# Interference Fit Calculation with Numerical and Simulation Methods

**Vi Phong Lam and Huu Loc Nguyen**

**Abstract** This study presents the steps of calculation and selection of interference fits. According to Lamé's formula, we can determine the stress and strain values generated when assembling the joint. We also show the modeling and simulation of interference fit for evaluating these same values. Then, we compare the calculated values with Lamé's formula and simulation results, which is the basis of further interference assembly experiments to find the load capacity for different pairs of assembly materials.

**Keywords** Interference fit · Simulation method · Finite element method (FEM) · Tolerance · Stress · Strain

## 1 Introduction

Researchers worldwide have conducted many studies on interference fits, focusing mainly on joint strength, selection of reasonable fit tolerance, simulation of concentrated stresses at the interface, and technological solutions to improve the workability and longevity of interference fits.

Traditionally, researchers have utilized Lamé's formula to develop analytical techniques to design the interference fits based on the analysis of plane stresses in the elastic region of the material. Following that, there are analyses of interference fit between shafts and spur gears, which apply analytical and numerical methods and conclude that, for highly complex geometry, Lamé's formula becomes up to 78% inaccurate in evaluating the actual contact pressure [1].

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Moreover, analysts also apply analytical models to mathematical tools in studying the interface pressure, the distribution of stresses, and the influence of surface conditions or thermal processes. Some studies emphasized the importance of the surface roughness value in fixed joints and reported that the load transmission wasting could be up to 300% with surface roughness values ranging from 0.24 to 6.82 μm [2]. In addition to determining the interference fit stresses with mathematical formulas, many studies applied the finite element method (FEM) with many different coupling 3D models. The experimental results are similar to Lamé's formula results [3].

This study introduces the calculation and selection steps of the regular interference fits. Then, by using mathematical calculation with Lamé's formula and simulating the interference joints with the finite element method on ANSYS Workbench 2019 R2 software, we determine the displacement in the radial direction, the stress values, and the residual interface stresses. Finally, we analyze the obtained results.

# 2 Basis of the Interference Fit Calculation

## *2.1 Selection of Interference Fit Tolerance*

The selection and calculation of interference fit usually follow a typical material strength problem called the Thick-wall cylinder theory, also known as Lamé's problem or Lamé's equations.

Usually, we carry out the calculation process according to the calculation model (see Fig. 1) and go along with the following steps [4–6]:

- Step 1: Calculation of the specific pressure $[p_{min}]$: The minimum interface pressure depends on the case of the applied load. This value depends on its loading condition: Torque transmission $T$; axial force transmission $F_a$; and sometimes both of them. In turn, we have:

$$[p_{min}] \geq \frac{2 \cdot K \cdot T \cdot 10^3}{\pi \cdot d_N^2 \cdot l \cdot f}; [p_{min}] \geq \frac{K \cdot F_a}{\pi \cdot d_N \cdot l \cdot f}; [p_{min}] \geq K \cdot \frac{\sqrt{F_t^2 + F_a^2}}{\pi \cdot d_N \cdot l \cdot f}$$

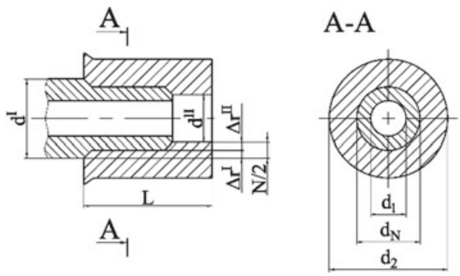


**Fig. 1** Basic calculation model of interference fit

whereas: $p$ is the interface pressure mean value (MPa); $K$ is the coefficient of contact safety ($K = 3 \div 4, 5$); $f$ is the coefficient of friction; $l$ and $d_N$, respectively, the fit length and nominal assembly diameter (mm).

- Step 2: Determination of Lamé's coefficients—$C_1$ and $C_2$, for the inner and outer parts.
- Step 3: Determination of the minimum theoretical interference value that is sufficient to power transmission:

$$N_{mintt} = [p_{min}] \cdot d_N \cdot \left( \frac{C_1}{E_1} + \frac{C_2}{E_2} \right) \quad (1)$$

where: $E_1$ and $E_2$ are Young's modulus of the inner and outer part material (MPa); $d_1$ and $d_2$ are the inner diameters of the inner and outer part (with a solid shaft, $d_1 = 0$) (mm); $\mu_1$ and $\mu_2$ are The Poisson's ratios of the inner and outer part material.

- Step 4: Range of the damaged asperities value $H$.
- Step 5: Determination of the minimum allowable interference value taking into account the effect of surface roughness:

$$N_{pmin} = N_{mintt} + H = N_{mintt} + 1.2 \cdot (R_{zD} + R_{zd}) \quad (2)$$

where: $H$ is the damaged asperities value (μm), $R_{zd}$ and $R_{zD}$ are the ten-point mean roughness values of the inner and outer parts (μm).

- Step 6: Determination of the maximum allowable specific pressure $[p_{max}]$:

$$[p_{max}] = \max[p_1; p_2] \quad (3)$$

$$\text{with} \quad p_1 = 0.58 \cdot \sigma_{y1} \cdot \left[ 1 - \left( \frac{d_1}{d_N} \right)^2 \right]; \; p_2 = 0.58 \cdot \sigma_{y2} \cdot \left[ 1 - \left( \frac{d_N}{d_2} \right)^2 \right] \quad (4)$$

where: $\sigma_{y1}$ and $\sigma_{y2}$ are the material yield strengths of the inner and outer parts (MPa).

- Step 7: Determination of the maximum theoretical interference value:

$$N_{maxtt} = [p_{max}] \cdot d_N \cdot \left( \frac{C_1}{E_1} + \frac{C_2}{E_2} \right) \quad (5)$$

- Step 8: Determination of the maximum allowable interference value taking into account the effect of surface roughness:

$$N_{pmax} = N_{maxtt} + 1.2 \cdot (R_{zD} + R_{zd}) \quad (6)$$

- Step 9: Selection of standard interference fits: According to Ref. [7], select the suitable assembly tolerance which can fulfill these conditions:

$$\begin{cases} N_{pmin} \le [N_{min}] \\ N_{pmax} \ge [N_{max}] \end{cases} \quad (7)$$

## 2.2 Calculation of Interference Fit Stress and Strain

We can evaluate the radial stress $\sigma_r$, the tangential stress $\sigma_t$, or the interface pressure $p$ with Lamé's equations, as follows:

$$\sigma_{r,t}(r) = A \pm \frac{B}{r^2};\ p = \frac{[N_{maxtt}]}{d_N \cdot \left(\frac{C_1}{E_1} + \frac{C_2}{E_2}\right)}(\text{MPa}) \quad (8)$$

where: $A$ and $B$ are the integral constants determined by the boundary conditions that depend on the under-consideration radius value $r$; $[N_{maxtt}]$ is the maximum actual interference value, with $[N_{max}]$ is the maximum interference value obtained according to the selected joint tolerance (µm). Considering the boundary conditions of the solid shaft, the hollow shaft and the outer part, we can get the equations in Table 1.

From Fig. 1, we can consider the interference value $N$ as the difference between the actual outer diameter of the inner part $d^I$ and the actual inner diameter of the outer part $d^{II}$. After assembling, $d^I$ will decrease by an amount of $\Delta r^I$, and $d^{II}$ will increase by an amount of $\Delta r^{II}$. Alternatively, we can present this as:

$$\frac{N}{2} = \left|\Delta r^I\right| + \Delta r^{II} = \left|\frac{p \cdot d^I}{2 \cdot E_1} \cdot (1 - \mu_1)\right| + \frac{p \cdot d^{II}}{2 \cdot E_2} \cdot \left(\frac{d^{II^2} + d_2^2}{d_2^2 - d^{II^2}} + \mu_2\right)(\text{mm}) \quad (9)$$

**Table 1** Boundary conditions and tangential stresses of the hollow shaft and the outer part

| | Boundary conditions | Tangential stresses |
|---|---|---|
| Solid shaft | $\begin{cases} \sigma_r^I(r = r_1 = 0) = -p \\ \sigma_r^I(r = r_N) = -p \end{cases}$ | $\begin{cases} \sigma_t^I(r = r_1 = 0) = -p \\ \sigma_t^I(r = r_N) = -p \end{cases}$ |
| Hollow shaft | $\begin{cases} \sigma_r^I(r = r_1) = 0 \\ \sigma_r^I(r = r_N) = -p \end{cases}$ | $\begin{cases} \sigma_t^I(r = r_1) = -\frac{2 \cdot p \cdot r_N^2}{r_N^2 - r_1^2} \\ \sigma_t^I(r = r_N) = -\frac{p \cdot \left(r_N^2 + r_1^2\right)}{r_N^2 - r_1^2} \end{cases}$ |
| Outer part | $\begin{cases} \sigma_r^{II}(r = r_N) = -p \\ \sigma_r^{II}(r = r_2) = 0 \end{cases}$ | $\begin{cases} \sigma_t^{II}(r = r_N) = \frac{p \cdot \left(r_N^2 + r_2^2\right)}{r_2^2 - r_N^2} \\ \sigma_t^{II}(r = r_2) = -\frac{2 \cdot p \cdot r_N^2}{r_2^2 - r_N^2} \end{cases}$ |

## 3 Calculation Results and Simulation of Interference Fit

We select the input parameters as follows: the nominal joint diameter $d_N = 30$ mm, the inner part's inner diameter $d_1 = 0$ mm, the outer part's outer diameter $d_2 = 50$ mm, the coupling length $l_2 = 55$ mm, the inner part's length $l_1 = 70$ mm; and the material properties: assembly parts elastic modulus $E = 2.06 \cdot 10^{11}$ Pa, Poisson's ratio $\mu = 0.3$, material yield strength $\sigma_y = 350$ MPa, coefficient of friction $f = 0.14$, coefficient of contact safety $K = 3$, inner part contact surface roughness value $R_{zd} = 0.8$ µm and outer part contact surface roughness value $R_{zD} = 1.6$ µm. We specifically investigate the loading condition for only torque transmission $T = 20$ Nm.

From the results between theorical and simulating results, there is a good agreement. So from here, we can use simulation to determine the coefficients considering the influence of roughness on the calculated interference fit with different diameter sizes and material pairs (Fig. 2; Table 2).

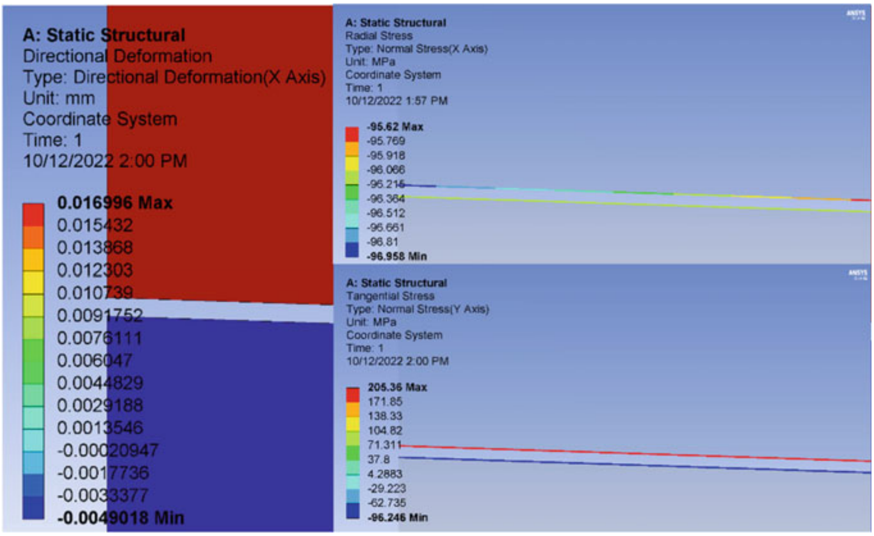


**Fig. 2** Stress and strain simulation results at the interface

**Table 2** Comparison of stress and strain

| | Section | Results | | Deviation (%) |
|---|---|---|---|---|
| | | Simulation | Theoretical calculation | |
| Radial stress | | (MPa) | (MPa) | |
| Shaft—$\sigma_r^I$ (MPa) | $r_I = 0$ | −96.180 | −98.5285 | + 2.38 |
| | $r = r_N$ | −96.207 | −98.5285 | + 2.36 |
| Hub—$\sigma_r^{II}$ (MPa) | $r = r_N$ | −96.958 | −98.5285 | + 1.59 |
| | $r = r_2$ | −0.0128 | 0 | |
| Tangential stress | | (MPa) | (MPa) | |
| Shaft—$\sigma_t^I$ (MPa) | $r_I = 0$ | −96.180 | −98.5285 | + 2.38 |
| | $r = r_N$ | −96.246 | −98.5285 | + 2.32 |
| Hub—$\sigma_t^{II}$ (MPa) | $r = r_N$ | 205.360 | 209.373 | + 1.92 |
| | $r = r_2$ | 108.280 | 110.845 | + 2.31 |
| Strain | | (mm) | (mm) | |
| Shaft—$\Delta r^I$ (mm) | | 0.0049018 | 0.00502210 | + 2.39 |
| Hub—$\Delta r^{II}$ (mm) | | 0.0169960 | 0.01739792 | + 2.31 |

## 4 Conclusion

The above comparison tables show that the calculated results according to Lamé's formula and the values inferred from the simulation by the finite element method have certain similarities (deviation less than 5%). Because of this similarity, we can test the fit strength by transferring the existing 3D-designed model to the finite element environment, which could reduce the calculation steps or shorten the design time.

In addition, when simulating interference fits using finite element software, we can consider more variables, including temperature effects, contact pressure, thermal conductivity, and even values that traditional calculations cannot achieve.

This study has succeeded in the strength analysis of the interference fits by applying modern methods of simulation based on the conditions to ensure the functional ability of the transmission system. We could conduct more experiments to check the calculation and simulation results for further research. From there, we could discover technical solutions to improve the strength of the interference fits, which might be about new assembly techniques or enhancing the contact surface properties by coating and plating technologies [8, 9].

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