obtained with LS share equal to 46.6%. (partial risk transfer). When a hedging strategy is introduced,

Table 1. Results with no hedging and swap strategies minimizing $CVaR_\alpha[TUL^{LS}]$: LCA scenario (second and third columns) and ATFLCA scenario (forth and fifth columns). Results with swap strategy minimizing $CVaR_\alpha(TUL^{LS})$ in presence of asymmetric information: ATFLCA scenario (sixth column).

	LCA scenario		ATFLCA scenario		Asymm. inf.
	No hedging	Hedging	No hedging	Hedging	Hedging
π^{LS}		0.421%		1.580%	0.421%
h^{LS}	0.0%	46.6%	0.0%	42.9%	100.0%
HC^{LS}	0.00	325.76	0.00	1,122.84	697.29
$\mathbb{E}[TUL]$	–707.71	-381.95	–2,647.99	–1,525.15	–1,950.70
$CVaR_{99.5\%}[TUL]$	8,239.21	4,490.62	29,712.79	17,277.74	–1,183.74
$\mathbb{E}[TPC]$	–96.00	–48.00	–357.91	–178.96	–1,465.93

$\mathbb{E}[TUL]$ is increased (but still negative) while the $CVaR_{99.5\%}[TUL]$ is almost halved. The expected total cost ($\mathbb{E}[TPC]$) is also increased, but still negative.

Adopting a demographic technical basis determined through the ATFLCA model (see Table 1 columns 4 and 5), the initial portfolio liabilities are: $V_0 = 167,595$ while the total portfolio single premium, P, are: 170,242. Without hedging the $\mathbb{E}[TUL]$ is negative, but the portfolio is characterized by a very high $CVaR_{99.5\%}[TUL]$, as a consequence of the high variability of the death probability simulated with the ATFLCA model. As a consequence, the $\mathbb{E}[TPC]$ is strongly increased even if still negative. Results show that the optimal strategy minimizing $CVaR_{99.5\%}[TUL^LS]$ is obtained with LS share equal to 42.9% (partial risk transfer). When a hedging strategy is introduced, $\mathbb{E}[TUL]$ is increased (but still negative) but the $CVaR_{99.5\%}[TUL]$ is drastically reduced. The expected total cost ($\mathbb{E}[TPC]$) is still negative but an half than in the absence of LS.

The last case we consider is one in which the protection seller (short position on LS) and the protection buyer (long position on LS) have different information on the insured population (see Table 1 column 6), so that the former considers appropriate to adopt the LCA model (and price the swap accordingly), while the latter considers the ATFLCA model more appropriate and determines the optimal de-risking strategy accordingly. Results show that the optimal strategy minimizing $CVaR_{99.5\%}[TUL]$ is obtained with LS share equal to 100% (total risk transfer). $\mathbb{E}[TUL]$ is increased but still negative (implying a profit)) but the $CVaR_{99.5\%}[TUL]$ is totally reduced and becomes negative. The expected total cost ($\mathbb{E}[TPC]$) is more negative, which implies an improvement of the annuity provider's position.

4 Conclusions

The aim of this paper is to analyse how de-risking strategies based on longevity swaps are affected by their cost, expressed as a fixed proportional risk premium that the counterparty requires to assume longevity risk, with a focus on the choice of the underlying mortality model. The mortality models considered are the Lee-Carter model and its extension including a frailty factor (ATFLCA model). Our results show that different mortality models imply a different risk assessment of an annuity portfolio. If both counterparties of the longevity swap assume the same mortality model, the cost of the de-risking strategy increases with the portfolio's riskiness and the optimal strategy for the annuity provider (protection buyer) is to transfer only part of the longevity risk to the protection seller, with the share depending on the model adopted. If we assume that the protection seller and the protection buyer have different information about the insured population, so that the former considers it appropriate to adopt the Lee-Carter model (and prices the swap accordingly) while the latter considers the ATFLCA model more appropriate, de-risking is more effective and optimal strategy is achieved through a total transfer of longevity risk.

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