A Port-Hamiltonian, Index ≤ 1, Structurally Amenable Electrical Circuit Formulation

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Abstract. We present a recently developed electrical circuit formulation that has port-Hamiltonian (pH) structure and results in a structurally amenable differential-algebraic equation (DAE) system of index ≤ 1. Being pH assures energy stability—the total energy of the system cannot increase. It also provides compositionality—larger pH models can be assembled from smaller ones in a standard way that facilitates building pH models in software. Structurally amenable and index ≤ 1 eliminate the phases of DAE index analysis and reduction, which are commonly used in circuit simulation software. Thus, standard numerical solvers can be applied directly to integrate the DAE. In addition, it has a known *a priori* block-triangular form that can be exploited for efficient numerical solution. A prototype MATLAB code shows high potential for development of this "compact port-Hamiltonian" (CpH) methodology.

1 Summary

Computer simulation of electrical circuits entails integrating systems of *differential-algebraic equations* (DAEs). Our work forms a DAE of remarkably simple structure promising faster numerics. It is a synergy of three oldish themes

- Energy-based *port-Hamiltonian* modelling, from ∼2000, [1–3]
- Structural analysis (SA) of *circuit topology*, from ∼1960, [4–7] but using ideas of Kron dating to the 1940s, [8]
- Structural analysis of *general DAEs*, from 1988, [9,10]

Port-Hamiltonian (pH) is a philosophy/technology for multi-physical system modelling, making *energy flow* central; while SA deploys combinatorial algorithms, inexpensive compared with the numerical ones, to *reveal structure* and thus speed up the numerics—cf. reordering algorithms in solving sparse linear systems.

2 Constituents

We take the three parts of the title in turn.

"Port-Hamiltonian". The universe runs on energy conservation across all physical domains. It is perilous for modelling to ignore this. The port-Hamiltonian (pH)

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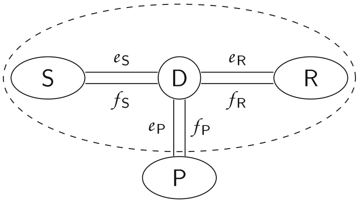


Fig. 1. pH system.

approach [3] splits a system into an *energy-storing* part S, a *resistive* or energy-dissipating part R, a *control or input-output part* P, and lossless *energy-distributing* elements D connecting them, Fig. 1. Stored energy in S is described by a *Hamiltonian* function $\mathscr{H}$. The e,f lines are *ports* where energy flows; $e \cdot f$ has dimension of power. For a circuit they are voltages and currents and D represents Kirchhoff's laws. Multi-physics models and numerics respecting this structure are energy-stable, e.g. perpetual motion is excluded.

"Index ≤ 1 ". Unlike ODEs, DAEs typically have *hidden constraints* found by differentiating one or more equations. As an example take the simple DAE

$$x_1 - g(t) = 0, \, \dot{x}_1 - x_2 = 0.$$

To find the solution $x_1 = g(t)$, $x_2 = \dot{g}(t)$, one *must* differentiate the first equation.

An *index* of the DAE measures how much difficulty this causes. There are several; we use the differentiation index [11], the largest number of times some equation must be differentiated so the resulting equations can be solved to give an ODE.

Index > 1 DAEs typically give numerical difficulties. In practice, some index-reduction procedure is applied to arrive at an index-1 DAE, e.g. Pantelides's algorithm [9].

Those of index ≤ 1 are solvable by standard codes e.g. DASSL [12], SUNDIALS [13], MATLAB's `ode15i` etc. Popular circuit models such as the modified nodal analysis (MNA) [14] can give index 2. That our CpH method is of index ≤ 1 was a surprise bonus, and as a consequence, no index reduction is necessary.

"Structurally Amenable". A DAE is structurally amenable (S-amenable) if analysing the sparsity pattern of its equations reveals exactly what differentiations of them are needed. This analysis is inexpensive, only needs be done once, and when successful, allows various efficient numerical methods to be used (e.g., Mattsson–Söderlind Dummy Derivatives [15] for reducing the DAE to index ≤ 1).

The original method to find if a DAE is S-amenable is in Pantelides's 1988 paper [9]. We use the 2001 Pryce Σ-method [10], which is equivalent and more direct.

Consider a DAE with N equations $f_i = 0$ in N variables $x_j(t)$ and some of their t-derivatives. In vector form we can write it as

$$f(t;\, x \text{ and derivatives}) = 0.$$

A sketch of the Σ-method's steps follows.

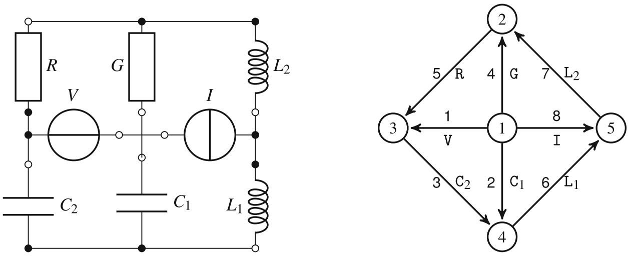
1. Form the $N \times N$ *signature matrix* $\Sigma = (\sigma_{ij})$, where σ_{ij} is the highest derivative order of x_j in f_i, or $-\infty$ if x_j is absent from f_i.
2. Find suitable *offsets* $c_i \geq 0$, $d_j \geq 0$ with $d_j - c_i \geq \sigma_{ij}$ $(i, j = 1, \dots, N)$, and equality on some *transversal*, a set of N positions (i, j) with one in each row and each column. This is a linear assignment problem, an efficiently solvable kind of linear programming problem [16].

3. Form the $N \times N$ *system Jacobian* $\mathbf{J}$ with $\mathbf{J}_{ij} = \partial f_i / \partial x_j^{(d_j - c_i)}$, or 0 if $d_j - c_i < 0$.
4. If $\mathbf{J}$ is nonsingular at some arbitrary point, the DAE is S-amenable. Then the offsets say what differentiations of equations are needed, and how to reduce the DAE to an implicit ODE.

3 Circuit Equations

Circuits. We consider circuits made of *2-pin elements* on graph *edges*, joined at *nodes*, see e.g. [7]. Key variables are *voltage drop* v over, and *current* ι in, an edge.

As an example, Fig. 3 shows an RLC circuit schematic as commonly drawn, with elements $V =$ voltage source, $I =$ current source, $R =$ resistor, $G =$ conductor[1], $C =$ capacitor, $L =$ inductor. Figure 3 is the corresponding mathematical graph, showing e.g. that the top ends of R, G, L_2 come together at a single node (Fig. 2(a) and Fig. 2(b)).



(a) Schematic.

(b) Circuit graph with edge orientations.

Fig. 2. RLC circuit example. On the right the edges have been numbered 1–8, with orientations shown, and the nodes 1–5.

In a circuit graph G, we allow multiple edges between two given nodes, but not edges from a node to itself (which have no electrical use). G is undirected but each edge has an *orientation* to say which direction of v and ι counts as positive (switching it doesn't change the physics). Henceforth, we assume G is connected.

Trees. Spanning trees (just called trees in circuit literature) in the graph are key to finding the right set of equations for the DAE. A tree T is a minimal subset of the m edges containing a path from any of the n nodes to any other. Necessarily it contains no cycles and has $n-1$ edges. The $m-n+1$ edges not in T are the cotree $\mathtt{T}^*$.

[1] A resistor with Ohm's law written as $\iota = vG$ instead of $v = \iota R$.

Each cotree edge specifies a *fundamental cycle*—that edge, plus the unique path between its ends via the tree. Each tree edge specifies a *fundamental cutset*—removing it splits the nodes into two nonempty subsets, the cutset is all edges between these subsets.

In Fig. 3, with tree $\{5,1,2,6\}$, cotree edge 4 specifies fundamental cycle $\{1,4,5\}$; tree edge 2 when removed splits the nodes into $\{1,2,3\}$, $\{4,5\}$, hence the fundamental cutset of edges between them is $\{2,3,7,8\}$.

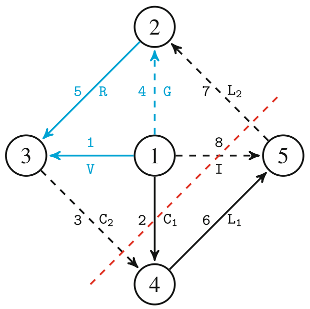
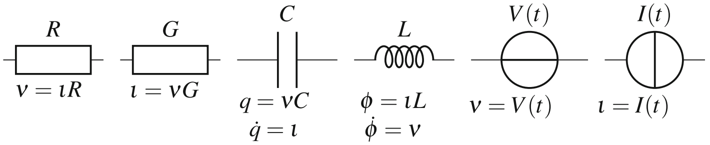


Fig. 3. Cycle and cutset example.

Physics. The physical assumptions on which the DAE is constructed are:

(a) *Constitutive relations*. If we assume standard linear elements, these are:



(b) *Kirchhoff's voltage and current laws*. Given T, it suffices to impose KVL (sum of voltages round a cycle is 0) round the $m-n+1$ fundamental cycles, and KCL (sum of currents across a cutset is 0) across the $n-1$ fundamental cutsets, since by linear combination these make all possible KVL and KCL equations. E.g., the cycle and cutset in Fig. 3 give $v_4 + v_5 - v_1 = 0$ and $\imath_3 + \imath_2 + \imath_8 - \imath_7 = 0$.

Each of the many circuit formulations combines the constitutive relations with selected Kirchhoff equations, to get a DAE $f(t,x,\dot{x}) = 0$ in some variables $x = x(t)$. Methods differ in what variables are in vector x—they can be voltages, currents, capacitor charges and inductor fluxes—and which KVL/KCL equations are used.

4 Graph Linear Algebra

Definition 1. G's *$n\times m$ incidence matrix A* has

$$a_{pj} = 1,\ a_{qj} = -1 \text{ if edge } j \text{ is from node } p \text{ to node } q$$

and zero elsewhere.

Assuming the graph is connected we have the well known facts:

Theorem 1. *A's column space (the linear span of its columns) is the hyperplane $x_1 + \cdots + x_n = 0$ in $\mathbb{R}^n$. A set of edges is a tree if and only if the corresponding columns of A are a basis of this column space.*

Consider such a graph, its incidence matrix A, a tree T and its cotree T*.

Definition 2. The $(m{-}n{+}1)\times(n{-}1)$ *Kron matrix* $F=(f_{ij})$ of $\mathtt{T}$ holds the unique representation of cotree columns of A as linear combinations of tree columns[2]:

$$a_i = -\sum_{j\in\mathtt{T}} f_{ij}a_j, \qquad \text{columns indexed by } \mathtt{T}\text{, rows by } \mathtt{T}^*.$$

Theorem 2. *Kirchhoff's laws for the graph can be written as* $\imath_{\mathtt{T}} = F^\top \imath_{\mathtt{T}^*}$, $v_{\mathtt{T}^*} = -F v_{\mathtt{T}}$ *or equivalently*

$$\begin{bmatrix} \imath_{\mathtt{T}} \\ v_{\mathtt{T}^*} \end{bmatrix} = \begin{bmatrix} 0 & F^\top \\ -F & 0 \end{bmatrix} \begin{bmatrix} v_{\mathtt{T}} \\ \imath_{\mathtt{T}^*} \end{bmatrix} \tag{1}$$

where $\imath_{\mathtt{T}}, \imath_{\mathtt{T}^*}$ *denote the vectors of currents on tree and cotree edges respectively, and similarly* $v_{\mathtt{T}}, v_{\mathtt{T}^*}$.

A non-obvious fact [17] is that all nonzeros of the Kron matrix are -1 or 1, and that these nonzeros encode the fundamental cycles and cutsets, with the orientation of each edge thereon. Our example circuit graph has the following 5×8 incidence matrix and 4×4 Kron matrix. In F for instance, the row labeled 4 encodes cycle $\{1,4,5\}$; the column labeled 2 encodes cutset $\{2,3,7,8\}$.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}, \qquad F = \begin{array}{c|cccc|c} & 1 & 2 & 5 & 6 & \\ \hline 3 & 1 & -1 & 0 & 0 & \mathtt{C}_2 \\ 4 & -1 & 0 & 1 & 0 & \mathtt{G} \\ 7 & -1 & 1 & 1 & 1 & \mathtt{L}_2 \\ 8 & 0 & -1 & 0 & -1 & \mathtt{I} \\ \hline & \mathtt{V} & \mathtt{C}_1 & \mathtt{R} & \mathtt{L}_1 & \end{array} \tag{2}$$

5 The Compact port-Hamiltonian DAE

Circuit formulations differ in what vector x of *DAE variables*, and which *Kirchhoff equations* they select. The CpH method in its simplest form chooses x with one x_j for each non-source edge, thus:

$$\begin{gathered} \text{charge } q \text{ for capacitors,} \qquad \text{flux linkage } \phi \text{ for inductors,} \\ \text{voltage } v \text{ or current } \imath \text{ at will for resistors,} \end{gathered} \tag{3}$$

and applies Kirchhoff's laws in the form in Theorem 2. A circuit must be *well posed*, meaning it has no cycles composed only of voltage sources and no cutsets composed only of current sources ("no V-cycles or I-cutsets"). Otherwise, since we have ideal circuit elements, Kirchhoff's equations are either underdetermined or contradictory.

Circuit SA says that a well-posed circuit has a *normal tree* containing all voltage sources, no current sources, and in a well defined sense the most possible capacitors and the fewest possible inductors. For a simple proof see [18, §2.3].

Constructing the CpH DAE then comprises the following steps.

[2] The $-$ is chosen to match the notation in [7].

1. **Input**: t and the DAE vector x defined in (3).
2. Evaluate the constitutive relations. Given the choice of variables (3), these make ν and ι on each edge an explicit function of x or $\dot{x}$ and (for source edges) t.
3. Substitute these ν and ι into Kirchhoff's equations in the Kron form (1). Each such equation "belongs" to a unique edge: e.g. each KVL equation is round a fundamental cycle, and belongs to the cotree edge that generates this cycle.
4. Separate out equations belonging to voltage and current source edges. They give control-output to be handled after the DAE is solved.
5. **Output**: the remaining equations as the DAE $f(t,x,\dot{x}) = 0$, of size N equal to the number of non-source edges.

We have assumed simple linear RLC circuit elements above. However this construction and the next theorem work more generally, for nonlinear elements and various kinds of coupling. Thus diodes, transistors, transformers, etc. are supported.

Theorem 3. *Subject to suitable* passivity *assumptions on the circuit elements, the CpH DAE is port-Hamiltonian, S-amenable and index* ≤ 1.

Sketch proof, showing the synergy of our three themes: see [18] for details.
Tree $\mathtt{T}$ being normal puts some blocks of zeros in the Kron matrix F … *[circuit-SA]*
… which improve the sparsity of the system Jacobian $\mathbf{J}$, making it block-triangular with three diagonal blocks *[DAE-SA]*

$$\mathbf{J} = \begin{bmatrix} \text{capacitor data} & & \\ & \text{inductor data} & \\ \times & \times & \text{resistor data} \end{bmatrix}.$$

The passivity assumptions mean *physically* that no circuit element except voltage and current sources can create energy in the system. They have the *mathematical* form that certain Jacobian matrices must be positive definite. *[pH]*

These Jacobians enter into $\mathbf{J}$'s diagonal blocks. Their positive definiteness implies each such block is nonsingular. So $\mathbf{J}$ is nonsingular, proving S-amenability.

Index ≤ 1 is a by-product, and pH structure is immediate from construction. □

For the example circuit's F in (2), the tree that produces it is normal; the blocks of zeros mentioned in the sketch proof are the three zeros in F's top right corner.

6 Conclusion

For details of what is said here, and proofs, see [18]. Other circuit formulations presented at the SCEE 2022 conference are in [20, 21], of which the first is port-Hamiltonian and the second is always index ≤ 1; but neither discusses SA-amenability.

CpH Advantages. CpH essentials are to be port-Hamiltonian and structurally amenable. Being pH is desirable *physics*: pH assures *energy-stability* of the mathematical DAE, and also of its numerical solution when suitable methods are used.

Being S-amenable is good for *numerics*. It makes possible, or inexpensive to implement, methods that in its absence are unavailable or expensive. Essentially equivalent for a DAE are that (a) it is S-amenable; (b) the Pantelides method (1988) works on it—e.g. to find consistent initial values; and (c) the Mattsson-Söderlind dummy derivatives method (1993) works on it—e.g. to reduce it to an implicit ODE.

Being pH is good *software engineering*, since pH models are compositional, i.e. can be assembled in a standard way to make larger pH models. This suits them to languages like Modelica, whose essence is to build systems from basic components.

Code Generation. We have implemented our theory in MATLAB, in principle supporting nonlinear dependent elements of the full generality in [18]. An object of a class `pHcircuit`, a "part", is specified by an incidence matrix, and type and parameters for each edge. We build larger circuits by combining existing parts; e.g. for the circuit in Fig. 4, the statement `P = [P0, BJT]/["a4b1", "a3b2", "a1b3"];` was used to join transistor `BJT` to the rest of the circuit `P0` by "soldering" pins 4, 3, 1 of `P0` to pins 1, 2, 3 of `BJT` respectively. Then we generate MATLAB code for the DAE function $f(t,x,\dot{x})$ and use `ode15i` to integrate the DAE. The generated code is readable and efficient. Generation is easily customised to make `C++` or `Fortran`.

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Appendix: Application, and Example CpH Code

We modelled the bipolar junction transistor (BJT) amplifier circuit example from Falaize & Hélie [19]. This shows the CpH method going beyond the linear elements assumed in Sect. 3.

The upper part of Fig. 4 is the circuit schematic. From the graph viewpoint, edges IN, 9V, OUT join to the "ground" node. We number the edges and assign orientations as marked by red arrows.

The MATLAB code was generated automatically, and the lower part shows the output from solving by `odei5i` with absolute and relative tolerances 10^{-3}, with input $V_{\text{in}}(t)$ as in [19], namely zero for 0.3 s to let a steady state be reached, followed by a sinusoidal oscillation of linearly increasing amplitude for 0.01 s. It agrees to graphical accuracy with [19, Fig. 6(b)].

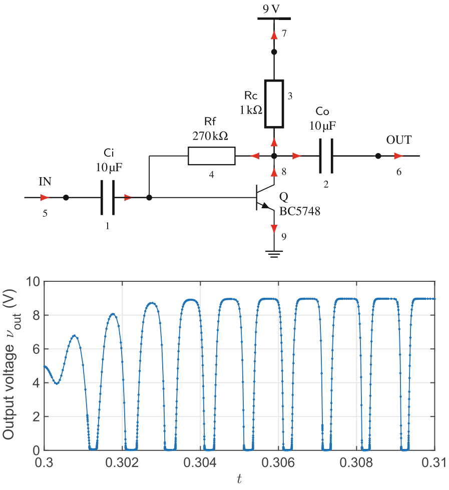

Fig. 4. BJT amplifier circuit, and output.

One could use tolerances down to around 10^{-12}, beyond which step size failure occurred. An upper step size limit was needed (0.005 worked), else the large steps built up in the initial 0.3 s were liable to make the solver go from $t = 0.3$ to 0.31 in one step, not noticing the changed behaviour at 0.3.

Below is the central part of the code—constitutive relations and Kirchhoff equations, expressed in mathematical notation. The physical parameters are in lines 5–11. The transistor is constructed on the Ebers–Moll model, of two Shockley diodes (nonlinear voltage-controlled resistors) with a linear dependence between them. In the code the diode is scalar function D on line 10; note $@(\dots)\dots$ is how MATLAB defines anonymous functions. The transistor is modelled by function BJT on line 11, with 2-vector input and output. Values $\alpha_F, \alpha_R > 1$ in matrix M derive from the Ebers–Moll forward and reverse current gains β_F, β_R.

Constitutive relations for independent edges are in lines 14–20; for the dependent transistor edges, in line 21. The Kirchhoff equations derive from the tree of edges $\{1,2,3,5,7\}$. Setting the currents of voltage sources and voltages of current sources to zero (lines 18–20 is a trick to simplify code generation. It makes the y's in lines 28–30 equal minus their correct output values, hence the sign reversal at line 33.

```
1   function $[f,y] = \mathsf{fcnBJTamplifier}(t,x,\dot{x})$
2   % DAE vector $x = (x_1,\dots,x_6)^T = (q_{C1},\ q_{C2},\ i_{R3},\ i_{R4},\ v_{G8},\ v_{G9})^T$

4   % Physical parameters
5   $C_i = 10^{-6},\ C_o = 10^{-6},\ R_c = 270\times 10^3,\ R_f = 10^3,\ V_{cc} = -9,\ I_{out} = 0$
6   $t_d = 0.3,\ t_{max} = t_d + 0.01, V_{max} = 0.2,\ \omega = 2\pi 10^3$
7   $V_{in} = @(t) - (t - t_d \geq 0) V_{max} \frac{t-t_d}{t_{max}-t_d} \sin(\omega(t-t_d))$
8   $I_s = 10^{-13},\ V_T = 0.025,\ \beta_F = 250,\ \beta_R = 10,\ \alpha_F = 1 + 1/\beta_F,\ \alpha_R = 1 + 1/\beta_R$
9   % Shockley diode D and Ebers–Moll transistor BJT
10  $D = @(v)\, I_s \cdot (e^{v/V_T} - 1)$ %It accepts vector input
11  $M = \begin{bmatrix} \alpha_F & -1 \\ -1 & \alpha_R \end{bmatrix}$,   $\mathrm{BJT} = @(v)\, M \cdot D(v)$ % $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

13  % Constitutive   relations
14  $v_1 = C_i^{-1} x_1$                 $i_1 = \dot{x}_1$
15  $v_2 = C_o^{-1} x_2$                 $i_2 = \dot{x}_2$
16  $v_3 = R_c x_4$                      $i_3 = x_4$
17  $v_4 = R_f x_3$                      $i_4 = x_3$
18  $v_5 = V_{in}(t)$                    $i_5 = 0$
19  $v_6 = 0$                            $i_6 = I_{out}$
20  $v_7 = V_{cc}$                       $i_7 = 0$
21  $\begin{bmatrix} v_8 \\ v_9 \end{bmatrix} = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}$        $\begin{bmatrix} i_8 \\ i_9 \end{bmatrix} = \mathrm{BJT}\left(\begin{bmatrix} x_5 \\ x_6 \end{bmatrix}\right)$

23  % Kirchhoff laws
24  $f_1 = i_1 + i_4 - i_8 - i_9$
25  $f_2 = i_2 - i_6$
26  $f_3 = i_3 + i_4 + i_6 - i_8$
27  $f_4 = v_4 - v_1 - v_3 - v_5 + v_7$
28  $y_1 = i_5 + i_4 - i_8 - i_9$
29  $y_2 = v_6 + v_2 - v_3 + v_7$
30  $y_3 = i_7 - i_4 - i_6 + i_8$
31  $f_5 = v_8 + v_1 + v_3 + v_5 - v_7$
32  $f_6 = v_9 + v_1 + v_5$
33  $y = -y$

35  % Return: $f = (f_1,\dots,f_6)^T$ and control–output $y = (y_1,y_2,y_3)^T = (i_{V5}, v_{I6}, i_{V7})^T$
```

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