Newton's Mechanics Lectures in Cambridge

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Abstract. The publication of the correspondence between Newton (1642–1727) and Roger Cotes (1682–1716), reviewer and editor of the second edition of the *Principia* (1713), is also accompanied by other important manuscripts and works, in the sense of a better evaluation and understanding of the immense intellectual construction that was carried out with the publication of the *Principia*, in 1687. The publication of this correspondence based on manuscripts belonging to the library of Trinity College, Cambridge, also includes the lectures given by Newton in the years 1684, 1685 and 1687, in Cambridge. In this paper, we will explore the manuscripts relating to these courses, trying to reconstruct the content of the mechanics lectures given by Newton in 1684, as we consider that they have a very great importance in the preparation of the *Principia* itself.

Keywords: Newton´s *Principia* · Newton´s Lectures · Orbital Motion · Newtonian Mechanics

1 Introduction

The courses given by Newton in Cambridge in 1684, 1685, and 1687, are of great importance, mainly when considered from a broader point of view, because they are part of the process of building his *magnum opus*. Before 1687, Newton published short works such as *De Motu Corporum in Gyrum* (1684), the contents of which would be incorporated into the larger work represented by the *Principia*. Probably, in 1679 or 1684 Newton composed *The Kepler-Motion Papers* while the first edition of the *Principia* was published in 1687. According to [1], the *De Motu* manuscript was subjected to many corrections, amendments, and additions, but its content is very close to what would be published in Book I of the *Principia*. To better understand Newton's courses program, we will use the *De Motu* manuscript presented by [1], as mentioned above, as well as the *Principia* [2] itself.

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The *De Motu* manuscript was delivered to the Royal Society in 1686, and according to [1], there are five versions of this treatise. Three of them are part of the Portsmouth Collection belonging to the Cambridge University Library, another version is in the archives of the Royal Society and, finally, the last version is part of the Macclesfield Collection. They are all practically identical, which ensures that they have a common origin.

In August 1684, Halley (1656–1742) visited Newton in Cambridge and received the good news that Newton had demonstrated the law governing the movement of celestial bodies. When searching his papers for this demonstration, Newton did not find it [3]. However, he redid the calculations made previously and in November sent them to Halley in the form of four theorems and seven problems. It is also important to emphasize that in the summer of 1685 Newton completed Book II of the *Principia*, but he needed to complete a full theory of the moon which had been left incomplete in the first edition of the *Principia.*

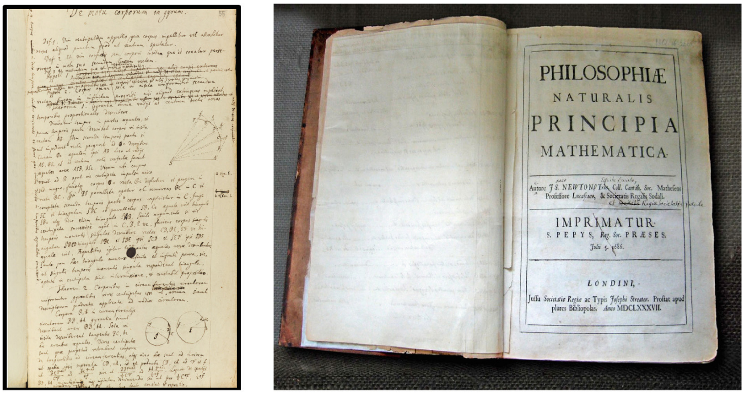
We believe that the courses given by Newton in Cambridge, which in addition to mechanics and celestial mechanics, also covered mathematics and optics, provide important elements to understand the accomplishment of his *Principia*, as well as indicating how this new theory of movement should be organized and transmitted it in the form of lectures.

Specifically in regard to mechanics, these lectures also help to study the transformations that Newtonian mechanics underwent, moving from a geometrical approach (Newton) to algebra and analysis formalization [4] with Lagrange (1736–1813) in 1788, passing through the differential form of the second law of motion by Euler (1707–1783), around 1750 [5].

2 Excerpts from Newton Lectures

Newton became Lucasian professor in 1669. From 1670 to 1672 he taught optics. His first lecture as a Lucasian professor was in January 1670. From 1673–74 he gave classes in algebra. In the period 1684–85 he lectured on theoretical mechanics and in 1687 concentrated on mechanics applied to astronomy. In 1696 Newton moved to London but would only leave his Lucasian professorship in 1701. There is no record of Newton giving other classes in other periods [6].

The two figures below (Figs. 1a and 1b) show the two works by Newton used in this investigation.



a: Newton´s *De Motu* **b**: Newton´s Principia

Fig.1. **a:** Newton´s De Motu **b**: Newton´s Principia

LECTURES ON THE MOTION OF BODIES

(MS. Univ., Libr. Mr. 9. 46.)

[The numbers on the left denote the pages in the 1st ed. of *Principia*: those on the right the leaves in the MS.]

The title is “De motu corporum Liber Primus.“ It forms the draft of the first book of the *Principia*, see p. 209, note.

October 1684

1 -11 Definitions... I composed the last treatise. Lect.1(1-9)

12-20 Axioms or Laws of Motion...and the movements among themselves.
Lect. 2(10-16)

20-29 Schol. So far, I have given the principles...
...in the final account of part to part. Lect. 3(16-20)

29 - 36 Lemma V. I had figured it out... diminishing without limit. Lect. 4(20-25)
37-49 Artic. II. On the Discovery of Centripetal Forces...
...turned into a centrifuge. Lect. 5(25-31)

50-56 Artic. III. The motion of the bodies in eccentric conical sections .... the major axes of the ellipsoid. Lect. 6(31-36)

56-63 Prop. XVI. Theorem VIII. Putting the same list... ...I do not attach too many obstacles. Lect. 7(36-39)

67 - Lemma XV. From the given tribe ... I will continue to explain. Lect. 8(39-42)

115 - Prop. XXI Prob. XIII Put what you want... Lect. 9(42—)
breaks off at fol. 44, in Prop. XXIV. with the words "turning, bow Kk."
Principia. p. 118.

October 1685.

79-88 Prop. XXII. Prob. XIV... The problem is impossible. Lect. 1(58-63)

89-98 Prop. XXVI. Prob. XVIII. Similar and Equal. Lect. 2(63-68)

98-107 Lemma XXVII.... irrational as follows. Lect. 3(68-73)

107-114 Prop. XXXI. Prob. XXIII....I will continue to explain. Lect. 4(73-76)

115-125 Artic. VII. Of the ascent of the bodies... to the area, Lect. 5(76-83)
(Whole of 7th section.)
125-131 Artic. VIII. Containing the Discovery of the Orbs in which...
...let's add a few things. Lect. 6(83-86)
(Whole of 8th section.)
132-13 Artic. VIII. [error for IX] To ascend obliquely. Lect. 7(86-89)

137-144 Prop. XLV. Prob. XXXI. Lect. 8(90-95)

145-152 Artic. X. The Movements of the Bodies... they always perform. Q. E. D.
Lect. 9(95-99)
153-Prop. LII. Prob. XXXIV... Lect. 10(99-)

The MS., it will be seen, is imperfect, ending abruptly at the second page of fol. 102. Fol. 37–44 are repeated, one set being the first draft, the other as printed in the *Principia,* pp. 57–73. The nature of the former will be understood from the following outline. After Corol. 6. In Parabola, &c., and the other corollaries, comes Prop. XVI Prob. VIII. Which is Prob. XVI Prob. IX of the *Principia.*

Then, Prop. XVIII …. X. … IX. …… Xi….
without demonstration.
Then Lem. XV……….. Lem. XVI. *Principia*.

Prop. XIX……… Prop. XXI…………..

The latter set and fol. 55–58 as far as "it is absurd. Q.E.D." (Princip. p. 79) are not divided into Lectures. Fol. 45 is numbered 55 apparently by a clerical error, which is propagated through the remainder of the MS.

In binding the volume, the sheets seem to have been taken at random. When the disjointed members are brought together, they form a whole, as follows:

1–57 On the motion of bodies…at tangent. (1–36)
57–73 Corol. 6… in the right way (37–44)
(The other 37–44 in the MS. is the rough draft of this.)
73–83 From any of the points… two will escape (55–62)
88–118 Parallel arc Kk (63–78)
118–133 describe…they can be moved (79–86)
133–144 it is in triplicate…we use plains (87–94)
1 44–159 parallel to these…height CT by (95–102).

Lectures on the System of the World
From a Copy in Cotes' hand in Trin, Coll. Library, (R. 16. 39).
De Motu Corporum Liber

Sept. 29, l687.

1-8 Fixed in the highest astronomers	Prelect. 1
8-16 Mars-double.	Lect. 2.
16—22 Stability—soot.	Lect. 3.
22-27 By analogy.	Lect. 4.
27-33 It will be designated-understood.	Lect. 5.

Here Cotes' copy ends. The remainder of the treatise, however, although not divided into Lectures, is bound in the same volume, and was probably obtained by Professor Smith from Charles Morgan of Clare Hall, for in the Library of that College there is a MS. Copy of the treatise which belonged to Morgan, who states in a note that the first five Lectures were communicated to him by Smith, and the remainder by Martin Folks.

3 Considerations on Mechanics Lectures

In this paper, we will restrict our study to mechanics lectures relating to the motion of bodies, which are part of the lectures given by Newton in October 1684, shown in the excerpts above and whose contents we can find in *De Motu* as well as in Book I of the *Principia*, focusing our attention on the following Lectures:

Lecture 1: Definitions …I composed the last treatise.
Lecture 2: Axioms and Laws of motion… And the motions among themselves.
Lecture 5: On the Discovery of Centripetal Forces…turned into a centrifuge.
Lecture 6: The Motion of the Bodies in Eccentric Conical Sections…The Major Axes of the Ellipsoid.

The method used here, in order to satisfactorily approach the content presented in the four Lectures mentioned above, will be to compare the items listed with what appears in *De Motu* and the *Principia*. In this way, we will try to compare both, namely what is contained in *De Motu* and what is contained in the *Principia* [7].

Regarding *De Motu*, we will use the manuscripts published by Herivel as MSS X_a and X_b, constituting drafts of the *Definitions* and *Axioms* parts of the same work (*De Motu*). Regarding the *Principia*, we will use the translation of the *Principia* from Latin, made by Andrew Motte and revised by Florian Cajori, published by the University of California Press, in 1934.

3.1 Lecture 1: Definitions

Definitions which appear in *De Motu*:

Definition 1: The quantity of matter is that arising conjointly from its density and magnitude.

Definition 2: The quantity of motion is that arising conjointly from the velocity and the quantity of matter.

Definition 3: The internal force of matter is the power of resistance by means of which any one body continues as far as it can in its state of rest or moving uniformly in a straight line.

Definition 4: Impressed force is an action exercised on a body to change its state of rest or motion.

Definition 5: Centripetal force is a certain action or power by which a body is impelled or drawn or in any way tends towards a certain point as if to a centre.

Definition 6: The density of a body is the quantity or bulk of matter compared with the quantity of space occupied.

Definition 7: By the heaviness of a body, I understand the quantity or bulk of matter moved apart from considerations of gravity as often as it is not a matter of gravitating bodies.

Definition 8: Position.

Definition 9: Rest.

Definition 10: Motion.

Definition 11: Velocity.

Definition 12: The exercised force of a body is that by which it attempts to preserve that part of its state of rest or motion.

Definitions which appear in the *Principia*:

Definition I: The quantity of matter is the measure of the same, arising from its density and bulk conjointly.

Definition II: The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

Definition III: The *vis insita*, or innate force of matter, is a power of resistance, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.

Definition IV: An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.

Definition V: A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

Definition VI: The absolute quantity of a centripetal force is the measure of the same, proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about.

Definition VII: The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.

Definition VIII: The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time.

Comparing the definitions above, we can observe that those that appear in the *De Motu* manuscript apply as a set of definitions that cover practically all aspects of general mechanics. Those that appear in the *Principia* are quite precise in their definitions of inertial force and external force and prepare an entire discussion and the foundations of centripetal force, with the purpose of studying orbital motion [8].

3.2 Lecture 2: Axioms and Laws of Motion

It is important to note, which can be corroborated below, that the changes made by Newton in the passage from *De Motu* to the *Principia* leave the formulation of the second law untouched. The discussion that exists among historians as to whether “variation of movement” or “variation of momentum” are equivalent for Newton, in addition to some studies carried out, seem to confirm the hypothesis of this equivalence [9].

Transcription from *De Motu*:

Law 1: *By reason of its innate force every body preserves in its state of rest or of moving uniformly in a straight line unless in so far as it is obliged to change its state by forces impressed on it.*

Law 2: *The change of motion is proportional to the force impressed and takes place along the straight line in which the force is impressed.*

Law 3: *As much as any body acts on another so much does it experience in reaction. Whatever presses or pulls another thing by this equally is pressed or pulled.*

Law 4: *The relative motion of bodies enclosed in a given space is the same whether that space rests absolutely or moves perpetually and uniformly in a straight line without circular motion.*

Law 5: *The common centre of gravity of [a number of] bodies does not change its state of rest or motion by reason of the mutual actions of the bodies. This law and the two above mutually confirm each other.*
Law 6: *The resistance of a medium is jointly proportional to the density of that medium, the area of the moved spherical body and the velocity. I do not assert this law to be exact. It suffices that it should be approximately true. I actually suppose the bodies spherical lest it be a question of considering the states of different figures.*

Transcription from the *Principia:*

LAW I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

LAW II

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

LAW III

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Comparing the laws of motion as they are presented in the *De Motu* and in the *Principia*, we can observe that the first three laws are very similar in the two texts. These three laws are what we know today as Newton's laws. In the text of *De Motu* Newton gives more emphasis to the phenomenon of inertia, the change in the state of the body, whether from rest or from uniform rectilinear movement, will only happen through the action of an external force that he calls impressed force [10].

The three additional laws in *De Motu* form the fundamental difference between the two texts. In the fourth law, Newton discusses what happens in the relative motion between bodies, stating that it does not change if the space (in current language, the reference frame) remains at rest or moves uniformly in a straight line. This is what we know today as Galilean relativity [11].

In the fifth law, Newton states that the center of gravity (today we would speak of the center of mass) of the system of bodies remains at rest or in uniform rectilinear motion due to the internal action between them. Obviously, the fourth and the fifth laws are contained in Newton's three laws stated in the *Principia.*

In the sixth law, Newton deals with the issue of resistance to the passage of a body. He states that this resistance is proportional to the density of the medium and the area of the spherical body, taken as an approximation. Because of this he also considers this law as an approximation.

3.3 Lecture 5: On the Discovery of Centripetal Forces

The *De Motu* manuscript is practically a treatise on the orbital motion of the planets, and, therefore, a study of the action of the centripetal force exerted by the sun, as a fixed center, located at the focus of the ellipse, determining the motion of a given planet in its orbital motion around the sun [12]. This manuscript is composed of four theorems, with their corollaries and seven problems with their scholia.

A summary of *De Motu* is shown as follows:

Theorem 1: *All bodies circulating about a centre [of force] sweep out areas proportional to the times [of description].*

In Fig. 2 below, we can see what in *De Motu* appears as *Theorem 1* and in the *Principia* appears as *Proposition 1.* Newton divides time into equal intervals and in the first-time interval, the body, due to its inertia, describes the line AB. Likewise, in the second time interval it continues in a straight line going to c describing Bc, equal to the length AB, so that the areas ASB and BSc are constructed equal. When the body arrives at B, the centripetal force acts in the form of an impulse, forcing the body to deviate from line Bc and continue along line BC. Constructing cC parallel to BS, cC meeting BC in C completing the second time step. Thus, equal areas are described in equal times, which configures Kepler's second law, better known as the law of areas [13].

Obviously, this construction by Newton is of Archimedean inspiration, and is therefore at the origins of integral calculus itself [14]. Both the time intervals and the arcs described are approaching each other into increasingly smaller subdivisions and the centripetal forces are applied in the form of small impulses directed to the center S.

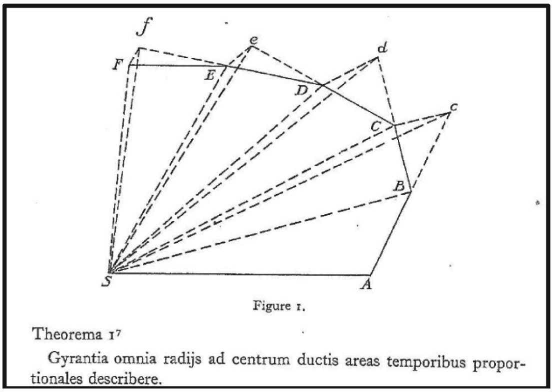


Fig. 2. Illustration of centripetal force towards the center S.

Theorem 2: *The centripetal forces of bodies revolving uniformly in the circumferences of circles are as the squares of the arcs described in the same time divided by the radii of the circles.*

Assuming bodies B and b (Fig. 3), located in different orbits of circles BD and bd, at the same time. Under the action of the inertia of these bodies, they will follow the tangents BC and bc, considered equal to these arcs. The centripetal forces acting on them pulls them back from the tangents to the circles, and, consequently, the distances CD and cd are those where the tangents exceed the circles, producing CD and cd for F and f, with BC^2/CF and bc^2/cf, or as $BD^2/1/2CF$ and $bd^2/1/2cf$. These spaces BD and bd are considered very small and infinitely decreasing, such that for 1/2CF and 1/2cf, we can write that SB and sb are the radii of the circles.

Theorem 3: *If the body P (Fig. 4) circulating about the centre S describes some curved line APQ, and if the straight line PR touches that curve in a certain point P, and if from any other point Q of the curve QR is drawn to the tangent parallel to SP and*

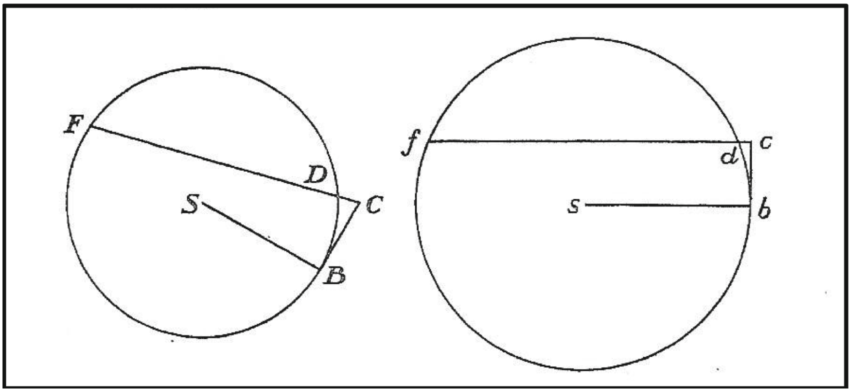


Fig. 3. Centripetal force varying with arc length squared and circle radius.

the perpendicular QT is dropped on the line SP: I say that the centripetal force will be inversely as the ratio SP^2 *x* QT^2/QR*, provided only the quantity of that ratio is always taken as that which it becomes in the limit when the points P and Q coincide.*

Given the infinitely small figure QRPT, the line QR varies in a given time with the centripetal force and with the square of the time, when the force is given; when nothing is given QR varies together with the centripetal force and the square of time, that is, with the centripetal force directly and the area SQP, proportional to the time, or twice the area $SP \times QT$, squared.

Considering each side of this proportionality applied to the QR line and the centripetal force is $SP^2 \times QT^2/QR$ together form the unit, which is the centripetal force which inversely is $SP^2 \times QT^2/QR$.

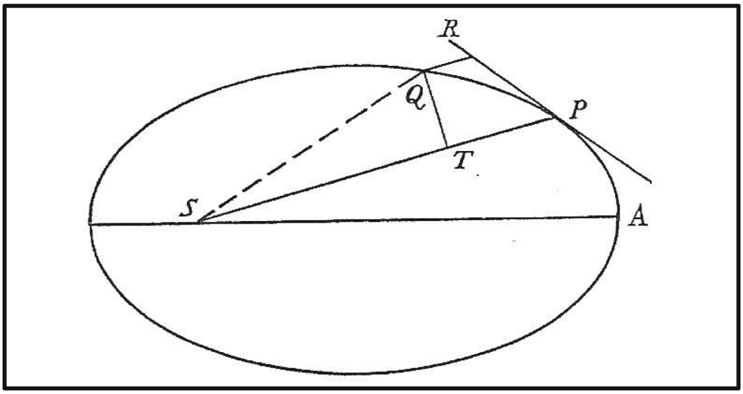


Fig. 4. Centripetal force with the passage to the limit of areas.

Problem 1: *If a body revolves in the circumference of a circle the law of centripetal force is required tending to a certain point in the circumference.*

Problem 2: *Given a body revolving in the ellipse of the ancients, there is required the law of centripetal force directed to the centre of the ellipse.*

Problem 3: *Given a body revolving in an ellipse there is required the law of centripetal force directed to a focus of the ellipse.*

Theorem 4: *Given that the centripetal force is inversely proportional to the square of the distance from the centre the squares of the periodic times in ellipses vary as the cubes of the transverse axes.*

Let AB (Fig. 5) be the transverse axis of the ellipse, PD the other axis, S one of the foci of the ellipse, and we suppose that the circle PMD with a center in S and the radius SP is constructed. Let us also suppose that bodies simultaneously describe the ellipse arc PQ and the circle arc PM. The centripetal force directed to the focus S, PR and PN touch the ellipse and the circle at P. Constructing QR and MN parallel to PS finding the tangents at R and N. But the figures PQR and PMN are infinitely small and so, using results deducted previously, we have:

$$\mathrm{L} \times \mathrm{QR} = \mathrm{QT}^2 \text{ and } 2\mathrm{SP} \times \mathrm{MN} = \mathrm{MV}^2$$

Thus, due to the common distance of SP from the center S, the resulting equality of the centripetal forces MN and QR occurs.

The area SPQ is to area SPM as the total area of the ellipse is to the total area of the circle. However, the positions of areas generated at individual moments are proportional to the SPQ and SPM areas, as well as the areas to the total areas. When multiplied by a certain number of moments it becomes equal to the total areas. We can conclude that complete revolutions in ellipses are carried out in the same period of time as in circles whose diameters are equal to the transverse axes of the ellipses. Using *Corollary 5* of *Theorem 2*, the squares of the periods in the arcs of circles are for the cubes of the diameters as well as in the ellipses.

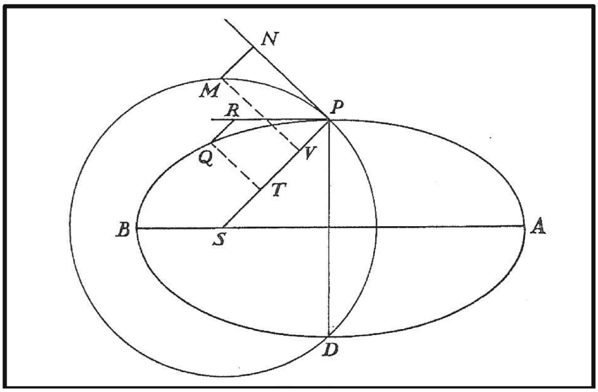


Fig. 5. Newton's demonstration of Kepler's Third Law.

Problem 4: *Given that the centripetal force is inversely proportional to the square of the distance, and knowing the magnitude of that force, required to find the ellipse* *which a body describes when projected from a given point with given velocity in a given straight line.*

Problem 5: *Given that the centripetal force is inversely proportional to the square of the distance from the centre to define the spaces described in given times by a body falling straight to the centre.*

Problem 6: *Define the motion of a body carried along by its innate force alone in a uniformly resisting medium.*

Problem 7: *Given a uniform centripetal force, define the motion of a body ascending and descending rectilinearly in a uniformly resisting medium.*

The Lecture we are studying, which in *De Motu* has the title *On the Discovery of Centripetal Forces*, appears in the *Principia* as *The Determination of Centripetal Forces.* It is found in Book I, Section II and encompasses ten Propositions, five Theorems, and five Problems, also containing some Lemmas and Scholia. Propositions appear in pairs either with a Theorem or a Problem, as we will have the opportunity to analyze below. It is important to note that the change of title for the same study from *De Motu* to the *Principia* highlights an interesting evolution, moving from a strategy of discovery to a determination.

Proposition 1. Theorem 1: *Areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes and are proportional to the times in which they are described.*

Proposition 2. Theorem 2: *Every body that moves in any curved line described in a plane, and by a radius drawn to a point either immovable, or moving forwards with an uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force directed to that point.*

Proposition 3. Theorem 3: *Every body, that by a radius drawn to the centre of another body, howsoever moved, describes areas about that centre proportional to the times, is urged by a force compounded of the centripetal force tending to that other body, and of all the accelerative force by which that other body is impelled.*

Proposition 4. Theorem 4: *The centripetal forces of bodies, which by equable motions describe different circles, tend to the centres of the same circles; and are to each other as the squares of the arcs described in equal times divided respectively by the radii of the circles.*

Proposition 5. Problem 1: *There being given, in any places, the velocity with which a body describes a given figure, by means of forces directed to the same centre: to find that centre.*

Proposition 6. Theorem 5: *In a space void of resistance, if a body revolves in any orbit about an immovable centre, and in the least time describes any arc just then nascent; and the versed sine of that are supposed to be drawn bisecting the chord, and produced passing through the centre of force; the centripetal force in the middle of the arc will be directly as the versed sine and inversely as the squares of the time.*

Proposition 7. Problem 2: *If a body revolves in the circumference of a circle, it is proposed to find the law of centripetal force directed to any given point.*

Proposition 8. Problem 3: *If a body moves in the semi-circumference; it is proposed to find the law of the centripetal force tending to a point S, so remote, that all the lines drawn thereto, may be taken for parallels.*

Proposition 9. Problem 4: *If a body revolves in a spiral, cutting all the radii, in a given angle; it is proposed to find the law of the centripetal force tending to the centre of that spiral.*

Proposition 10. Problem 5: *If a body revolves in an ellipse; it is proposed to find the law of the centripetal force tending to the centre of the ellipse.*

Comparing the two studies of orbital motion made by Newton in *De Motu* and the *Principia*, we can see a great evolution in the treatment and scientific rigor of the same problem. *De Motu's Theorem 1* appears in the *Principia* subdivided into two parts in the form of *Proposition 1* together with *Theorem 1*, considering a fixed center around which the body describes its motion and in *Proposition 2* together with *Theorem 2*, considering the center of motion in a straight line and uniform motion. All demonstrations made by Newton in the ten Propositions, five Theorems, and five Problems are anchored in his laws of motion and Euclidean geometry (*Elements of Geometry*), cited throughout the demonstrations [15].

The most characteristic feature of the passage from *De Motu* to the *Principia*, even though all demonstrations are geometric, is the calculation of the value of the centripetal force, as clear in *Proposition 4* together with *Theorem 4*, where this value is proportional to v^2/r, where v is the tangential velocity and r is the radius of the circle [16]. The same concern appears in other subsequent propositions. Newton's strategy in determining the value of the centripetal force is to generalize the demonstrations of Kepler's laws, which are thus contained in this determination [17].

3.4 Lecture 6: The Motion of the Bodies in Eccentric Conical Sections

The content of the lecture for this item can be found in Book I, Section III of the *Principia.* It generalizes the study of the previous item, dedicating itself to the analysis of the motion of bodies in conic sections. Conic sections have been important in history since the Greek mathematicians, as in the case of Apollonius of Perga [18] and many others. It is also important to point out that other bodies in the solar system, such as the comets, have orbits that are conic sections.

This item contains seven Propositions, three Theorems and four Problems, sometimes with other complements such as Lemmas, Corollaries or Scholia.

Proposition 11. Problem 6: *If a body revolves in an ellipse; it is required to find the law of the centripetal force tending to the focus of the ellipse.*

Let us suppose the ellipse in Fig. 6 below, whose focus is on S. Drawing the line SP cutting the diameter DK of the ellipse in E and observing that the ordinate Qv in x, with the parallelogram QxPR, can prove that EP is equal to major semiarch AC. HI is drawn from the other focus H of the ellipse parallel to EC, because CS and CH are equal. ES and EI are also equal, so that EP is the half-sum of PS and PI, because of the parallelism between HI and PR and the equal angles IPR and HPZ. PS and PH are the same as the 2AC axis. Drawing QT perpendicular to SP we will have $2BC^2/AC$. By Corollary II, Lemma 7, when the points P, Q, coincide, $Qv^2 = Qx^2$ and Qx^2 or Qv^2 are equivalent, and we will have the following proportions: $QT^2 = EP^2 : PF^2 = CA^2 : PF^2$ and, by Lemma 12, it is equal to $CD^2 : CB^2$. When points Q and P coincide, 2PC and Gv will be equal. Using Corollaries I and V and Proposition 6 from the *Principia*, we will have that the centripetal force is inversely as the square of the distance SP.

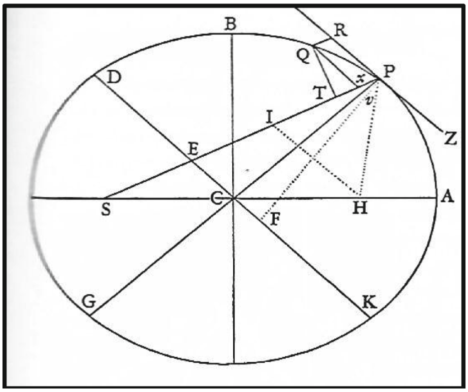


Fig. 6. Centripetal force for elliptical orbits.

Proposition 12. Problem 7: *Suppose a body moves in a hyperbola; it is required to find the law of the centripetal force tending to the focus of that curve.*

Proposition 13. Problem 8: *If a body moves in the perimeter of a parabola; it is required to find the law of the centripetal force tending to the focus of that figure.*

Proposition 14. Theorem 6: *If several bodies revolve about one common centre, and the centripetal force is inversely as the square of the distance of places from the center: I say, that the principal latera recta of their orbits are as the squares of the areas, which the bodies by radii drawn to the centre describe at the same time.*

Proposition 15. Theorem 7: *The same things being supposed, I say, that the periodic times in ellipses are as the 3/2th power (in ratione sesquiplicata) of their greater axes.*

Proposition 16. Theorem 8: *The same things are assumed, and right lines drawn to the bodies that shall touch the orbits, and perpendiculars let fall on these tangents from the common focus: I say, that the velocities of the bodies vary inversely as the perpendiculars and directly as the square roots of the principal latera recta.*

Proposition 17. Problem 9: *Supposing the centripetal force to be inversely proportional to the squares of the distances of places from the centre, and that the absolute value of that force is known; it is required to determine the line which a body will describe that is let go from a given place with a given velocity in the direction of a given right line.*

According to Westfall [10], the central problem to be solved in Newtonian dynamics and which the first version of *De Motu* still leaves unresolved is how to solve the contradiction between two different types of force. The inherent or innate force, which is the force of inertia that maintains uniform motion and the centripetal force that changes this condition. Therefore, in this item we only present the solution provided by Newton and which appears in the *Principia.*

4 Final Remarks and Conclusion

This paper attempts to reconstruct the content of the mechanics lectures given by Newton as a Lucasian Professor at Cambridge. Firstly, we tried to select some of the most important topics from these lectures for the composition of Book I of the *Principia.* Purposely, the lectures chosen are those from the year 1684 when the courses began.

In this sense, we begin with the fundamental *Definitions* on which mechanics rests. Then the *Principles* or the *Laws of Motion* were looked at, where the improvement between *De Motu* and the *Principia* are clear. The remaining two topics are closely related and concern the determination of the centripetal force for orbital motion with a central force [19]. As we have seen, Newton generalizes and inscribes Kepler's Laws in the conceptual framework of his study of the mechanics of orbital motion. If the mathematical model of the central force is known, the orbits are automatically determined. Conversely, Newton also determines the value of the centripetal force for a known orbit [20].

The similarity between the structure of the *Principia* and Euclid's *Elements of Geometry* is clear. In Book I, of the *Elements*, we can see that it begins with 23 Definitions, while in the *Principia* we have 13 Definitions. Next, we see that the *Element*s contain the five famous *Postulates* which correspond in the *Principia* to the three *Axioms* or *Laws of Motion*. The solutions to *Problems* in the *Elements* can be compared to *Propositions* or *Problems* in the *Principia.*

This proposal for a structure that appears in the *Elements* would be a proposal for a paradigm in which all knowledge would be structured in the form of *Definitions*, *Postulates* or *Laws* and knowledge would be obtained and problems solved in a deductive way. This paradigm was seriously shaken with Kurt Gödel's theorems in 1931[21].

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