




# Approaches for Addressing Spatial Connectivity of Final Harvests Within Forest Harvest Scheduling Algorithms

Pete Bettinger<sup>(✉)</sup> 

University of Georgia, Athens, GA 30602, USA  
pbettinger@warnell.uga.edu

**Abstract.** Harvest scheduling, or the scheduling of management activities within a forest for a given period of time, is an important aspect of forest planning. Often, harvest scheduling results in a tactical plan that allows forest managers the ability to understand where to go, and what to do, at different points in time. In the development of a harvest schedule, an objective is optimized and constraints are satisfied. As examples, an objective may be to maximize wood produced or revenue obtained over time, or to minimize environmental impact over time. Examples of constraints include restrictions on the flow of wood produced over time, the condition of the standing inventory (uncut forests), the amounts of areas of different management activities, and the location and timing of specific management activities. In many cases, these mathematical problems are formulated either with exact methods (linear or mixed-integer programming) or heuristic methods (simulated annealing, tabu search, genetic algorithms, etc.). This work describes the manner in which the connectivity of final harvests is assessed and controlled in both types of approaches. This work also explores how the control of activities differs between cases (a) when the focus is on controlling the timing of the final harvest of adjacent pairs of forest management units, and (b) when the focus is on controlling how large a collective area might become when multiple adjacent forest management units are scheduled for a final harvest within a given time window.

**Keywords:** Connectivity · Spatial analysis · Forestry

## 1 Introduction

Forests provide society numerous benefits, including historically the means by which society builds infrastructure, heats homes and businesses, and cooks food. The benefits derived from forests have been categorized into four common sets of ecosystem services: provisional, regulating, supporting, and cultural [18]. The locations where forest management activities are placed can contribute positively or negatively to the attainment of different ecosystem services, therefore within the scope of forest planning a schedule of future management activities is an important consideration. The management activities

that are considered can be influenced by laws and regulations (e.g., [11]), forest certification guidelines (e.g., [8]) and the interests of the forest landowner. Geography can also influence the management activities considered, as some management actions may be better suited to different landscape features, and as some management actions may be limited based on other nearby management actions (scheduled or implemented).

Forest planning efforts have benefitted greatly from the advancements made in both computer technologies and geographical information systems (GIS) in the last 40 years. The mathematical models concerning the management of forested landscapes that can now be formulated can address very large landscapes and very complex management actions. The ability to mathematically connect the functional relationships between different ecosystem services (e.g., wood production and wildlife habitat) has also been facilitated with these advances. One area of harvest scheduling that has attracted a lot of attention involves the geographic concept of *adjacency*, and concerns over whether similar management activities are (or will be) placed on the landscape that result in forest conditions that are too large. For example, the size of final harvest (clearcut) activities is now regulated in some U.S. states, and therefore two or more final harvests that are in close proximity (adjacent) may create, ecologically, one rather large area of early successional forest condition that violates the regulations. Therefore it has become common to integrate adjacency relationships into tactical forest planning efforts to devise a plan that suggests these types of conditions will not be developed.

Aside from adhering to laws and regulations, there are many other reasons why a forest landowner would want to pursue the development of a tactical forest plan that includes issues addressing the spatial adjacency of management activities. These can include concerns about the cumulative effects of forest management and the ability to develop and maintain suitable wildlife habitat conditions [3, 7] or the size of the cumulative activities [17]. However, the development of a tactical forest plan is based on a model of a real world system. The closer the model reflects real world conditions and issues of concern, the more likely the resulting plan will be implemented successfully. Unfortunately, some issues, such as the adjacency of management activities, can be rather difficult to address in a mathematical model. In many cases, the amount of spatial relationships that need to be recognized can increase exponentially as the size of forest management units decreases, and as the scope of the analysis increases. In sum, the mathematical approaches that could be used to represent important spatial relationships in a harvest schedule may become burdensome, and tax the abilities of both the planner and the data development processes that are employed.

In this work, some common approaches for addressing the adjacency of forest management activities, specifically final harvests, within mathematical models employed to develop a harvest schedule are described. These approaches are described for both exact and heuristic methods of solving mathematical problems. Further, these approaches are described for two types of adjacency relationships, one that focuses only on controlling management activities between pairs of adjacent forest management units, and another that focuses on a maximum size of management activity, and the adjacent management units that could be scheduled for final harvest activity at the same time without exceeding the maximum size.

## 2 Methods

In this work, the methodological concepts relate to the development of a tactical forest plan, which serves to provide land managers with an idea of where and when forest management activities should be implemented. In the mathematical algorithms associated with contemporary forest management planning, it is possible to control the scheduling of management activities based on their proximity in both space and time. Geographically, the idea that two places are adjacent in space is often based on whether those two places share an edge (line, arc). However, some organizations have defined adjacency based on whether two places share only a single point in geographical space. And, further, if the edges of two places are simply within some distance of each other (yet not physically touching), this can serve as a definition of an adjacent relationship.

In order to set the stage for the work illustrated below, a few definitions are necessary. In forestry, a *management unit* (i.e., stand, polygon) is often defined as a contiguous area of land that will likely be managed as a whole when management activities are implemented. The boundaries of management units are defined using roads, streams, topography, and changes in timber type (age, species, etc.). These features are maintained in a geographic information system (GIS) database, and the adjacency relationships amongst them can be extracted using algorithms that understand the connection or proximity of the edges that form the polygons. Along these lines, *adjacency* refers to the proximity of each management unit to other management units. As noted earlier, two management units might have an adjacency relationship, in spatial terms, when they (a) only share a point (vertex), (b) share an edge, or (c) have edges that are near each other in geographical space. Within quantitative forest harvest scheduling, this information can be of value to prevent the scheduling of two management activities that will conceptually result in a single, larger management activity. The most common example involves final harvests (clearcuts). When two adjacent management units are scheduled for a final harvest during the same time period, the outcome is one larger (the sum of the area of the two management units) final harvest, which may be too large with respect to policies guide forest management. The two types of adjacency relationships commonly recognized in quantitative forest harvest scheduling to control the timing and placement of final harvests are the unit restriction model and the area restriction model [14].

### 2.1 Unit Restriction Adjacency

In forest management planning, the concept of *unit restriction adjacency* refers to the relationship between only two management units. This relationship notes that one management unit is adjacent to another, and it can be used to control (constrain) the assignment of forest management activities to only one of the two that constitute the pair. For example, if a final harvest were scheduled for one of the two management units, a final harvest would be disallowed for the other during the same period of time. The period of time which is used to disallow a management activity varies from one organization to the next, and perhaps from one set of regulations to another set. Often this period of time is referred to as the *green-up period*, which indicates the amount of time (years) that separate the final harvests of two adjacent management units to allow the new trees in one (the first management unit to be harvested) to grow to a desired height (to allow

the management unit to *green up*). In some United States (Oregon and Washington), the green-up period is often assumed to be 4 or 5 years for privately owned forests. On public lands, the green-up period can be much longer. For example, on certain Crown forest lands in Alberta, the green-up period can extend 30 years [9].

**Exact Approach.** Exact approaches for solving harvest scheduling problems are those that can guarantee that an optimal solution has been located. These include linear, goal, and mixed-integer programming methods among others. For addressing the unit restriction adjacency constraints within a forest harvest scheduling problem, *pairwise constraints* are developed, these types of constraints limit the ability of the optimization process to schedule the same type of management activities to two adjacent management units within a certain period of time. For example, assume there are two adjacent management units, *MU1* and *MU2*. Assume further that for an exact approach decision variables are created to indicate whether management unit 1 or management unit 2 are assigned final harvest actions in period 1 (*MU1P1*, *MU2P1*). With respect to potential actions in subsequent time periods, *Px* will change. For example, *P1* may change to *P2* to represent those activities possible in period 2. Finally, assume that these variables can only be represented by binary integer values in the final solution to the scheduling problem (e.g.,  $MU1P1 = 1$  or  $MU1P1 = 0$ ). This would indicate whether a final harvest has been scheduled (1) or not (0) for the management unit during the time period. To prevent the scheduling of final harvests within both management unit 1 and management unit 2 during the same period of time (e.g., time period 1) a pairwise constraint would be developed:

$$MU1P1 + MU2P1 \leq 1 \quad (1)$$

Only one of the two choices is possible when using this type of constraint, limiting actions amongst adjacent neighbours. When there are multiple time periods to consider (when a green-up period is longer than a single time period), additional pairwise constraints are likely necessary.

$$MU1P1 + MU2P2 \leq 1 \quad (2)$$

$$MU1P1 + MU2P3 \leq 1 \quad (3)$$

$$MU1P1 + MU2P3 \leq 1 \quad (4)$$

In the example above, when management unit 1 is scheduled for a final harvest during time period 1 (e.g.,  $MU1P1 = 1$ ), management unit 2 will not be allowed a final harvest during time periods 1, 2, and 3. These types of equations representing the constraints must be constructed prior to using an exact approach algorithm (e.g., branch and bound, cutting plane, etc.) to solve the harvest scheduling problem. When the definition of adjacency changes, the constraints must be re-constructed. When the green-up assumption changes, the constraints must also be re-constructed.

**Heuristic Approach.** As a heuristic approach such as simulated annealing or tabu search is being applied to a forest harvest scheduling problem, computer logic is employed to assess resource and policy constraints in real time. For example, if a heuristic attempts to schedule a final harvest for a management unit (say, management unit 1 during time period 1) it will assess potential constraint violations before formally assigning the final harvest period. In other words, when attempting to change *MUIPI* to 1, rather than 0 (previous value where the harvest was not scheduled for time period 1) all potential wood flow, habitat, adjacency (and other) constraints are assessed using computer logic (If-Then-Else blocks of code and others). To facilitate the assessment of adjacency constraints, an adjacency list is needed. This list indicates the neighbours (in geographic space) of every management unit. The adjacency list is stored in the memory of the computer and accessed when it is needed. An example list below suggests that management unit 1 is adjacent to management units 2, 3, and 4.

```

management unit, adjacent management unit
1, 2
1, 3
1, 4
2, 1
3, 1
3, 4
4, 1
4, 3
...
```

As you might notice, this list is redundant, which is important when one desires to improve the overall efficiency of the heuristic search process. When the list of adjacency relationships is sorted by management unit number (first value on each line), pointers can be developed to facilitate fast access to only the pertinent information in the list. The pointers for management unit 3, for instance, are 5 (the beginning line number) and 6 (the ending line number). The pointers then serve to direct the heuristic to only the information related to management unit 3 (e.g., the adjacent neighbours of management unit 3 begin on line 5 in the list and end on line 6).

A heuristic process that is designed to assess the final harvest adjacency constraints in a forest management problem would assume first that the time period assigned to a management unit (e.g., management unit 1) is temporarily assigned. Then the status of all adjacent neighbours to the management unit would be assessed to determine whether any one of them is also scheduled for a final harvest during the same time period. If this is the case, a constraint violation is noted, and the temporary assignment of the management unit to the time period is dismissed in subsequent processes of the heuristic (i.e., the final harvest is not allowed.).

```

Constraint violation = 0
For a = Beginning pointer(Management Unit 2) to Ending pointer(Management
unit 2)
  If (Potential harvest period(Management unit 2) = Scheduled harvest
period(Adjacency list(a))) Then
    Constraint violation = 1
  End If
Next a
```

If the green-up period were assumed to be longer than a single time period, the logic would be enhanced:

```

LowerPeriod = Potential harvest period(Management unit 2) - (Greenup win-
dow - 1)
UpperPeriod = Potential harvest period(Management unit 2) + (Greenup
window - 1)
Constraint violation = 0
For a = Beginning pointer(Management Unit 2) to Ending pointer(Manage-
ment unit 2)
  If (Scheduled harvest period(Adjacency list(a)) >= LowerPeriod AND
Scheduled harvest period(Adjacency list(a)) <= UpperPeriod) Then
    Constraint violation = 1
  End If
Next a

```

In contrast to the exact approach, when using this logic there would be no need to re-construct the process if the green-up length assumption changes. Here, the *LowerPeriod* and the *UpperPeriod* represent bounds (in terms of time periods) on the range of the green-up period. Some additional logic would seem necessary to ensure that the computations of the *LowerPeriod* and *UpperPeriod* are valid for the problem that is being solved (i.e., the lower period is greater than 0, and the upper period is less than or equal to the number of time periods within the time horizon).

## 2.2 Area Restriction Adjacency

In contrast to unit restriction adjacency, which focuses on only two adjacent management units, an area restriction adjacency issue can involve many management units, depending on the contiguous area of concern. For example, if the contiguous area of final harvest activities is limited to 40 ha, then any number of adjacent management units can be scheduled for a final harvest during the same period of time as long as their total area does not exceed 40 ha. This model for controlling the timing and placement of forest management activities on a landscape is more closely aligned with common forestry practices than is the unit restriction model [2]. Further, since the GIS databases that support forest management often contain management units (polygons) of various sizes, when an area restriction for management activities guides the actions of forest managers, some adjacent management units may be combined for simultaneous treatment to improve the efficiency of logging operations (and other processes).

Area restriction adjacency constraints therefore are designed to (a) allow the assignment of similar management activities to two or more adjacent management units during a specific period of time, and (b) disallow this to occur when the total size of the potential block of management units exceeds the maximum area assumed. If the maximum area assumed is relatively small, the number of adjacent management units that might be scheduled for simultaneous activities will also be small. Conversely, when the maximum area assumed is relatively large, the number of adjacent management units that might be scheduled for simultaneous activities may also be large. Assessing the large blocks of similarly treated (in action and in time) management units is the main challenge when using this approach. The area restriction model has been used for controlling the size of final harvests and for building minimum-sized habitat patches [16]. When

final harvest sizes are being controlled in a harvest scheduling model, the length of the green-up period complicates the assessment, as the constraint on final harvest size must be viewed from the perspective of each individual management unit. Therefore, the final harvest (clearcut) area may look different from the perspective of each management unit, depending on the time period in which each management unit is scheduled for harvest.

**Exact Approach.** A number of different methods have been described for controlling the potential assignment of forest harvest activities to multiple management units within a given time frame, while allowing several adjacent management units to be scheduled at the same time as long as the total area does not exceed some maximum area (e.g., [13, 15]). In this work, as in previous work [2], we use the path model [12] since it concisely described an exact approach for addressing area restriction adjacency issues in forest management planning.

Given some maximum area (*MaxArea*) for final harvests (clearcuts) of forests, an exact approach would embark on the development of equations (constraints) that prevent any cluster of adjacent management units from being scheduled for a final harvest at the same time. Conceptually, the equations begin with an adjacent pair of management units. If the total size of these exceeds *MaxArea*, then a simple pairwise constraint (described earlier) suffices to control how large the potential final harvest might become. If the total size of the two management units is less than *MaxArea*, a third adjacent management unit (adjacent to either of the two initial management units) is added to the equation. If the sum of all three management units exceeds *MaxArea*, then a constraint is developed to prevent all three from begin scheduled for harvest during the same time period.

$$MU1P1 + MU2P1 + MU3P1 \leq 2 \quad (5)$$

As you can see in this equation, only two of the three management units are allowed to be scheduled for harvest during time period 1, since harvesting all three would exceed the *MaxArea* assumption for final harvests. The process of constructing the constraints continues with all possible combinations of adjacent management units (and their neighbours, and so on), until the *MaxArea* assumption has been exceeded, which then prompts the development of a constraint. Some constraints are redundant.

$$MU3P1 + MU1P1 + MU2P1 \leq 2 \quad (6)$$

And some constraints are dominated by others. For example,

$$MU1P1 + MU2P1 \leq 1 \quad (7)$$

dominates the previous constraint, since if the result of (7) is true, then the result of (6) will also be true, therefore Eq. 6 is not necessary.

One challenge with this approach for solving a harvest scheduling problem is that all of the constraints must be constructed prior to supplying the problem formulation to a solver (e.g., LINGO 20, CPLEX®, etc.). If the green-up time period assumption is altered (increasing or shortening the time assumed for forests to green up) or if the *MaxArea* assumption is altered, a new set of constraints is needed.

**Heuristic Approach.** Within a heuristic search process, area restriction adjacency constraints can be assessed in real time. In this case, there is no need to construct the adjacency relationships *a priori*. Each potential final harvest, following the example of this work, would undergo an assessment process before the harvest activity would be formally accepted into a solution. For example, assume that management unit 1 was potentially being scheduled a final harvest during time period 1. A set of logic would be employed to check all adjacent neighbours of management unit 1 for a similar management action during time period 1. If an adjacent management unit is also scheduled for a final harvest during time period 1, the total size of the two management units is determined. If the total size of the two management units does not exceed the *MaxArea* assumption, then management unit 1 can also be scheduled for a final harvest during time period 1. However, most importantly, all adjacent neighbours of management unit 1, as well as all adjacent neighbours of the second management unit that is scheduled for a final harvest during the same time period (and their neighbours, and so on) must be assessed to determine how large the resulting final harvest block might become. The process described below (first offered in [2]) might be used to conduct this analysis.

```

Constraint violation = 0
Block size = Size(Management Unit 1)
Queued(1) = Management Unit 1
Do While Queued (1) > 0
For a = Beginning pointer(Queued(1)) to Ending pointer(Queued(1))
  If (Potential harvest period(Queued(1)) = Scheduled harvest period(Adjacency list(a))) Then
    Place Adjacency list(a), the adjacent neighbour, in the next empty cell of the Queued array.
    Block size = Block size + Size(Adjacency list(a))
    If (Block size > MaxArea) Then
      Constraint violation = 1
    Exit Loop
  End If
End If
Next a
"Seated" Management unit = Queued(1)
Adjust Queued array
Loop

```

This process sets the initial block size as the area of management unit 1 (line 2). Management unit 1 is then “queued” for assessment. As long as there is a management unit in the first cell of the queued array, the process continues. Obviously, at the beginning of this process management unit 1 is in the first cell of the queued array. Pointers to the places in the adjacency list where adjacent neighbours of management unit 1 can easily be found are then used as the beginning and ending points of a For-Next loop (line 5). A question is then asked: if the potential harvest period of management unit 1 is the same as the scheduled harvest period of one of its adjacent neighbours, the adjacent neighbour is then placed into the next empty cell of the queued array. The block size is then increased using the size of the adjacent neighbour, and the constraint is assessed. If the total block size exceeds the *MaxArea* assumption, a constraint violation is noted, and the process terminates. Later, in the heuristic process, this constraint violation is recognized, and management unit 1 is prevented from being scheduled during time period 1. However,



if the block size is less than the *MaxArea* assumption, the process checks all other neighbours of management unit 1. When all other neighbours have been checked, and if the block size still has not been exceeded (exiting successfully the For-Next loop) management unit 1 is seated (my term), removed from the queued array, and all other management units in the queued array are shifted one position upward. This suggests that the adjacent neighbours of the adjacent neighbours of management unit 1 will then be assessed. If the entire process does not result in an adjacency violation, scheduling management unit 1 during time period 1 will not result in an area restriction adjacency constraint violation.

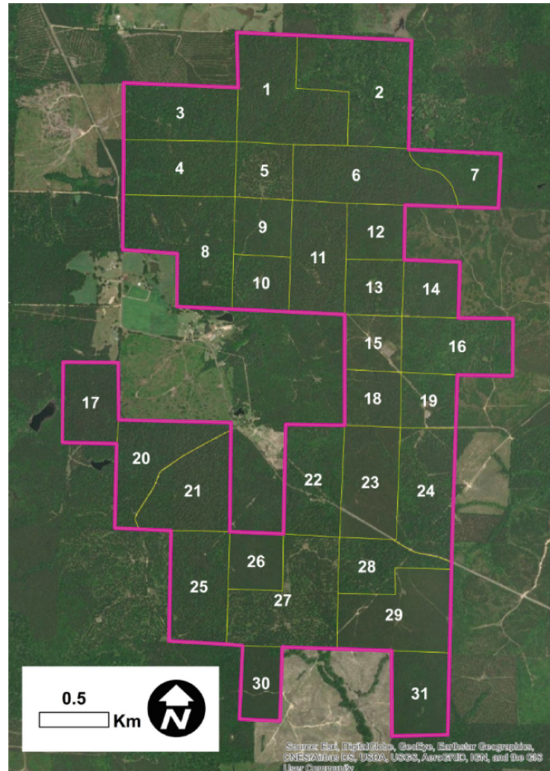
The main disadvantage of this heuristic approach is the complex computer logic that is required to efficiently and correctly assess the size of a harvest block. Additional computer logic would also be required when the green-up period is assumed to be longer than one time period of the time horizon. In undergoing this process, it is important to note that the harvest period of the focal management unit (the management unit in position Queued(1)) defines the *LowerPeriod* and *UpperPeriod*.

### 2.3 Case Study

The case study for this work involves a forested tract of land that is situated in the southern United States, in the southern-most region of Arkansas. The forest (872.7 hectares) is formed as a contiguous tract of land through a collection of 31 management units (Fig. 1) which contain stands of trees of different ages, and contain pine tree species (e.g. *Pinus taeda*, *P. echinata*, etc.) and various deciduous trees (e.g., oaks or *Quercus* spp.) native to the eastern United States.

The tactical forest management plan devised for this property had a 15-year time horizon that consisted of 15, 1-year long time periods. The objective of the tactical plan was to provide a relatively even flow of wood products from the forest, and it was measured in a goal programming sense by minimizing the deviations from a harvest target. The harvest target was defined as an amount less than that suggested using the Meyer amortization volume control method for the forest [4]. For this tract of land, the pre-defined desired sustainable flow of wood products was assumed to be 18,850 tons (2,000 lb per ton) per year. The maximum final harvest size constraint for the area restriction adjacency issue was assumed to be 48.6 hectares (120 acres). Management units that share an edge were assumed to be adjacent. The green-up period was assumed to be 2 years (current year + one additional year). The minimum average harvest age for the trees in each management unit was assumed to be 22 years.

The problem formulations for the exact approaches were developed as mixed integer quadratic programming models which were intended to be solved using LINGO Extended 20.0 [10]. The algorithms for the URM and ARM cases were embedded into a tabu search heuristic that employed search reversion and 2-opt moves [5, 6].



**Fig. 1.** The case study forest area.

### 3 Results

For the exact method, 690 non-redundant pairwise adjacency constraints were necessary to address final harvest adjacency restrictions within a single time period when using the unit restriction adjacency model. As the length of the green-up period increased, the number of pairwise constraints increased to 1,978 for two years of green-up (current year + one additional year before and after a scheduled final harvest), 3,174 for three years (current year + two additional years before and after a scheduled final harvest), and 4,278 for four years (current year + three additional years before and after a scheduled final harvest). The increase in necessary adjacency-related constraints was not necessarily linear, since when the green-up period surrounding a proposed harvest (measured in years) extends backwards in time before the initial time period, or extends forward in time beyond the final time period, fewer pairwise adjacency constraints were needed to address these harvest restrictions.

In contrast, when addressing the unit restriction model of adjacency, the heuristic method required no pre-defined adjacency constraints. In this case, the potential adjacency and green-up constraint violations were assessed in real time with each attempted move (shift from one feasible solution to a neighbouring solution) within tabu search. The logic employed to address unit restriction adjacency and green-up constraints within

a heuristic can be as minimal as the code that was presented earlier in this work. However, additional computer code is necessary to read, store, and access the list of adjacent management units. To increase the efficiency of this process, pointers (information indicating where the pertinent information begins in the adjacency list) would need to be developed. For the case study forest, an example tactical harvest schedule when employing the unit restriction model, which illustrates the planned harvest period for each management unit, is found in Fig. 2. As you can see, there are two management units in the upper right part of the property that are scheduled for a final harvest during time periods (years) 13 and 15. Since the green-up period is 2 years, this schedule of harvests two years apart is the closest (temporally) possible option when employing the unit restriction adjacency approach, given the assumptions of this harvest scheduling problem.

Again for the exact method, 600 non-redundant, non-dominated adjacency constraints were necessary to address final harvest adjacency restrictions within a single time period when using the area restriction adjacency model. As the length of the green-up period increased, the number of non-redundant, non-dominated adjacency constraints increased substantially to 2,294 constraints for two years of green-up, 4,552 constraints for three years of green-up, and 7,234 constraints for four years of green-up. Like the previous case, the increase was not necessarily linear, and seemed somewhat more exponential in nature than in the unit restriction case. The cause of the increase is based on the number of adjacent management units that can be scheduled for harvest at the same time (relatively speaking) and not exceed the maximum final harvest area assumption. Unlike in previous similar work [2], where one non-dominated area restriction constraint contained six management unit decision variables when the maximum area size was 48.6 ha, and eight non-dominated area restriction constraints contained five management unit decision variables, in this case study there were only seven non-dominated area restriction constraints that contained only three management units.

As suggested earlier, the task of eliminating redundant and dominated constraints from the problem formulation of an exact method can be cumbersome. For example, the area restriction adjacency constraint that allows (at most) only two management units to be scheduled for harvest in a single time period

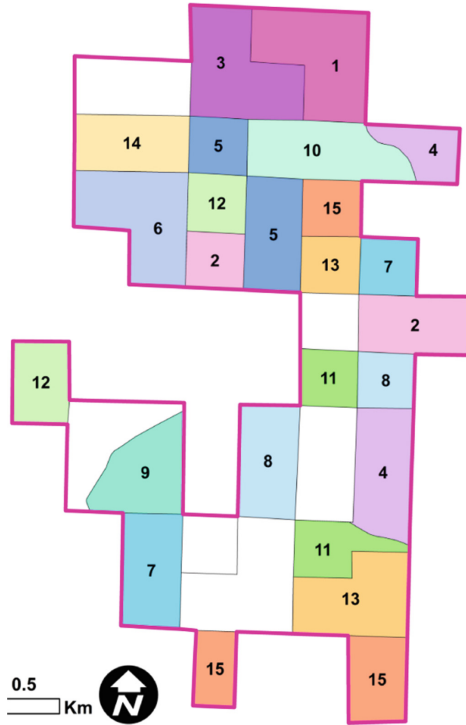
$$MU1P1 + MU2P1 + MU3P1 \leq 2 \quad (8)$$

is dominated by an equation that relates to only two of the management units

$$MU1P1 + MU2P1 \leq 1 \quad (9)$$

since if the latter (Eq. 9) is true, then the former must also be true. Therefore, the former (Eq. 8) is not necessary as long as the latter is present in the problem formulation.

A unit restriction adjacency model devised for an exact method only requires pairwise adjacency constraints. These are relatively easy to develop (a) when the adjacency relationships are known) and (b) when the green-up is one time period long (the current period of interest). When the green-up period extends beyond the current time period, careful consideration should be applied to the development of pairwise constraints that prevent two or more adjacent management units from being scheduled for harvest within the green-up period. An area restriction adjacency model is more complex in this regard when there are more than two management units within the constraint, and the green-up



**Fig. 2.** A forest plan that indicates the time period of final harvests, while accommodating unit restriction adjacency constraints with a green-up length of two time periods.

period is longer than one time period. All possible combinations of potential harvest periods within the guide of the green-up period need to be recognized to prevent the development of a final harvest that in effect is larger than the assumed maximum size. For example, consider three management units (1, 2, and 3). They are each adjacent to each other, and their total size exceeds an assumed maximum final harvest size. If management unit 1 was were to be scheduled in time period 5, and the green-up period was 2 years (years 4, 5, and 6), the following constraints would be necessary to prevent all three management units from being scheduled for harvest in years 4–6:

$$MU1P5 + MU2P4 + MU3P4 \leq 2 \quad (10)$$

$$MU1P5 + MU2P4 + MU3P5 \leq 2 \quad (11)$$

$$MU1P5 + MU2P4 + MU3P6 \leq 2 \quad (12)$$

$$MU1P5 + MU2P5 + MU3P4 \leq 2 \quad (13)$$

$$MU1P5 + MU2P5 + MU3P5 \leq 2 \quad (14)$$

$$MU1P5 + MU2P5 + MU3P6 \leq 2 \tag{15}$$

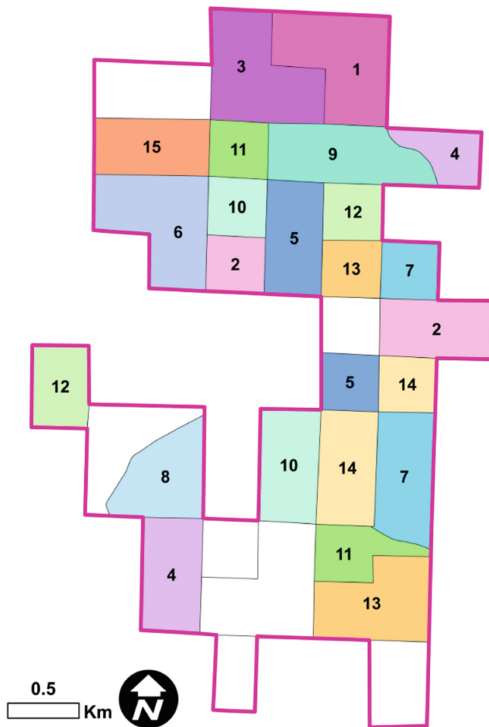
$$MU1P5 + MU2P6 + MU3P4 \leq 2 \tag{16}$$

$$MU1P5 + MU2P6 + MU3P5 \leq 2 \tag{17}$$

$$MU1P5 + MU2P6 + MU3P6 \leq 2 \tag{18}$$

So as it was noted earlier, one of the main challenges for employing an exact method and constraining the timing and placement of forest management activities involves re-constructing the necessary constraint-related equations. One simple change in the assumptions regarding the forest management situation (change in maximum harvest area, change in green-up period assumed) requires re-constructing the constraints.

As with the unit restriction adjacency constraints, the heuristic method has no need for pre-defined area restriction adjacency constraints. An example tactical forest plan representing the scheduled harvest year for each management unit, recognizing area restriction constraints, is found in Fig. 3. As you can see, there are two management units in the upper right part of the property that are scheduled for a final harvest during



**Fig. 3.** A forest plan that indicates the time period of final harvests, while accommodating area restriction adjacency constraints with a green-up length of two time periods.

time periods (years) 12 and 13. Since the green-up period is 2 years, this schedule of harvests would not be possible when using the unit restriction adjacency approach. However, since these two management units, in sum, represent an area smaller than the *MaxArea* assumption, they are allowed to be scheduled for a final harvest only one year apart in time.

The potential constraint violations are assessed in real time during an optimization process. As one might imagine however, the computer code and logic required to assess the area restriction adjacency constraints within a heuristic method can be cumbersome. In the example provided earlier in this work, extensive logic would need to be designed to manage the so-called *queued* and *seated* arrays containing management units surrounding a proposed final harvest activity.

## 4 Conclusions

The approaches for addressing spatial connectivity of forest final harvests within harvest scheduling algorithms have focused here on exact (mixed-integer) and heuristic (e.g., simulated annealing, tabu search, etc.) methods for developing a feasible and efficient tactical harvest schedule. These methods (exact and heuristic) are two lines of inquiry that have captured the attention of researchers over the last 3 decades. While the processes for assessing the unit restriction model of adjacency are relatively straightforward, the exact methods for assessing area restriction final harvest adjacency issues have been well described in the literature (e.g., [12]). And while the processes for assessing unit restriction adjacency within heuristics have been provided in several published papers (e.g., [1, 3, 6]), the logic for assessing area restriction adjacency has only been described theoretically until recently [2].

With exact approaches for assessing final harvest adjacency, the constraints must be developed prior to solving the problem. This is a disadvantage to the approach, and it is further complicated by the fact that the constraints need to be re-constructed if the *MaxArea* or green-up assumptions change. Further, the number of constraints necessary to describe the management problem may grow exponentially depending on the character of the problem (number of management units, size of management units, maximum size assumption, green-up length assumption). With heuristic approaches for assessing final harvest adjacency, the logic employed to correctly assess constraint violations can be cumbersome to develop and time-consuming to assess during the operation of the heuristic. These are disadvantages to the approach. However, constraints do not need to be re-constructed when assumptions regarding the management problem change.

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